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LIQUIDITY CRISES, LIQUIDITY LINES AND SOVEREIGN RISK

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Liquidity Crises, Liquidity Lines and Sovereign Risk *

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Abstract

This paper quantitatively investigates the trade-offs of introducing an extra line of credit in an emergency situation. I show that temporary access to these lines for up to 3 percent of mean annual income during low liquidity periods yields long-term effects with a lower cost of borrowing but with incentives to accumulate higher debt. Permanent access, however, has only short-lived effects because temporal arrangement better completes the markets and induces market discipline as the government worries about rollover risk once the low liquidity period ends. I also present in an event analysis that Mexico's arrangement of swap lines with the Federal Reserve amid the global financial crisis in 2008 helped avoid a potential debt crisis.

Keywords: sovereign default, liquidity shocks, swap lines, sudden stops
JEL Codes: F30, F34

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1 Introduction

Since the onset of the global financial crisis, and more recently with the COVID-19 pandemic, a number of countries — including advanced and emerging economies — have established liquidity lines for utilization in times of low liquidity. Having a facility that provides resources during capital flight can provide relief to financial markets during times of stress.[1] Following the global financial crisis, debates revived among policy makers and platforms such as G20 concerning the necessity of liquidity lines and improving their accessibility in order to contain the debt crisis.[2] Yet, as a counterargument, some views claim that the existence of such lines might lead to excessive borrowing by the government and render both lenders and the government worse off over the long run (see IMF (2006)). This paper investigates the trade-offs of introducing an extra line of credit in an emergency situation.

Following the classic setup of Eaton and Gersovitz (1981), I study a small open economy model in which a benevolent government maximizes the utility of the representative household. In this endogenous sovereign default framework, the government can borrow in two ways: (*i*) by issuing long-term non-contingent (defaultable) bonds and (*ii*) by exercising one-period liquidity lines (non-defaultable) during a liquidity crisis. Baseline policy analysis sets a low limit for liquidity lines (3 percent of mean annual income). A low limit is anticipated to lower the moral hazard concerns of line providers and alleviate political objections by reducing the government's incentive to default on these loans. A standard assumption in the literature is that a defaulting government is temporarily excluded from financial markets and cannot issue new debt during its exclusion. Furthermore, a defaulting government has to honor its debt accrued from liquidity lines but does not repay its dues from long-term bonds. To capture the essence of the discussions that allow

[1] For empirical evidence on the positive impact of liquidity lines for managing and resolving a liquidity crisis, see Obstfeld et al. (2009), Goldberg et al. (2010), Aizenman and Pasricha (2010), Fleming and Klagge (2010), and Rose and Spiegel (2012), to name a few. Even though swap lines are mainly arranged between central banks, it is not hard to envision that some of these resources end up in government securities. For instance, consider longer-term refinancing operations (LTRO) of the European Central Bank which provide financing to credit institutions. Krishnamurthy et al. (2018) document that a significant amount of these funds end up being used to purchase government securities.

[2] G20, which is an international forum for the governments from 20 major economies, endorses the importance of swap lines and supports the IMF for using its liquidity lines to help emerging economies that have strong fundamentals but are facing exogenous shocks (see G20 (2011)). Liquidity lines such as precautionary and liquidity lines (PLLs) and flexible credit lines (FCLs) offered by the International Monetary Fund (IMF) are essentially designed for emerging market economies that are solvent but have liquidity problems. For terms and conditions as well as qualification criteria of FCLs and PLLs, see IMF (2014b). Table A.1 in the Appendix summarizes the list of countries that have arranged and withdrawn from the IMF's liquidity lines, and Table A.2 provides Mexico's swap line arrangements to date.

governments to borrow through liquidity lines during a liquidity crisis, a liquidity shock is introduced in the baseline model. Liquidity shocks induce a change in lenders' risk aversion and entail an income cost. Later, I drop the income cost of the liquidity shock assumption and explore the model dynamics.

I show that temporary liquidity lines are good ex-post because they generate welfare gains and entail an ex-ante cost in the form of increased borrowing. I then explain that the availability of the liquidity lines is important when considering whether they are accessible all the time as opposed to only during liquidity shocks. Accessing them only during liquidity shocks provides the correct covariance: the government receives fresh and cheap financing precisely during risk-off episodes. If lines are accessible all the time the government taps these cheaper funds to the limit as soon as they are introduced and essentially the insurance feature of liquidity lines is lost.

I find that liquidity lines eradicate 70 percent of defaults driven by low liquidity. For an initial spread of 7.2 percent, the inception of liquidity lines immediately reduces spreads to 2.6 percent. However, a decline in spreads induces the government to accumulate more debt in the long run, as it knows that it will have access to emergency lines of credit during low liquidity periods. Yet, the long-run spreads remain lower than the baseline economy, and the government smooths its consumption better during low liquidity periods as it transfers resources from normal times to low liquidity periods. Strikingly, I show that these benefits are short-lived only if liquidity lines are arranged to be permanently available, as in Hatchondo et al. (2017). Intuitively, introducing a state-contingent asset completes markets, though not perfectly. The main friction in endogenous sovereign default models is the lack of commitment and market incompleteness. Liquidity lines partially complete the markets if used only during liquidity shocks. A temporal arrangement induces market discipline because the government worries about rollover risk when exiting the low liquidity period. At that time, the government has fewer resources available to pay for defaultable debt compared to the baseline economy. If these lines can be arranged to be issued at any time, the government always uses these lines to the limit in each period as they are a cheaper source of funding.[3]

Regarding the analysis above, I compute that the welfare gains from the introduction of liquidity lines are equivalent to a permanent consumption increase of 0.3 percent, which is driven by the lower interest paid on defaultable debt. I quantify the capital gains for

[3] I explore the multiplicity dynamics within my framework and check whether the implications of this paper are a by product of hidden equilibrium selection, as in Önder (2016), which initiates the value function iteration from two different initial conditions and confirms that the results are the same.

holders of previously issued defaultable debt during the introduction of liquidity lines. At the time of the arrangement, the value of the long-term debt surges, and thus debt holders enjoy capital gains.

Even though default risks decline the first time liquidity lines are arranged, the number of defaults in both economies becomes the same in the long run with the government's higher debt accumulation. However, default dynamics change. The economy with a liquidity line either defaults the first time it receives a liquidity shock without exercising these lines or almost never defaults in a low liquidity period. The baseline economy, on the other hand, can default at any point in time during a liquidity crisis. Intuitively, the marginal benefit of using liquidity lines falls if a severe income shock overlaps with a low liquidity period. Once liquidity lines are exercised, defaulting becomes excessively costly as the government is not allowed to default on liquidity lines.

I also investigate whether liquidity lines entail a moral hazard channel through larger debt accumulation in two ways: (i) by increasing the liquidity line limits from 3 percent to 10 percent and (ii) by exploring whether larger debt accumulation amplifies the debt dilution problem which refers to the decline in the value of existing defaultable debt because of the issuance of new defaultable debt. I show that if a moral hazard channel exists, it is likely to be small compared to the gains obtained from the inception of liquidity lines. Lastly, I extend the paper by investigating the model implications using the liquidity crisis in 2008 and 2009 in an event study analysis. I compare the time paths predicted by my model with the event in Mexico. Output fell 6 percent at the peak of the crisis, and spreads more than doubled. I feed the identical income shocks observed in the data into the model and introduce 3 percent liquidity lines in the first quarter of 2009 as this was the period during which swap lines were arranged between Mexico and the Federal Reserve (FED). The model delivers time paths of spreads that resemble those in Mexico and shows that Mexico's liquidity lines in 2009 helped avoid a potential debt crisis.

