WORKING PAPER

DEMOGRAPHIC CHANGE, SECULAR STAGNATION AND INEQUALITY: AUTOMATION AS A BLESSING?

Arthur Jacobs
Freddy Heylen

October 2021
2021/1030

GHENT UNIVERSITY

Department of Economics

Faculty of Economics and Business Administration
Campus Tweekerken, St.-Pietersplein 5, 9000 Ghent - BELGIUM

Demographic change, secular stagnation and inequality: automation as a blessing?

Arthur Jacobs*, Freddy Heylen
Department of Economics, Ghent University

October 2021

Abstract: We construct and calibrate an overlapping generations model incorporating demographic change and the possibility to automate the production process to test the hypothesis put forward by Acemoglu and Restrepo (2017). In line with their hypothesis, we find that ageing is a powerful force stimulating the adoption of automation technologies in OECD economies. Ageing-induced automation is found to soften the negative effects of labour scarcity and rising old-age dependency rates on per capita growth, but the compensation is incomplete. One important reason is that automated tasks are far from perfect substitutes for tasks executed by human labour. A second reason is that ageing-induced automation reduces the intensity of positive behavioural reactions to ageing in the form of retiring later and investing more in human capital. Moreover, the partial compensation comes at the price of rising wage and welfare inequality between individuals of different innate ability level and a fall in the net labour share of income. Compared to existing literature, we pay special attention to the theoretical and empirical foundations of the modelling of automation. Theoretically, our work is the first one testing this hypothesis that relates the approach to automation rigorously to the state-of the-art conception by Acemoglu and Restrepo (2018a; 2018b). Empirically, we tested and largely confirmed the validity of our approach and calibration by comparing model predictions of (changes in) automation density to actual data on robotization in a cross-country fashion.

Keywords: Automation, Demographic change, Secular stagnation, Overlapping generations model, Robotics, Factor shares

JEL-Classification: E22, E27, J11, J23, J24, J31

*Correspondence:

Arthur Jacobs, Ghent University, Department of Economics, Sint-Pietersplein 6, 9000 Gent, Belgium. Tel: +32495799407; arthur.jacobs@ugent.be (corresponding author)

1 Introduction

A large literature has studied the macroeconomic effects of demographic change in high income countries. The decline of fertility and the continuous rise of life expectancy during the last decades, together with the retirement of the baby boom generation, have been shown by many to cause lower per capita output and growth, lower productivity and investment, and lower real interest rates (e.g., Feyrer, 2007; Bloom et al., 2010; Ludwig et al., 2012; Gordon, 2016; Aksoy et al., 2019; Eggertsson et al., 2019; Devriendt & Heylen, 2020). Many therefore consider demographic change as the principal driving force behind 'secular stagnation'. On the other hand, some of the research mentioned above also demonstrated the existence of positive behavioural reactions of individuals to ageing: they will build more human capital, work more, and retire later (e.g., Ludwig et al., 2012; Cervellati & Sunde, 2013; Devriendt & Heylen, 2020). These endogenous reactions could counteract the negative effects of ageing on investment and growth. Recently, others have argued that demographic change may provide strong incentives to automation which could offset the negative effects on productivity and growth, although it may also eliminate jobs (Abeliansky & Prettner, 2017; Acemoglu & Restrepo, 2018c; Abeliansky et al., 2020).

Not only the net effects of demographic change on per capita output remain a matter of debate, so are those on inequality. Although there is general agreement that scarcity of labour due to a declining population at working age raises aggregate wages (e.g., Ludwig et al., 2012), the displacement of workers by robots may simultaneously erode the labour share of income (Dauth et al., 2017; Prettner, 2019; Acemoglu and Restrepo, 2020; Prettner and Strulik, 2020; Stähler, 2021). Moreover, differences between individuals in the capacity to build more human capital and in the degree to which the tasks they execute are automatable, may imply falling relative (and even absolute) wages for workers with low innate ability (Prettner & Strulik, 2020). The costs and benefits of both demographic change and automation may thus be very unequally distributed.

Our main objective and research question is to assess the net effects of demographic change on economic growth, the labour share of income, and wage and welfare inequality, when the response of automation and its induced effects are also taken into account. As such, our work can also be viewed as a theoretical test of the hypothesis in Acemoglu and Restrepo (2017), who suggest that the baseline negative effect of ageing on output per capita is neutralized or even reversed when considering that ageing endogenously triggers the adoption of automation technologies. Achieving this goal requires a quantitative general equilibrium model which is realistic in the modelling of automation, and which also gives room to the other behavioural reactions to demographic change highlighted by the literature (in particular labour supply at older age and human capital formation). Moreover, as to inequality, a promising analysis requires modelling individuals with heterogeneous skills.

Our approach is to construct and simulate a 5-period overlapping generations model that incorporates heterogeneity in innate ability. We have three active generations (the young, the middle-aged and the older) and two generations of retired. Fertility and life expectancy are time-varying. Individuals enter the model at the age of 20 with either high, medium, or low ability and productivity of schooling. Those with high or medium ability will allocate their time when young to either work or education. Due to very low productivity of learning, young individuals with low ability do not study, they only work. When middle aged, individuals devote their time fully to work, regardless of their innate ability level. In the third and last period of active life, i.e., between the ages of 50 and 65, individuals may choose the fraction of time they still work. The alternative is to choose leisure (which could include early retirement). Demographic perspectives and the degree of automation may affect all these choices. In the last two periods of life, i.e., from age 65 onwards, if they survive, individuals are retired and only have leisure.

To model automation, we start from the task-based approach to automation of Acemoglu and Restrepo (2018b) and modify it to isolate demographic change from technological drivers of automation. The production of output requires the execution of both automatable and non-automatable tasks. Both types are imperfect substitutes in production. Automatable tasks can be executed by either human labour or automation capital, which are perfect substitutes in the execution of these tasks. Since using automation capital will be more cost-effective, all automatable tasks will be automated, mirroring the simplifying assumption in Acemoglu and Restrepo (2019). Non-automatable tasks will be executed by labour. We will enrich this approach in several ways, however. In addition to automation capital, the production of goods in our model also requires the input of traditional physical capital, which is a complement to the execution of tasks. Next, as mentioned above, we distinguish three types of workers by ability. Depending on their level of education, they are capable of performing tasks of different complexity levels: high, medium, or low. The higher the complexity of tasks, i.e., the higher the required ability if they were executed by humans, the lower the fraction of tasks of that type that is automatable. Not only low-educated workers will thus face the probability of job destruction due to automation, but also the high-educated. For the latter, however, this probability is much smaller since fewer of their tasks are automatable routine tasks. In this, our modelling of automation is in accordance with earlier work finding that, for high-educated individuals too, the risk of job displacement by automation technologies is non-zero (Acemoglu & Restrepo, 2020; Frey & Osborne, 2017; Popescu et al., 2018). At the same time, we acknowledge that the risk is higher for low ability individuals such that automation is, in general, a force that increases the inequality between different education levels, as found in empirical research (Dauth et al., 2017; Graetz & Michaels, 2018).

As we highlighted above, many researchers have studied the impact of demographic change on per capita economic growth, wages, and the interest rate. Some have integrated the endogenous response of automation and enriched the analysis of the labour share of income and inequality. The papers to which ours is most closely related are those of Basso and Jimeno (2021), Stähler (2021), Irmen (2021) and Zhang et al. (2021). Compared to these studies, we contribute to the literature in three major ways. First, we model both the education decision of young people and the labour supply (and early retirement) of older workers endogenously. In extensions of their model, Basso and Jimeno (2021) demonstrate the major importance of doing this for key outcomes, while Irmen (2021) explicitly indicates the endogenization of educational investment as a promising way forward. Second, we build our framework of automation based on the seminal task-based framework of Acemoglu and Restrepo (2018a; 2018b), which allows us to be more explicit about the precise channels through which ageing stimulates automation. More specifically, we set up our model such that ageing-induced automation takes the form of automated tasks substituting for non-automated tasks in a less-than-perfect manner, rather than capital substituting for labour perfectly in the execution of any given task. As the interest rate falls and aggregate wages rise as a result of ageing, firms will make more use of automated tasks, at the expense of non-automated tasks, but, since firms are technologically constrained, the share of tasks that are automated will not rise. At the same time, our model acknowledges the typical role of capital as a complement for all types of labour by distinguishing between ‘traditional capital’ and ‘automation capital’. Moreover, our set-up allows us to verify a crucial assumption on which the economic conception of automation rests: automation capital should only feature in the production function if the use of automation capital is more cost-effective than the use of human labour for the execution of automatable tasks. Our third major contribution is that we explicitly test and show the empirical validity of the way we model automation, labour supply at older age and education. We see it as an important contribution that the calibrated model that we use for our simulations can explain a very large fraction of the cross-country differences in the level of robotization in OECD countries in 2019, as well as in the increase in robotization during the last two or three decades. Our model also explains a large fraction of cross-country differences in employment rates among older individuals and education rates among young individuals. Stokey and Rebelo (1995) have shown how sensitive the predictions of nicely calibrated models can be to the choice of key parameters. Before simulating our model, we therefore test (and show) that it translates observed cross-country differences in demography and policy into realistic performance differences with regard to automation, employment, and education.

Our main findings and answers to our research question are as follows. Ageing strongly stimulates the adoption of automation technologies, as found in earlier empirical and theoretical work, and this ageing-induced automation can contribute to the growth performance of ageing economies. Given the current level of development of automation technologies, however, demographic change will still constitute a force weighing down per capita growth in the foreseeable future of the US, as old-age dependency starts to rise. Likewise, the fall in the interest rate that ageing induces, is softened by ageing-induced automation, but not halted. We thus consider our results to be *cautiously* supportive of the hypothesis of Acemoglu and Restrepo (2017), since the mitigation is only partial. One explanation is that automated tasks are far from perfect substitutes for tasks executed by human labour. A second explanation for this "*only partial*"-finding is that, as ageing-induced automation softens the relative shortage of human labour, it also reduces the strength of behavioural reactions to this relative shortage. Without ageing-induced automation, the incentives to retire later and invest more in human capital accumulation would have been even stronger. Moreover, the partial mitigation also comes at the cost of heightened inequalities. First, ageing-induced automation generates a fall in the labour share of income thus benefiting capital-owners. Second, ageing-induced automation is likely to increase the wage and welfare inequality between individuals of different innate ability levels.

The remainder of this paper is structured as follows. Section 2 sets out our model. In Section 3, we describe the parameterization of the model, and we demonstrate its empirical relevance. In Section 4, we simulate the impact of demographic change for the US in our baseline model and we investigate a counterfactual scenario to see how aggregate per capita income and intra-generational welfare inequality would have evolved in the absence of ageing-induced automation. Section 5 concludes the paper and summarizes our main findings.

2 The model

Our framework consists of a five-period overlapping generations model for a closed economy where hours worked at older age, human capital formation and the degree to which the production process is automated are endogenously determined. The set-up of the model also accommodates the study of inequality by allowing for heterogeneity in the innate ability of individuals within each generation. More specifically, differences in innate ability are reflected in varying degrees of human capital upon entering the model and differences in the returns to schooling. Furthermore, our model incorporates the empirical finding that the automatability of tasks falls unambiguously in the educational attainment of the individuals executing these tasks (Arntz et al., 2016; Frey & Osborne, 2017).

With regard to notation, we use superscript t to refer to the time period in which individuals enter the model. Individuals entering at time t will further on be called individuals 'of generation t'. Subscript j is used to indicate that the generation is in the j-th period of their life and thus denotes the model age of an individual. Subscripts L, M and H refer to the three levels of innate ability: low, medium and high, respectively. Finally, time subscripts t that are added to aggregate variables indicate historical time periods.

2.1 Demography

In each modelling period, five different generations are alive: three active adult generations representing young, middle-aged and older workers and two generations of retired individuals. Individuals enter the model when they become 20 years old and each period of life lasts for fifteen years. Model ages $j = 1, 2, 3, 4 \text{ and } 5$ thus correspond to actual ages 20-34, 35-49, 50-64, 65-79 and 80-94. Demographic change in the model is captured by time-varying fertility and life expectancy. Equation (1) indicates how the size of the young generation of 20- to 34-year-olds alive at time t (N_1^t) relates to the size of the young generation at time $t-1$ (N_1^{t-1}), where n_t is the time varying fertility rate in the model. This approach follows, among others, de la Croix et al. (2013).

$$(1)\ N_1^t = (1 + n_t)N_1^{t-1}$$

The survival of an individual from one period into the following is uncertain. We denote by sr_j^t (< 1) the time-varying and age-dependent probability that an individual of generation t experiences utility in the j-th period of life, conditional upon having been alive in period $j-1$. The size of generation t then evolves as described by equation (2).

$$(2)\ N_j^t = sr_j^t N_{j-1}^t \qquad \forall\, j\ =\ 2, 3, 4, 5$$

The unconditional probability for an individual of generation t to reach the age group j is simply the product of the relevant conditional survival rates, as indicated in equation (3).

$$(3)\ \pi_j^t = \prod_{i=2}^{j} sr_i^t \qquad \forall\, j\ =\ 2, 3, 4, 5 \text{ and } \pi_1^t = 1$$

Individuals in the fifth period of their life, representing those aged 80 to 94, die with certainty at the end of the period.

Each generation consists of individuals of low, medium and high innate ability. It is assumed that survival and fertility rates do not vary over ability types and, in equation (4) that each ability group represents an equal share of one third of each generation at every point in time.

$$(4)\ N_{ja}^{t} = \frac{1}{3} N_{j}^{t} \qquad \forall\, j = 1,2,3,4,5\ ;\ \forall\, a = L, M, H$$

Because of the long duration of each period, the modelling of actual demographic change is somewhat stylised, but it still captures the main demographic trends in terms of life expectancy and fertility. The fertility and survival rates follow exogenous, country-specific trajectories throughout this study. Details on data sources and construction of the demographic parameters can be found in Appendix C. There we also show that cross-country differences in the old-age dependency ratio are captured quite well by our model.

2.2 Individuals

2.2.1 Preferences

A representative individual of ability a and generation t experiences utility in the j-th period of life through the instantaneous utility function described in equation (5).

$$(5)\ u\left(c_{ja}^{t}, l_{ja}^{t}\right) = \ln\left(c_{ja}^{t}\right) + \gamma_j \frac{(l_{ja}^{t})^{1-\theta}}{1-\theta} \qquad \forall\, a = L, M, H \text{ and } \forall\, j = 1, 2, 3, 4, 5$$

with $\gamma_1 = \gamma_2 = 0,\ \gamma_j > 0\ \forall\, j = 3, 4, 5$ and $\theta > 0\ (\theta \neq 1)$

Instantaneous utility is thus increasing in consumption c_{ja}^{t} and leisure time l_{ja}^{t} experienced in that period. Preferences are logarithmic in consumption, such that the intertemporal elasticity of substitution in consumption is 1. Additionally, preferences are iso-elastic in leisure with the intertemporal elasticity of substitution in leisure being $1/\theta$. γ_j indicates the age-dependent utility value of leisure relative to consumption. Young and middle-aged individuals do not value leisure and, as a result, they will opt to not allocate any time to leisure.

Each individual in the model maximises his/her expected lifetime utility, described by equation (6). In this equation, β is the discount factor determining the present value of future utility.

$$(6)\ U^{t} = \sum_{j=1}^{5} \beta^{j-1}\, \pi_j^{t}\, u\left(c_{ja}^{t}, l_{ja}^{t}\right)$$

The maximisation of expected lifetime utility is subject to both time and budget constraints.

2.2.2 Time constraints

Every period in our model is of the same fifteen-year length. We normalize this length to 1. Depending on the specific age and ability of an individual, time is allocated to either work (n), education (e) or leisure (including useful activities at home) (l). Equations (7) to (10) state the time constraints in each period.

$$(7)\ n_{1a}^{t} = 1 - e_{1a}^{t} \text{ with } e_{1L}^{t} = 0$$

$$(8)\ n_{2a}^{t} = 1$$

$$(9)\ l_{3a}^{t} = 1 - n_{3a}^{t}$$

$$(10)\ l_{ja}^{t} = 1 \text{ with } j = 4,5$$

Young individuals in equation (7) allocate their time to either work (n) or education (e), at least when they have medium or high ability. Individuals of low innate ability do not study when young ($e_{1L}^{t} = 0$). They are assumed to have zero productivity of schooling at the tertiary level. In later periods no one studies. Whereas young and middle-aged individuals are assumed to have no leisure, retired individuals in their fourth and fifth period of life have only leisure, as expressed in equation (10). The statutory retirement age in our model is 65. The generation of age 50 to 64 ($j = 3$) is the only generation in our model that is able to choose in equation (9) what share of time $l_{3,a}^t$ they allocate towards non-productive activities. Leisure time when older l_{3a}^t can be considered attainable through either reducing hours worked while still employed or entering into early retirement schemes.

2.2.3 Budget constraints

Each individual enters the model with zero assets. Equations (11) and (12) state the budget constraints with which individuals of generation t are confronted in the different periods of their life.

$$(11)\ \ (1+\tau_c)c_{ja}^t + \omega_{ja}^t = w_{a,t+j-1}h_{ja}^t n_{ja}^t\left(1-\tau_{wja,t+j-1}\right) + (1+r_{t+j-1}(1-\tau_{ci}))\omega_{j-1,a}^t + iht_{t+j-1} + tra_{t+j-1} \text{ for } j = 1,2$$

$$\begin{aligned}(12)\ \ (1+\tau_c)c_{3a}^t + \omega_{3a}^t \\ &= w_{a,t+2}h_{3a}^t n_{3a}^t\left(1-\tau_{w3a,t+2}\right) + b\, w_{a,t+2}h_{3a}^t(1-n_{3a}^t)\left(1-\tau_{w3a,t+2}\right) \\ &+ (1+r_{t+2}(1-\tau_{ci}))\omega_{2a}^t + iht_{t+2} + tra_{t+2}\end{aligned}$$

$$(13)\ \ (1+\tau_c)c_{ja}^t + \omega_{ja}^t = pp_{ja}^t + (1+r_{t+j-1}(1-\tau_{ci}))\omega_{j-1,a}^t + iht_{t+j-1} + tra_{t+j-1} \text{ for } j = 4,5$$

Individuals allocate their disposable resources to either consumption c_{ja}^t or the accumulation of non-human wealth. We denote by ω_{ja}^t the stock of wealth held by an individual of ability a at the end of the j-th period of his/her life. Consumption is taxed at rate τ_c. The right-hand sides of equations (11) and (12) show individuals' available resources during their active life. These include after-tax labour income, non-employment benefits (only when older), non-human wealth accumulated in the previous period and the after-tax return on it, accidental bequests or inheritances (iht_t) and lump sum transfers from the government (tra_t). After-tax labour income rises in the real remuneration w_a per unit of effective human labour provided by an individual of ability a, the human capital of that individual h_{ja}^t, and the fraction of time spent working n_{ja}^t. It falls in the average tax rate on labour income τ_{wja}. Since we model a progressive labour income tax system (cf. infra), tax rates depend on the ability and age of individuals. When older, at model age $j = 3$, individuals may choose to reduce their working time. In line with reality in many countries, they may then receive a benefit that is proportional to the time not spent working. This benefit can be thought to reflect both early retirement benefits (for those no longer working) and benefits in the context of phased retirement schemes (for older employees reducing work hours). The level of the benefit is a fraction of the net labour income an individual would receive if (s)he worked. The policy parameter b indicates the net replacement rate.

If an individual survives to the next period, his/her unconsumed resources are lent out to firms or the government. The individual is paid back at the end of that period and remunerated at the after-tax real interest rate $r(1-\tau_{ci})$, with τ_{ci} a proportional capital income tax. At the end of the fifth period, all surviving individuals fully consume their remaining resources. This way, individuals in the final period of life do not die with debt nor do they willingly leave bequests. Although in other periods individuals are allowed to have negative values ω_{ja}^t and to finance consumption by incurring debt, we observe such borrowing only for young medium and high ability individuals spending time in education and already anticipating high income levels in the future.

During retirement, the first two sources of income are no longer available, but the individual then receives old-age pension benefits, pp_{4a}^{t} and pp_{5a}^{t} respectively. The pension system in the model is of a pay-as-you-go (PAYG) type. Equation (14) describes the formation of these benefits as a function of the individual's net labour income in the past, when (s)he was still active. The replacement rate is denoted by ρ. We impose that each of the three periods of active life are equally important for the calculation of the pension assessment base. We note that, for given tax rates, an increase in this net pension replacement rate encourages individuals to work longer hours when older and increase education efforts through positive substitution effects, but also discourages work and education through an income effect.

$$(14)\ \ pp_{4a}^{t} = pp_{5a}^{t} = \rho\left\{\frac{1}{3}\sum_{j=1}^{3}\left[w_{a,t+j-1}h_{ja}^{t}n_{ja}^{t}\left(1-\tau_{wja,t+j-1}\right)\right]\right\}$$

Finally, if individuals do not survive the transition to the following period, the unconsumed part of their disposable resources is not saved but immediately passed on as a source of disposable income that is equally divided among all individuals in the population. In equation (15), IHT_t indicates the complete mass of unconsumed resources at the end of time period t of individuals who do not reach the following period of their life. Note that individuals dying with debt negatively contribute to the total inheritance mass IHT_t. This total inheritance mass IHT_t will be immediately distributed equally among all individuals surviving period t in the form of a lump sum transfer. The inheritance per person at the end of time t (iht_t) is given by equation (16)[1].

$$(15)\ \ IHT_t = \sum_{j=1}^{4}\sum_{a=L,M,H}\left(1-sr_{j+1}^{t+1-j}\right)\pi_{j}^{t+1-j}N_{1a}^{t+1-j}\ \omega_{ja}^{t+1-j}$$

$$(16)\ \ iht_t = \frac{IHT_t}{N_1^t + \pi_2^{t-1}N_1^{t-1} + \pi_3^{t-2}N_1^{t-2} + \pi_4^{t-3}N_1^{t-3} + \pi_5^{t-4}N_1^{t-4}}$$

2.2.4 Human capital formation

Individuals of different ability enter the model with different initial levels of human capital. The human capital of young individuals of the high ability type is normalized to h_0. Young individuals of medium and low ability dispose of only a fraction ε_a of this level.

$$(17)\ \ h_{1,a}^{t} = \varepsilon_a h_0 \quad \text{with } \varepsilon_L < \varepsilon_M < \varepsilon_H = 1$$

This heterogeneity in individuals upon entering the model can be thought to reflect both differences in innate ability at birth and heterogeneous learning outcomes in primary and

[1] Note that the remaining wealth of those dying at the end of the period t is already viewed as a source of income at time t by all potential survivors. The intuition is as follows. There is no aggregate uncertainty in this model such that individuals perfectly anticipate both the number of people dying at the end of t and the magnitude of their unconsumed resources. They realise that they can already choose to either consume out of it or invest it at the end of t, precisely as they allocate income generated in period t. Even individuals who do not survive to the next period $t+1$ and whose unconsumed resources are the source of the inheritance viewed this inheritance as an income source at time t.

secondary education. Neither h_0 nor ε_a varies across generations such that equation (17) is not a source of long-term growth or of fluctuations in the skill premium.

In the first period of their lives, medium and high ability individuals can allocate time to increasing their level of human capital. We have in mind that individuals spend time studying in tertiary education between the ages of 20 and 34. Equation (18) describes the human capital production function. It is identical to that of Bouzahzah et al. (2002) and Buyse et al. (2017), among others. For more details, we refer to their work. Equations (19) and (20) indicate that human capital never depreciates. We have in mind that learning by doing while at work may counteract depreciation.

$$(18)\ h_{2a}^{t} = h_{1a}^{t}(1+\phi(e_{1a}^{t})^{\sigma}) \qquad \forall\, a = M,H \text{ with } \phi > 0, 0 < \sigma \leq 1$$

$$(19)\ h_{2L}^{t} = h_{1L}^{t}$$

$$(20)\ h_{3a}^{t} = h_{2a}^{t} \qquad \forall\, a = L,M,H$$

We abstain from shocks to individual human capital and productivity during individuals' life. Our set of assumptions seems to offer the best match to findings by Huggett et al. (2006, 2011) and Keane and Wolpin (2007) that heterogeneity in human capital endowment at young age and learning abilities, rather than shocks to human capital, account for most variation in lifetime utility.