Related Literature. This paper builds on the quantitative endogenous sovereign default literature that follows Aguiar and Gopinath (2006) and Arellano (2008). The closest study to this paper is Hatchondo et al. (2017). To formally evaluate the impact of Eurobonds proposal, they propose a model in which the sovereign issues both defaultable and non-defaultable debt with which governments can borrow up to 10 percent of mean annual income at every period. The rationale of the proposal is to form a risk-free bond so that the European governments can borrow at a lower cost to contain the debt crisis. In contrast, this paper assumes that the arrangement of non-defaultable debt is conditioned on low liquidity periods. Unlike Hatchondo et al. (2017), this paper shows that an emergency credit line is successful in mitigating the cost of debt service in the long run. To this end, this paper shows that the design of emergency lines is crucial: governments should have access to such lines only during low liquidity periods. If governments are provided with permanent access to liquidity lines, then this new source of financing fails to mitigate the cost of debt service and simply serves as a free lunch in the first period of its inception. The intuition for that outcome relies on the increased cost of defaulting after using liquidity lines; the loans cannot be extended indefinitely. The government knows that its access expires when the liquidity shock is over; thus, it cannot simply roll over the existing debt by tapping the emergency lines, unlike Hatchondo et al. (2017). The increased cost of defaulting leads to a permanent decline in spreads and enables the government to support a higher debt-to-GDP ratio with a relatively low limit of liquidity lines (3 percent of mean annual income during low liquidity periods). In terms of welfare comparisons, liquidity lines sustain welfare gains similar to those of Hatchondo et al. (2017) with its 3 percent limit of non-defaultable debt as opposed to the 10 percent limit in Hatchondo et al. (2017). This subtle contractual difference generates dramatic differences in the implications and highlights that access to liquidity lines should be granted only during emergencies. This paper would thus help policy makers in the design of such contracts.

Other related papers are Boz (2011) and Fink and Scholl (2016). They both introduce non-defaultable debt obtained from an international financial institution (IFI) into a one-period canonical Eaton and Gersovitz (1981) sovereign default model. Their interpretation of non-defaultable debt, however, entails conditionality modeled as restrictions on the government's choices. The former study focuses on the cyclical properties of the IFI lending with one-period defaultable debt over the business cycle. The latter study finds that these funds generate higher default risk and spreads. This stems from the initial decline in spreads at the time of the policy introduction, which induces the sovereign to accumulate larger debt levels, leading to higher spreads and defaults. These studies also come with the assumption that IFI funding is available all the time.

This paper also contributes to the discussions on whether such emergency lines can be considered as an alternative to reserve accumulation (see IMF (2013) and the references therein). Liquidity lines are loans provided by foreign institutions, while reserves are the savings of a sovereign. If such emergency loans are successfully arranged, then liquidity lines would do a better job than reserves in terms of mitigating default risks. Bianchi et al. (2018), Önder (2017), and Alfaro and Kanczuk (2009) indeed show that reserves transfer resources from high-income states to low-income states including default states, which may increase default frequency by lowering the cost of defaulting. This is not to say that accumulating reserves is not optimal; Bianchi et al. (2018) show that sovereigns in equilibrium still accumulate reserves.

Related empirical studies on liquidity lines are the works of Obstfeld et al. (2009) and Goldberg et al. (2010). Obstfeld et al. (2009) discuss that liquidity lines can work as an alternative to international reserves. Goldberg et al. (2010) find that swap lines can be an essential tool for managing and resolving a liquidity crisis.

The rest of the paper proceeds as follows. Section 2 presents the model, and Section 3 discusses the calibration strategy. Section 4 features the simulation results and extends the analysis on the dimensions of permanency of liquidity lines, the income cost of liquidity shocks, the moral hazard channel, and the event analysis. Finally, Section 5 concludes.

2 Model

This section develops a dynamic small open economy model inhabited by a large number of identical consumers and a benevolent sovereign in which the government issues non-state-contingent, defaultable long-term debt as well as non-defaultable, one-period debt during a liquidity crisis.

2.1 Model Timing

The timing of events can be summarized as follows:

1. Period t starts, and the government first learns its endowment y and the state of global liquidity g, which are both public information.
2. The government decides whether to default:
   - If the government repays, it may choose to issue more debt or purchase back some of its outstanding debt holdings. If a government is hit by a global liquidity shock, then it also has access to liquidity lines and can borrow through these facilities.
   - If the government reneges on its debt obligations, it does not have access to credit markets including liquidity lines for a stochastic number of periods with an income cost during financial exclusion. The government returns to the credit markets with zero debt.
3. Period $t + 1$ starts.

2.2 Environment

Preferences and endowment. The benevolent government maximizes the utility of the representative household whose preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2.1}$$

where E denotes the expectation operator, $0 < \beta < 1$ is the discount factor, and c_t denotes aggregate consumption at time t. The utility function $u(.)$ belongs to the class of CRRA utility functions and reads as

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}. \tag{2.2}$$

The utility function $u(.) : [0, \infty) \rightarrow \mathcal{R}$ is increasing, strictly concave, continuous, and bounded above by the quantity U, and γ is the risk aversion parameter. The single tradable good of the endowment economy is denoted by $y \in \mathcal{Y} \subset \mathbb{R}_{++}$, which follows an AR(1) process:

$$\log(y_t) = (1 - \rho)\,\mu + \rho \log(y_{t-1}) + \varepsilon_t,$$

with $|\rho| < 1$ and $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$ denoting innovations to the income process.

Asset spaces. As in Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012), and Arellano and Ramanarayanan (2012), I assume that the long-term bond issued in period t carries out an infinite stream of coupon payments that decrease at a rate $\delta \in (0, 1]$, which is a fixed parameter. In particular, the long-term bond issued in period t promises to pay $(1 - \delta)^{j-1}$ units of the consumption good in period $t + j$, conditional on not defaulting, for all $j \geq 1$. The obvious advantage of this formulation is to avoid keeping track of the entire distribution of different bond maturities.

The government also has access to a new source of financing s_t such as swap lines, FCLs or PLLs during a global liquidity shock. Since these debt instruments can be provided by the FED, IMF or ESM (European Stability Mechanism), saving is not allowed. The interest rate for this new source of financing is assumed to be priced at 1 percent (equivalent to the risk-free rate r) since these funds are provided at a significantly lower rate than the market by the FED and the IMF. The access to liquidity lines takes place only during a liquidity shock as that is the nature of the arrangements Mexico had with the FED or with the IMF. The access is provided only when the government runs into liquidity problems. For instance, the swap line between the US and Mexico was not extended after its expiration on February 1, 2010, up until the COVID-19 pandemic even though it was extended with European countries. The IMF extended its FCL arrangement with Mexico in 2014 for precautionary purposes given the global downside risks (IMF (2014a)). The borrowing through liquidity lines is assumed to have one-period maturity because the maturity of the debt that Mexico had with the FED was within a quarter. Also, the duration of the liquidity line arrangements with the IMF was relatively short as well. The appendices contain details about the arrangements and transactions. Throughout the model, swap lines will be referred to as the new source of funding for notational purposes. The budget constraint of the economy corresponding to a period with current access to liquidity lines conditional on having access to credit markets is given by

$$c_t = y_t - \phi^g(y) - s_t + \frac{1}{1+r}s_{t+1} - b_t + q_t(b_{t+1} - (1-\delta)b_t),$$

where q_t denotes the price of the defaultable debt issued by the government and depends on the borrowing rules of the long-term debt issuance and line arrangements as well as the income and global liquidity shocks in equilibrium. Furthermore, s_t denotes the amount of liquidity line payments accrued by liquidity line arrangements due this period, s_{t+1} denotes the amount of borrowing through liquidity lines to be repaid at time $t+1$, $\phi^g(y)$ denotes the income cost of a global liquidity shock, and r is the risk-free interest rate at which international lenders can borrow or lend.

Defaults. In addition to being a contractual obligation; a typical assumption in the literature is that all future debt payment promises become due if a government misses a payment. As in most of the studies in the sovereign default literature, the assumption is that the fraction of the defaulted debt lenders can recover is zero.

As is commonly assumed in the sovereign default literature, a default event leads to the exclusion from credit markets for a stochastic number of periods so that the government gains access to the markets with an exogenous probability of $\psi \in [0,1]$. The government also suffers an income loss of ϕ^d for every period during an exclusion. Upon default, I assume that the government does not have access to new emergency lines and that the government repays all of its outstanding debt obligations accrued from liquidity lines. The assumption of full repayment for the debt through liquidity lines is made because no country has defaulted on liquidity lines arranged by the IMF, and the limit a sovereign can exercise through these lines is relatively small. In practice, a small limit is likely to eliminate a country's incentive to default on these loans and mitigate the moral hazard concerns of line providers. Furthermore, these lines are granted to economies that are financially sound but having liquidity issues. For instance, the Greek default on IMF debt in July 2015 was not arranged through liquidity lines but instead a Stand-By arrangement. Thus, the budget constraint of an economy in a default becomes

$$c_t = y_t - \phi^d(y) - s_t.$$

Global liquidity shock. To capture the essence of the introduction of liquidity lines, I introduce a global liquidity shock g_t. This shock drives both the access to liquidity lines and lenders' risk aversion (described below). Formally, during such an episode, the government suffers an income loss $\phi^g(y)$. The global liquidity shocks are prevalent in the literature, and my assumption of global liquidity shocks leading to output losses is consistent within the empirical and quantitative literature as these periods are often linked to currency and banking crises as well as deep recessions. More details are provided under calibration in Section 3.[4]

One may consider an alternative modeling option for liquidity lines such that whenever the sovereign's current income realization is below a threshold y^*, access to these lines is triggered. Thus, access to liquidity lines can simply be contingent on an income cutoff. However, this type of formulation is problematic and does not guarantee a fixed point as it leads to non-monotonicity in the prices. Non-monotonicity arises because for an endowment realization that is slightly below the triggering threshold y^*, the government's consumption increases following its access to these lines. To see this formally, let c_0 and c_1 denote consumption of the economies with lines such that $c_0 = y^* - b + q(b', s', y)[b' - (1 - \delta b)]$ and $c_1 = y^* - \varepsilon - b + q(b', s', y)[b' - (1 - \delta b)] + \frac{1}{1+r}s'$ with ε>0 such that an income cutoff to access liquidity lines is just met. Without loss of generality, consider that b' is very small so as not to worry about the effect of s' on prices. One can see that $c_1 > c_0$ for s' big enough. To this end, the strict monotonicity of the utility and value functions is not ensured as these functions are not necessarily monotonic with respect to income. Introducing a global liquidity shock is one way to avoid non-monotonicity as a way to quantitatively investigate the effects of introducing liquidity lines during low liquidity periods.

The assumption of costly liquidity crises also intends to capture the cost of local disturbances caused by a difficulty in accessing international credit markets. This assumption

[4] Chari et al. (2005), Caballero and Panageas (2007), and Mendoza (2010) quantitatively study the external shocks and sudden reversals as well as their adverse impact on governments' output. Similar to Hatchondo et al. (2020) and Hatchondo et al. (2019), a global liquidity shock g follows a Markov process such that the probability of a shock π is $\in$ [0,1], and it persists with probability $\psi^g \in$ [0,1].

allows the baseline model to capture that some, but not all, sudden stops prompt defaults. With only a higher risk aversion assumption during a liquidity crisis, one may not be able to generate defaults triggered by a liquidity shock as risk aversion induces discipline on the government's borrowing behavior. Here, I should note that I am abusing the market discipline term as its definition changes with risk-averse lenders. In the baseline scenario, it refers to the fact that the government needs to pay back all of the liquidity line when the crisis episode ends and this makes the government behave ex-ante. With risk-averse lenders, however, it refers to the fact that default risk is much more expensive and the government optimally chooses to reduce its debt balances to lower the expected default risk (Aguiar et al. (2016)). Thus, two disciplining devices arise: (i) temporariness of liquidity lines and, (ii) risk-averse lenders. I drop the income cost of liquidity shocks and explore the model dynamics in Section 4.4.

I resort to using the term "global" liquidity shocks instead of "local" liquidity shocks as several variables could be used as a measure of a global liquidity crisis, such as a jump in VIX, CISS (as proposed by Hollo et al. (2012)), or a one standard deviation decline in capital inflows (as in Calvo et al. (2008) and Eichengreen and Gupta (2016)), to trigger the inception of liquidity lines. It is not immediate to determine what these variables would be for local liquidity shocks. Besides, changes in local liquidity conditions can be driven by political factors, which may be deemed ineligible for the inception of these lines.

Lenders' risk aversion. Global liquidity shocks may also amplify lenders' risk aversion so that lenders might ask for higher yields during such episodes. Several studies document that volatility observed in the data is often associated with global factors (see Calvo and Reinhart (2014) and Verdelhan and Borri (2010)). Aguiar et al. (2016) also document that sovereign risk premia are not solely determined by default risk, so they model risk-averse lenders by explicitly outlining their portfolio problem. Following Hatchondo et al. (2016) and Arellano and Ramanarayanan (2012), the bond price is determined by the no-arbitrage condition with the stochastic discount factor $\mathcal{G}(y', y, g) = exp(-r - g\left[\alpha\epsilon' + 0.5\alpha^2\sigma_\epsilon^2\right])$, where α denotes the risk aversion parameter for lenders, g is a binary variable with which $g = 1$ indicates that an economy is hit by a global liquidity shock, and $g = 0$ indicates otherwise. This specification states that lenders ask for a higher sovereign risk premium during a liquidity shock and lenders are risk-neutral when the economy is not in such a state (as assumed in the majority of sovereign default studies). This formulation has advantages for replicating the spreads observed in the data during a liquidity shock without increasing the number of state variables, and it is documented that this additional risk premium offered by the above specification accounts for the significant spread volatility in the data (see Verdelhan and Borri (2010)).

2.3 Recursive Formulation

Let $\bar{s}$ be the limit to the number of assets that can be issued through swap lines during a liquidity shock, and let v denote the value function of a government that is not currently in default. For any price function q, the function v satisfies the following functional equation:

$$v(b,s,y,g) = \max\left\{v_R(b,s,y,g), v_D(s,y,g)\right\}, \tag{2.3}$$

where the government's value of repaying is given by

$$v_R(b,s,y,g) = \max_{b'\geq 0,\, s'\geq 0}\left\{u\left(c\right) + \beta\mathbb{E}_{y',g'|y,g} v(b',s',y',g')\right\}, \tag{2.4}$$

subject to

$$c = y - g\phi^g(y) - s + g\frac{1}{1+r}s' - b + q(b',s',y,g)\left[b' - (1-\delta)b\right],$$

$$s' \leq \bar{s}.$$

Recall that g is a binary variable and becomes unity if the economy is hit by a global liquidity shock during which the economy can access liquidity lines and borrow $\frac{1}{1+r}s'$ units of goods to be repaid as s' units in the next period.

The value of default v_D does not depend on the long-term debt level as the government does not make any payments to private creditors when it defaults and regains access to credit markets with a zero recovery rate. However, the value of default depends on the amount of borrowing that is accrued through lines since the government has to honor all these obligations. It also follows that the government cannot borrow in an additional emergency facility if it declares default. Thus, the value of defaulting is given by

$$v_D(s,y,g) = u\left(c\right) + \beta\mathbb{E}_{y',g'|y,g}\left[(1-\psi)v_D(0,y',g') + \psi v(0,0,y',g')\right], \tag{2.5}$$

subject to

$$c = y - \phi^d(y) - s.$$

The solution to the government's problem implies a default decision rule $\hat{d}(b,s,y,g) \in \{0,1\}$, 1 if the government defaults, and 0 otherwise; borrowing rules through swap lines $\hat{s}$ and long term debt $\hat{b}$. In equilibrium, defined in Section 2.4, lenders use these decision rules to price contracts. With the zero-profit assumption, the price function q solves the following functional equation:

$$q(b',s',y,g) = \mathbb{E}_{y',g'|y,g}\left[\mathcal{G}\left(y',y,g\right)\left(1-\hat{d}(b',s',y',g'\right)\left[1+(1-\delta)q\left(b'',s'',y',g'\right)\right]\right] \quad (2.6)$$

where

$$\begin{aligned} b'' &= \hat{b}(b',s',y',g'), \\ s'' &= \hat{s}(b',s',y',g'). \end{aligned}$$

Equation (2.6) indicates that the value of selling a bond today (left-hand side of the equation) has to be equal to the expected value of keeping the bond (right-hand side of the equation) in equilibrium. If a lender keeps the bond and the government does not default next period ($\hat{d}(b',s',y',g') = 0$), she first receives a one-unit coupon payment and then sells the bond at the market price, which is equal to $(1-\delta)$ times the price of a bond issued next period ($q\left(b'',s'',y',g'\right)$), both of which are discounted by $\mathcal{G}$. Recall that through the function $\mathcal{G}\left(y',y,g\right)$, lenders discount expected flows at a higher rate when a government is hit by a liquidity shock.