2.2.5 Optimality conditions for savings, education and work

The rational individuals in the model maximise their expected lifetime utility (equation (6)) subject to the budget and time constraints (equations (7) to (20)) by optimally choosing consumption in each period of their active life and the share of time spent working in the third period of life. Medium and high ability individuals also choose the amount of time spent at educational activities when young. The relevant optimality conditions are described and explained in Appendix E.

2.3 Production and the modelling of automation

A large number of identical firms operate on competitive markets for final goods, labour and capital[2]. The production function in equation (21) exhibits constant returns to scale in traditional physical capital K_t and effective labour in efficiency units A_tH_t. The model is neoclassical in nature in that the sole source of long run growth lies in labour-augmenting technical progress that is assumed to grow at a constant and exogenous rate x. Note that this exogenous growth result depends crucially on imposing that automation capital does not substitute perfectly for human labour in all tasks (in contrast to the work of Abeliansky and Prettner (2017)).

$$(21)\ Y_t = K_t^{\alpha}(A_tH_t)^{1-\alpha} \quad \text{with } A_t = A_{t-1}(1+x)$$

In equation (22), aggregate effective labour H_t is defined as a CES composite of effective labour supplied by each of the three ability groups ($H_{L,tot}$, $H_{M,tot}$ and $H_{H,tot}$) where s is the elasticity of substitution between the ability types and η_L, η_M and η_H are the share parameters. Equation (23) clarifies how the effective labour supplied by each ability type is in its turn also

[2] Note that this assumption of competitive markets implies that all movements in the model's factor shares are solely attributable to capital-labour substitution. Two key drivers of the fall in the labour share according to Manyika et al. (2019) - the effects of globalisation on workers' bargaining power, and increasing mark-ups due to a consolidation on product markets - play no role in our model.

a CES composite of labour provided by humans of that ability type (H_L, H_M and H_H) and automation capital P. The elasticity of substitution between human labour and automation capital is a non-varying κ for each ability type and the share parameter of human labour in total effective labour of type a is ξ_a. Last but not least, equation (24) describes total human labour of a specific ability type. Human labour of different age groups within an ability type is assumed to be perfectly substitutable.

$$(22)\ \ H_t = \left(\eta_L H_{L,tot,t}^{\frac{s-1}{s}} + \eta_M H_{M,tot,t}^{\frac{s-1}{s}} + \eta_H H_{H,tot,t}^{\frac{s-1}{s}}\right)^{\frac{s}{s-1}}$$

$$(23)\ \ H_{a,tot,t} = \left(\xi_a^{\frac{1}{\kappa}} H_{a,t}^{\frac{\kappa-1}{\kappa}} + (1-\xi_a) J P_t^{\frac{\kappa-1}{\kappa}}\right)^{\frac{\kappa}{\kappa-1}} \qquad \forall\, a = L, M, H, \text{ with } 0 < \xi_a < 1$$

$$(24)\ \ H_{a,t} = \sum_{j=1}^{3} \pi_j^{t+1-j} N_{1a}^{t+1-j} n_{ja}^{t+1-j} h_{ja}^{t+1-j} \qquad \forall\, a = L, M, H$$

Equation (23) mirrors the task-based approach of Acemoglu and Restrepo (2018b). In the spirit of their work, this equation can be derived from an underlying basic specification $H_{a,tot,t} = (\int_0^1 t_{a,i,t}^{\frac{\kappa-1}{\kappa}} di)^{\frac{\kappa}{\kappa-1}}$ with $t_{a,i,t} = h_{a,i,t}\ , \forall i < \xi_a$ and $t_{a,i,t} = jP_t + \lambda h_{a,i,t}\ , \forall i > \xi_a$ where the effective amount of work of ability type a is executed by combining the execution of a continuum of tasks $t_{a,i,t}$ with an elasticity of substitution κ. In Appendix A (Part 1), we show the validity of equation (23). In this underlying equation, $h_{a,i,t}$ then represents the amount of human labour of ability a that is devoted to the execution of task $t_{a,i,t}$ and P_t is a general purpose automation technology contributing to the execution of all automatable tasks. The parameter j (closely related to J) expresses the efficiency of automation capital in the execution of automatable tasks, while λ stands for the efficiency of human labour in the execution of automatable tasks.

More revealingly, one can consider the CES in (22) to distinguish between low ability, medium ability and high ability tasks which have to be executed for the production of final output Y. From this perspective, η_a indicates the share of total tasks that are of the ability type a and s then indicates how easily performing tasks of an ability type different from a substitutes for tasks of ability type a in the production of final goods. The CES in equation (23) then indicates how each of the three types of tasks consists of both technologically non-automatable tasks, which only human labour can execute, and technologically automatable tasks, which both human labour and automation capital can execute. We follow the common assumption in the task-based literature on automation that automation capital and human labour substitute perfectly in the execution of technologically automatable tasks (Acemoglu & Restrepo, 2018b). It is ξ_a that indicates the share of total tasks of type a that are not technologically automatable and that can only be performed by humans. Finally, the elasticity of substitution κ determines how well automated tasks can substitute for non-automated tasks, and vice versa. It is assumed to be non-varying over ability types.

Note that human labour does not feature in the part of the CES indicating the execution of automatable tasks in equation (23). This is because the parameter J will be calibrated in the next section such that all automatable tasks are more cost-effectively executed by automation capital. All *automatable* tasks will therefore also be *automated* in practice. This is not an uncommon simplifying assumption in task-based models of automation (Acemoglu & Restrepo, 2019) and it implies that the share of tasks that is automated is constrained by technology (What tasks are automatable?) and not by the optimal choice of firms (Should I use capital or labour for the execution of this automatable task?). By imposing this, we adopt the same view of automation as in Acemoglu and Restrepo (2018b), but extend it by imposing a high elasticity of substitution κ between automated and non-automated tasks of type a and distinguishing between two types of capital. In this approach, as in Acemoglu and Restrepo (2018b), two technological processes embodying 'automation' coexist: automation at the extensive margin implies that more tasks become technologically automatable and this is captured by a rise in $1 - \xi_a$ in the model. Such automation at the extensive margin will displace workers in that the marginal product of human labour will fall (infra: equation (27)). Automation at the intensive margin represents a rise in the productivity j of automation capital P at already automated tasks, for a given share of automatable (and automated) tasks $1 - \xi_a$. Firms will then substitute automated tasks for tasks executed by human labour in their production process. Contrary to the two-factor framework of Acemoglu and Restrepo (2018b), this automation at the intensive margin has the potential to decrease the demand for labour in our model with two types of capital (DeCanio, 2016).

When exogenous evolutions such as demographic change alter the cost of capital and/or labour, the reaction of firms to the changing cost structure can imply processes of non-technological automation. Automation of a non-technological nature can be divided into the same two types as technological automation. Automation at the extensive margin then takes the form of firms using automation capital for the execution of automatable tasks that were executed by humans before. This option is excluded in our model, since firms are technologically constrained in the sense that all automatable tasks are already automated. On the other hand, automation at the intensive margin implies that firms choose to make more use of automatable tasks, and less of non-automatable tasks in their production process. For the low ability individuals, this non-technological automation at the intensive margin will also lower the real hourly wage[3].

In our model, it is this non-technological automation at the intensive margin that plays a crucial role, since it is an important channel through which firms react to the change in factor costs implied by demographic change. This channel is very distinct from the true replacement of humans by automation capital in the execution of a given task, since it are *tasks*, not production factors that substitute for one another in the case of automation at the intensive margin. An example might clarify this. The adoption of cost-effective computer technology entirely displaced the human computing profession. This is the automation at *extensive* margin: a drop in ξ_a implies that tasks which capital could not perform in the past because they were technologically unautomatable, are now *only* executed by capital due to the perfect substitutability between capital and labour for the execution of that task and the lower cost of executing that task using capital. Computers can do exactly the same computations, but in a

[3] This is shown in sub-section 3.1.6. The main driver of this result will be the larger $(1 - \xi_a)$ for the low ability type. It implies that the "substitution of automated tasks for non-automated tasks" will play a larger role for low ability labour than for other ability types. For individuals of medium and high ability, this task substitution effect will also be negative (since the sign of this effect is mostly determined by s and κ, which are parameters common across ability types), but the overall effect will be less strong for them and more than fully offset by the fact that automation also implies that tasks of other ability types are executed more. In sub-section 3.1.6 we expand further on this by explicitly stating expressions determining the sign of the mixed second order derivatives of final output Y with respect to automation capital and human labour of a certain ability type. We will find that, given the calibrated parameter values, automation capital is a q-substitute for low ability labour and a q-complement for medium and high ability labour.

far more cost-effective way than humans. Automation at the *intensive* margin happens when those tasks now performed by computers are increasingly executed and substitute, rather than complement for the tasks still performed by humans. High-speed computers still do not deliver the mail - this task itself is not automated -, but demand for this task itself has diminished due to the performance of tasks like e-mailing substituting for it. Of course, one could argue that the "delivering mail" task has in fact become automated, but this would not be entirely accurate. Delivering mail and delivering electronic messages are distinct tasks that require very distinct actions and skills for their execution, but they just substitute for one another relatively well. Important to note is that, while we assume perfect substitutability between automation capital and human labour for the execution of automatable tasks (as in Acemoglu and Restrepo (2018b)), we consider automatable and non-automatable tasks of the same ability type a to substitute for one another in a less-than-perfect way. Sending an e-mail is an alternative to sending a postcard, but they are surely not interchangeable in all circumstances.

The high elasticity of substitution between (tasks executed by) capital and tasks executed by labour is surely not warranted in any situation. In fact, it is one of the great assets of the task-based approach to automation that the accumulation of capital is allowed to play its traditional role of increasing the demand for human labour. By distinguishing between two types of capital in our model, automation capital explicitly represents those instances in which the work performed by capital makes the work performed by labour less relevant for the production of final output. Following Acemoglu and Restrepo (2018b), we mainly have computer-assisted machines, robotics, and artificial intelligence in mind when referring to automation capital. Traditional capital (e.g., infrastructure) then represents those, more typical, instances where capital empowers the relevance of human work and increases the demand for it. Our approach allows this distinction between automation capital and traditional capital, although it is not typical in task-based models of automation. We borrowed it from other frameworks of automation (Abeliansky & Prettner, 2020; Cords & Prettner, 2018; Lankisch et al., 2019; Prettner, 2019). It is because of this distinction between two types of capital that the traditional labour-empowering role of traditional capital accumulation can be preserved, while also allowing for q-substitutability between (tasks executed by) automation capital and (tasks executed by) certain types of human labour. Namely, DeCanio (2016) shows that q-substitutability is impossible in any framework with only two production factors (and constant returns to scale). Furthermore, it is important to note that there is only one type of automation capital P present in our framework and it executes all tasks, even of different ability types simultaneously. It is for this reason that P_t, and not $P_{a,t}$, features in equation (23). This approach mainly proxies well for general purpose automation technologies such as computerisation, which are capable of performing sets of tasks of very heterogeneous ability levels and which are not solely devoted to the execution of a limited amount of tasks. More details on the technical implications of this approach can be found in Appendix A.

On a final note, the labour-augmenting technical change in our model is consistent with stable factor shares, just as in many representations of the neoclassical growth model. Technical progress in our model can be considered 'total labour'-augmenting in the sense that it lifts the degree to which the execution of tasks (by both humans and automation capital) contributes to the production of final goods. The ratio of the marginal products of automation capital and human labour is thus left unaffected by the technical change in the model: the marginal products of both factors increase in the same proportion, as under Hicks-neutral technical progress. Furthermore, total effective labour H and traditional capital K are combined in a

Cobb-Douglas production function such that the labour augmenting technical progress is not labour-biased.[4]

2.4 Firm optimisation

Equation (25) expresses the standard first-order condition that firms invest in traditional capital up to the point where its marginal product net of depreciation (δ_k) is equal to the interest rate.

$$(25)\quad \left[\alpha\left(\frac{A_t H_t}{K_t}\right)^{1-\alpha} - \delta_k\right] = r_t$$

In equation (26), we impose that investment in traditional capital and investment in automation capital yield precisely the same after-tax rate of return. This is what Abeliansky and Prettner (2017), Cords and Prettner (2018) and Lankisch et al. (2019) refer to as the "no-arbitrage condition". In this equation, τ_p indicates the tax rate ($\tau_p > 0$) or subsidy rate ($\tau_p < 0$) that is applied to the marginal product of automation capital, while δ_p is the depreciation rate of automation capital.

$$(26)\quad \left[(1-\alpha)A_t^{1-\alpha}\left(\frac{K_t}{H_t}\right)^{\alpha} \sum_{a=L,M,H}\left\{\eta_a\left(\frac{H_t}{H_{a,tot,t}}\right)^{\frac{1}{s}}(1-\xi_a)\left(\frac{H_{a,tot,t}}{P_t}\right)^{\frac{1}{\kappa}} J\right\} - \delta_p\right](1-\tau_p) = \left[\alpha\left(\frac{A_t H_t}{K_t}\right)^{1-\alpha} - \delta_k\right]$$

Labour markets too are assumed to be perfectly competitive such that firms employ human labour of ability type a up to the point where the marginal product of effective human labour of type a equals the real hourly wage per unit of human capital of individuals of that ability level. This condition is expressed in equation (27). Note that τ_{wjat} does not feature in this expression, since all labour taxes are assumed to be paid by the employee.

$$(27)\quad \left[(1-\alpha)A_t^{1-\alpha}\left(\frac{K_t}{H_t}\right)^{\alpha}\eta_a\left(\frac{H_t}{H_{a,tot,t}}\right)^{\frac{1}{s}}\xi_a^{\frac{1}{\kappa}}\left(\frac{H_{a,tot,t}}{H_{a,t}}\right)^{\frac{1}{\kappa}}\right] = w_{a,t} \qquad \forall\, a = L, M, H$$

2.5 Government

The government budget constraint is set out in equation (28). Government spending consists of government consumption G_t, non-employment benefits to older workers B_t, expenditures on the PAYG pension system PP_t, interest payments on public debt $r_t D_t$ and lump-sum transfers to all living individuals TRA_t. The government levies taxes on labour income $T_{n,t}$, on consumption $T_{c,t}$, on the return to non-human wealth $T_{ci,t}$ and on the returns of automation capital $T_{p,t}$. Government consumption G_t is wasteful: it does not enter in the production function nor in the individuals' utility function. The share of final output that is allocated to consumption G_t follows an exogenous path defined by the evolution of g. If the government sets specific targets on the evolution of public debt, it can adjust lump-sum transfers. Also note our assumption that the pension system is fully integrated into government accounts. We do not impose a specific financing of the PAYG pension plan. The government can use resources from the general budget to finance pensions.

[4] With the assumption of a Cobb-Douglas production function and labour-augmenting technical change, our model conforms to two features central in neoclassical growth models. Jones (2005) explains their pervasiveness in the literature by highlighting their analytical convenience (they lead to stable steady-state growth (Uzawa, 1961)), but also finds microfoundations justifying both modelling choices.

$$(28)\ G_t + B_t + PP_t + r_t D_t + TRA_t - T_{n,t} - T_{c,t} - T_{ci,t} - T_{p,t} = D_{t+1} - D_t$$

$$(29)\ T_{n,t} = \sum_{j=1}^{3} \sum_{a=L,M,H} \tau_{wjat}\, \pi_j^{t+1-j} N_{1a}^{t+1-j} w_a^t\, h_{ja}^{t+1-j} n_{ja}^{t+1-j}$$

$$(30)\ T_{c,t} = \tau_c \sum_{j=1}^{5} \sum_{a=L,M,H} \pi_j^{t+1-j} N_{1a}^{t+1-j} c_{ja}^{t+1-j}$$

$$(31)\ T_{ci,t} = \tau_{ci} \mathrm{r}_t Z_t$$

$$(32)\ T_{p,t} = \tau_p \left[(1-\alpha) A_t^{1-\alpha} \left(\frac{K_t}{H_t}\right)^{\alpha} \sum\nolimits_{a=L,M,H} \eta_a \left(\frac{H_t}{H_{a,tot,t}}\right)^{\frac{1}{s}} (1-\xi_a) \left(\frac{H_{a,tot,t}}{P_t}\right)^{\frac{1}{\kappa}} J - \delta_p \right] P_t$$

$$(33)\ B_t = b \sum_{a=L,M,H} \pi_3^{t-2} N_{1a}^{t-2} (1 - n_{3a}^{t-2})\, w_a^t h_{3a}^{t-2} (1 - \tau_{wa})$$

$$(34)\ PP_t = \sum_{j=4}^{5} \sum_{a=L,M,H} \pi_j^{t+1-j} N_{1a}^{t+1-j} pp_{ja}^{t+1-j}$$

$$(35)\ G_t = g_t Y_t$$

$$(36)\ TRA_t = \sum_{j=1}^{5} \sum_{a=L,M,H} \pi_j^{t+1-j} N_{1a}^{t+1-j} tra_t$$

Following Guo and Lansing (1998) and Boone and Heylen (2019), the average tax rates on labour income τ_{wja} are progressively determined by equation (37).

$$(37)\ \tau_{wjat} = \Gamma \left(\frac{w_{at} h_{ja}^{t+1-j} n_{ja}^{t+1-j}}{\bar{y}_t^{lab}} \right)^{\psi} \qquad \text{with } \psi \geq 0 \text{ and } 0 < \Gamma \leq 1$$

Here, $w_{at} h_{ja}^{t+1-j} n_{ja}^{t+1-j}$ is the total pre-tax labour income of the individual at time t, and $\bar{y}_t^{lab}$ is the average pre-tax labour income in the economy at time t. Furthermore, Γ represents the average labour tax rate for an individual whose labour income is at the economy-wide average and ψ determines the progressivity of the tax system. Both tax parameters are time-invariant throughout this study. The relevant labour tax rates for the decisions of individuals are the *marginal* tax rates, however. As long as $\psi > 0$, the marginal tax rate (in equation (38)) is higher than the average tax rate.

$$(38)\ \tau_{wjat}^m = (1+\psi) \Gamma \left(\frac{w_{at} h_{ja}^{t+1-j} n_{ja}^{t+1-j}}{\bar{y}_t^{lab}} \right)^{\psi}$$

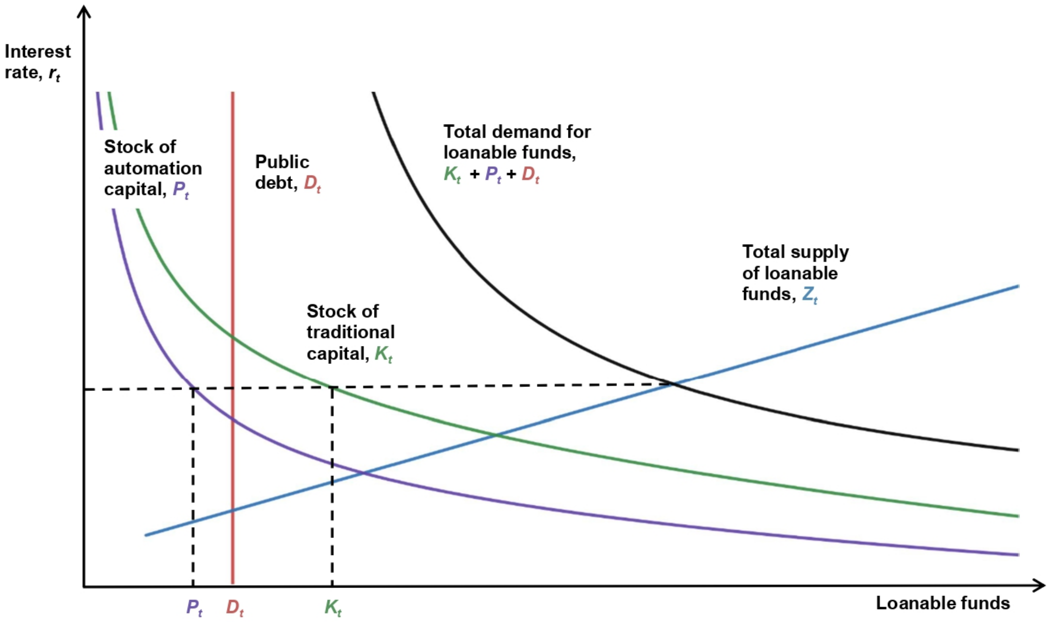
2.6. Aggregate equilibrium

Aggregate equilibrium on the market for final goods is ensured by the equilibrium on the market for loanable funds expressed in equation (39). More precisely, equation (39) imposes that the aggregate stock of non-human wealth held by individuals in the economy Z_t coincides with the total stock of traditional capital, automation capital and government debt. The interest rate r_t ensures that this equilibrium on the market for loanable funds is reached (see Figure 1).

$$(39)\ Z_t = K_t + P_t + D_t$$

$$(40)\ Z_t = \sum_{j=1}^{4} \sum_{a=L,M,H} \pi_j^{t-j} N_{1a}^{t-j} \omega_{ja}^{t-j}$$

Figure 1 *Visual representation of the market for loanable funds*



For simplicity's sake, the supply of loanable funds is represented linearly here. Note that it is upward-sloping in spite of the intertemporal elasticity of substitution of one, following Backus et al. (2014).

3 Parameterization and empirical relevance

3.1 Parameterization

In order to evaluate the fit of the model's predictions with the actual data and to simulate the impact of exogenous shocks like demographic change or policy initiatives, numeric values have to be assigned to the model's parameters. All parameter values in our baseline model for the US can be found in Table 1. Nine parameters were calibrated by imposing that the model perfectly replicates recent actual data for the US. In Table 1, these calibrated parameters are marked in bold. The other parameter values were taken from the literature. For more information on the construction of the calibration targets, we refer to Appendix D.

Table 1*: Parameterization and target values for calibration*

Technology and preference parameters	
Productions of final goods	$\alpha = 0.25$, $s = 1.5$, $\boldsymbol{\eta_L = 0.244}$, $\boldsymbol{\eta_M = 0.313}$, $\eta_H = 0.443$
Automation of tasks	$1 - \xi_L = 0.250$, $\boldsymbol{1 - \xi_M = 0.211}$, $\boldsymbol{1 - \xi_H = 0.120}$, $\boldsymbol{\kappa = 3.28}$, $\boldsymbol{J = 9.26}$
Exogenous technology growth	$\boldsymbol{x = 0.244}$
Human capital production	$\boldsymbol{\phi = 1.27}$, $\sigma = 0.3$
Initial human capital distribution	$\varepsilon_L = 0.67$, $\varepsilon_M = 0.84$, $h_0 = 1$
Preference parameters	$\beta = 0.8$, $\theta = 2$, $\gamma_1 = \gamma_2 = 0$, $\boldsymbol{\gamma_3 = 0.156}$
Capital depreciation rate	$\delta_k = \delta_p = 0.714$
Policy parameters (United States, 2005-2019)*	
$\Gamma = 0.307$, $\psi = 0.272$, $\tau_{ci} = 0.47$, $\tau_p = 0$, $\tau_c = 0.033$, $b = 0.063$, $\rho = 0.505$, $g = 0.151$, $D/Y = 0.928$	
Target values for calibration (United States, 2005-2019)	
Relative wages of young individuals by education: $\frac{w_L h_{1L}}{w_H h_{1H}} = 0.565$, $\frac{w_M h_{1M}}{w_H h_{1H}} = 0.645$	
Relative share of automatable tasks: $\frac{1-\xi_M}{1-\xi_L} = 0.847$, $\frac{1-\xi_H}{1-\xi_L} = 0.480$	
$\frac{\text{Automation capital per worker in the US with German demography in 2019}}{\text{Automation capital per worker in the US in 2019}} = 1.214$	
Gross labour share in national income = 70.1%	
Annual growth of potential GDP per person of working age = 1.47%	
Participation in tertiary education: $\frac{e_{1L} + e_{1M} + e_{1H}}{3} = 15.2\%$	
Hours worked when older $n_3 = 59.0\%$	

* Lump sum transfers adjust as the residual category in equation (28).

The parameters that were assigned a value through calibration on target values are marked in bold. The target values are ordered in a logical way: the first target is most relevant in assigning a value to the first calibrated parameter, etc. For details on the definition, the sources and the construction of the target values and the policy parameters, we refer to Appendix D.

3.1.1 Technology and preference parameters

The rate of physical capital depreciation is assumed to be the same for traditional capital and automation capital. We impose $\delta_k = \delta_p = 0.714$, which implies a yearly depreciation rate of around 8% because of the fifteen-year length of one model period. Similarly, we impose $\beta = 0.8$ which reflects a rate of time preference of 1.5% per year. We assume the share parameter α for traditional capital in the production function for final goods to be equal to 0.25. The idea is that before tasks were technologically automatable ($\xi_a = 1$, $\forall a = L, M, H$), the share of capital in national income was constant and equal to the share parameter α. This is what Samuelson (1964) labelled 'Bowley's Law' and which Kaldor (1961) referred to as the steadiness of the wage share. Keynes (1939) also acknowledged the existence of a constant labour share and famously labelled it “a bit of a miracle” (p. 49). With regard to the *level* of the constant initial wage share, our value of 0.75 is consistent with the findings of Johnson (1954) and Gollin (2002).