2.4 Equilibrium

This paper focuses on the Markov perfect equilibrium (MPE) concept wherein the government's equilibrium default and borrowing decisions for long-term debt as well as the utilization of liquidity lines depend only on payoff relevant state variables.

Definition 1. *A Markov perfect equilibrium is characterized by*

1. *a collection of value functions v, v_R, and v_D;*
2. *default rule $\hat{d}$ and borrowing rules $\hat{s}$ and $\hat{b}$;*
3. *and a bond pricing functional q;*

such that:

i. *given a price function q, the value functions $\{v, v_R, v_D\}$ and policy functions $\left\{\hat{d}, \hat{b}, \hat{s}\right\}$ solve the dynamic programming problem characterized by (2.3), (2.4), and (2.5).*

ii. *given policy rules $\left\{\hat{d}, \hat{b}, \hat{s}\right\}$, the pricing functional q satisfies condition (2.6).*

3 Calibration

The model economy is calibrated at a quarterly frequency. For most parameters, I use conventional parameter values that are reported in the quantitative business cycle and sovereign default studies. For the remaining ones, I resort to the data. The baseline model is calibrated to replicate the business cycle and debt statistics characteristics of Mexico, a common representative economy for emerging market studies. Table 1 summarizes the parameters used in the paper.

Following Arellano (2008), this paper assumes that the cost of defaulting increases more than proportionally with income. Mendoza and Yue (2009) present a model in which endogenous default costs are larger during high income episodes. As in Chatterjee and Eyigungor (2012), I assume a quadratic income loss function during a default episode $\phi^d(y) = d_0 y + d_1 y^2$. As the quadratic loss function ensures that the cost of defaulting is higher during good times, it is also natural to assume that the cost of liquidity shocks is likely to be higher during upturns. Thus, I assume that the income loss generated by a liquidity shock is equal to a fraction of the loss of output, that is, $\phi^g = \omega\phi^d$. With that assumption, I need only one more parameter to determine the cost of liquidity shocks. I target an average accumulated income loss of 12.8 percent during a liquidity shock, following Calvo and Reinhart (2000), as they document an income decline of 13.3 (12.3) percent for countries that experienced banking (currency) crises. This number is also similar to the estimates of Calvo et al. (2008) and Jeanne and Ranciere (2011). By setting $\omega = 0.3$, I obtain an average decline in income of 12.6 percent during a liquidity crisis.

The parameter capturing the probability of regaining access after a default episode, ψ, is set at 0.282, following Arellano (2008) and a number of other studies. The probability of entering a liquidity shock π and the probability of remaining in a global liquidity shock ψ^g are obtained from existing studies. Jeanne and Ranciere (2011) document the annual probability of having a sudden stop shock to be 10 percent and Calvo et al. (2008) present that it may take up to two years to return to normal after a sudden stop shock. Eichengreen and Gupta (2016) report that a sudden stop episode lasts an average of 4.5 quarters. Thus, I set $\pi = 0.025$ and $\psi^g = 0.75$ accordingly. With these values, we obtain one episode of a global liquidity shock every 10 years, with an average duration of each shock lasting 4.6 quarters in simulations.

The stochastic process of Mexico's GDP is estimated from the period of 1980Q1-2014Q3 using the data obtained from the Federal Reserve Bank of St. Louis. The estimated parameters from the stochastic process are similar if different end periods such as 2007Q1 are used, prior to the inception of liquidity lines. The cyclical component of GDP from the estimated AR(1) process yields $\rho = 0.94$ and $\sigma_\epsilon = 0.15$. For duration, δ is set to 0.033, which delivers an average duration of 5 years in simulations, consistent with emerging market economies' sovereign debt duration documented by Cruces et al. (2002). As is standard in the long-term sovereign debt literature (see Hatchondo and Martinez (2009)), the Macaulay definition of duration is used in this paper and is given by $D = \frac{1+r^*}{\delta+r^*}$ where r^* denotes the periodic yield delivered by the bond. Data for spreads are obtained using the quarterly Emerging Market Bond Index (EMBI) spreads to determine spreads during a liquidity shock for which I have EMBI spread data, I computed the historical average of VIX and noted the quarters that are two standard deviations above their historical mean. One can identify two episodes of high VIX episodes across 1995-2015: the period 1998-2000 (debt crisis in emerging markets) and the 2009 global financial crisis.[5] The average EMBI spread is 2 percentage points higher in those episodes than in normal periods. This is also consistent with the moment reported in Bianchi et al. (2018).

The parameters d_0 and d_1 denote the income cost of defaulting and are used to match the equilibrium mean levels of sovereign debt and spreads. To match the moments, I set d_0 and d_1 at -0.49 and 0.845, respectively. In simulations, I obtain a mean debt-to-income ratio of 42.6 percent and a mean spread of 3.9 percent (1.9 percent) when the economy is (is not) in a low liquidity state.[6]

At the time of the liquidity arrangement that Mexican Central Bank had with the Federal Reserve, or the subsequent arrangement of Mexican government with the IMF, the cap that Mexico could draw through these lines was equivalent to 3 percent of its GDP. Thus, the limit to the number of assets that can be issued through liquidity lines, $\bar{s}$, is set to be 0.12 (3

[5] VIX misses the 1994-1995 Tequila crisis.

[6] To compute the sovereign spreads in the bond price, I first compute the yield on a bond. To calculate the yield on long-term bonds in simulations, it is defined as the discounted value of coupons given that the government does not default, and the coupons are held until maturity (infinity in this case). That is, given price q, the yield r^* satisfies

$$q = \sum_{n=1}^{\infty} \frac{(1-\delta)^{n-1}}{(1+r^*)^n}.$$

So,

$$r^* = \frac{1}{q} - \delta.$$

The sovereign spread is defined as the difference between the yield r^* on long-term bonds and the default-free rate r and can be calculated as

$$r_s = \left(\frac{1+r^*}{1+r}\right)^4 - 1,$$

where r_s is the annualized spread reported in the tables. Debt levels reported in simulations are calculated as the present value of future debt obligations discounted at the default-free rate r, that is, $\frac{b'}{\delta+r}$.

percent $\times$ 4 $\times$ mean quarterly income, which is normalized to one). Table A.2 summarizes the Mexican experience.