The elasticity of substitution s between labour (tasks) of different ability types is set equal to 1.5. The empirical labour literature consistently documents values between 1 and 2 (Caselli & Coleman, 2006). For the value of the intertemporal elasticity of substitution in leisure $(1/\theta)$ we follow Rogerson (2007). He puts forward a reasonable range for θ in macro studies from 1 to 3. In line with this, we impose θ to be equal to 2. This choice implies an elasticity of labor supply which is much higher than the very low elasticities typically found in micro studies. Given our macro focus, however, these micro studies may not be the most relevant ones (Rogerson & Wallenius, 2009; Fiorito & Zanella, 2012). Several parameters in our model relate to human capital production. For the elasticity of human capital with respect to education time (σ) we choose a conservative value of 0.3. This value is within the range considered by Bouzahzah et al. (2002) and Docquier and Paddison (2003). For the values of the relative initial human capital of medium and low ability individuals (relative to the initial human capital of high ability individuals, ε_M and ε_L), we follow Buyse et al. (2017). They looked at the distribution of PISA science test scores in OECD countries. From the robust pattern they observed in relative scores of weaker and median performers relative to better performers, they derived $\varepsilon_L = 0.67$ and $\varepsilon_M = 0.84$. The initial level of human capital with which high ability individuals enter the model is normalized to one in our model ($h_0 = 1$). Finally, the efficiency parameter ϕ in the human capital production function has been determined by a calibration procedure that we discuss now.

We calibrated all remaining parameters by imposing that the model matches key data for the US. The US provide a good source of data for the calibration exercise since two crucial assumptions of the model are relatively justifiable in the US context. The most basic openness indicator - trade as a percentage of GDP - views the US as the most closed OECD economy. Furthermore, the very low ‘strictness of employment protection’ indicator of the OECD and very low union density show that our assumption of a perfectly competitive labour market holds up the most for the US.

The relative taste for leisure of individuals during the final period of active life (γ_3) is set to generate an employment rate among older workers, averaged over the three ability groups (n_3) of 0.59. This is the fraction of potential hours that were actually worked by all individuals aged 50 to 64 in the US 2005-2019 (for more details, see Appendix D). The exogenous growth rate of A_t is the only source of long-term per capita growth in the model, which is why x is set to match the average yearly growth rate of potential GDP per person of working age. This was 1.47% in the US for the years between 2005 and 2019, leading to a value of x of 0.244. We calibrate the efficiency parameter in the human capital production function ϕ such that the model accurately predicts the 2005-2019 data on the average aggregate participation in education of individuals between 20 and 34.

For the calibration of the share parameters of the three different ability types of labour (tasks) relevant to the production of final output (η_L, η_M, η_H), our target values are the pre-tax wages of young workers of low and medium education relative to the wages of young workers of high education. More specifically, we target data published by the OECD (Education at a Glance 2020) for the wages of 25- to 34-year old individuals whose highest degree is of the upper secondary level or lower (ISCED 3 or lower) and of individuals with short-cycle tertiary education (ISCED 5), relative to the wages of individuals with at least a bachelor's degree (ISCED 6 or higher)[5,6]. This results in values for η_L and η_M respectively. The value for η_H then follows as $1 - \eta_L - \eta_M$. This approach approximates best our modelling assumptions that individuals with low ability do not participate in tertiary education, while those of medium and high ability do. We focus on the skill premium for young individuals since this better reflects the differences in intrinsic demand for different types of abilities due to differences in the importance of tasks in the production of final goods.

3.1.2 Automation parameters: general

We identify five parameters relating to automation. These are the shares of automatable tasks by ability $1 - \xi_a$ (for $a = L, M, H$), the elasticity of substitution between automatable and non-automatable tasks κ, and the productivity of automation capital J. They are determined such that our model replicates or confirms five facts or well-informed hypotheses. A first one is that 25% of tasks of low ability are automatable. A second and third are that the fractions of automatable tasks of medium and high ability equal respectively 85% and 48% of the fraction of automatable tasks of low ability. A fourth one is that due to automation the labour share in the US fell from 75% to about 70% in 2005-19. The fifth one is Acemoglu and Restrepo's (2018c) claim that if the demographic structure in the US were the same as in Germany, robot density in the US would be 21% higher. We now clarify these facts or hypotheses in greater detail.

3.1.3 Shares of automated tasks: $1 - \xi_a$ for $a = L, M, H$

The basis of our calibration is the work of Arntz et al. (2016) and Popescu et al. (2018). Both studies reveal clear heterogeneity between ability types in the share $1 - \xi_a$ of tasks that are automatable. Unlike most other studies, Arntz et al. (2016) adopt a task-based approach to estimate the share of jobs and individuals at high risk of automation. This makes their results a more reliable point of reference for us to start from. More precisely, they report for the three groups in the US with the lowest ISCED levels[7], estimated shares of workers at high risk of automation equal to 100%, 44% and 19% respectively. Weighing these shares with the relative size of these three groups (US Census Bureau, Current Population Survey), we obtain that 25.3% of what we label low ability individuals are employed in highly automatable occupations. If next we assume a one-to-one relationship between the task content of jobs and the education level of those who execute them, our projection follows that a share $1 - \xi_L$ of 25% of low ability tasks are automatable.

Taking this 25% for $1 - \xi_L$ as our benchmark, similar shares of tasks of medium and high ability that are automatable can be derived fairly easily from Popescu et al. (2018). They build on the

[5] See Appendix D. Due to a lack of data on the wages of individuals with a post-secondary, non-tertiary degree (ISCED 4) in the US, we have to assume that the wages of individuals with short-cycle tertiary education (ISCED 5) are representative for the whole medium ability group.
[6] The US Census Bureau defines these categories of educational attainment as 'high school degree or lower', 'associate's degree of some college education' and 'bachelor's degree or higher', respectively. A natural question is whether our model's assumption of an equal size of each ability group holds up. In 2019, this assumption seems to approximate reality quite well: 33% of the US population between 25 and 34 had a high school degree or lower as their highest diploma (ISCED 3 or lower), 27% had an associate's degree or did some college education (ISCED 4 and 5) and 40% had a bachelor's degree or higher (ISCED 6 or higher) (U.S. Census Bureau, Current Population Survey, 2019 Annual Social and Economic Supplement).
[7] These are the three ISCED levels that we see as representative for low ability: primary education or less, lower secondary education or higher secondary education.

work of Frey and Osborne (2017) among others, and also estimate the probability of job automation by education level.[8] We impose that the relative levels of future automation probabilities that they report, are also reflected in the share of tasks of each ability that are already automated. In practice, this results in two conditions demanding that the share of medium ability (high ability) tasks that are automated is 84.7% (48.0%) of the share of low ability tasks that are automated. In absolute terms, it then follows that we impose a value for $1-\xi_M$ equal to 21% and a value for $1-\xi_H$ equal to 12%. We thus assume that the same ability bias expected in future automation has been present in the automation technologies up to now. Compared to the findings of Arntz et al. (2016), Popescu et al. (2018) put forward relatively small differences in job automatability between education levels. By opting for their estimates to serve as calibration targets, our model is less likely to overestimate the inequality-enhancing impact of automation.

3.1.4 Efficiency parameter of automation capital J

Together with the three shares of automated tasks $(1-\xi_a)$ fixed above, it will be the efficiency parameter J that determines the share of income that is a remuneration for automation capital (equation (41)).

$$(41)\quad (1-\alpha)A_t^{1-\alpha}\left(\frac{K_t}{H_t}\right)^{\alpha}\sum_{a=L,M,H}\left\{\eta_a\left(\frac{H_t}{H_{a,tot,t}}\right)^{\frac{1}{s}}(1-\xi_a)\left(\frac{H_{a,tot,t}}{P_t}\right)^{\frac{1}{\kappa}}J\right\}\frac{P_t}{Y_t}$$

Since traditional capital K_t and aggregate effective labour H_t are combined in a Cobb-Douglas production function with an output elasticity of traditional capital of 0.25, the share of human labour in the national income is 0.75 minus automation capital's income share. As indicated earlier, before tasks became technologically automatable ($\xi_a = 1$, $\forall a = L, M, H$), automation capital's share of income was zero (equation (41)) and the labour share of income was a constant 0.75, which is thought to reflect the constancy of the labour share that Keynes noted (1939). We then calibrate the constant parameter value J such that – with given values of $1-\xi_a$ and the demographic parameters – the labour share that our model produces in the US for the 2005-2019 model period is lower than this original 0.75 due to automation. More specifically, we will target a value for the US labour share in this period of 0.701. That is precisely half of the fall from the initial 0.75 to the level of 0.652 that Gutiérrez (2017)[9] finds for the US labour share (excluding the real estate, finance and non-business sectors) for 2010-2014. In imposing that the automation of tasks was the driving force behind 50% of the fall in the labour share, we follow the findings of Karabarbounis and Neiman (2014) and Dao et al. (2017).[10] As one can see in Table 1, this approach yields a value for J equal to 9.26.

[8] In line with our approach in the previous section, we equate the low ability type with the 'less than high school' and 'high school' attainment, the medium ability type with the 'some college' and 'associate' attainment and the high ability type with the 'bachelors', 'masters' and 'doctorate' attainment. We weigh each attainment by the percentage of jobs with this education level as reported in Popescu et al. (2018). This approach results in automation probabilities that unambiguously fall with the educational level, consistent with the original findings of Frey and Osborne (2017) and the work of Arntz et al. (2016).

[9] Gutiérrez and Piton (2020) report similar data for the US labour share (excluding real estate) for 2010-2015 in Figure D.2 of their appendix. In assuming that, in total, the gross US labour share for the private business sector declined by almost 10 percentage points (from its 'constant' level until 2005-2019), we follow Manyika et al. (2019) and Karabarbounis and Neiman (2012).

[10] An alternative approach could be to target the stock of automation capital, for example robot capital in the US in the 2005-2019 period. Doing that, however, would inevitably result in a vision of automation that is too narrow: in reality, there are a multitude of automation technologies contributing to the fall in the labour share.

We verified and confirmed that this parameterization leads to an efficiency of automation capital that is sufficiently high such that our simplifying assumption that 'all automatable tasks are automated' holds throughout all simulations. Following Acemoglu and Restrepo (2018b), the condition that has to be satisfied such that it is strictly cheaper to produce automated tasks with automation capital is $w_t > \frac{r_t + \delta_p}{j}$ where $(r_t + \delta_p)/j$ represents the real cost of producing a task with automation capital and w_t the real cost of producing the same task with human labour[11]. Given the general-purpose nature of our automation technology, the condition is more complex in our model. The inequality condition that has to hold such that automatable tasks of the low ability type are automated is expressed by equation (42). This is derived in appendix A, part 2. Mutatis mutandis, this condition also applies to medium and high ability tasks.

$$(42)\quad \frac{\frac{\partial Y_t}{\partial P_t}}{\frac{\partial Y_t}{\partial h_{L,k,t}}} = \frac{(1-\alpha)(\frac{K_t}{H_t})^{\alpha}\left[\sum_{a=L,M,H}\eta_a(\frac{H_t}{H_{a,tot,t}})^{\frac{1}{s}} H_{a,tot,t}{}^{\frac{1}{\kappa}}\,\frac{\kappa-1}{\kappa}(1-\xi_a)JP_t{}^{\frac{-1}{\kappa}}\right]}{(1-\alpha)(\frac{K_t}{H_t})^{\alpha}\eta_L(\frac{H_t}{H_{L,tot,t}})^{\frac{1}{s}} H_{L,tot,t}{}^{\frac{1}{\kappa}}\,\frac{\kappa-1}{\kappa}J^{\frac{-1}{\kappa-1}}P_t{}^{\frac{-1}{\kappa}}\frac{1}{3}} > \frac{(r_t+\delta_p)(1-\tau_p)^{-1}}{w_{L,t}}$$

The expression in (42) will always hold if J is set sufficiently high. For the value that we obtained, that is the case. The corresponding conditions for medium and high ability tasks are also satisfied.

3.1.5 Elasticity of substitution between automated and non-automated tasks: κ

Our model and calibration impose constant parameter values for $1-\xi_a$ and J over all periods. This choice reflects our focus on non-technological automation at the intensive margin, as we have argued in sub-section 2.3. We emphasize as an important channel in response to ageing (increased life expectancy, scarcity of young workers) that firms make more use of automated tasks in their production and less use of tasks executed by humans. The elasticity of substitution between automated and non-automated tasks κ is crucial here. We calibrate this parameter such that ageing induces automation to the extent that is found by the empirical study of Acemoglu and Restrepo (2018c). They find that if they increase their ageing variable from the US level to the German level, keeping all other things equal[12], the induced increase in robots per worker "is about 25% of the Germany-US difference in the adoption of robots" (p. 22)[13]. Practically, we use the data for 2014 that Acemoglu and Restrepo (2018c) present on the size of the robotics gap between Germany and the US: Germany's relative lead in robotics was approximately 85.5%. Acemoglu and Restrepo (2018c) thus more generally find that if the US had the demographic structure of Germany, their robot density would be 21.4% higher (a quarter of 85.5%). For our calibration, we then impose that, just like in Acemoglu and Restrepo (2018c), applying German demography to the US and keeping all other things equal leads to

---

[11] We assume that workers have no preference with regard to executing automatable tasks or non-automatable tasks such that they will only execute the former if both tasks pay the same hourly wage.

[12] In the work of Acemoglu and Restrepo (2018c), this implies that the control variables remain at US level. In our case, "keeping all other things equal" means that all non-demographic parameters are kept at US level.

[13] We are aware of the fact that in the newest version of this study, Acemoglu and Restrepo (2021) report a stronger link between ageing and robot adoption, with ageing explaining 50% of the Germany-US robotics gap. The authors convincingly show that ageing significantly stimulates robot adoption, but the slope of the relationship is (in both studies) somewhat dependent on the precise specification. We opt to follow the more cautious finding of 25% in Acemoglu and Restrepo (2018c). In appendix F, we show that if we double the calibration target, which implies (requires) a much higher κ, we would strongly overestimate the cross-country differences in robot density.

a rise in the baseline automation density of 21.4%. In the counterfactual case with German demographics, the US level of automation capital per worker has to be 1.214 times the level of automation capital per worker in the baseline case for the US with regular demographics, when both are evaluated at the end of the 2005-2019 period. This is the target value for our calibration. It yields a value of κ equal to 3.28.[14]

The explanation for why the longer life expectancy and the scarcity of young workers in Germany relative to the US contributes to the higher adoption of robotics in Germany is twofold. Note that as automated and non-automated tasks substitute better for one another (a higher κ), both explanations for why ageing stimulates automation gain in strength.

First, Carvalho et al. (2016), among many others, argue that increased longevity and reduced fertility have a net positive effect on total savings. In our closed economy model, increased national savings will lead to lower interest rates and capital deepening for all types of capital. As noted in Palivos and Karagiannis (2010), however, a higher elasticity of substitution with human labour counteracts diminishing returns to capital. This implies that the ageing-induced capital deepening will not stimulate the accumulation of the two capital types equally. A proportional increase in K and P would imply a stronger fall in the marginal product of traditional capital. Due to the no-arbitrage condition in equation (26), this is not allowed and, compared to the situation of proportional increases, P will increase even more and K will fall until the equality of returns has been re-established. Irmen (2021) too finds that, in the long run, a rise in longevity stimulates automation. Second, Abeliansky and Prettner (2017) outline how a fall in fertility can generate a relative shortage of human labour supply that can encourage the adoption of automation technologies. While automation capital is only a q-substitute for low ability labour (cfr. infra), it can be shown that an equal fall in the labour supply of all ability types positively affects the marginal product of automation capital (while, of course, reducing the marginal product of traditional capital) for the final parameter values. Given the no-arbitrage condition, a fall in fertility will thus stimulate the accumulation of automation capital.[15] Both the automation-enhancing effect of a rise in longevity and of a fall in fertility will be stronger in case of a higher elasticity of substitution between automated and non-automated tasks κ, since the derivative of the marginal product of automation capital with respect to the human labour input will be lower in case of a higher κ. This is shown in Appendix B.

[14] Ultimately, we judge this calibration approach for κ to be more sensible than imposing that the model mimics the wage effect of increases in robot capital found in empirical studies. An important reason is the wide variety in estimated effects. Regarding the impact of increasing robot density on aggregate wages, Dauth et al. (2017) find insignificant effects, Graetz and Michaels (2018) find significantly positive effects and Acemoglu and Restrepo (2020) find significantly negative effects. In our model, the effects of a rise in automation density vary depending on the margin along which the increase was generated. In a general equilibrium model, one does not have the luxury to abstract from what has caused the increased automation. Automation at the intensive margin increases the aggregate wage moderately, automation at the extensive margin strongly decreases aggregate wages. The literature does not provide clear guidance on what type of technological progress is responsible for the increase in robot density. A varying importance of each type of progress could explain the wide variety in estimated wage effects of robotics.

[15] This is closely related to the work of Irmen (2021) who finds that a fall in fertility leads to a rise in aggregate wages, which provides incentives for firms to automate (here: the substitution of automated tasks for non-automated tasks).

3.1.6 Automation at the intensive margin and the effect on wages: q-substitutes?

Finally, we note that our model succeeds in creating heterogeneous effects from investment in automation capital on the different types of human labour despite the common elasticity of substitution κ. The main driver of this result is the difference in the shares of total tasks that are automated. Similar to the findings of Acemoglu and Restrepo (2020) with regard to robotics, the effects of any increase in the stock of automation capital are far more benign for the high ability workers. This can be seen in equation (43), derived in Appendix B.

$$(43)\ sign\left(\frac{\partial w_{a,t}}{\partial P_t}\right) = sign\left[\left\{(1-\xi_a)\left(\frac{H_{a,tot,t}}{P_t}\right)^{\frac{1}{\kappa}}\right\}\left\{\left(\frac{1}{s}-\alpha\right)\eta_a H_t^{-1}\left(\frac{H_t}{H_{a,tot,t}}\right)^{\frac{1}{s}} + \left(\frac{1}{\kappa}-\frac{1}{s}\right)H_{a,tot,t}^{-1}\right\} + \left(\frac{1}{s}-\alpha\right)H_t^{-1}\sum_{j\neq a}\left\{\eta_j(1-\xi_j)\left(\frac{H_t}{H_{j,tot,t}}\right)^{\frac{1}{s}}\left(\frac{H_{j,tot,t}}{P_t}\right)^{\frac{1}{\kappa}}\right\}\right]$$

The equation determines whether a rise in the input of automation capital in the production function has a positive or negative effect on the real hourly wage per unit of human capital for an individual of ability a. The continuity condition for symmetry of the mixed second order derivatives is met, such that the equation above also indicates whether the marginal product of automation capital is decreasing or increasing in the amount of human labour of type a.

The two terms that play a role have distinct interpretations. The first term indicates the effect of a rise in P on the wage of workers performing tasks of type a through the increase in *total* effective labour performing tasks of type a. The degree to which automated tasks can substitute for non-automated tasks of the same ability type a - embodied by the elasticity of substitution κ - plays a crucial role here. It can easily be seen that, when the elasticity of substitution κ is sufficiently high, this first effect always turns negative. For our calibrated parameter values, this is the case for all ability types. It will be more negative for the low and medium ability workers, however, since a larger share of tasks performed by them are technologically automatable (larger $1-\xi_a$). This is not the whole story though. The second term indicates the effect of an increase in automation capital on the wage of workers performing tasks of type a through the increase in total effective labour that performs tasks that are different from a.[16] The elasticity of substitution s between the different ability types plays a large role in determining the sign of this second effect. If performing tasks of an ability level different from a substitutes for performing tasks of type a - s is large -, this effect will turn negative. For our calibrated parameter levels, however, tasks of different ability types are q-complements and the effect will be positive.

The total effect of this non-technological automation at the intensive margin is negative for low ability workers, but positive for medium and high ability workers for the calibrated parameter

[16] Note that the presence of this second term depends crucially on a specific modelling choice. In our framework, it is the same stock of automation capital P that substitutes for all human labour, regardless of the type of task a. There is only one automation technology and the representative firm increases its input of this technology by investing in P. This automation capital then contributes to the execution of tasks of all types proportional to the share parameter $1-\xi_a$ for that task type. This modelling choice reflects the nature of more general-purpose automation technologies such as computerisation better than technologies that focus solely on tasks performed by blue-collar, low ability workers (of which robotisation might be a more suitable example).

values. Strictly speaking, automation is thus only a q-substitute for low ability human labour, since, for workers of higher ability, the negative displacement effect of automation is more than fully compensated by the increased execution of complementary tasks.

3.2 Empirical test of the model

In this section we confront our model's predictions with the data on cross-country differences in automation, employment among older workers, and investment in human capital by the young generation. Our calibration implies that the model's predictions match the data in the US exactly. A minimal test of the model's validity and empirical relevance is whether it can also match the data for other OECD countries and (especially) the size of the cross-country differences. To do this test, we basically impose the same preference and technology parameters reported in Table 1 on all countries. Only the exogenous demographic variables and policy parameters, and one 'technology' parameter, differ. In Appendix C and D, we describe the demographic variables and the policy parameters in greater detail. We also show the data per country. The one technology parameter that differs is the efficiency parameter ϕ in the production of human capital. Here we follow the approach in Boone and Heylen (2019) and allow differences across four country-groups to capture the effects of differences in institutions that may affect the characteristics and the quality of tertiary education[17]. All in all, this confrontation with the data in Figure 2 is encouraging. Our model seems able to translate observed differences in demography and policy into realistic performance differences.