Table 1: Parameter values used in the paper

	Symbol	Value	Description
Discount factor	β	0.973	Literature
Risk aversion of households	γ	2	Literature
Income autocorrelation coefficient	ρ	0.94	Estimated
Standard deviation of innovations	σ_ϵ	0.015	Estimated
Mean log income	μ	$(-1/2)\sigma_\epsilon^2$	
Risk-free rate	r	0.01	Literature
Debt duration	δ	0.033	Debt maturity = 5 years
Liquidity debt limit	$\bar{s}$	0.12	Swap limit arranged by Mexico
Probability of re-entry after default	ψ	0.282	Literature
Probability of a global liquidity shock	π	0.025	Calvo et al. (2008); Jeanne and Ranciere (2011)
Prob of a remaining in global liq. shock	ψ^g	0.75	Calvo et al. (2008); Eichengreen and Gupta (2016)
Jointly calibrated parameters			
Income cost of defaulting	d_0	-0.49	Mean debt/GDP = 42.2 percent
Income cost of defaulting	d_1	0.845	Mean EMBI spread = 2.3 percent
Income cost of liquidity shocks	ω	0.3	Income decline = 12.3-13.3 percent
Risk premium	α	15	Spread increase with liq. shock = 2 percent

3.1 Computation Algorithm

The numerical algorithm requires iterating on value functions v_R and v_D and price functional q until convergence is obtained. I solve for the equilibrium of the finite-horizon version of the model economy; that is, the interpolated value and price functions of the first and second period are sufficiently close (smaller than 10^{-5}) after the iterations. I then use the first-period equilibrium functions as the infinite-horizon-economy equilibrium functions. For the long-term debt holdings and income realizations, I use 21 grid points, for liquidity line holdings 5 grid points, and to calculate the expectations 50 Gauss-Legendre quadrature points are used. Details of the algorithm are delegated to the Appendix.

4 Simulation Results

I initially solve the baseline model by ruling out liquidity lines ($\bar{s} = 0$) while keeping liquidity shocks in place. Then I introduce liquidity lines to the baseline economy; that is, during a global liquidity shock, a government can borrow up to 3 percent of mean annual income if the government is not currently in default.

Table 2 presents debt and long-run business cycle moments obtained from model simulations of the baseline and liquidity line economies. The last two columns show that the baseline economy matches the target moments with reasonable accuracy. Column 1 of panel A shows that when the model is extended to account for liquidity lines, the borrowing costs with liquidity lines are significantly reduced. In particular, the average spreads fall from 3.9 to 2.7 percent during low liquidity periods. The table also shows that during a liquidity shock, the government utilizes this extra source of funding to the 3 percent limit.

Panel B displays that the baseline economy matches the excess volatility of consumption relative to income observed in the data and that consumption volatility falls with liquidity lines as the government can better smooth its consumption during low liquidity periods. To see how liquidity lines affect default dynamics, Table 3 is provided. The table shows that the long-run total number of defaults in both economies remains unchanged, but the timing of default dynamics does change.

Table 2: Long-run statistics: Effects of introducing liquidity lines

	(1)	(2)	(3)
	Liquidity Lines	Baseline	Data
Panel A: Debt Statistics			
Mean debt-to-GDP ratio (in percent)	44.4	42.6	42.2
Mean r_s during a global liq. shock	2.7	3.9	4.0
Mean r_s excluding a global liq. shock	1.9	1.9	2.3
$\sigma(r_s)$	0.5	0.7	0.8
Average debt duration (in years)	5.2	5.3	5.0
Mean liquidity shock inc. cost (in percent)	12.6	12.6	12.8
Duration of the liquidity shock (in quarters)	4.6	4.6	4.5
Mean liq. lines to GDP (in percent)	0.3	*n.a.*	0.3
Mean liq. lines to GDP during a liq. shock (in percent)	3	*n.a.*	3
Panel B: Business Cycle Statistics			
$\sigma(c)/\sigma(y)$	1.0	1.1	1.1
$\sigma(tb)$	1.3	0.8	1.4
$\rho(c,y)$	0.9	0.9	0.9
$\rho(r_s,tb)$	0.4	0.6	0.6
$\rho(r_s,y)$	-0.7	-0.8	-0.5

The Standard deviation of a variable is denoted by σ, and the coefficient correlation between variables is denoted by ρ. Consumption and income are reported by natural logs. Details on how the moments are obtained are provided under the numerical solution algorithm in Appendix B.

The second row in Table 3, number of defaults triggered by a liquidity crisis, is computed as the total number of defaults that would not have taken place had the government not hit by a liquidity shock, given that the last period was free of a liquidity crisis. The third row in the table, number of defaults occurred in a liquidity crisis, is computed as the total number of defaults during a liquidity crisis had the government not defaulted within the first period of receiving the liquidity shock (remember that liquidity shocks are persistent). Now observe that if the economy with liquidity lines defaults in a low liquidity period, it does so within the first period during which it is hit by a liquidity shock. Thus, the government prefers defaulting to using liquidity lines. This actually happens only when the income shock is severe. Even though liquidity lines may smooth consumption in the first period, the marginal benefit of utilizing the lines is limited with severe income shocks as income shocks follow an AR(1) process. That is, even though the economy may avoid defaulting in the first period, the government is likely to draw another severe income shock in the next period. This time, the government can roll over the lines only if it is also in a low liquidity period. Thus, defaulting in such a situation would be costlier as the government is required to repay its dues from liquidity lines. This explains why the government is generally either defaulting the period of the liquidity shock or not defaulting at all during the low-liquidity regime. In the baseline economy, 67 percent (0.6/0.9) of defaults occur in the first period of a liquidity crisis, while 33 percent (0.3/0.9) of defaults happen during a liquidity crisis because of the persistent nature of the liquidity shock.

The same number of total default incidences with liquidity lines can be attributed to overborrowing. The first time the government is introduced with these lines, the default risk per one unit of debt declines. The economy with liquidity lines then starts issuing higher debt by taking advantage of lowered interest rates. I explore more of this phenomenon when discussing how liquidity lines affect prices. Turning again to Table 3, we can see that liquidity lines can alleviate 75 percent (0.6/0.8) of liquidity-related defaults the first time they are arranged. Yet, as the economy starts overborrowing, this benefit fades away. This also explains why the second row in Table 3 is higher in an economy with liquidity lines.

The impact of liquidity lines on bond prices. Figure 1 presents the government's equilibrium pricing schedule in an economy with and without liquidity lines when the government is in a low liquidity period (left panel) and when it is in a normal period (right panel). The figure plots the equilibrium price functions for two different states of the world: high and low income correspond to income realizations that are two standard deviations above and below the mean income, respectively. In the left panel, the cost of borrowing is cheaper in an economy with liquidity lines because the probability of defaulting is smaller for a

Table 3: Long-run default statistics

	(1) Liquidity Lines	(2) Baseline
Number of defaults in 100 years	0.9	0.9
Number of defaults triggered by liq. crisis	0.8	0.6
Number of defaults that occurred in a liq. crisis	0	0.3
Number of liquidity-triggered defaults that could have been avoided	0.6	

Column 1 reports default frequencies that are obtained in the economy with liquidity lines, while column 2 reports default frequencies that are obtained in the economy without liquidity lines.

given level of income and debt doubles (the interest paid on the debt is inversely related to the price). As in the left panel, the prices under the economy with liquidity lines remain higher than that of its no liquidity line counterpart in the right panel because default risk depends on the government's future default probabilities under long-term debt.

The impact of global liquidity shocks on spreads is also visible in Figure 2. Each panel presents the equilibrium spread levels for 300 samples of 100 periods for each income realization of the economies with and without liquidity lines. Each dot shows a single simulation outcome. The panels display two clusters of dots. The blue cluster on the left side of the panels shows spreads during a liquidity shock, and the red-orange cluster shows spreads without a liquidity shock. With a visual investigation, both the level and volatility of the spread are lower in the left panel, which plots the simulation outcomes of an economy that has access to liquidity lines.

4.1 Liquidity Lines and Debt Crises

This section provides a quantitative account of how liquidity lines might play a pivotal role in maintaining debt sustainability in EMEs who face a liquidity shock. The analysis entails feeding into the baseline and liquidity line model economies the same hypothetical path of income and liquidity shocks and analyzes how the endogenous variables of the model compare in the two specifications.