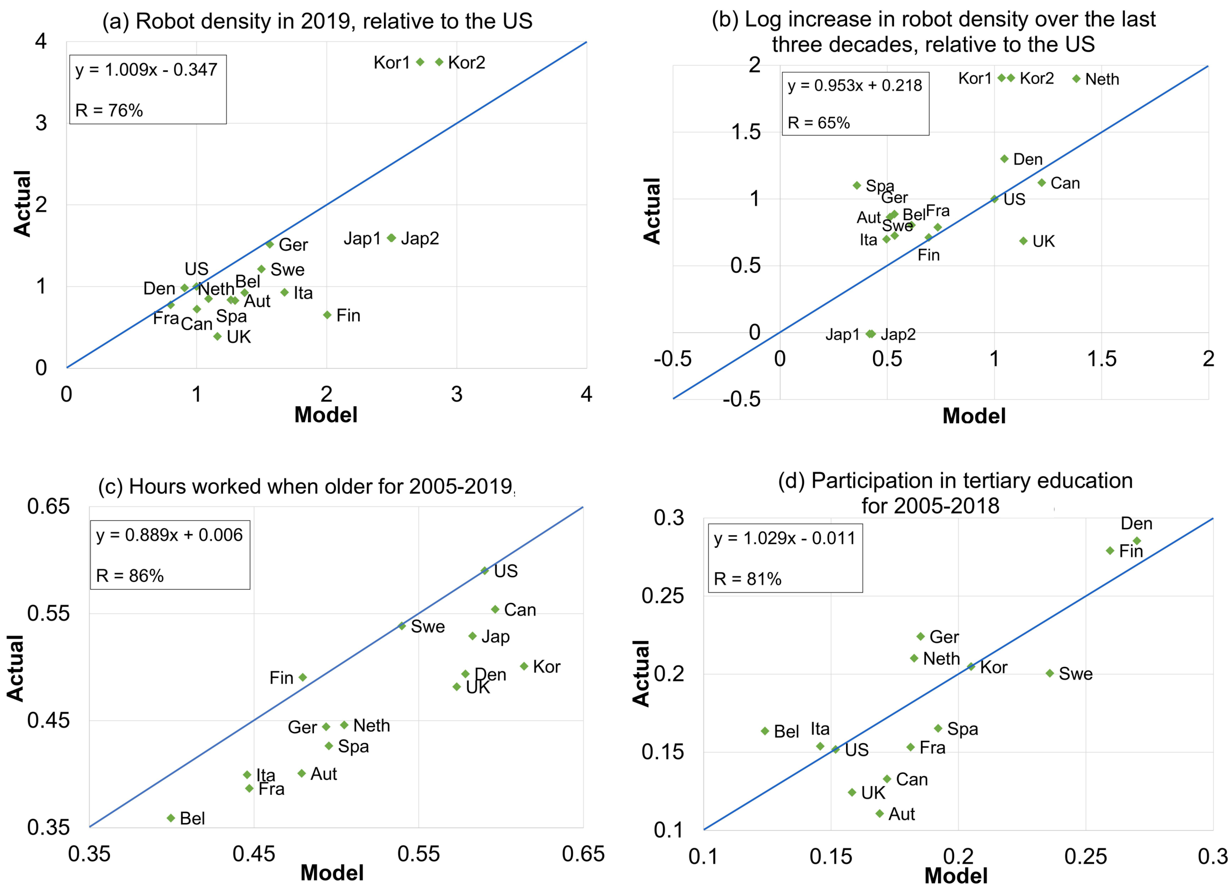
The blue line in each panel of Figure 2 is the 45°-line. In the upper left corner of each panel, we also report the specification of the regression line that would provide the best fit between the model's predictions and the data, as well as the correlation coefficient R. The regression line itself is not drawn. Figures 2(a) verifies whether the model can accurately reproduce differences in automation between countries. The actual data used comes from the International Federation of Robotics (2005; 2014; 2020) (IFR) reports on the most robotized countries in 2019.[18] The IFR presents data on robot density in the form of "number of industrial robots per 10 000 employees in the manufacturing industry", out of which we select fifteen OECD economies. The vertical axis of Figure 2(a) shows the actual robot density in 2019, expressed relatively to the US. The model's indicator of automation density - set out on the horizontal axis - is the ratio of the amount of automation capital P_t at the end of the 2005-2019 period relative to the size of the three active generations in this period. Since everyone on the labour market actually works in the model (there is no unemployment), this indicator reflects the amount of automation capital per worker (of which some workers work full time and some

[17] The four country groups we consider are euro area countries (Austria, Belgium, France, Germany, Italy, the Netherlands, Spain), Nordic countries (Denmark, Finland, Sweden), East-Asian countries (Korea, Japan) and Anglo-Saxon countries (Canada, UK, US). For the Anglo-Saxon countries, we impose the calibrated $\phi = 1.27$ of the US. For the other country groups, the parameter ϕ is calibrated such that the average share of time spent studying when young across the country block accurately predicts the recent average participation in education of individuals between 20 and 34 in these countries. These averages are respectively 16.9%, 25.5% and 20.5% yielding $\phi_{euro} = 1.49$, $\phi_{nordic} = 2.78$ and $\phi_{east\ asia} = 1.37$. For Japan, OECD.stat does not include data on the enrolment rate of 20- to 34-year-olds. As a result, the East Asian calibration is solely based on Korea.

[18] The data thus only represents one automation technology. Due to its focus on manual labour tasks, it might not fully be in line with our model's automation technology which is inherently general-purpose in nature. Our focus on robotics is however very much in line with the literature, and is the result of the relative lack of data on the adoption of other automation technologies (Martens & Tolan, 2018).

part-time, as in reality). The values of this indicator are, for all fifteen OECD economies, also expressed relative to the value for the US.[19]

Figure 2: *Model predictions (horizontal) against actual data (vertical)*



In panel (a) and (b), Kor2 and Jap2 denote the result for South Korea and Japan respectively, when taking into account the historical and future tax incentives for investment in robotics which the countries have offered. Kor1 and Jap1 denote the result for both countries when ignoring these tax incentives. Underlying the model predictions for each country is the assumption that lump sum transfers adjust in equation (30) to keep the predicted public debt-to-GDP ratio equal to its actual level.

The baseline correlation between the model's predictions and the actual values in panel (a) is 76%, when taking into account the past and future tax credits[20] for robotics in Japan and Korea. The slope of 1.01 is extremely close to the 'optimal' value of 1. In words, our model does not systematically overestimate the effects of demographic and policy differences (slope below 1), nor does it systematically underestimate these effects (slope above 1). When we ignore the

.

[20] Japan and Korea are, to the best of our knowledge, the only two countries in our sample who had special tax measures for robotics. In Japan, the tax credit rate related to investment in robotics was 3-5% under the 'Connected Industries tax system'. The system was in place from June 2018 until March 2020. More details can be found at https://www.jetro.go.jp/en/invest/support_programs/incentive. In the Republic of Korea, the Restriction on Special Taxation Act defines a tax credit related to investment in robotics from January 1994 onwards. The tax credit rate was initially 3% to 7% depending on firm size, but the tax credit rate was lowered by 2% from 2017 onwards. Korean legal records on the evolution of the tax credit for robotics can be found on https://www.law.go.kr/LSW/main.html. From 2020 onwards, the tax credit rate will be 3%, 5% or 12% for large, medium-size and small companies respectively (https://assets.kpmg/content/dam/kpmg/kr/pdf/2020/korea-tax-brief-202008-v2-eng.pdf). We model the tax credit rates through negative values for τ_p dependent on the size of investment in automation capital.

tax incentives for robotics investment in Japan and Korea, correlation falls slightly to 73%. Even when we remove Korea from the sample, correlation is still a respectable 52%. The model accurately captures the high degree of automation in the two East Asian economies relative to the other nations, but the size of the robotics lead of Japan is somewhat overestimated.[21]

The model mainly relies on exogenous differences in ageing and fiscal policy between countries to reproduce actual differences in robot adoption. Ageing stimulates the adoption of automation capital for the two reasons highlighted in sub-section 3.1.5. High taxes on low ability labour and high benefit replacement rates for low ability individuals all induce a relative shortage in the labour supply of low ability labour leading to an increased effectiveness of automation capital. High taxes on the return to savings, on the other hand, increase the cost of capital and lower the profitability of automated tasks substituting for non-automated tasks. High government debt functions in a similar way by raising the interest rate. Finally, generous old-age pension systems reduce aggregate savings and thus increase the interest rate and the cost of (automation) capital (see for instance Rachel and Summers (2019)), making automation capital less cost-effective.

Figure 2(b) verifies whether the model correctly reproduces cross-country differences in recent trends in robot adoption. On the vertical axis of this panel, the change in the log of robots per 10.000 workers in the manufacturing sector is set out for each country, relative to the change in the log in the US in the same period. For most countries, the period over which the log change is calculated starts in 1993 (the earliest data the IFR provides) and ends in 2019. For Belgium, Canada, the Netherlands, Korea and Japan, we are limited to 2005-2019 for reasons of data availability and quality. On the horizontal axis of panel (b), the log increase in our model's proxy for automation density is set out, again relative to the log increase in the proxy for the US in that same period.[22] Focusing on changes relative to the change in the US is desirable since it allows to abstract from common factors driving automation, such as technological improvements in the quality of robots. This approach matches the set-up in our model to hold technology constant over time and between countries.

The correlation between actual values and the model's predictions in panel (b) is 65% in the baseline sample, when taking into account the robot tax incentives in Japan and Korea. The slope of 0.95 is, again, very close to 1, indicating that our model neither over- nor underestimates the degree to which ageing causes differences in the evolution of automation. When ignoring the robot tax incentives for Japan and Korea, correlation is still 64%. In general, the model performs quite well on this front and the bulk of its explanatory power in this regard has to be ascribed to demographic change, since, except for the size of government consumption and the evolution of the debt-to-GDP ratio, policy parameters are kept constant

---

[21] One remaining concern might be that some countries have a higher robot density because they are historically specialized in sectors which are very suitable to automation (e.g., Germany in the automobile industry). When we correct the actual values on the vertical axis of Figure 2(a) based OECD data on the historical sector composition of manufacturing employment in 1995 and IFR data on the robot density per sector in 2019, we still find the correlation in panel (a) to be 67% with a slope of 0.69.

[22] Given the fifteen-year length of periods in the model, the log change between 1993 and 2019 is proxied by the log increase from the end of the 1975-1989 period to the end of the 2005-2019 period. For the five countries above, our model indicator is the log increase in automation density from the end of the 1990-2004 period to the end of the 2005-2019 period.

throughout the simulations. This result is thus in line with the claim of Acemoglu and Restrepo (2018c) that (expected) ageing is a significant determinant of the adoption of robots.

We also check the model's performance with respect to hours worked among older individuals in panel (c). The actual employment rate is constructed based on OECD data for 2005-2019 and considers both the intensive and extensive margin of employment (more details in Appendix D). It is proxied by the average share of time spent working by individuals aged 50 to 64 in the 2005-2019 modelling period. Although we observe a clear overestimation of the employment rate in most European nations, the correlation between actual values and model predictions is very high and the slope of the regression line is again close to 1. The overestimation of the employment rate in Europe may follow from imposing the calibrated taste for leisure of the US. Europeans may have a higher taste for leisure as suggested by Blanchard (2004).

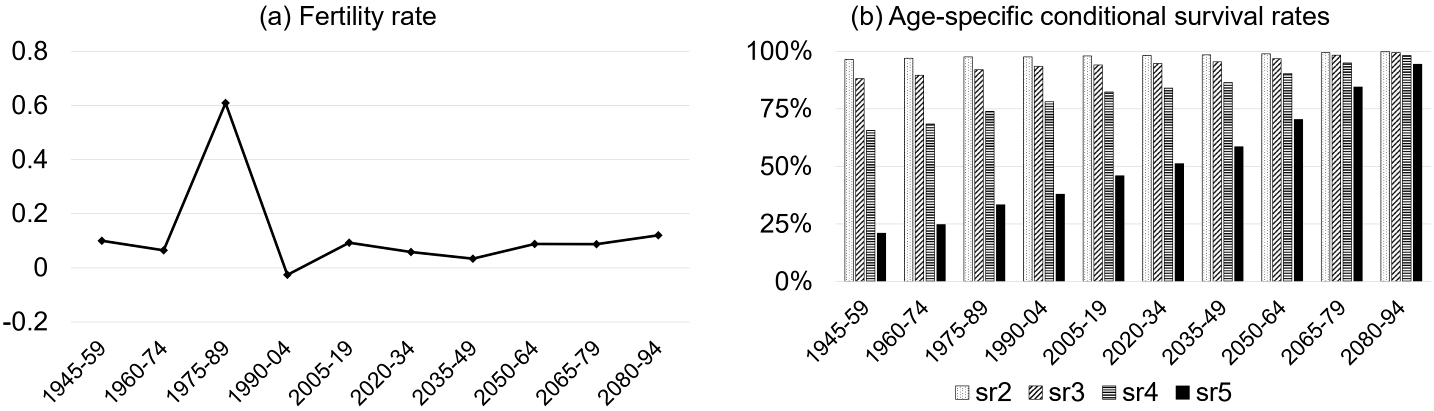
Finally, we check the model's performance with respect to participation in tertiary education in panel (d). The actual tertiary education participation rate is constructed based on OECD data for 2005-2018 and considers both part- and full-time students (more details in Appendix D). It is proxied by the average share of time spent studying of an individual aged 20 to 34 in the 2005-2019 period. The correlation between actual values and model predictions is 81% and the slope is 1.03. Of course, the region-specific value of ϕ contributes to the very good result here.

After comparing the model's predictions with key actual data on four fronts across OECD economies, we conclude that it is meaningful to use our model to evaluate the automation effects of ageing and to simulate policy shocks. The cross-country differences in automation density are realistically captured by the model in panel (a) of Figure 2, while panel (b) shows that the model can also account for differences in the evolution of automation density during recent decades. Furthermore, the model's specification and parameters seem capable of translating observed differences in policy and demography into realistic differences in labour supply at older age and education when young (panels (c) and (d)). This is crucial since, in addition to automation, the reaction in these two variables plays a major role in the relationship between ageing and the possibility of secular stagnation. Despite the obvious limitations of our test in Figure 2, its outcome clearly raises confidence in the reliability of our calibration, and our simulations in the next section.

4 Ageing, automation and inequality: the effects of ageing-induced automation

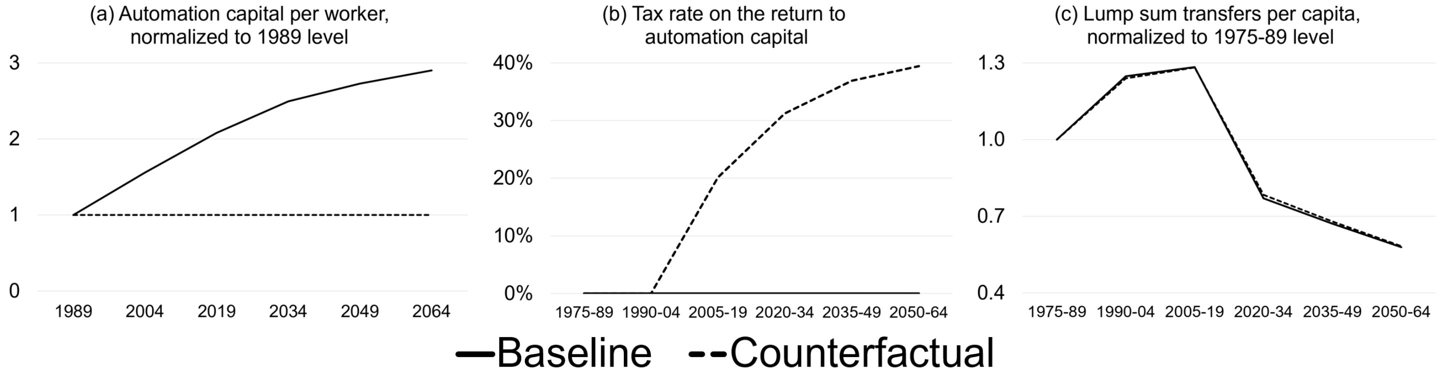
For the purpose of this section, we simulate the deterministic model for the US using Dynare 4.6.2 (Adjemian et al., 2011). Just as before, we impose exogenous paths for the five demographic parameters, which are fully known beforehand by the individuals in the model (more details in Appendix C and D). The exogenous path of the demographic parameters in the US is shown in Figure 3. In our baseline simulation of the future, all policy parameters and the public debt ratio are kept at their level of 2005-2019. Lump sum taxes adjust such that the government budget constraint always holds.

Figure 3: *Path of exogenous demographic parameters for the US*



In this baseline simulation, we observe that the exogenous rise in life expectancy and the retirement of the baby boom generation generate a shift in factor prices facilitating the adoption of automation capital (see Figure 4, panel (a)). As wages increase and the interest rate falls, substituting automated tasks for non-automated tasks is the logical thing to do for cost-minimizing firms. To evaluate the impact of the ageing-induced automation, we compare the baseline results with the counterfactual scenario in which the government raises taxes on the return to automation capital such that the level of automation capital per worker does not increase through time (see Figure 4, panel (b)). More precisely, the tax rate on the return to automation capital $\tau_{p,t}$ varies endogenously over time to ensure that the decisions of rational investors lead to a level of automation density that is constant to the 1989 level. Lump sum transfers per capita adjust to absorb the budgetary impact of this automation tax, but, in per capita terms, transfers are not materially different from the baseline case (Figure 4, panel (c)). The first taxation of automation capital is announced in the 1990-2004 model period and implemented in the 2005-2019 model period (see Figure 4, panel (b)). In our counterfactual scenario, we thus look at *the consequences of ageing when the incentive to automate that ageing implies is neutralized.* This differs from the approach taken by Stähler (2021) whose counterfactual looks at the consequences of ageing in the absence of any automation technology. We argue that since ageing directly impacts factor prices and not technology, our approach based on a scenario with counterfactual costs rather than automatability offers a more suitable comparison.

Figure 4: *Evolution of automation density, automation tax and lump sum transfers*[19]



In Figure 4(a), 'automation capital per worker' relates to the ratio of the stock of automation capital at the end of the relevant model period to the size of the workforce in that relevant period. The same applies to Figure 5(d) and (e).

4.1 Output, capital formation and labour

Figure 5 summarizes the dynamic simulation results of both the baseline and the counterfactual scenario from the 1975-89 period until the 2050-64 model period.[23] Given our explanation above, the difference between the baseline and the counterfactual reveals the impact of the ageing-induced automation. Based on panel (a), we can thus conclude that the additional automation generated by demographic change has been a factor contributing to per capita growth in the past and it will be a factor softening the negative output per capita effect of a rising old-age dependency ratio in the future.[24] Since the output effect of ageing is thus somewhat less negative when taking into account that ageing stimulates automation, we consider our results as evidence that is *cautiously* supportive of the hypothesis of Acemoglu and Restrepo (2017): automation mitigates the negative effects of ageing, but only partially. The intuitive reason behind this mitigation is that, when additional automation is allowed, the increased execution of automated tasks can compensate for the relative shortage of human workers executing non-automated tasks. In short, ageing-induced automation can soften the decline of the labour supply per capita and thus slow down the 'headwind' that ageing creates from the supply side (Gordon, 2014). Automated and non-automated tasks are imperfect substitutes, however, such that the compensation is only partial. As of now at least, the scope of tasks in which automation technologies can help cushion the shortage of human workers is found to be insufficient. The degree of substitutability between automated and non-automated tasks seems to be a crucial factor. When κ is raised sufficiently such that automated and non-automated tasks substitute better for each other, it is theoretically possible that ageing-induced automation entirely neutralizes the negative effects of future demographic change in the US on growth. In Appendix F, we explore this scenario further, but also explain why we judge this theoretical case to be empirically unrealistic.

In Figure 5(b), one can observe that demographic change is a factor drastically lowering the interest rate, consistent with earlier analyses of demographic change in an OLG context

[23] In Figure 4, 5 and 6, we detrend aggregate variables (such as wage rates, output levels, capital stocks) that increase by the rate of technical progress x from period to period. These figures thus indicate how these macroeconomic variables deviate from their rising trend. As a result, our 'constant automation capital per worker' counterfactual is, in fact, a scenario in which the growth of automation capital is limited to the rate of technical progress x.

[24] In appendix C, we show that the old-age dependency ratio of the US only starts to rise strongly from the 2020-34 model period onwards. Before that, the growth effect of demographic change is positive (even in the counterfactual without ageing-induced automation) since the primary reason ageing could weigh down growth - what Bloom et al. (2010) labelled the 'accounting effect' - is absent.

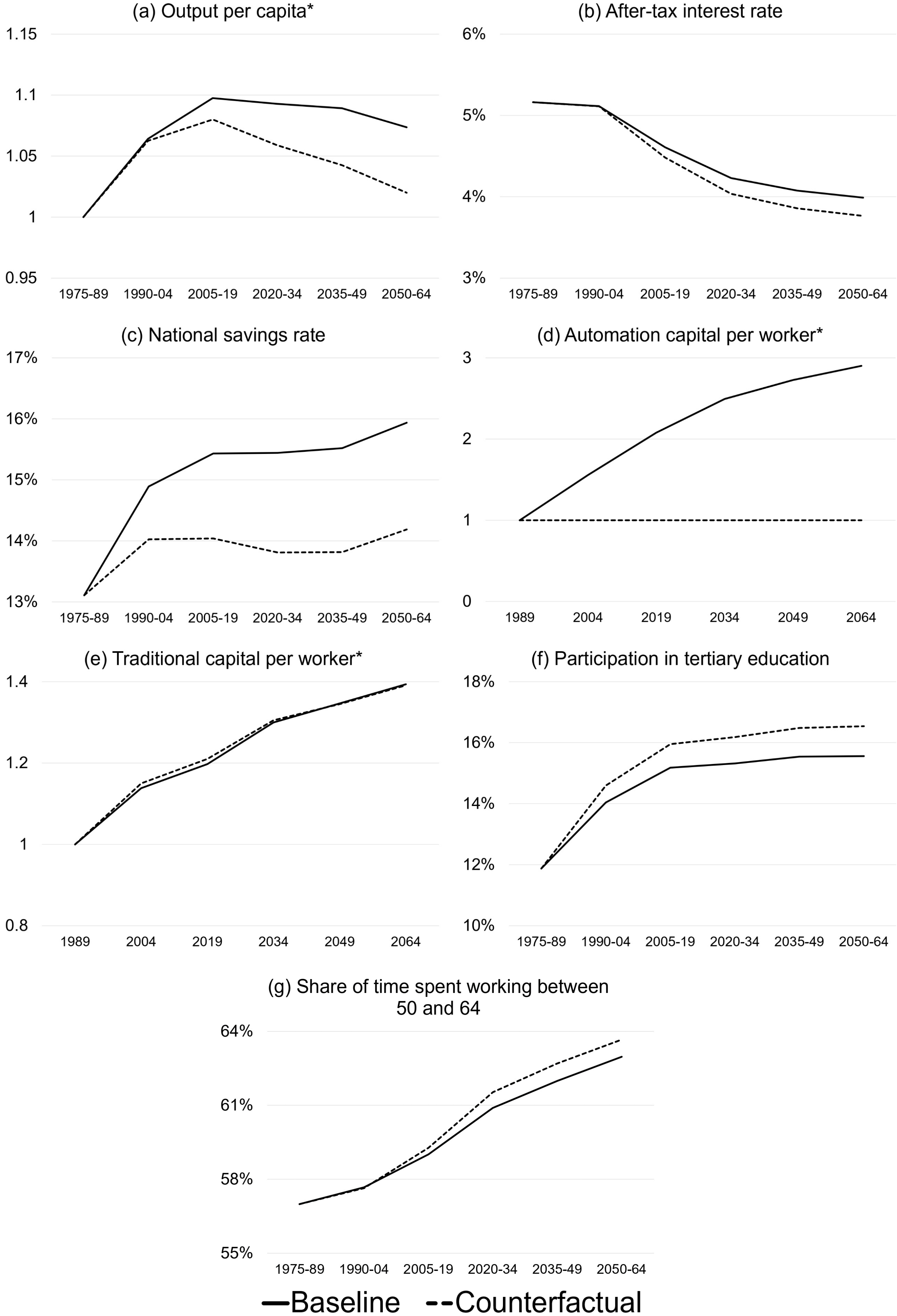
(Ludwig et al., 2012; Eggertson et al., 2019; Gagnon et al., 2021). When the rise in automation capital is artificially suppressed by levying an automation tax, the fall in the interest rate is even more pronounced. Ageing-induced automation thus partially counteracts the decline in the interest rate generated by demographic change. The main reason for this is that traditional capital and automation capital complement one another such that allowing investment in automation capital can help to keep the productivity of traditional capital high in times of rising capital intensity.[25] As in Eggertson et al. (2019), we interpret the decline in the return to (traditional) capital as an indicator for the new secular stagnation hypothesis and thus conclude that ageing-induced automation might also constitute a force mitigating demand-side secular stagnation. It is also noteworthy that, when allowed, the rise in automation capital density is far more pronounced than for traditional capital (see Figure 5, panel (d) and (e)). The intuitive explanation behind this is that, because of its less complementary relation to human labour, automation capital thrives in an ageing economy where labour becomes scarcer. Because investors demand the same return on both types of investments, automation density has to rise more than traditional capital intensity.

In line with the findings of Heijdra and Romp (2009) and Ludwig et al. (2012), rising life expectancy stimulates investment in human capital considerably (Figure 5, panel (f)) since individuals have a higher chance of being alive in stages of life where one can benefit from this investment in human capital. The anticipation of a longer life as pensioner (without labour income) likewise leads individuals to work more when older (Figure 5, panel (g)). This too is in line with the findings of the OLG literature studying the effects of ageing (e.g., Heijdra & Reijnders, 2018; Devriendt & Heylen, 2020). Following Bloom et al. (2010) working longer and studying more can be labelled important “behavioural effects of ageing” mitigating the negative impact of an older population. In Figure 5 panel (f) and (g), one can observe that both behavioural effects are less strong when ageing-induced automation is allowed to take place. The main driver of this result is the higher interest rate in this scenario. When the interest rate is higher, working when young and transferring income to the future through the accumulation of non-human wealth becomes more interesting relative to investing in the accumulation of human capital. Given the higher return on savings, individuals also don’t have to work as hard when old to achieve sufficient resources during retirement (income effect) and, given the lower amount of built up human capital, the financial return to working is also less worthwhile (substitution effect). The negative linkage between the automation effect of ageing and the behavioural effects provides a second explanation for why the mitigating effect of ageing-induced automation is only partial. In short, typical theoretical models studying the growth effects find a negative net effect linked to ageing because *behavioural* effects compensate incompletely for the negative accounting effect of a rising old-age dependency. We find that, even when adding the possibility for capital - not just labour - to react endogenously to counter the relative labour shortage, the net growth effect of ageing remains negative.

---

[25]Note that allowing both capital types to complement one another generates, what is effectively, an upward shift in capital demand. In combination with an upward-sloping capital supply curve, this leads to a rise in the national savings rate ($=\frac{Y_t-C_t-G_t}{Y_t}$) (see Figure 5, panel (c)). Intuitively, one might have thought that restricting the investment in automation capital in the counterfactual would generate a larger traditional capital density in the counterfactual (as investors are forced into traditional capital investments). However, this is not the case: the positive productivity shock to traditional capital allows traditional capital intensity to be equally high in the baseline as in the counterfactual. Therefore, allowing investment in labour-saving technologies does not come at the expense of investment in labour-augmenting technologies.

Figure 5: *Evolution of seven selected indicators in the baseline scenario and the counterfactual scenario without ageing-induced automation. Indicators marked with * are normalized to the level in the 1975-1989 model period. The other variables are in level (%).*



4.2 Wage formation, labour share and inequality

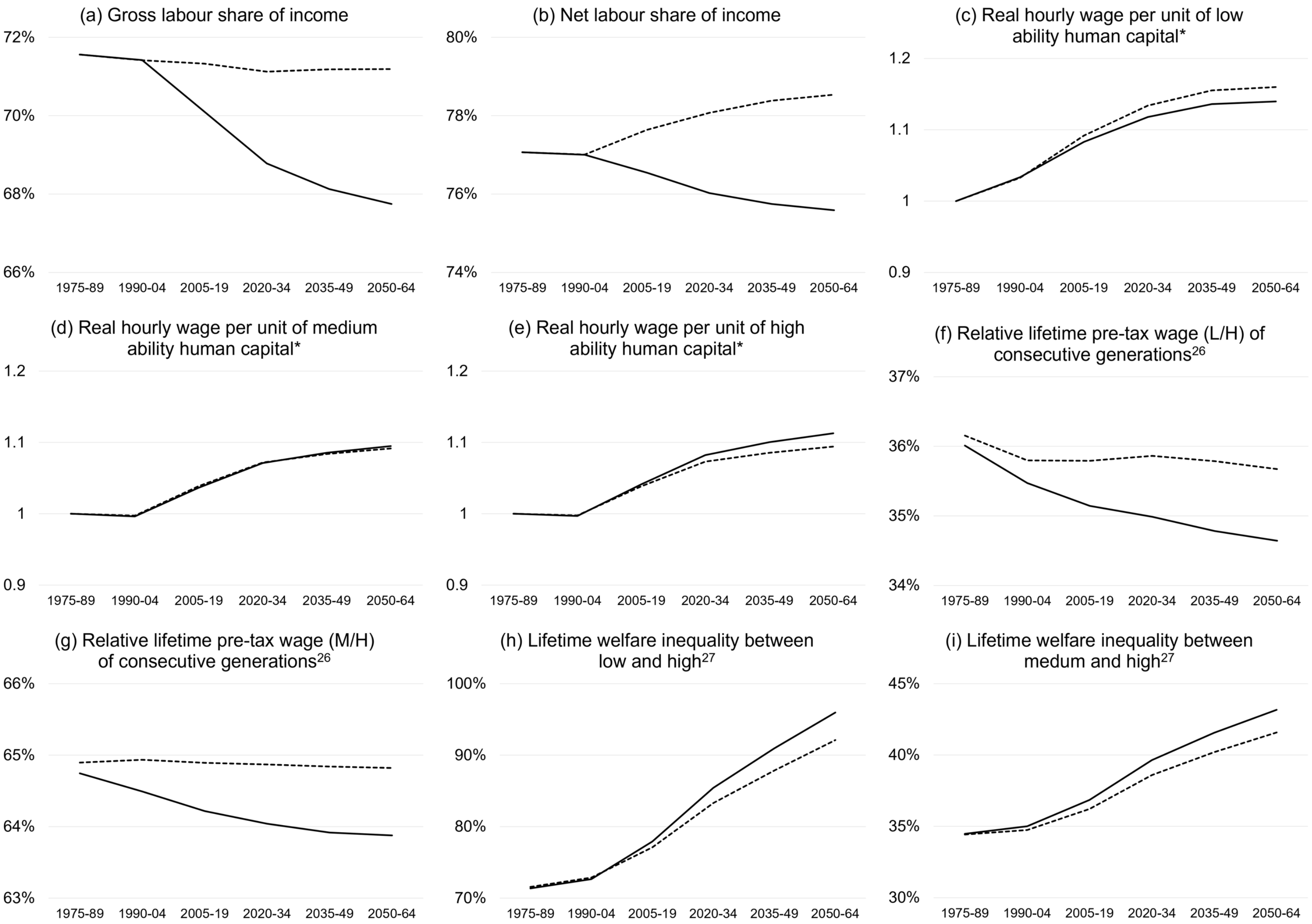
The strong, ageing-induced increase in the adoption of automation capital generates a fall in the labour share due to the high elasticity of substitution between automated and non-automated tasks κ (Figure 6, panel (a)). This is in line with the findings of Karabarbounis and Neiman (2014) and Dao et al. (2017) who respectively regard capital-labour substitution and automation of routine tasks as important drivers of the fall in the labour share. Given that the elasticity of substitution between automated and non-automated tasks κ is higher than 1, non-technological automation at the intensive margin will lead to a wedge between the evolution of the average productivity and the marginal productivity (or, on competitive labour markets, the remuneration) of human labour. Rognlie (2014) rightly points out that it is the evolution of the *net* labour share that has distributional consequences for capital-owners and non-capital-owners. Since capital accumulation also implies additional depreciation, the ageing-induced automation in our model leads to a smaller decline in the *net* labour share (Figure 6, panel (b)). Nevertheless our results imply that automation will be a factor strongly favouring capital owners.

As indicated in sub-section 3.1.6, automation capital is a q-substitute for low ability labour, but a q-complement for (medium and) high ability labour. As a result, ageing-induced automation generates downward pressure on low ability hourly wages per unit of human capital w_L (Figure 6, panel (c)), while high ability wages (w_H) are positively impacted (Figure 6, panel (e)). The dominant, ageing-driven effect is one of rising wages through time, however, as human labour becomes relatively scarcer. Unsurprisingly, this results in an increase in lifetime wage inequality between the ability types (Figure 6, panels (f) and (g)).[26] As a result, ageing-induced automation will lead low ability individuals to reduce their workload when older *more* than their medium and high ability counterparts due to substitution effects of the changing wages (results not shown). The opposite effects of automation on wages of different ability types thus also lead to diverging employment trends. Not just wage, but also welfare inequality increases as a result of ageing-induced automation (Figure 6, panels (h) and (i).[27] Note that, even when ignoring the automation impact, ageing worsens wage and welfare inequality because of a strong increase in education by medium and high ability individuals and the fall in the interest rate. The falling interest rate is not inconvenient for studying individuals who borrow when young and save close to retirement. It is more damaging for low ability individuals with low productivity of schooling, however, who are (relative to their lifetime labour income) most dependent on interest income throughout their life.

[26] The indicator in Figure 6(f), is the average hourly wage of low ability individuals entering the model at time t throughout their lifetime, relative to the average lifetime hourly wage of high ability individuals of the same generation t. That time t is indicated on the horizontal axis of the panel (f). In the same way, Figure 6(g), indicates the wage gap between medium and high ability individuals.

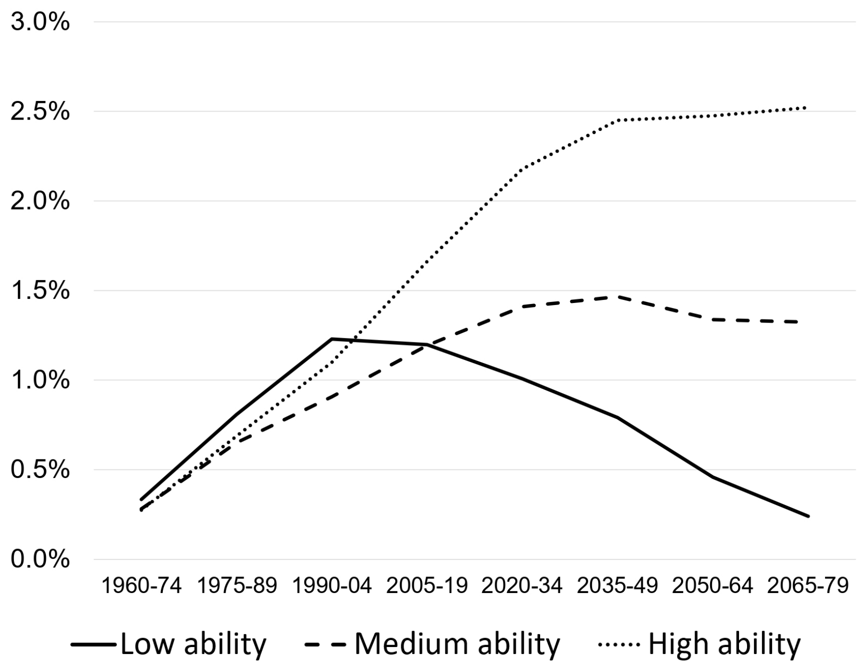
[27] The indicator in Figure 6(h), is the percentage increase in consumption in each period of life of low ability individuals of generation t that is necessary to raise their expected lifetime utility to the counterfactual expected lifetime utility they would have enjoyed if they experienced the consumption and leisure inputs of high ability individuals of that same generation t. The generation t, being the period in which individuals enter the model, is indicated on the horizontal axis of panel (h). For example, it would only be after a 90% rise in their actual consumption levels that low ability individuals, young in the 2035-49 model period, would be indifferent between entering the baseline model as a high ability individual or as a low ability individual. In the same way, Figure 6(i), indicates the welfare gap between medium and high ability individuals.

Figure 6: *Evolution of nine selected indicators in the baseline scenario and the counterfactual scenario without ageing-induced automation. Indicators marked with * are normalized to the level in the 1975-1989 model period. The other variables are in level (%).*



The absolute welfare effects of ageing-induced automation are positive for individuals of all ability and all generations, although the positive welfare effects are spread unevenly (see Figure 7).[28] For earlier generations, it is actually the low ability individuals who benefit slightly more from the automation that ageing elicits: since the bulk of the rise in automation density happens after they retire, they barely suffer from the lower demand for low ability labour that automation implies, but they do benefit from the higher interest rate. Given their smaller retirement income relative to higher ability individuals (and the decreasing marginal utility of consumption), the extra interest income has the largest positive effect for them. For later generations, however, the labour market effects during active life start to dominate and low ability individuals have the least to gain from automation (while high ability individuals have the most). Nevertheless, the benefits of automation (in the form of a higher interest income and more leisure) still outweigh the negative wage effects for low ability individuals.

Figure 7: *Absolute welfare effect of ageing-induced automation, by generation and ability type*[28]



[28] In Figure 7, we consider the absolute welfare impact of ageing-induced automation. The indicator considers the percentage increase in consumption in each period of life that is necessary to raise an individual's expected lifetime utility under the constant automation density counterfactual to the utility level of an individual of the same generation t and same ability level a under the baseline scenario. The generation t, being the period in which individuals enter the model, is indicated on the horizontal axis of Figure 7. Note that, since the evolution of the share of output that is absorbed by wasteful government spending is identical in both scenarios, a welfare analysis is appropriate.

5 Conclusion

In this study, we built and simulated a stylized computable overlapping generations model that incorporates automation of the production process and demographic structure with the aim to test the dual hypothesis of Acemoglu and Restrepo (2017) that (1) ageing can be a factor stimulating automation and (2) that this ageing-induced automation compensates for the typical negative growth effects of ageing that theoretical models find. We paid special attention to the theoretical and empirical foundations of the modelling of automation in this paper. Theoretically, our work is the first one testing this hypothesis that relates the approach to automation rigorously to the state-of the-art conception of automation by Acemoglu and Restrepo (2018a; 2018b). Empirically, we tested and largely confirmed the validity of our approach and calibration by comparing model predictions of automation density to actual data on robotization in a cross-country fashion. An additional important contribution to the literature consists of the fact that, while our model is quite small-sized, employment at older age and human capital investment are endogenous. This allows us to examine how automation interacts with other (more behavioural) effects which compensate for the typical negative effect of ageing on growth. The rapidly growing recent literature (e.g. Stähler, 2021; Basso & Jimeno, 2021; Irmen, 2021; Zhang et al., 2021) has largely neglected empirical verification as well as the key role of both labour supply at older age and investment in education. Finally, we add to the literature by calculating the welfare impact of ageing-induced automation for individuals with different innate ability.

Our main findings are as follows. Ageing strongly stimulates the adoption of automation technologies in our model, as found in earlier empirical and theoretical work and this ageing-induced automation can improve the growth performance of ageing economies. Given the current level of development of automation technologies, however, demographic change will still constitute a force weighing down per capita growth in the foreseeable future of the US, as old-age dependency starts to rise. Likewise, the fall in the interest rate that ageing induces, is softened by ageing-induced automation, but not halted. We thus consider our results to be only *cautiously* supportive of the hypothesis of Acemoglu and Restrepo (2017), since the mitigation is only partial. Given the current state of automation technologies, the extent to which automation can negate the shortage of human labour is found to be insufficient for complete mitigation. An additional explanation for this "*only partial*"-finding is that, as ageing-induced automation softens the relative shortage of human labour, it also reduces the strength of behavioural reactions to this relative shortage. Without ageing-induced automation, the incentives to retire later and invest more in human capital accumulation would have been even stronger. Moreover, the partial mitigation also comes at the cost of heightened inequalities. First, ageing-induced automation generates a fall in the labour share of income (not only the *gross*, but also the *net* share) and higher interest rates, thus benefiting capital-owners. Second, it is found to increase the wage and welfare inequality between individuals of different innate ability levels. While the real wage of high ability individuals rises, ageing-induced automation is a factor reducing the real wage of low ability individuals. Since low ability individuals benefit from the rise in the interest rate, however, ageing-induced automation may also make them better off in absolute welfare terms.

Our findings regarding the effects of ageing-induced automation on the labour share of income and inequality largely confirm the results in the recent literature mentioned above. When it comes to effects on per capita growth, however, the conclusions of several studies (e.g. Irmen, 2021; Zhang et al., 2021) may be too optimistic. Only when we impose unrealistically high elasticities of substitution between automated tasks and tasks executed by human labour, we also find that ageing-induced automation may fully offset the negative per capita growth effects of demographic change. This observation underscores the importance of empirical verification of calibrated theoretical models.

An important caveat to this study is of course its ‘constant level of development in automation technologies’ assumption. It falls beyond the scope of this work to study how ageing affects the returns to automation-related R&D activities, but it is clear that, as the cost of labour rises and the cost of capital falls, the incentives to automate currently non-automatable tasks will rise. A more optimist interpretation of the results therefore presents itself: even without any progress in robotics, AI or computer-assisted machines, these technologies can, as they are increasingly used, play a crucial role in limiting the negative consequences of ageing. Nuance is required here, however. This study suggests that, even without technical progress, the social effects of ageing-induced automation will already be very disruptive. It will be policymakers’ challenge to create sufficient public support for automation to play its crucial role in the next decades. Clearly communicating that embracing automation technologies goes hand in hand with policy initiatives redistributing income from the winners to the losers of automation, could prove to be key.

References

Abeliansky, A. L., Algur, E., Bloom, D. E., & Prettner, K. (2020). The future of work: Meeting the global challenges of demographic change and automation. *International Labour Review*, *159*(3), 285-306. https://doi.org/10.1111/ilr.12168

Abeliansky, A., & Prettner, K. (2017). Automation and demographic change. *Hohenheim Discussion Papers in Business, Economics, and Social Sciences*. https://doi.org/10.2139/ssrn.2959977

Acemoglu, D., & Restrepo, P. (2017). Secular stagnation? The effect of aging on economic growth in the age of automation. *American Economic Review*, *107*(5), 174-179. https://doi.org/10.1257/aer.p20171101

Acemoglu, D., & Restrepo, P. (2018a). The race between man and machine: Implications of tech-nology for growth, factor shares, and employment. *American Economic Review*, *108*(6), 1488-1542. https://doi.org/10.1257/aer.20160696

Acemoglu, D., & Restrepo, P. (2018b). Modeling automation. *AEA Papers and Proceedings,* 108, 48-53. https://doi.org/10.1257/pandp.20181020

Acemoglu, D., & Restrepo, P. (2018c). Demographics and automation. *NBER Working Paper,* National Bureau of Economic Research, N° 24421. https://doi.org/10.3386/w24421

Acemoglu, D., & Restrepo, P. (2019). Artificial Intelligence, Automation, and Work, in A. Agrawal, J. Gans & A. Goldfard (Eds),*The Economics of Artificial Intelligence*, University of Chicago Press, Ch. 8, pp. 197-236. https://doi.org/10.7208/chicago/9780226613475.001.0001

Acemoglu, D., & Restrepo, P. (2020). Robots and jobs: Evidence from US labor markets. *Journal of Political Economy*, *128*(6), 2188-2244. https://doi.org/10.1086/705716

Acemoglu, D., & Restrepo, P. (2021). Demographics and automation. *Review of Economic Studies*. Advance online publication. https://doi.org/10.1093/restud/rdab031

Adjemian, S., Bastani, H., Juillard, M., Karamé, F., Maih, J., Mihoubi, F., Mutschler, W., Perendia, G., Pfeifer, J., Ratto., M, & Villemot, S. (2011). Dynare: Reference Manual - Version 4. *Dynare Working Papers*.

Aksoy, Y., Basso, H. S., Smith, R. P., & Grasl, T. (2019). Demographic structure and macroeconomic trends. *American Economic Journal: Macroeconomics*, *11*(1), 193-222. https://doi.org/10.1257/mac.20170114

Arntz, M., Terry G., & Ulrich Z. (2016). The Risk of Automation for Jobs in OECD Countries. *OECD Social, Employment, and Migration Paper,* N° 189. https://doi.org/10.1787/5jlz9h56dvq7-en

Backus, D., Cooley, T., & Henriksen, E. (2014). Demography and low-frequency capital flows. *Journal of International Economics*, *92*, 94-102. https://doi.org/10.1016/j.jinteco.2014.01.006

Basso, H. S., & Jimeno, J. F. (2021). From secular stagnation to robocalypse? Implications of demographic and technological changes. *Journal of Monetary Economics*, *117*, 833-847. https://doi.org/10.1016/j.jmoneco.2020.06.004

Blanchard, O. (2004). The economic future of Europe. *Journal of Economic Perspectives*, *18*(4), 3-26. https://doi.org/10.1257/0895330042632735

Bloom, D. E., Canning, D., & Fink, G. (2010). Implications of population ageing for economic growth. *Oxford Review of Economic Policy*, *26*(4), 583-612. https://doi.org/10.1093/oxrep/grq038

Boone, B., & Heylen, F. (2019). Cross-Country Differences in Unemployment: Fiscal Policy, Unions, and Household Preferences in General Equilibrium. *The Scandinavian Journal of Economics*, *121*(3), 1270-1302. https://doi.org/10.1111/sjoe.12302

Bouzahzah, M., de la Croix, D. & Docquier, F. (2002). Policy reforms and growth in computational OLG economies, *Journal of Economic Dynamics and Control*, *26*(12), 2093-2113. https://doi.org/10.1016/S0165-1889(01)00023-9

Buyse, T., Heylen, F., & Van de Kerckhove, R. (2017). Pension reform in an OLG model with heterogeneous abilities. *Journal of Pension Economics & Finance*, *16*(2), 144-172. https://doi.org/10.1017/S1474747215000281

Carvalho, C., Ferrero, A., & Nechio, F. (2016). Demographics and real interest rates: Inspecting the mechanism. *European Economic Review*, *88*, 208-226. https://doi.org/10.1016/j.euroecorev.2016.04.002

Caselli, F., & Coleman II, W. J. (2006). The world technology frontier. *American Economic Review*, *96*(3), 499-522. https://doi.org/10.1257/aer.96.3.499

Cervellati, M., & Sunde, U. (2013). Life expectancy, schooling, and lifetime labor supply: theory and evidence revisited. *Econometrica*, *81*(5), 2055-2086. https://doi.org/10.3982/ECTA11169

Cords, D., & Prettner, K. (2018). Technological unemployment revisited: Automation in a search and matching framework. *Hohenheim Discussion Papers in Business, Economics and Social Sciences*.

Dao, M. C., Das, M. M., Koczan, Z., & Lian, W. (2017). Why is labour receiving a smaller share of global income? Theory and empirical evidence. *IMF Working Paper Series*, N° 169.

Dauth, W., Findeisen, S., Südekum, J., & Woessner, N. (2017). German Robots - The Impact of Industrial Robots on Workers. *CEPR Discussion Paper*, N° DP12306.

de la Croix, D., Pierrard, O., & Sneessens, H. R. (2013). Aging and pensions in general equilibrium: Labor market imperfections matter. *Journal of Economic Dynamics and Control*, *37*(1), 104-124. https://doi.org/10.1016/j.jedc.2012.06.011

DeCanio, S. J. (2016). Robots and humans – complements or substitutes?. *Journal of Macroeconomics*, *49*, 280-291. https://doi.org/10.1016/j.jmacro.2016.08.003

Devriendt, W., & Heylen, F. (2020). Macroeconomic and distributional effects of demographic change in an open economy – the case of Belgium. *Journal of Demographic Economics*, *86*(1), 87-124. https://doi.org/10.1017/dem.2019.14

Dhont, T., & Heylen, F. (2009). Employment and growth in Europe and the US—the role of fiscal policy composition. *Oxford Economic Papers*, *61*(3), 538-565. https://doi.org/10.1093/oep/gpn034

Docquier, F., & Paddison, O. (2003). Social security benefit rules, growth and inequality. *Journal of Macroeconomics*, *25*(1), 47-71. https://doi.org/10.1016/S0164-0704(03)00006-5

Eggertsson, G. B., Mehrotra, N. R., & Robbins, J. A. (2019). A model of secular stagnation: Theory and quantitative evaluation. *American Economic Journal: Macroeconomics*, *11*(1), 1-48. https://doi.org/10.1257/mac.20170367

Feyrer, J. (2007). Demographics and productivity. *The Review of Economics and Statistics*, *89*(1), 100-109. https://doi.org/10.1162/rest.89.1.100

Fiorito, R., & Zanella, G. (2012). The anatomy of the aggregate labor supply elasticity. *Review of Economic Dynamics*, *15*(2), 171-187. https://doi.org/10.1016/j.red.2012.01.002

Frey, C. B., & Osborne, M. A. (2017). The future of employment: How susceptible are jobs to computerisation?. *Technological Forecasting and Social Change*, *114*, 254-280. https://doi.org/10.1016/j.techfore.2016.08.019

Gagnon, E., Johannsen, B. K., & Lopez-Salido, D. (2021). Understanding the new normal: The role of demographics. *IMF Economic Review*, *69*, 357-390. https://doi.org/10.1057/s41308-021-00138-4

Gollin, D. (2002). Getting income shares right. *Journal of Political Economy*, *110*(2), 458-474. https://doi.org/10.1086/338747

Gordon, R. J. (2014). The demise of US economic growth: restatement, rebuttal, and reflections. *NBER Working Paper*, National Bureau of Economic Research, N° 19895. https://doi.org/10.3386/w19895

Gordon, R. J. (2016). *The rise and fall of American growth*. Princeton University Press. https://doi.org/10.1515/9781400873302

Graetz, G., & Michaels, G. (2018). Robots at work. *Review of Economics and Statistics*, *100*(5), 753-768. https://doi.org/10.1162/rest_a_00754

Guo, J. T., & Lansing, K. J. (1998). Indeterminacy and Stabilization Policy. *Journal of Economic Theory, 82*(2), 482-490. https://doi.org/10.1006/jeth.1997.2446

Gutiérrez, G. (2017). Investigating global labor and profit shares. *NYU Working Paper*. https://doi.org/10.2139/ssrn.3040853

Gutiérrez, G., & Piton, S. (2020). Revisiting the global decline of the (non-housing) labor share. *American Economic Review: Insights*, *2*(3), 321-38. https://doi.org/10.1257/aeri.20190285

Heijdra, B. J., & Reijnders, L. S. (2018). Longevity shocks with age-dependent productivity growth. *Journal of Pension Economics & Finance, 17*(2), 200-230. https://doi.org/10.1017/S147474721600024X

Heijdra, B. J., & Romp, W. E. (2009). Human capital formation and macroeconomic performance in an ageing small open economy. *Journal of Economic Dynamics and Control*, *33*(3), 725-744. https://doi.org/10.1016/j.jedc.2008.09.003

Huggett, M., Ventura, G., & Yaron A. (2006). Human capital and earnings distribution dynamics. *Journal of Monetary Economics*, *53(2)*, 265-290. https://doi.org/10.1016/j.jmoneco.2005.10.013

Huggett, M., Ventura, G., & Yaron, A. (2011). Sources of lifetime inequality. *American Economic Review*, *101*(7), 2923-2954. https://doi.org/10.1257/aer.101.7.2923

International Federation of Robotics. (2005). *World Robotics 2005*.