The left panel of Figure 3 visualizes the time-series paths for defaultable debt as a percentage of income for the baseline economy (dashed black line), liquidity lines economy (solid blue line) and percent deviation from trend income on the right axis (yellow dashed-dotted line) for 100 quarters. The right panel plots spreads for both economies. The gray area

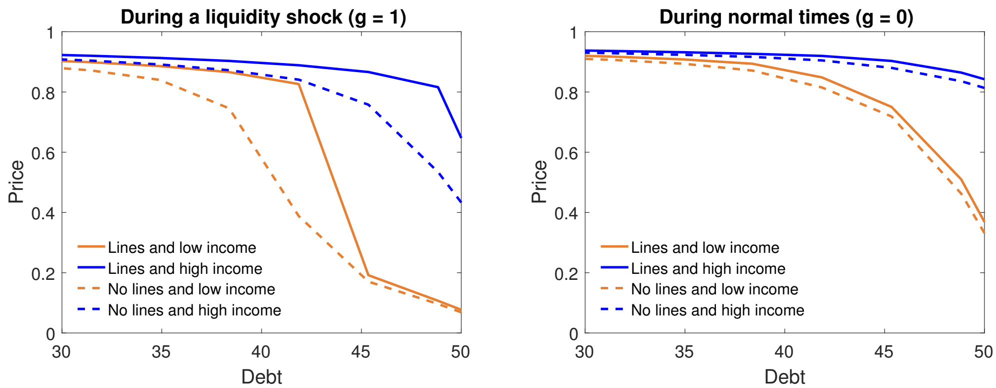


Figure 1: The panels show equilibrium price functions with and without a liquidity shock for two different states of the world: high and low income correspond to the income levels that are two standard deviations above and below the mean income, respectively, the x-axis denotes the level of debt to annual mean income, which is normalized to one. In all cases, the liquidity line utilization ($s = 0$) is zero. The left panel plots pricing functions when the government faces a liquidity shock, and the right panel plots the price function when a government is free from liquidity shocks.

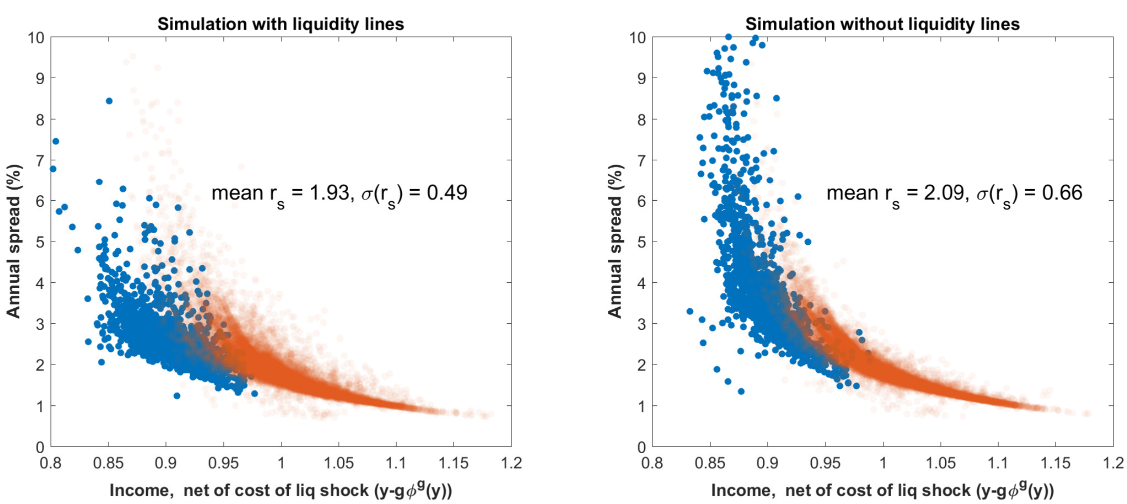


Figure 2: Each panel shows income realizations and corresponding spread observations in simulations. Both panels include 300 samples of 100 periods before a default. Blue dots are obtained during the periods of low liquidity, while red-orange dots are obtained during normal times. The left (right) panel presents the simulation outcomes with (without) liquidity lines.

denotes the low liquidity periods. A few observations stand out. The economy with liquidity lines, with an unanticipated announcement explaining that the government can now access liquidity lines, avoids defaulting while the baseline economy defaults. Our baseline figure thus plots the time series had the government not defaulted. I plot "no lines economy had it not defaulted" so that one can compare how the introduction of a liquidity line leads to different time-series paths. The left panel shows that the economy with liquidity lines deleverages defaultable debt as $b' - (1-\delta)b$ is negative at the time of stress with the availability of a new source of financing and then quickly builds it up. During this time, the government immediately taps into its liquidity lines to the limit and keeps rolling over its liquidity lines during the crisis episode. Along the time-series path, the debt ratio of the economy with liquidity lines remains higher than that in the baseline economy.

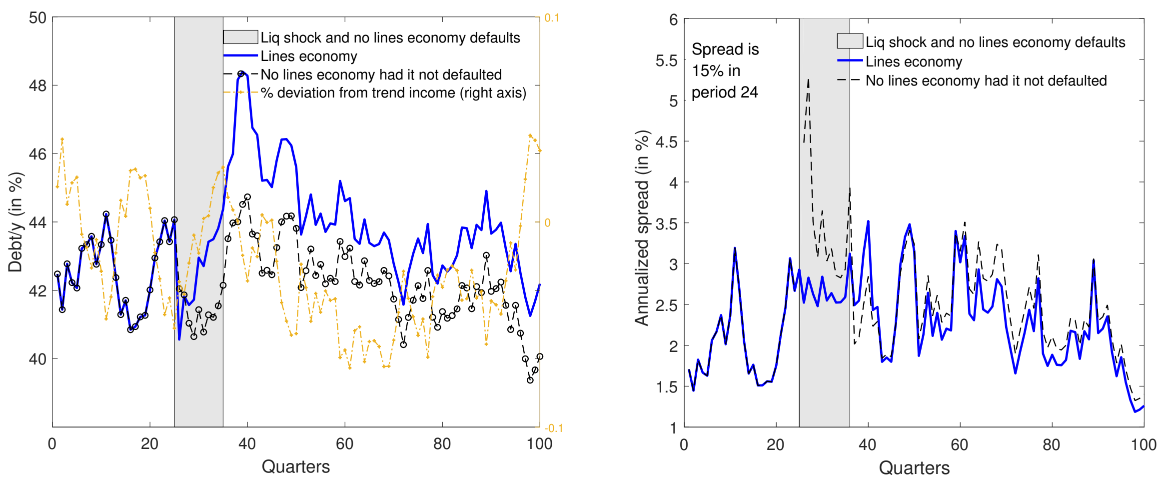


Figure 3: The economy is unexpectedly introduced with liquidity lines when it is hit by a liquidity shock (shaded gray area). The baseline economy (the economy without liquidity lines) defaults in period 24 when it is hit by a liquidity shock. However, the economy without liquidity lines avoids default. Thus, the baseline economy shows how the debt-to-income ratio (left panel) and spreads (right panel) would have evolved had it not defaulted with the shock. The spread in the baseline economy jumps in period 24. Had the baseline economy not defaulted, the spread would have been 15 percent.

The right panel deserves some attention as it reveals some important dynamics.[7] The economy with liquidity lines displays a relatively stable spread path during the low liquidity period. However, when the duration of the liquidity shock is over, the spread shoots over the baseline economy for 6 quarters and then remains below the baseline

[7] The first spread observation is not plotted as it corrupts the image with its 15 percent value.