International Federation of Robotics. (2014). *World Robotics 2014 – Industrial Robots*.

International Federation of Robotics. (2020). *World Robotics 2020 – Industrial Robots*.

Irmen, A. (2021). Automation, growth, and factor shares in the era of population aging. *Journal of Economic Growth*, 1-39. https://doi.org/10.1007/s10887-021-09195-w

Johnson, D. G. (1954). The functional distribution of income in the United States, 1850-1952. *The Review of Economics and Statistics*, *36*(2), 175-182. https://doi.org/10.2307/1924668

Jones, C. I. (2005). The shape of production functions and the direction of technical change. *The Quarterly Journal of Economics*, *120*(2), 517-549. https://doi.org/10.1093/qje/120.2.517

Kaldor, N. (1961). Capital accumulation and economic growth. In F. A. Lutz & D. C. Hague (Eds.),*The theory of capital* (pp. 177-222). Palgrave Macmillan. https://doi.org/10.1007/978-1-349-08452-4_10

Karabarbounis, L., & Neiman, B. (2012). Declining labor shares and the global rise of corporate saving. *NBER Working Paper*, National Bureau of Economic Research, N° 18154. https://doi.org/10.3386/w18154

Karabarbounis, L., & Neiman, B. (2014). The global decline of the labor share. *The Quarterly Journal of Economics*, *129*(1), 61-103. https://doi.org/10.1093/qje/qjt032

Keane, M. P., & Wolpin, K. I. (2007). Exploring the usefulness of a nonrandom holdout sample for model validation: Welfare effects on female behavior. *International Economic Review*, *48*(4), 1351-1378. https://doi.org/10.1111/j.1468-2354.2007.00465.x

Keynes, J. M. (1939). Relative movements of real wages and output. *The Economic Journal*, *49*(193), 34-51. https://doi.org/10.2307/2225182

Lankisch, C., Prettner, K., & Prskawetz, A. (2019). How can robots affect wage inequality?. *Economic Modelling*, *81*, 161-169. https://doi.org/10.1016/j.econmod.2018.12.015

Ludwig, A., Schelkle, T., & Vogel, E. (2012). Demographic change, human capital and welfare. *Review of Economic Dynamics*, *15*(1), 94-107. https://doi.org/10.1016/j.red.2011.07.001

Manyika, J., Mischke, J., Bughin, J., Woetzel, J., Krishnan, M., & Cudre, S. (2019). A new look at the declining labor share of income in the United States. *McKinsey Global Institute Discussion paper.*

Martens, B., & Tolan, S. (2018). Will this time be different? A review of the literature on the Impact of Artificial Intelligence on Employment, Incomes and Growth. *JRC Working Papers on Digital Economy.* https://doi.org/10.2139/ssrn.3290708

Prettner, K. (2019). A note on the implications of automation for economic growth and the labor share. *Macroeconomic Dynamics*, *23*(3), 1294-1301. https://doi.org/10.1017/S1365100517000098

Popescu, G. H., Petrescu, I. E., Sabie, O. M., & Muşat, M. (2018). Labor-displacing technological change and worldwide economic insecurity: How automation and the creation of innovative tasks shape inequality. *Psychosociological Issues in Human Resource Management*, *6*(2), 80-85. https://doi.org/10.22381/PIHRM6220188

Prettner, K., & Strulik, H. (2020). Innovation, automation, and inequality: Policy challenges in the race against the machine. *Journal of Monetary Economics*, *116*, 249-265. https://doi.org/10.1016/j.jmoneco.2019.10.012

Rachel, Ł., & Summers, L. H. (2019). On secular stagnation in the industrialized world, *Brookings Papers on Economic Activity*, Spring, 1-54. https://doi.org/10.3386/w26198

Rogerson, R. (2007). Taxation and market work: is Scandinavia an outlier?. *Economic Theory*, *32*(1), 59-85. https://doi.org/10.1007/s00199-006-0164-9

Rogerson, R., & Wallenius, J. (2009). Micro and macro elasticities in a life cycle model with taxes. *Journal of Economic theory*, *144*(6), 2277-2292. https://doi.org/10.1016/j.jet.2008.05.011

Rognlie, M. (2014). A note on Piketty and diminishing returns to capital. Unpublished manuscript. MIT. Retrieved from: http://gesd.free.fr/rognlie14.pdf.

Samuelson, P. (1964). *Economics: An Introductory Textbook*. McGraw-Hill.

Stähler, N. (2021). The impact of aging and automation on the macroeconomy and inequality. *Journal of Macroeconomics*, *67*, 103278.

Stokey, N. L., & Rebelo, S. (1995). Growth effects of flat-rate taxes. *Journal of Political Economy*, *103*(3), 519-550. https://doi.org/10.1086/261993

Uzawa, H. (1961). Neutral inventions and the stability of growth equilibrium. *The Review of Economic Studies*, *28*(2), 117-124. https://doi.org/10.2307/2295709

Zhang, X., Palivos, T., & Liu, X. (2021). Aging and automation in economies with search frictions. *Journal of Population Economics*, 1-22. https://doi.org/10.1007/s00148-021-00860-3

Appendix A: Task-based framework of automation

Part 1 - Proof of the validity of equation (23)

Our starting point is the following expression for $H_{a,tot,t}$:

$H_{a,tot,t} = (\int_0^1 t_{a,i,t}{}^{\frac{\kappa-1}{\kappa}} di)^{\frac{\kappa}{\kappa-1}}$ with $t_{a,i,t} = h_{a,i,t}$, $\forall i < \xi_a$ and $t_{a,i,t} = jP_t + \lambda h_{a,i,t}$, $\forall i > \xi_a$

$$\Leftrightarrow H_{a,tot,t} = \left[\left(\int_0^{\xi_a} h_{a,i,t}{}^{\frac{\kappa-1}{\kappa}} di\right) + \left(\int_{\xi_a}^{1} (jP_t + \lambda h_{a,i,t})^{\frac{\kappa-1}{\kappa}} di\right)\right]^{\frac{\kappa}{\kappa-1}} \qquad (1)$$

Given our calibration, it is at any point cost-effective for firms to only use automation capital to execute automatable tasks (see the second part of this Appendix A). We can therefore state that

$\forall i > \xi_a : t_{a,i,t} = jP_t \qquad (2)$

$$(1) \,\&\, (2) \Rightarrow H_{a,tot,t} = \left[\left(\int_0^{\xi_a} h_{a,i,t}{}^{\frac{\kappa-1}{\kappa}} di\right) + \left(\int_{\xi_a}^{1} (jP_t)^{\frac{\kappa-1}{\kappa}} di\right)\right]^{\frac{\kappa}{\kappa-1}}$$

$$\Leftrightarrow H_{a,tot,t} = \left[\left(\int_0^{\xi_a} h_{a,i,t}{}^{\frac{\kappa-1}{\kappa}} di\right) + (1-\xi_a)(jP_t)^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}} \qquad (3)$$

Given that the productivity of effective human labour at the execution of any task $i < \xi_a$ is identical and individuals have no preference between different tasks, the identical amount of human labour $h_{a,t}$ will be used for any task $i < \xi_a$ such that $h_{a,i,t} = h_{a,t}$ (4)

$$(3) \,\&\, (4) \Rightarrow H_{a,tot,t} = \left[h_{a,t}{}^{\frac{\kappa-1}{\kappa}} \xi_a + (1-\xi_a)(jP_t)^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}} \qquad (5)$$

The total amount of human labour provided by individuals of a particular ability level $H_{a,t}$ is allocated over the different tasks such that $H_{a,t} = \int_0^{\xi_a} h_{a,t}\, di = \xi_a h_{a,t} \quad (6)$

$$(5) \,\&\, (6) \Rightarrow H_{a,tot,t} = \left[(H_{a,t}/\xi_a)^{\frac{\kappa-1}{\kappa}} \xi_a + (1-\xi_a)(jP_t)^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}}$$

$$\Leftrightarrow H_{a,tot,t} = \left[\xi_a{}^{\frac{1}{\kappa}} H_{a,t}{}^{\frac{\kappa-1}{\kappa}} + (1-\xi_a)\,(jP_t)^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}}$$

$$\Leftrightarrow H_{a,tot,t} = \left[\xi_a{}^{\frac{1}{\kappa}} H_{a,t}{}^{\frac{\kappa-1}{\kappa}} + (1-\xi_a)\, JP_t{}^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}} \text{ with } J = j^{\frac{\kappa-1}{\kappa}}$$

Which results in equation (23).

Part 2 - Derivation of the necessary condition that has to hold such that all automatable tasks are fully automated

Deriving Y_t with regard to $h_{L,k,t}$, which is the human labour devoted to the execution of an automatable task $t_{L,k,t}$ of the low ability type (with $k > \xi_L$), results in:

$$\frac{\partial Y_t}{\partial h_{L,k,t}} = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} \eta_L \left(\frac{H_t}{H_{L,tot,t}}\right)^{\frac{1}{s}} \frac{\partial H_{L,tot,t}}{\partial h_{L,k,t}} \quad (1)$$

Deriving Y_t with regard to P_t, which is - given the general-purpose nature of the automation technology - the automation capital devoted to the execution of an automatable task $t_{L,k,t}$ (with $k > \xi_L$), results in:

$$\frac{\partial Y_t}{\partial P_t} = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} \sum_{a=L,M,H} \eta_a \left(\frac{H_t}{H_{a,tot,t}}\right)^{\frac{1}{s}} \frac{\partial H_{a,tot,t}}{\partial P_t} \quad (2)$$

Starting out from the most general expression, and setting the productivity λ of human labour at the execution of automatable tasks (relative to its productivity at the execution of non-automated tasks) at 1/3, the executed work of the ability type L can be written as[29]:

$$H_{L,tot,t} = \left[\left(\int_0^{\xi_L} h_{L,i,t}{}^{\frac{\kappa-1}{\kappa}} di\right) + \left(\int_{\xi_L}^{1} (jP_t + \frac{1}{3}h_{L,i,t})^{\frac{\kappa-1}{\kappa}} di\right)\right]^{\frac{\kappa}{\kappa-1}} \quad (3)$$

When applying the Leibniz integral rule for definite integrals with constant lower and upper limits, we get:

$$(3) \Rightarrow \frac{\partial H_{L,tot,t}}{\partial h_{L,k,t}} = H_{L,tot,t}{}^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} \frac{1}{1-\xi_L} \int_{\xi_L}^{1} (jP_t + \frac{1}{3}h_{L,i,t})^{\frac{-1}{\kappa}} \frac{1}{3} di \quad (4)$$

$$(3) \Rightarrow \frac{\partial H_{L,tot,t}}{\partial P_t} = H_{L,tot,t}{}^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} \int_{\xi_L}^{1} (jP_t + \frac{1}{3}h_{L,i,t})^{\frac{-1}{\kappa}} j\, di \quad (5)$$

As a result,

$$(1) \,\&\, (4) \Rightarrow \frac{\partial Y_t}{\partial h_{L,k,t}} = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} \eta_L \left(\frac{H_t}{H_{L,tot,t}}\right)^{\frac{1}{s}} H_{L,tot,t}{}^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} \frac{1}{1-\xi_L} \int_{\xi_L}^{1} (jP_t + \frac{1}{3}h_{L,i,t})^{\frac{-1}{\kappa}} \frac{1}{3} di \quad (6)$$

$$(2) \,\&\, (5) \Rightarrow \frac{\partial Y_t}{\partial P_t} = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} \left[\left\{\eta_L \left(\frac{H_t}{H_{L,tot,t}}\right)^{\frac{1}{s}} H_{L,tot,t}{}^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} \int_{\xi_L}^{1} (jP_t + \frac{1}{3}h_{L,i,t})^{\frac{-1}{\kappa}} j\, di\right\} + \left\{\sum_{a=M,H} \eta_a \left(\frac{H_t}{H_{a,tot,t}}\right)^{\frac{1}{s}} H_{a,tot,t}{}^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} (1-\xi_a) J P_t{}^{\frac{-1}{\kappa}}\right\}\right] \quad (7)$$

[29] We judge a value for λ of 1/3 to be not unreasonable given the exponential comparative advantage schedule that Acemoglu and Restrepo (2018a) impose on the productivity of labour.

We now proceed by evaluating the ratio $\frac{\partial Y_t}{\partial P_t} / \frac{\partial Y_t}{\partial h_{L,k,t}}$ for $h_{L,k,t} = 0$ and we verify that this ratio is, throughout our simulations, larger than $\frac{(r_t + \delta_p)(1-\tau_p)^{-1}}{w_{L,t}}$ (being the cost of capital relative to human labour). Note that evaluating this inequality for $h_{L,k,t} = 0$ is sufficient since the ratio $\frac{\partial Y_t}{\partial P_t} / \frac{\partial Y_t}{\partial h_{L,k,t}}$ will be the lowest for $h_{L,k,t} = 0$. This is the case because any increase in $h_{L,k,t}$ would leave the ratio $\frac{\partial H_{L,tot,t}}{\partial P_t} / \frac{\partial H_{L,tot,t}}{\partial h_{L,k,t}}$ unchanged (at a constant value of $(1-\xi_a)3j$), but it would increase the productivity of automation capital in the execution of tasks of ability types different from L by increasing H_t. As such, if the inequality holds for $h_{L,k,t} = 0$, it also holds for higher values of $h_{L,k,t}$.

When evaluating $\frac{\partial Y_t}{\partial h_{L,k,t}}$ for $h_{L,k,t} = 0$, we find:

$$\frac{\partial Y_t}{\partial h_{L,k,t}} = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} \eta_L \left(\frac{H_t}{H_{L,tot,t}}\right)^{\frac{1}{s}} H_{L,tot,t}{}^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} j^{\frac{-1}{\kappa}} P_t{}^{\frac{-1}{\kappa}} \frac{1}{3} \quad \text{(8) (Based on (6))}$$

And since $J = j^{\frac{\kappa-1}{\kappa}}$

$$(8) \Rightarrow \frac{\partial Y_t}{\partial h_{L,k,t}} = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} \eta_L \left(\frac{H_t}{H_{L,tot,t}}\right)^{\frac{1}{s}} H_{L,tot,t}{}^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} J^{\frac{-1}{\kappa-1}} P_t{}^{\frac{-1}{\kappa}} \frac{1}{3} \quad (10)$$

When evaluating $\frac{\partial Y_t}{\partial P_t}$ for $h_{L,k,t} = 0$, we find:

$$\frac{\partial Y_t}{\partial P_t} = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} \left[\left\{\eta_L \left(\frac{H_t}{H_{L,tot,t}}\right)^{\frac{1}{s}} H_{L,tot,t}{}^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa}(1-\xi_L) j^{\frac{\kappa-1}{\kappa}} P_t{}^{\frac{-1}{\kappa}}\right\} + \left\{\sum_{a=M,H} \eta_a \left(\frac{H_t}{H_{a,tot,t}}\right)^{\frac{1}{s}} H_{a,tot,t}{}^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa}(1-\xi_a) J P_t{}^{\frac{-1}{\kappa}}\right\}\right] \quad \text{(9) (Based on (7))}$$

And since $J = j^{\frac{\kappa-1}{\kappa}}$

$$(9) \Rightarrow \frac{\partial Y_t}{\partial P_t} = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} \left[\sum_{a=L,M,H} \eta_a \left(\frac{H_t}{H_{a,tot,t}}\right)^{\frac{1}{s}} H_{a,tot,t}{}^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa}(1-\xi_a) J P_t{}^{\frac{-1}{\kappa}}\right] \quad (11)$$

We find that, given our parameterization, the following inequality holds throughout all of our simulations for any country:

$$(10)\ \&\ (11) \Rightarrow \frac{\frac{\partial Y_t}{\partial P_t}}{\frac{\partial Y_t}{\partial h_{L,k,t}}} = \frac{(1-\alpha)(\frac{K_t}{H_t})^{\alpha}\left[\sum_{a=L,M,H} \eta_a (\frac{H_t}{H_{a,tot,t}})^{\frac{1}{s}} H_{a,tot,t}{}^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa}(1-\xi_a) J P_t{}^{\frac{-1}{\kappa}}\right]}{(1-\alpha)(\frac{K_t}{H_t})^{\alpha} \eta_L (\frac{H_t}{H_{L,tot,t}})^{\frac{1}{s}} H_{L,tot,t}{}^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} J^{\frac{-1}{\kappa-1}} P_t{}^{\frac{-1}{\kappa}} \frac{1}{3}} > \frac{(r_t + \delta_p)(1-\tau_p)^{-1}}{w_{L,t}}$$

The same inequality holds true for medium and high ability tasks. As a result, it is strictly cheaper for firms to use automation capital for the execution of automatable tasks of any ability type. As a result, our simplifying assumption that all automatable tasks are automated (and that no human labour is used to execute automatable tasks) is valid.

Appendix B: Evaluating the sign of mixed second order derivates; q-substitutability?

The real wage per unit of human labour of individuals of ability type a equals the marginal product of effective human labour of type a. The equation below is equation (27) in the main text.

$$w_{a,t} = \left[(1-\alpha) A_t^{1-\alpha} \left(\frac{K_t}{H_t}\right)^{\alpha} \eta_a \left(\frac{H_t}{H_{a,tot,t}}\right)^{\frac{1}{s}} \xi_a{}^{\frac{1}{\kappa}} \left(\frac{H_{a,tot,t}}{H_{a,t}}\right)^{\frac{1}{\kappa}} \right]$$

After rearranging, a non-negative expression E_t that is independent of P_t can be put in front.

$$w_{a,t} = \left[(1-\alpha) A_t^{1-\alpha} K_t^{\alpha} \left(H_{a,t}\right)^{\frac{-1}{\kappa}} \eta_a \, \xi_a{}^{\frac{1}{\kappa}} \right] H_t{}^{\left(-\alpha+\frac{1}{s}\right)} \left(H_{a,tot,t}\right)^{\left(-\frac{1}{s}+\frac{1}{\kappa}\right)}$$

$$= E_t H_t{}^{\left(-\alpha+\frac{1}{s}\right)} \left(H_{a,tot,t}\right)^{\left(-\frac{1}{s}+\frac{1}{\kappa}\right)}$$

This simplified expression for the real wage per unit of human capital is now derived with respect to the input of automation capital P_t.

$$\frac{\partial w_{a,t}}{\partial P_t} = E_t \left[\left(-\alpha + \frac{1}{s}\right) H_t{}^{\left(-\alpha+\frac{1}{s}-1\right)} \sum_{j=L,M,H} \left\{ \eta_j \left(\frac{H_t}{H_{j,tot,t}}\right)^{\frac{1}{s}} (1-\xi_j) \left(\frac{H_{j,tot,t}}{P_t}\right)^{\frac{1}{\kappa}} J \right\} \left(H_{a,tot,t}\right)^{\left(-\frac{1}{s}+\frac{1}{\kappa}\right)} \right.$$
$$\left. + \left(-\frac{1}{s}+\frac{1}{\kappa}\right) \left(H_{a,tot,t}\right)^{\left(-\frac{1}{s}+\frac{1}{\kappa}-1\right)} (1-\xi_a) \left(\frac{H_{a,tot,t}}{P_t}\right)^{\frac{1}{\kappa}} J (H_t)^{\left(-\alpha+\frac{1}{s}\right)} \right]$$

Common terms are put in front and it is identified that they form a non-negative expression F_t.

$$\frac{\partial w_{a,t}}{\partial P_t} = E_t \left[\left(H_{a,tot,t}\right)^{\left(-\frac{1}{s}+\frac{1}{\kappa}\right)} H_t{}^{\left(-\alpha+\frac{1}{s}\right)} J \right] \left[\left(-\alpha + \frac{1}{s}\right) H_t^{-1} \sum_{j=L,M,H} \left\{ \eta_j \left(\frac{H_t}{H_{j,tot,t}}\right)^{\frac{1}{s}} (1-\xi_j) \left(\frac{H_{j,tot,t}}{P_t}\right)^{\frac{1}{\kappa}} \right\} \right.$$
$$\left. + \left(-\frac{1}{s}+\frac{1}{\kappa}\right) H_{a,tot,t}^{-1} (1-\xi_a) \left(\frac{H_{a,tot,t}}{P_t}\right)^{\frac{1}{\kappa}} \right]$$

$$\frac{\partial w_{a,t}}{\partial P_t} = E_t F_t \left[\left(-\alpha + \frac{1}{s}\right) H_t^{-1} \sum_{j=L,M,H} \left\{ \eta_j \left(\frac{H_t}{H_{j,tot,t}}\right)^{\frac{1}{s}} (1-\xi_j) \left(\frac{H_{j,tot,t}}{P_t}\right)^{\frac{1}{\kappa}} \right\} \right.$$
$$\left. + \left(-\frac{1}{s}+\frac{1}{\kappa}\right) H_{a,tot,t}^{-1} (1-\xi_a) \left(\frac{H_{a,tot,t}}{P_t}\right)^{\frac{1}{\kappa}} \right]$$

Only the final expression is relevant when determining the sign of the derivative:

$$sign(\frac{\partial w_{a,t}}{\partial P_t}) = sign\left[\left(-\alpha + \frac{1}{s}\right) H_t^{-1} \sum_{j=L,M,H} \left\{\eta_j \left(\frac{H_t}{H_{j,tot,t}}\right)^{\frac{1}{s}} (1-\xi_j) \left(\frac{H_{j,tot,t}}{P_t}\right)^{\frac{1}{\kappa}}\right\} + \left(-\frac{1}{s} + \frac{1}{\kappa}\right) H_{a,tot,t}^{-1} (1-\xi_a) \left(\frac{H_{a,tot,t}}{P_t}\right)^{\frac{1}{\kappa}}\right]$$

$$= sign\left[\left\{(1-\xi_a)\left(\frac{H_{a,tot,t}}{P_t}\right)^{\frac{1}{\kappa}}\right\}\left\{\left(\frac{1}{s} - \alpha\right)\eta_a H_t^{-1}\left(\frac{H_t}{H_{a,tot,t}}\right)^{\frac{1}{s}} + \left(\frac{1}{\kappa} - \frac{1}{s}\right) H_{a,tot,t}^{-1}\right\} + \left(\frac{1}{s} - \alpha\right) H_t^{-1} \sum_{j \neq a} \left\{\eta_j (1-\xi_j)\left(\frac{H_t}{H_{j,tot,t}}\right)^{\frac{1}{s}} \left(\frac{H_{j,tot,t}}{P_t}\right)^{\frac{1}{\kappa}}\right\}\right]$$

Based on this final expression, we can draw conclusions with regard to why the effects of an increase in automation capital vary over the different ability types. For this interpretation, we refer to sub-section 3.1.6 of the main text.

Appendix C: Modelling of demographic change

For each country, we use data from the Human Mortality Database (HMD) to construct time-varying survival rates and fertility rates. For many OECD countries that we include to evaluate the empirical value of our model, high-quality time series do not start until 1945. We calculate the demographic parameters such that the modelling periods are aligned with fifteen-year blocks starting in 1945: 1945-1959, 1960-1974, 1975-1989, 1990-2004, 2005-2019, 2020-2034, 2035-2049, 2050-2064, 2065-79 and 2080-2094. The survival rates are based on the mortality rates of each year in these 15-year time periods, which are subsequently averaged. Since individuals in the model can only die when making the transition from one fifteen-year block to another (e.g., when going from 20-34 to 35-49), our model parameters do not reflect reality perfectly. Real-world survival rates are approximated by calculating the probability of reaching the mean age of every period conditional upon having reached the mean age of the previous period. The sr_2 of the model is thus computed as (1-death$_{27}$)*(1-death$_{28}$)*…*(1-death$_{41}$). This is then calculated for every year in the 15-year historical time period and averaged. The historical survival rates for all age groups, all time periods and all countries were constructed in this fashion. Fertility data was constructed on the basis of the number of 27-year-olds in a country at each year and averaged across fifteen years. The fertility rate is then calibrated such that the relative sizes of the young generations through time fit the real data on the evolution of these 15-year averages. Following Devriendt and Heylen (2020), we do not explicitly account for immigration and emigration in our model. Migration movements of individuals up to 27 years old are taken into account in the computation of the fertility rate, however.