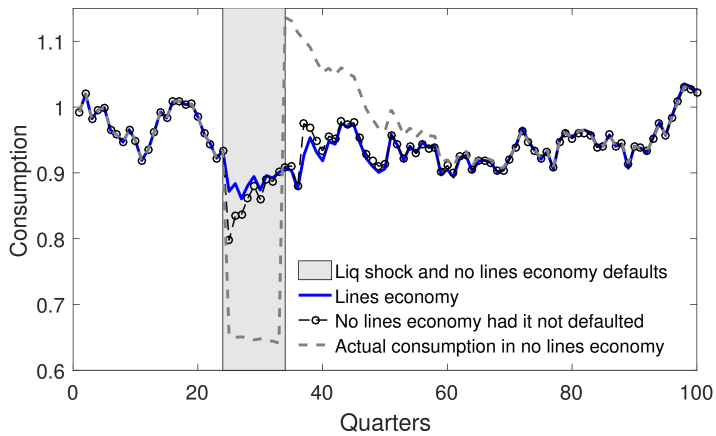


Figure 4: The economy is unexpectedly introduced with liquidity lines when it is hit by a liquidity shock (shaded gray area). The baseline economy (the economy without liquidity lines) defaults around period 24, when it is hit by a liquidity shock. However, the economy without liquidity lines avoids the default.

economy again. This result stems from the fact that the government cannot roll over its liquidity lines and has to repay the debt in full as the liquidity crisis is over. Besides, the economy also accumulates higher debt than the baseline economy during the low liquidity period. Thus, the economy with liquidity lines has fewer resources available to repay after exiting the crisis. This mechanism, the fact that the government will not be able to roll over liquidity lines, does in fact discipline the government to not build up excessive debt so as to be able to repay the defaultable asset once the liquidity crisis is over.

Figure 4 plots the-time series evolution of consumption for three economies, adding an economy that defaults with the liquidity shock (dashed gray line). It is noticeable that liquidity lines play a pivotal role in consumption smoothing during liquidity shocks. Actual consumption in the baseline economy is subject to the income cost of defaulting during exclusion and starts with zero debt upon regaining access. Emergency lines, in essence, transfer resources to periods of low liquidity. Thus, liquidity lines can achieve a significant reduction in consumption volatility, as shown in Figure 4, which is consistent with the decline in consumption volatility presented in Table 2. Now it remains to explore how the long-run dynamics evolve with liquidity lines.

Figure 5 plots the debt and spread dynamics 500 quarters after the initiation of liquidity lines so that the long-run averages for all studied economies can be attained. I also include a liquidity line economy with a 10 percent line limit, as in Hatchondo et al. (2017), to improve the understanding of model dynamics. When liquidity lines are arranged to be permanently accessible, as in Hatchondo et al. (2017), I obtain a perfect overlap with the economy with no liquidity lines in Figure 5. I further explore why permanent arrangements fail featuring long-run effects in Section 4.3. Turning to Figure 5, we see that the economy with liquidity lines with a 10 percent limit displays a slightly higher defaultable debt-to-mean annual income (normalized to one) ratio relative to the economy with liquidity lines with 3 percent limit during normal times. Both liquidity line economies initially rely on the resources provided by liquidity lines while deleveraging defaultable debt during a liquidity crisis (shaded regions). The defaultable debt levels of the economy with 10 percent liquidity line limit, however, remain considerably lower relative to other economies, which can be attributed to the temporary nature of the lines. The government will not be able to roll over the lines and will now have fewer resources to pay for defaultable debt at the time of exit. This mechanism can also explain why the 10 percent limit economy only borrows slightly more than the 3 percent limit economy during normal times, given that the 3 percent limit economy borrows significantly more than the baseline economy. Figure 5 plots the debt and spread dynamics 500 quarters after the initiation of liquidity lines so that the long-run averages for all studied economies can be attained. I also include a liquidity line economy with a 10 percent line limit, as in Hatchondo et al. (2017), to improve the understanding of model dynamics. When liquidity lines are arranged to be permanently accessible, as in Hatchondo et al. (2017), I obtain a perfect overlap with the economy with no liquidity lines in Figure 5. I further explore why permanent arrangements fail featuring long-run effects in Section 4.3. Turning to Figure 5, we see that the economy with liquidity lines with a 10 percent limit displays a slightly higher defaultable debt-to-mean annual income (normalized to one) ratio relative to the economy with liquidity lines with 3 percent limit during normal times. Both liquidity line economies initially rely on the resources provided by liquidity lines while deleveraging defaultable debt during a liquidity crisis (shaded regions). The defaultable debt levels of the economy with 10 percent liquidity line limit, however, remain considerably lower relative to other economies, which can be attributed to the temporary nature of the lines. The government will not be able to roll over the lines and will now have fewer resources to pay for defaultable debt at the time of exit. This mechanism can also explain why the 10 percent limit economy only borrows slightly more than the 3 percent limit economy during normal times, given that the 3 percent limit economy borrows significantly more than the baseline economy.

The right panel of Figure 5 displays that spreads are significantly lower in economies with liquidity lines during low liquidity periods as the economies are now less likely to default. The average spread for the 10 percent limit economy during normal (low-liquidity) periods is 1.76 percent (1.92 percent), and the average spread volatility across the sample period is

0.44.

Notice that the baseline economy defaults around the 105^{th} quarter. This default behavior is accounted for by the third row in Table 3, the number of defaults that occurred in a liquidity crisis. It shows that the baseline economy skipped defaulting the first time it experienced the shock but then defaulted with the persistence of liquidity shocks. Had the economy liquidity lines defaulted, it would have defaulted the first period it received the shock before utilizing the lines.

Figure 6 plots the equilibrium response of consumption for all of the three studied economies.[8] The solid (dashed) lines represent the economies under low (normal) liquidity periods. The initial stock of defaultable debt holdings is set to the ergodic level of the defaultable debt stock of each economy. This figure reveals that the economies with liquidity lines transfer resources from normal times to low liquidity periods. Liquidity lines provide better consumption smoothing during low liquidity periods. It is evident that consumption during low liquidity periods is consistently higher relative to the baseline economy. However, particularly during lower income periods, consumption levels are slightly lower in normal times for economies with liquidity lines because of overborrowing.

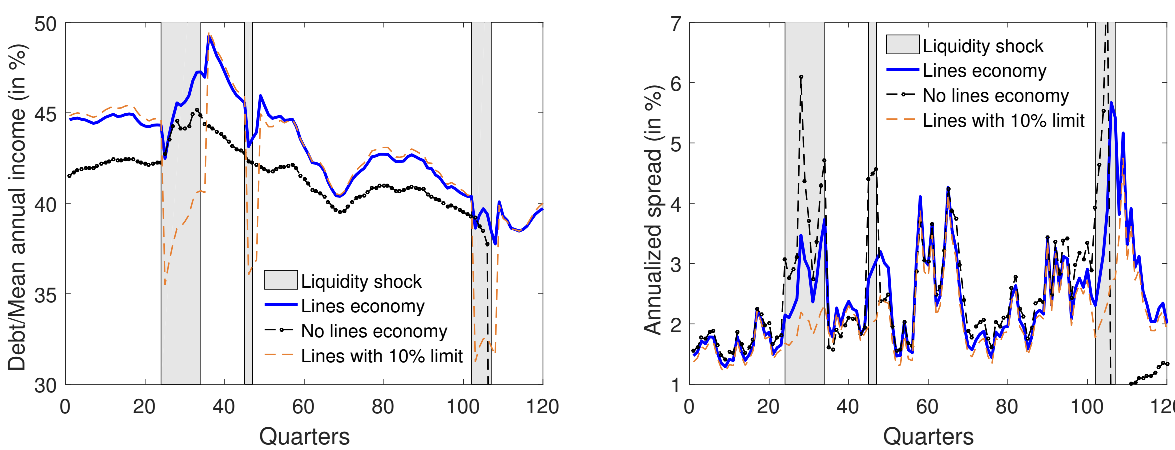


Figure 5: Evolution of debt and spread dynamics in the baseline economy and in economies with liquidity lines 3 percent and 10 percent limits. The first period in the figure corresponds to the 501^{st} quarter after the initiation of liquidity lines. Mean annual income is normalized to one.

[8] Plotting the time-series evolution of consumption does not provide a clean picture. It is provided in Figure B.1.