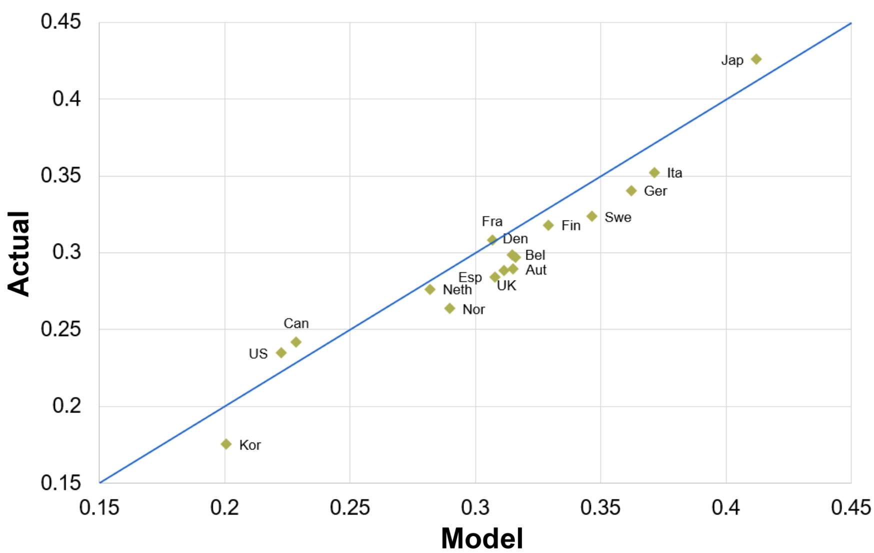
The HMD only provides historical data. For long term projections regarding the evolution of survival rates and fertility rates in modelling periods in the future, we use the projections of the United Nations Population Division (UNPD).

For the relative sizes of new cohorts entering the model in the future, we based ourselves on the medium-variant projections of the UNPD for the total population by five-year age group. We use fifteen-year averages of the 25- to 29-year old population. The projections are available until 2100 such that the size of new cohorts relative to the previous cohort size can be calculated for five 15-year periods after the 2005-2019 period. From the 2095-2109 period onwards, our simulations assume that each cohort of 20- to 34-year olds is of identical size as the previous one ($n = 0$). This approach was followed for all countries.

For the evolution of future survival rates, we impose that the model accurately replicates the growth in the old-age dependency ratio based on the UNPD's medium variant projections until 2100. The old-age dependency ratio is defined here as the number of individuals older than 65 divided by the population between 20 and 64, just like we will do for our model. Since the evolution of the old-age dependency ratio does not provide sufficient guidance to determine four survival rates, we have to make assumptions. We impose that the proportions of the different mortality rates are fixed at their 2005-2019 level for the US throughout the future. For instance, the probability of not surviving the transition from the 65-79 age group to the 80-94 age group for individuals who were in the 65-79 age group in the 1990-2004 period was around 54%. For individuals in the 35-49 group in the 1990-2004 period, the probability of not reaching the next period was around 6%. The survival rates in future periods will increase in a way that this factor 9 ratio is maintained. This approach delivers quite intuitive results: the largest absolute gains in the reduction of mortality rates in the future will be made for the ages at which this mortality rate is highest. On the other hand, the probability of dying will never fully go to zero, even for low ages (see Figure 3 in the main text). This approach allows us to come up with survival rates for five 15-year periods after the 2005-2019 period. From the 2095-2109 period onwards, the survival rates are kept constant in the simulation, at their respective 2080-2094 levels. Note that for the other countries, we impose that future mortality rates decrease at the same rate as for the US, while the proportions of the different mortality rates are also fixed at their 2005-2019 level. A fictitious country in which the probability to not survive the transition to the third period of life $(1 - sr_2)$ is only half as large as the probability in the US in 2005-2019 will thus also have a mortality probability $(1 - sr_2)$ that is half as large as the US in all future periods. If the same country exhibits a probability to not survive the transition to the *fourth* period of life $(1 - sr_3)$ that is 25% smaller than the probability the US in 2005-2019, this 25% lead relative to the US will also be kept in all future periods. For Korea and Germany, the historical survival rates which the HMD provides do not start sufficiently early. As a result, we constructed the survival rates before the 2005-2019 period for these two countries by imposing that the evolution of the life expectancy at age 20 (by the UNPD) is replicated by the model (based on the same assumptions as earlier). An approach based on the old-age dependency ratio is not possible here, since we cannot replicate the dependency ratio when fertility rates are only imposed from 1945-1959 onwards. Furthermore, since the HMD only provides data for 2005-2017, we also performed a correction such that Germany's lead in life expectancy at age 20 to the US is of the level indicated by UNPD data. For future survival rates, the same approach as for the other countries was used.

We verify whether our method allows us to accurately reproduce evolutions in the old-age dependency ratio in various OECD countries. To match the specific age group split in our model, we define the old-age dependency ratio as the number of individuals older than 65 as a share of the number of individuals between 20 and 64. The actual dependency ratio was calculated based on the OECD Historical Population and Population Projections datasets. Cross-sectionally, the model's dependency ratio for the 2005-2019 period (on the horizontal axis) matches the average value of the actual dependency ratio over that period (on the vertical axis) quite nicely (see Figure C.1).

Figure C.1: *Old-age dependency ratio (65+/20-64) in 15 OECD countries, 2005-2019*



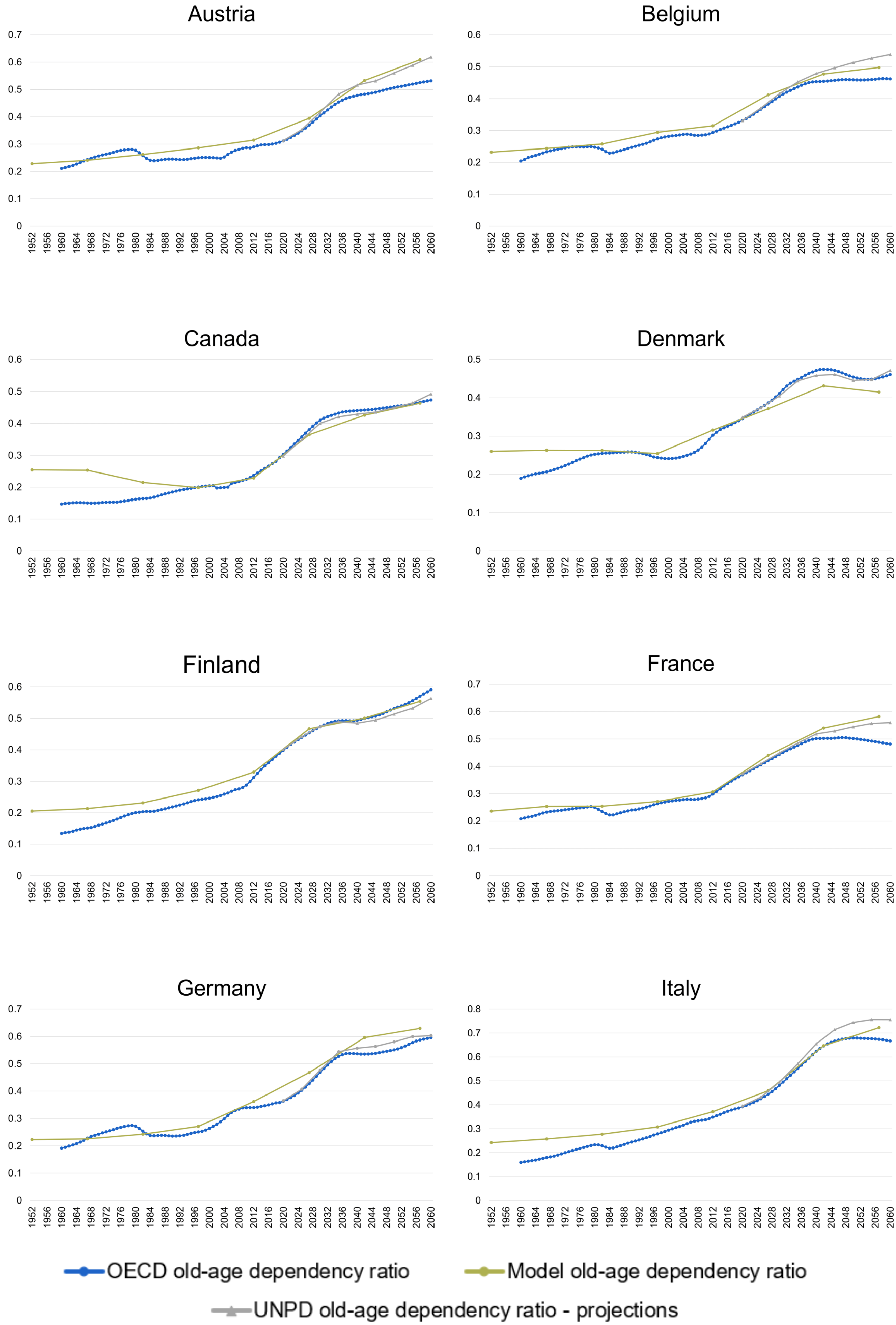
The solid blue line is the 45°-line.

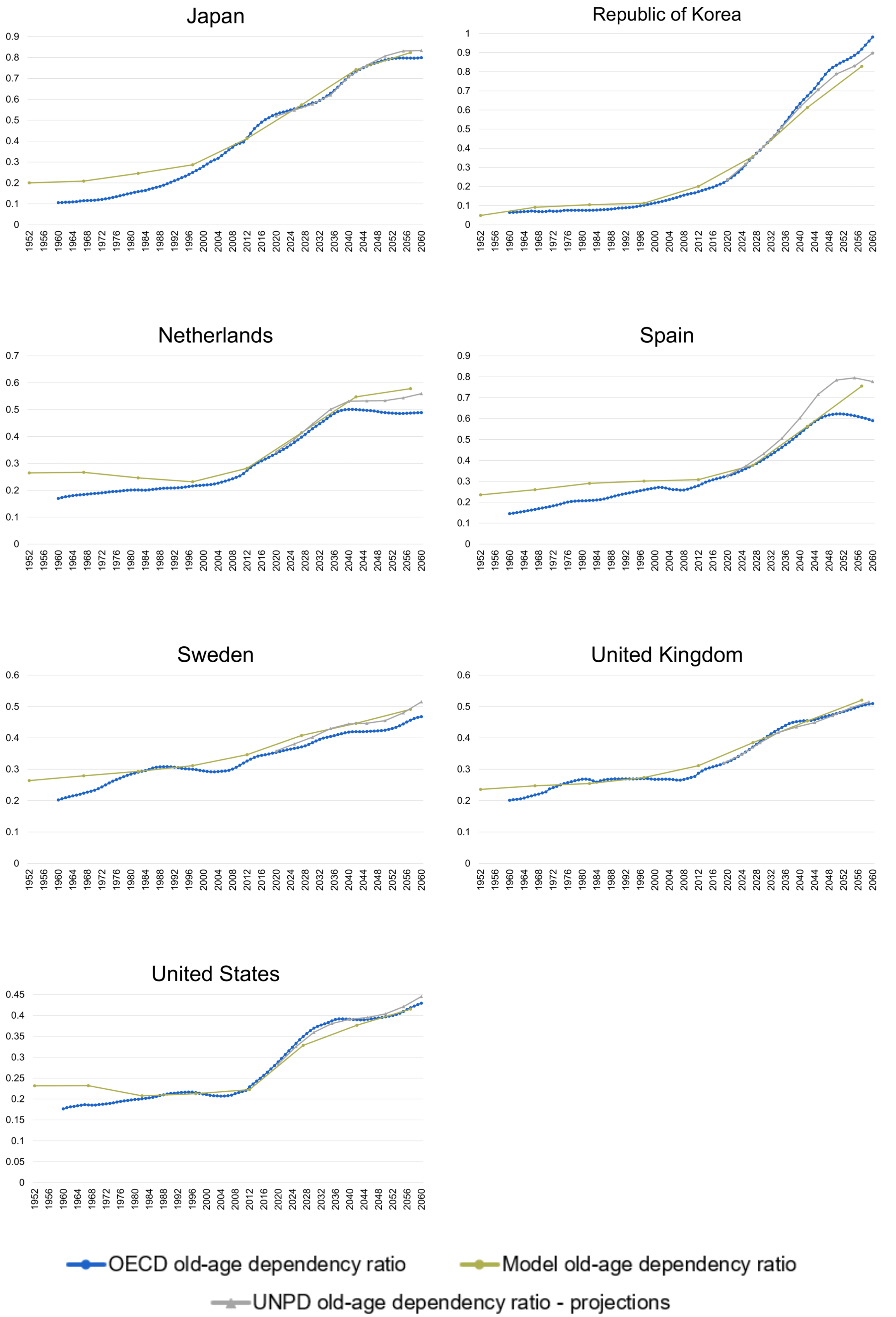
The correlation between the actual data and the model values is 96%. For most countries, there is a slight overestimation of the actual old-age dependency ratio. This is most likely the result of our simplified representation of survival rates where reaching the median age of the next period in real life is set equal to surviving the transition to the next period in the model. For younger ages, the fact that some people did not live the entire part of the next period (but are still categorised as 'survived the whole period') is balanced out quite well by the fact that some individuals lived for some time during the next period (but are still categorised 'as 'did not live in the whole period'). At older ages however, the first inaccuracy becomes far stronger since the actual probability of dying starts to increase faster throughout one's lifetime. This leads to a slight overestimation of the amount of people between the ages of 80 and 95, which is not entirely counterbalanced by the exclusion of the possibility to survive past the age of 95 in the model. The effect is very small, however, and the model captures cross-country differences very well overall.

When the evolution of the dependence ratio is set out for each country individually, it becomes clear that the degree of future ageing is reproduced quite well (see Figure C.2). Some remarkable differences between the OECD's projections and our model values are, however, apparent for Austria, France, Germany, the Netherlands and Spain in 2052. These differences are mostly the result of OECD and UNPD projections on future fertility being in disagreement.

Moreover, the model's old-age dependency ratios in the period before 2005-2019 are unlikely to capture the full reality, since - for most countries - the data on population size was only available from 1945 onwards. For countries such as Canada, Spain and the US, the dependency ratios in the beginning of the displayed period are large overestimations. This is mainly due to the rapid population growth these countries witnessed before the period of 1945, which increased the size of their working-age population relative to earlier generations. For the US, we use data on population growth from the 1930-1944 model period onwards, such that the fertility rate series starts in the 1945-1959 period for that country. Note that including pre-1945 data on population growth does not materially affect the model's predictions for the 2005-2019 modelling period whatsoever, since these generations are no longer alive in the 2005-2019 period.

Figure C.2: *The evolution of the actual and model old-age dependency ratio (65+/20-64), 1952-2060*




Japan
Republic of Korea
Netherlands
Spain
Sweden
United Kingdom
United States
OECD old-age dependency ratio
Model old-age dependency ratio
UNPD old-age dependency ratio - projections

Appendix D: Construction of data and data sources

In this appendix, we indicate how the actual values of key performance variables and policy parameters were constructed. The data source and the relevant period are always indicated. Several variables were constructed using an approach that is identical to Buyse et al. (2017) and we refer to their work for more details on the data construction. Any deviation from their approach is mentioned in the text below.

Performance variables:

Employment rate in hours when old (n_3) (for the age group 50-64, 2005-2019)

Definition: total actual hours worked by individuals in the age group / potential hours worked.

Total actual hours worked = total employment in persons x average hours worked per week x average number of weeks worked per year

Potential hours worked = total population in the age group x 2080 (where 2080 = 52 weeks per year x 40 hours per week)

Data sources:

* Total employment and total population by age group: OECD Stat, Labour Force Statistics by Sex and Age. Data are available for 50-54, 55-64. We constructed the data for our group of 50-64 as a weighted average.

* Average hours worked per week: OECD Stat, Labour Force Statistics, Hours worked, Average usual weekly hours worked on the main job. These data are available only for age groups 25-54 and 55-64. We use the OECD data for the age group 25-54 as a proxy for our age subgroup 50-54. We constructed the data for our group of 50-64 as a weighted average. The data considers dependent employment. Data is lacking here for Canada, Korea and Japan such that we used the average value of the remaining countries in our sample.

* Average number of weeks worked per year: OECD Stat, Labour Force Statistics, Hours worked. The average number of weeks worked per year has been approximated by dividing average annual hours actually worked per worker (dependent employment) by average usual weekly hours worked on the main job by all workers (dependent employment). Data is lacking here for Canada, Korea and Japan such that we used the average value of the remaining countries in our sample.

Education rate of young individuals (e) (for the age group 20-34, 2005-2018)

Data sources: OECD.Stat, Education and Training, Students enrolled by age; OECD.Stat, Labour Force Statistics by Sex and Age, Population; OECD.Stat, Education and Training, Enrolment rate by age.

For the 2005-2012 period, our approach is identical to Buyse et al. (2017). The 'Students enrolled by age' dataset is discontinued in 2012, however, such that for the 2013-2018 period we use 'Enrolment rate by age' data (all levels of education). Since this data does not distinguish between full-time and part-time students, we preferred not to use this dataset for the entire period. Enrolment rates for the age groups 20-24, 25-29 and 30-39 were used. We assumed that for 2013-2018, the share of part-time students for each specific age category was identical to the share of part-time students in that age category for the 2005-2012 period and we applied the same weighting of part-time students. Since each of the three age groups represents five years in the model, the final education rate of young individuals is the unweighted average of the three. The results for 2013-2018 align closely with what one expects based on the 2005-2012 period. The final education rate of a country is the average over the complete 2005-2018 period

Annual real potential per capita GDP growth rate (x) (2005-2019)

Data source for the real potential GDP: OECD Statistical Compendium, Economic Outlook Statistics and Projections, Supply Block, Potential Output of Total Economy, Volume.

Data source for the population at working age: OECD Statistical Compendium, Short-term Labour Market Statistics, Labour Force Statistics - Quarterly levels, Active Population, Aged 15-64.

The rate of technical progress x is calculated as the average annual growth rate of real potential GDP per person of working age, as in Buyse et al. (2017).

Wages of young, low and medium ability individuals, relative to the wage of young, high ability individuals ($\frac{w_L h_{1L}}{w_H h_{1H}}$ and $\frac{w_M h_{1M}}{w_H h_{1H}}$) (2018-2019)

Data source: OECD Education at a Glance 2020, Educations and earnings, relative earnings by educational attainment (The data considers 2018); OECD Education at a Glance 2020, Educational attainment of 25-64 year-olds (The data considers 2019).

We only consider full-time, full-year earners and the age category of 25- to 34-year-olds. For the data construction, we considered ‘below upper secondary education’ and ‘upper secondary education’ to be representative for the low ability type, ‘short-cycle tertiary education’ as representative for the medium ability type and ‘bachelor’s’ and ‘master’s or doctoral’ as representative for the high ability type. For ability types represented by multiple educational attainments, we weighted each attainment by the percentage of adults with that level of education as the highest level attained (Education at a Glance 2020, Educational attainment of 25-64 year-olds (2019)).

Policy parameters:

The values of the different policy parameters are indicated in Table D.1 below for the different countries in this study.

Table D.1: *Policy parameters in 2005-2019 for 15 OECD countries*

	Average labour tax rate at average labour income Γ (in %)	Progressivity parameter of labour income taxation ψ	Consumption tax rate (in %)	Tax rate on the return on savings (in %)	Public debt (in % of GDP), 2005-2019	Government consumption (in % of GDP), 2005-2019	Non-employment benefit when old (net replacement rate, in %)	Net own-income related pension replacement rate (in % of average earned net labor income)
Symbol in model	**Γ**	**ψ**	**τ_c**	**τ_{ci}**	**D/Y = d**	**G/Y = g**	**b**	**ρ**
Austria	48.4	0.17	13.5	34.5	95.0	19.7	34.0	89.7
Belgium	55.0	0.21	11.6	28.0	114.6	23.3	43.2	61.7
France	49.0	0.19	11.1	52.5	104.7	23.5	48.5	71.3
Germany	50.2	0.12	11.1	37.5	77.7	19.4	25.4	53.1
Italy	47.3	0.23	11.2	23.0	136.0	19.5	50.1	92.7
Netherlands	38.3	0.25	12.2	42.5	68.8	24.7	45.9	78.9
Spain	39.5	0.19	10.9	38.0	86.3	19.2	43.8	81.6
Core Eurozone average	**46.8**	**0.20**	**11.7**	**36.6**	**97.6**	**21.3**	**41.6**	**75.6**
Denmark	36.6	0.32	17.4	51.5	49.9	25.4	7.4	79.6
Finland	43.2	0.28	15.6	35.5	59.8	23.1	39.4	64.7
Sweden	43.8	0.28	13.3	39.0	51.3	25.5	15.9	61.0
Nordic average	**41.2**	**0.29**	**15.4**	**42.0**	**53.6**	**24.7**	**20.9**	**68.4**
Canada	31.1	0.18	8.3	49.0	87.3	20.5	10.8	49.6
United Kingdom	32.1	0.31	8.9	50.5	89.6	20.0	16.2	33.2
United States	30.7	0.27	3.3	47.0	92.8	15.1	6.3	50.5
Anglo-Saxon average	**31.3**	**0.25**	**6.8**	**48.8**	**89.9**	**18.6**	**11.1**	**44.4**
Japan	30.9	0.15	6.9	31.5	196.7	19.2	23.4	38.7
Republic of Korea	20.5	0.26	6.9	32.5	37.3	14.7	5.4	45.6
East Asia average	**25.7**	**0.21**	**6.9**	**32.0**	**117.0**	**16.9**	**14.4**	**42.2**
Total average	**39.8**	**0.23**	**10.8**	**39.5**	**89.9**	**20.8**	**27.7**	**63.5**

Public debt as a share of total output (d) (2005-2019)

Data source: OECD Statistical Compendium, Economic Outlook Statistics and Projections, Government Accounts, General Government Gross Financial Liabilities as a Percentage of GDP; IMF Historical Public Debt Database (HPDD).

The data in the table above represent averages for the 2005-2019 levels of gross public debt as a percentage of GDP. For earlier periods, starting from 1945-1959, fifteen-year averages of the level of public debt as a percentage of GDP are constructed based on the IMF HPDD.

Government consumption spending as a share of total output (g) (2005-2019)

Data source: OECD Economic Outlook Statistics and Projections, Expenditure and GDP, Government final consumption expenditure, nominal value, GDP expenditure approach; OECD Economic Outlook Statistics and Projections, Expenditure and GDP, Gross domestic product, nominal value, market prices

Dividing the yearly volume of final government consumption by the GDP data and subsequently averaging across fifteen-year periods results in the time-varying value for g. Due to limited data availability for some countries in our sample, we only let g vary over the three time periods 1975-1989, 1990-2004 and 2005-2019.

Consumption tax (τ_c) (2005-2018)

Data source: OECD.Stat, Annual National Accounts, Supply and Use Tables, SUT Indicators, Taxes less subsidies on products in percentage of final consumption expenditure by households (total product, total activity)

The aggregate consumption tax rate is calculated by deducting total subsidies on final products from the total taxes on final products and then expressing the result as a percentage of final consumption expenditures. Data is for the 2005-2018 period, but for several countries data availability is limited to a sub-period. For Spain and Germany, no data is available and we use the consumption tax rates calculated by Dhont and Heylen (2009), which consider the 1995-2001 period. For Japan, the Korean consumption tax rate was imposed. Assuming this low Korean rate of 6.9% for Japan seems warranted since, before April 2014, the official value-added tax rate was only 5% and, from April 2014 until October 2019, the rate was 8% (reported on the website of the Japanese National Tax Agency).

Tax rate on the return to savings (τ_{ci}) (2012)

Tax rates on the return to savings are proxied by the average of tax rates applied to interest and dividend payments as calculated by the OECD study of Harding (2013) (Table 16 of the study). The data considers the tax systems as they were in July 2012.

Average labour income tax rate, at average income level (Γ) (2005-2019)

Data source: OECD.Stat, Public Sector, Taxation and Market Regulation, Taxation, Tax Database, Table I.5. Average personal income tax and social security contribution rates on gross labour income, Total Tax Wedge.

The parameter Γ is given a country-specific value based on average OECD data over the 2005-2019 period on the average tax wedge on labour income including employer social security contributions (Table I.5). Consistent with our characterisation of the tax system in equation (40), Γ is proxied by the average labour income tax at 100% of the average wage over the model period.

Progressivity parameter of labour income tax system (ψ) (2005-2019)

Data source: OECD.Stat, Public Sector, Taxation and Market Regulation, Taxation, Tax Database, Table I.5. Average personal income tax and social security contribution rates on gross labour income, Total Tax Wedge.

The parameter ψ is set in a country-specific way such that for four different levels of income (67%, 100%, 133% and 167% of the average wage) the model's average tax rates, as stated by equation (40), proxy for the actually observed average tax rates as well as possible (minimization of squared errors).

Net pension replacement rates (ρ) (2018)

Data source: OECD, Pensions at a Glance 2019: OECD and G20 Indicators, Net pension replacement rates by earnings, Table 5.5.

The net replacement rates are defined as the individual's net pension entitlement as a percentage of the individual's net pre-retirement earnings. The replacement rates of all types of individuals are assumed to be identical to the unweighted average of the net replacement rates reported for individuals at 50%, 100% and 150% of average earnings. The calculations reflect the situation of individuals entering the labour market in 2018 and onwards.

Appendix E: Optimality conditions for behaviour of the household

Equation (E1) expresses a standard Euler equation for consumption. It is clear that rising survival rates will encourage individuals to save more at a given interest rate since this increases the probability that the individuals can make use of these savings in the next period.

$$(E1)\ \frac{c_{j+1,a}^{t}}{c_{ja}^{t}} = sr_{j+1}^{t}\beta\left(1 + r_{t+j}\right)\ \ \forall\, j\ =\ 1, 2, 3, 4$$

Equation (E2) states the optimality condition that determines labour supply in the final period of active life. The LHS indicates the loss in utility related to having one less unit of leisure in that period. The RHS of the equation describes the return of providing an additional unit of labour in that period. It consists of a part related to the extra consumption possibilities in the period in which more labour is provided itself and another part that is related to the increase in consumption possibilities when retired. The latter is the result of the construction of the pension system in which retirement benefits are a function of labour income during active life. The return to an extra hour of work also rises when it is likelier that the individual will live to enjoy this increased retirement benefit: this is the positive substitution effect that one can directly observe in equation (E2). As higher survival rates also imply that individuals will set more aside for when they become old so they can consume more (c_{4a}^{t} and c_{5a}^{t}), there is also a negative income effect of increased life expectancy on labour supply when old.

$$(E2)\ \frac{\gamma_3}{(l_{3a}^{t})^{\theta}} = \frac{w_{a,t+2}h_{3a}^{t}(1-\tau_{w3a}^{m})(1-b)}{c_{3a}^{t}(1+\tau_c)} + \frac{\pi_4^t}{\pi_3^t}\,\beta\,\frac{\rho p_3 w_{a,t+2}h_{3a}^{t}(1-\tau_{w3a}^{m})}{c_{4a}^{t}(1+\tau_c)} + \frac{\pi_5^t}{\pi_3^t}\,\beta^3\,\frac{\rho_{wa} p_3 w_{a,t+3}h_{3a}^{t}(1-\tau_{w3a}^{m})}{c_{5a}^{t}(1+\tau_c)}$$

Finally, equation (E3) gives the condition related to the optimal time spent at education when young for the medium and high ability individuals. The LHS indicates the marginal utility loss from higher investment in human capital when young, related to the fall of hours worked when young that it implies. It takes the form of a decrease in consumption possibilities that is composed of both the direct loss of income when young and the lower pension benefits later as a result of the lower hours worked when young. The RHS then indicates the expected discounted gain in utility that stems from earning a higher wage because of an increased human capital stock and, as a result of that, also higher pension and non-employment benefits. Once again, the rise in life expectancy creates a direct positive effect on the returns to education through the increased probability of being alive in stages of life where one can benefit of this investment in human capital. On the other hand, negative income effects lowering the marginal utility of consumption at later stages of life are also present.

$$(E3)\ \frac{w_{a,t}h_{1a}^{t}(1-\tau_{w1a}^{m})}{c_{1a}^{t}(1+\tau_c)} + \beta^3\pi_4^t\,\frac{\rho_a p_1 w_{a,t}h_{1a}^{t}(1-\tau_{w1a}^{m})}{c_{4a}^{t}(1+\tau_c)} + \beta^4\pi_5^t\,\frac{\rho_a p_1 w_{a,t}h_{1a}^{t}(1-\tau_{w1a}^{m})}{c_{5a}^{t}(1+\tau_c)} =$$

$$\beta\,\pi_2^t\,\sigma\phi(e_{1a}^{t})^{\sigma-1}\,\frac{b w_{a,t+1}h_{1a}^{t}(1-\tau_{w1a}^{m})}{c_{2a}^{t}(1+\tau_c)} + \beta^2\pi_3^t\,\sigma\phi(e_{1a}^{t})^{\sigma-1}\,\frac{b w_{a,t+2}h_{1a}^{t}(1-\tau_{w2a}^{m})(n_{3a}^{t}+b(1-n_{3a}^{t}))}{c_{3a}^{t}(1+\tau_c)} +$$

$$\beta^3\pi_4^t\,\rho\,\sigma\phi(e_{1a}^{t})^{\sigma-1}\,\frac{\sum_{j=2}^{3} p_j n_{ja}^{t} w_{a,t+j-1}h_{1a}^{t}(1-\tau_{wja}^{m})}{c_{4a}^{t}(1+\tau_c)} + \beta^4\pi_5^t\,\rho\,\frac{\sum_{j=2}^{3} p_j n_{ja}^{t} w_{a,t+j-1}h_{1a}^{t}(1-\tau_{wja}^{m})}{c_{5a}^{t}(1+\tau_c)}$$

Appendix F: Alternative calibration with higher elasticity of substitution κ

In section 3.1.5, we calibrated the elasticity of substitution between tasks κ by imposing that differences in demography account for 25% of the relative gap in automation density between Germany and the US. Given the substantial variation around the estimated effect of ageing on robotics, it is appropriate to consider alternative calibration targets for κ. By following the 25% found by Acemoglu and Restrepo (2018c), we are relatively conservative regarding the strength of the relationship. In an updated version of the same study, Acemoglu and Restrepo (2021) revised their conclusion by stating that ageing explains 50% of Germany's robotics lead over the US. The underlying range of estimates that the two studies produce are similar, but the final conclusion depends on the preferred specification. It is because of this uncertainty surrounding the precise value of our calibration target for κ and our earlier choice for a target value near the lower end of the range, that we here briefly consider an alternative parameterization of our model in which the target value is doubled. In the counterfactual case with German demographics, the US level of automation capital per worker now has to be *1.428* times the level of automation capital per worker in the baseline US model with regular demographics, when both are evaluated at the end of the 2005-2019 period.

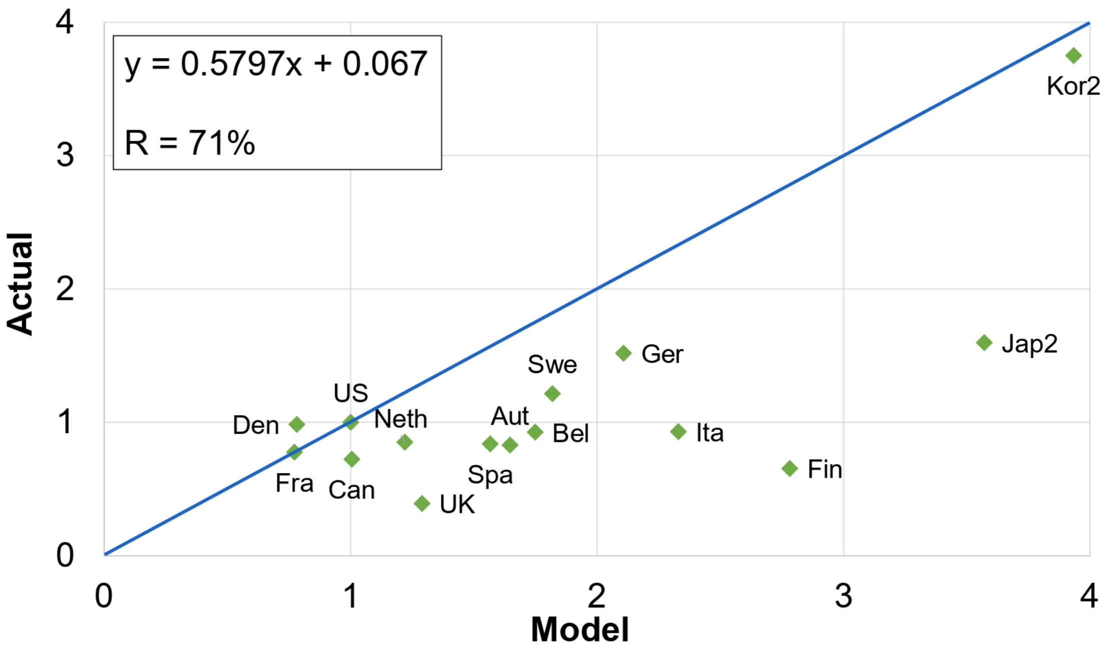
Table F.1: *Parameterization and target values for calibration*

Technology and preference parameters	
Productions of final goods	$\alpha = 0.25$, $s = 1.5$, $\boldsymbol{\eta_L = 0.242}$, $\boldsymbol{\eta_M = 0.312}$, $\eta_H = 0.446$
Automation of tasks	$1 - \xi_L = 0.250$, $\boldsymbol{1 - \xi_M = 0.211}$, $\boldsymbol{1 - \xi_H = 0.120}$, $\boldsymbol{\kappa = 8.8}$, $\boldsymbol{J = 22.7}$
Exogenous technology growth	$\boldsymbol{x = 0.244}$
Human capital production	$\boldsymbol{\phi = 1.33}$, $\sigma = 0.3$
Initial human capital distribution	$\varepsilon_L = 0.67$, $\varepsilon_M = 0.84$, $h_0 = 1$
Preference parameters	$\beta = 0.8$, $\theta = 2$, $\gamma_1 = \gamma_2 = 0$, $\boldsymbol{\gamma_3 = 0.155}$
Capital depreciation rate	$\delta_k = \delta_p = 0.714$
Policy parameters (United States, 2005-2019)*	
$\Gamma = 0.307$, $\psi = 0.272$, $\tau_{ci} = 0.47$, $\tau_p = 0$, $\tau_c = 0.033$, $b = 0.063$, $\rho = 0.505$, $g = 0.151$, $D/Y = 0.928$	
Target values for calibration (United States, 2005-2019)	
Relative wages of young individuals by education:	$\frac{w_L h_{1L}}{w_H h_{1H}} = 0.565$, $\frac{w_M h_{1M}}{w_H h_{1H}} = 0.645$
Relative share of automatable tasks:	$\frac{1-\xi_M}{1-\xi_L} = 0.847$, $\frac{1-\xi_H}{1-\xi_L} = 0.480$
$\frac{\text{Automation capital per worker in the US with German demography in 2019}}{\text{Automation capital per worker in the US in 2019}} = 1.428$	
Gross labour share in national income = 70.1%	
Annual growth of potential GDP per person of working age = 1.47%	
Participation in tertiary education:	$\frac{e_{1L} + e_{1M} + e_{1H}}{3} = 15.2\%$
Hours worked when older N_3 = 59.0%	

* Lump sum transfers adjust as the residual category in equation (28).

As can be seen in Table F.1, this alternative calibration leads to a much higher value of κ (8.8 instead of 3.28) and some changes in the other calibrated parameters. We note that condition (42) also holds under this new parameterization such that all automatable tasks are cost-effectively automated. As in section 3.2, we can check the empirical relevance of this parameterisation by comparing the model's predictions regarding automation density with the actual data on robot density. Figure F.1 shows the new version of Figure 2, panel (a).

Figure F.1: *Model predictions (horizontal) against actual data (vertical) on robot density in 2019 relative to the US*



We argue that the model now strongly overestimates the cross-country differences in automation in general and the impact of ageing on automation in particular. The slope is now 0.58 in the baseline case (versus 1.009 under the normal parameterization). The fact that the new slope is close to 0.5 implies that doubling the calibration target for κ leads us to overestimate the differences in automation by a factor two. This result legitimises our choice to calibrate based on the more cautious conclusion of Acemoglu and Restrepo (2018c).

Finally, we note that under this alternative parameterization some of the main findings of our study in Section 4 would have to be adjusted. As automated tasks can substitute a lot better for non-automated tasks when κ is raised to 8.8, the rise of the old-age dependency in the US over the next thirty years *does* become an engine of growth. Ageing then stimulates the adoption of automation technologies even more powerfully and automation is allowed to better compensate for the scarcity of humans of working age. These findings underscore the importance of the work of Stokey and Rebelo (1995): one theoretical model can lead to entirely different conclusions depending on the precise calibration targets. For this reason, it is especially important to verify the empirical implications of the model's parameterization. As a result, we conclude that it is theoretically possible for our model to find that ageing-induced automation completely neutralizes the negative effect of ageing on growth, but that the empirical relevance of the parameterization necessary for this result is highly questionable.

Appendix G: Robustness check - What if government consumption adjusts?

In Section 4, we outlined the general equilibrium effects of ageing-induced automation under the assumption that government consumption as a share of GDP g follows an exogenous path, while lump sum transfers adjust to maintain a constant public debt to GDP ratio at the level of 2005-2019. For all periods after 2005-19, the 2005-19 value was imposed on g. Details on construction and sources can be found in Appendix D. In this appendix, we test whether our main conclusions also hold true when the government keeps lump sum transfers per capita constant at the 1975-89 level, while g adjusts. The government now absorbs the costs of ageing by reducing government consumption. Everything else is identical to our approach in Section 4. Note that in the constant automation density counterfactual too, lump sum transfers per capita are kept constant at the 1975-89 level, while g adjusts endogenously. Figures G.1 to G.4 are the new versions of Figure 4 to 7 of the main text.

For the most part, results do not materially deviate from our findings in Section 4 of the study. The most eye-catching difference Is the lower welfare inequality between the ability types in Figure G.1, panels (h) and (i). This is mainly because, in Section 4, lump sum transfers are reduced (Figure 4, panel (c)), while, in this robustness test, the government does not have to reduce lump sum transfers since it is wasteful government consumption that absorbs the costs of ageing. These lump sum transfers are most important for low (and medium) ability individuals with fewer resources. On the other hand, the somewhat erratic evolution of government consumption is also reflected in more fluctuation in national savings and eventually output per capita. Here too, however, the main trends described in Section 4 remain valid.

Figure G.1: *Evolution automation density, automation tax and lump sum transfers*

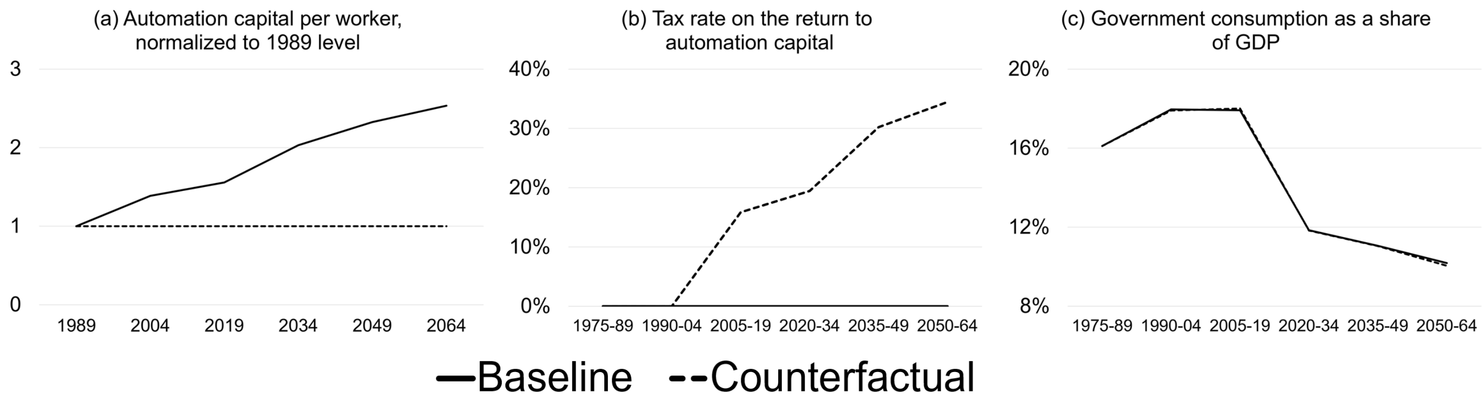


Figure G.2: *Evolution of seven selected indicators in the baseline scenario and the counterfactual scenario without ageing-induced automation. Indicators marked with * are normalized to the level in the 1975-1989 model period. The other variables are in level (%).*

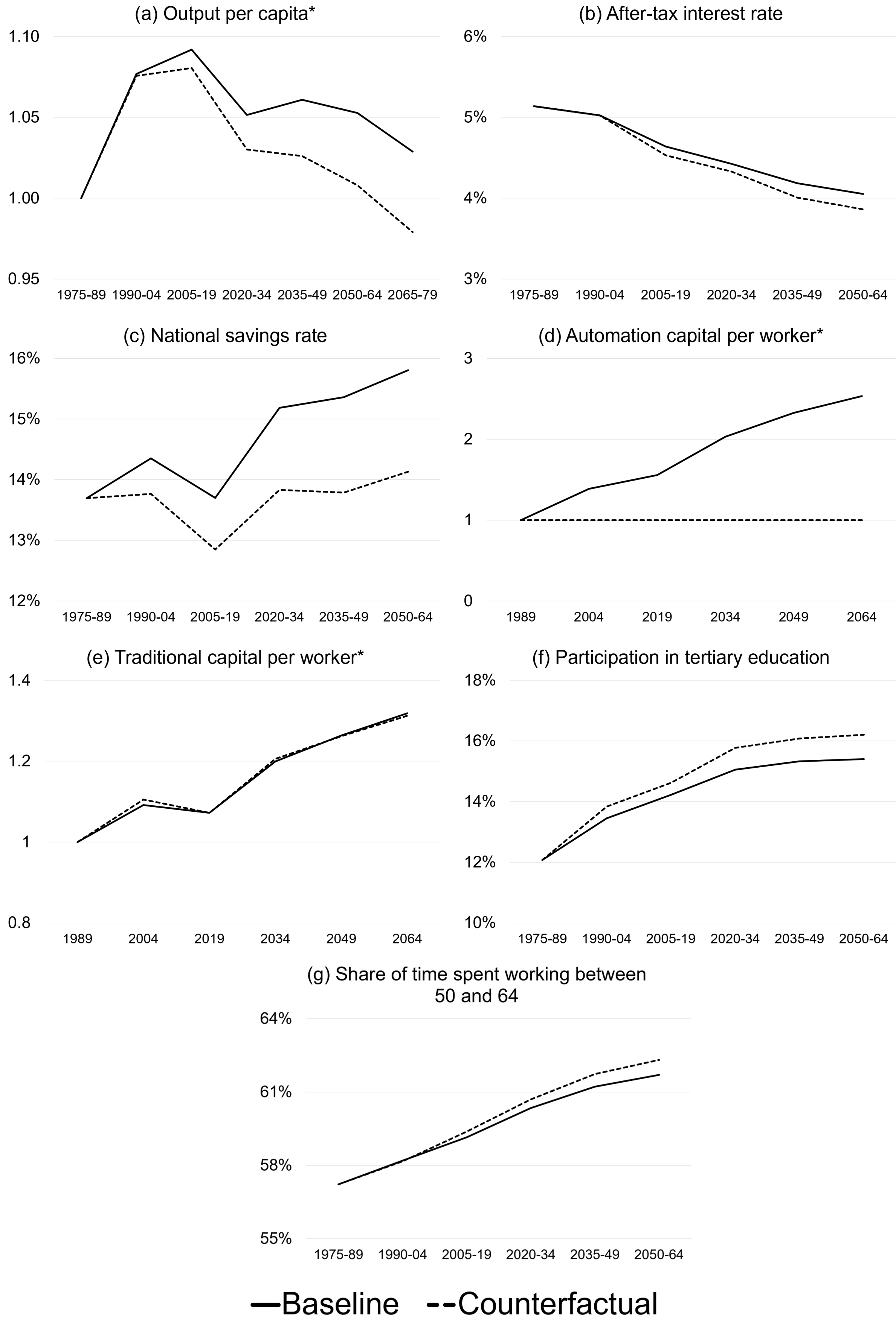


Figure G.3: *Evolution of nine selected indicators in the baseline scenario and the counterfactual scenario without ageing-induced automation. Indicators marked with * are normalized to the level in the 1975-1989 model period. The other variables are in level (%).*

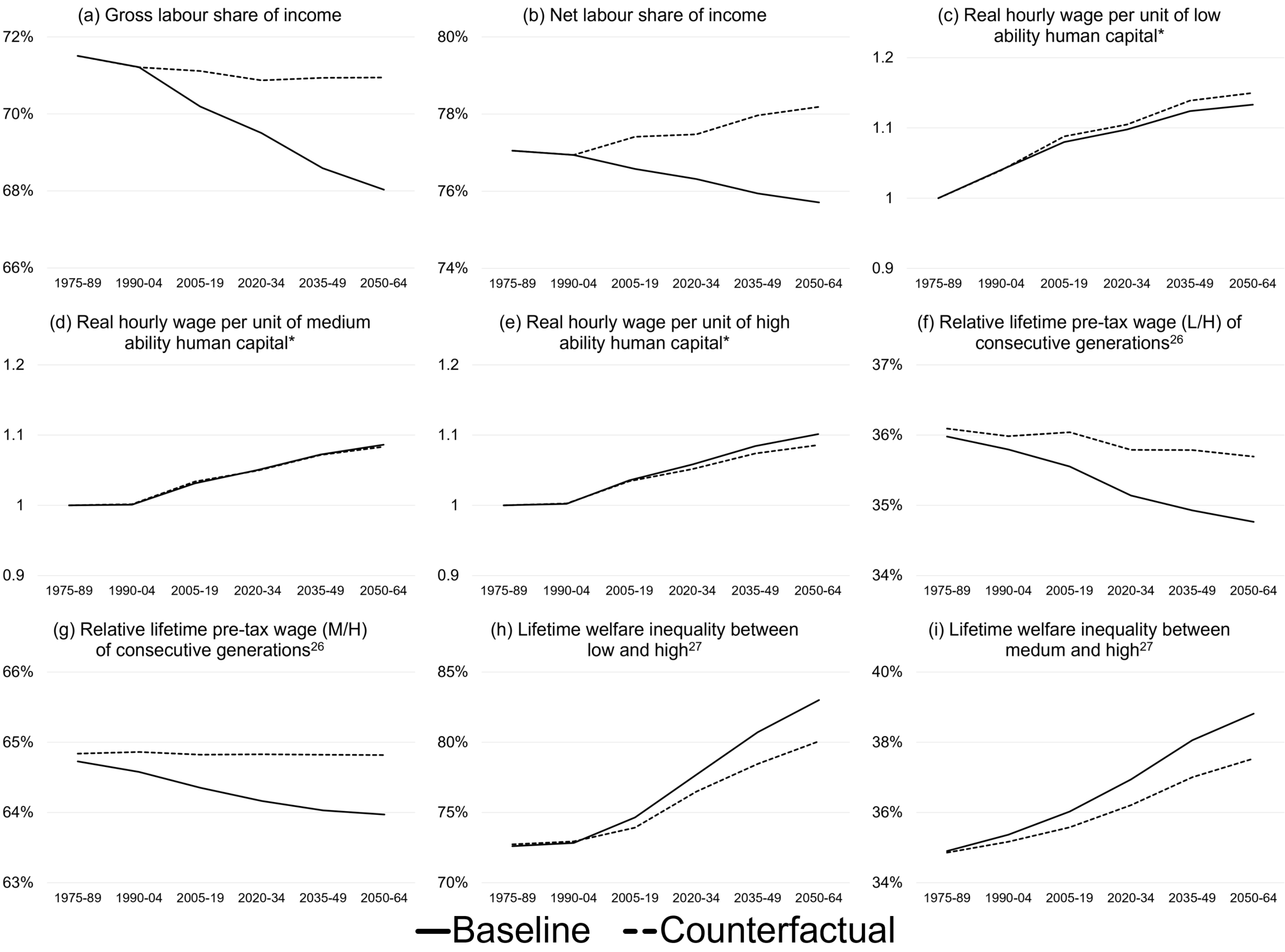


Figure G.4: *Absolute welfare effect of ageing-induced automation, by generation and ability type[28]*