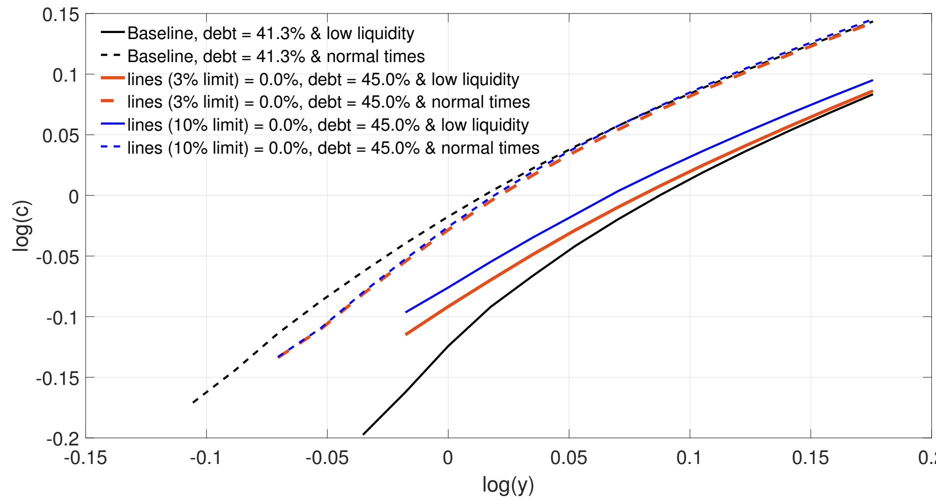


Figure 6: Response of consumption to a liquidity shock in the baseline economy, and in economies with liquidity lines with 3 percent and 10 percent limits. The initial stock of defaultable debt holdings is set to the ergodic level of the defaultable debt stock of each economy.

4.2 Transition Dynamics

In this section, I analyze an unanticipated announcement such that from now on, the government would have access to liquidity lines during a global liquidity shock. In this way, I can explore how long it takes for the liquidity line economy to attain its long-run average. To be specific, the constraints on the amount of debt a government can borrow through liquidity lines change from $\bar{s} = 0$ to $\bar{s}$ = 0.12 when $g = 1$.

First, I present the transitional dynamics when the government is in a global liquidity shock. Then I show the transitional dynamics in which liquidity lines are established when the government is not hit by a liquidity shock.

Having access to a new source of funding during times of stress is key to avoiding significant borrowing costs and default. As shown in Table 4 and Figure 7, liquidity lines provide immediate relief and significantly reduce borrowing costs when the economy faces capital flight. The table presents results with three different income levels before and after the introduction of liquidity lines. For all three cases, the initial debt level is equal to 42.6 percent of mean annual income. High and low income cases correspond to the income levels of two and a half standard deviations above and below the mean, respectively. Since the impact of introducing liquidity lines presented in Table 4 is calculated before any of the government's borrowing action, these outcomes represent lenders' expectations of the government's future optimal decision rules. Importantly, given that a government issues long-term debt, the current spread levels depend on the current and future coupon payments as well as on future default and borrowing rules.[9]

Table 4: Effects of introducing liquidity lines during a global liquidity shock

	(1) Low income	(2) Mean income	(3) High income
Spread before the intro of liq. lines (in %)	7.2	3.4	2.4
Spread after the intro of liq. lines (in %)	2.6	2.0	1.7

High and low income columns correspond to the income levels of two and a half standard deviations above and below the mean income, respectively.

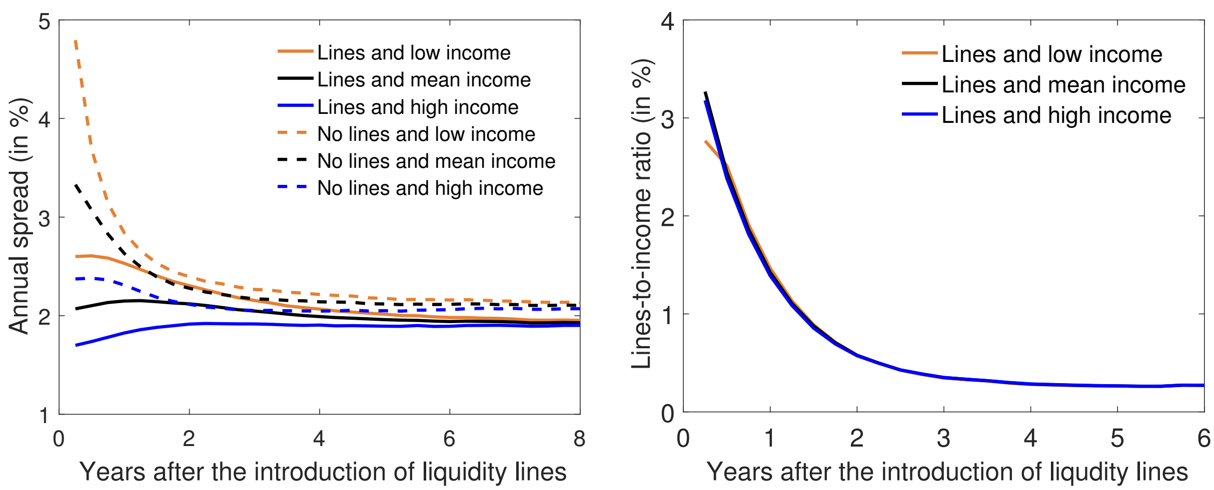


Figure 7: The left panel shows the sovereign spread with and without liquidity lines for samples without defaults. The right panel represents the mean line-to-income ratio ($\frac{b'/(\delta+r)}{4y}$) during transitions following the introduction of liquidity lines. Solid lines represent the means of transition paths for an economy with liquidity lines, and dashed lines represent the means of transition paths for economies without liquidity lines. Both panels represent the transitional dynamics for which the liquidity lines are established during a global liquidity shock.

Figure 7 shows the impact of introducing liquidity lines when these lines are established during a global liquidity shock in all three different states of the world. The left panel shows that spreads drop substantially because the establishment of liquidity lines reduces the default risk of defaultable debt. The right panel shows that as the government gains

[9]Moments in Table 4 are computed with their corresponding initial income states before the government's borrowing, while Figure 7 plots debt and spread dynamics after the introduction of policy and the government's borrowing. Also, the first-period income level in the figure is computed according to the income process described in Section 2.2. This is to explain why the spreads before and after the introduction of liquidity lines in Table 4 do not necessarily coincide with the first period of Figure 7.

access to liquidity lines during a liquidity shock, the government starts utilizing these lines. As the persistence of the liquidity shock fades away, the utilization rate of liquidity lines moves to its long-run mean. On average, it takes around seven years for the government to attain its long-run spread levels.

Table 5: Effects of introducing liquidity lines during normal times

	(1) Low income	(2) Mean income	(3) High income
Spread before the intro of liq. lines (in %)	2.5	1.8	1.5
Spread after the intro of liq. lines (in %)	2.0	1.5	1.2

High and low income columns correspond to the income levels of two and a half standard deviations above and below the mean income, respectively.

Now I present the outcomes for an unanticipated introduction of liquidity lines when the economy is not hit by a liquidity shock ($g = 0$). Table 5 shows that the introduction of liquidity lines produces an immediate reduction in the sovereign spread, and this decline is bigger when default risk is higher among the three different states of the world. Since the government does not have access to liquidity lines at the time of the arrangement, the fall in borrowing costs represents the government's ability to engineer liquidity lines when it is hit by a liquidity shock. Thus, it is important to model the economy with long-term debt.

Figure 8 depicts an immediate decline in spreads for all three different income levels right after the introduction of liquidity lines. Such a fall in borrowing costs induces the government to borrow more, and the government ends up with a higher debt-to-income ratio in the long run. Spreads start rising as the government borrows more. However, the long-term spreads still stay lower than the economy without liquidity lines. I plot the same figures with a 10 percent liquidity line limit in Figure B.2 and shows that the long-run decline in spreads is steeper with a higher limit.

To gauge the impact of this policy change on welfare, Figures 9 and 10 present the welfare gains from introducing liquidity lines. Welfare gains are measured as the constant proportional change in consumption that would leave a consumer indifferent between living in the economy without liquidity lines and living in the economy with liquidity lines. This consumption change is given by

$$\left(\frac{v_S(b,0,y,g)}{v_N(b,y,g)}\right)^{\frac{1}{1-\gamma}} - 1,$$

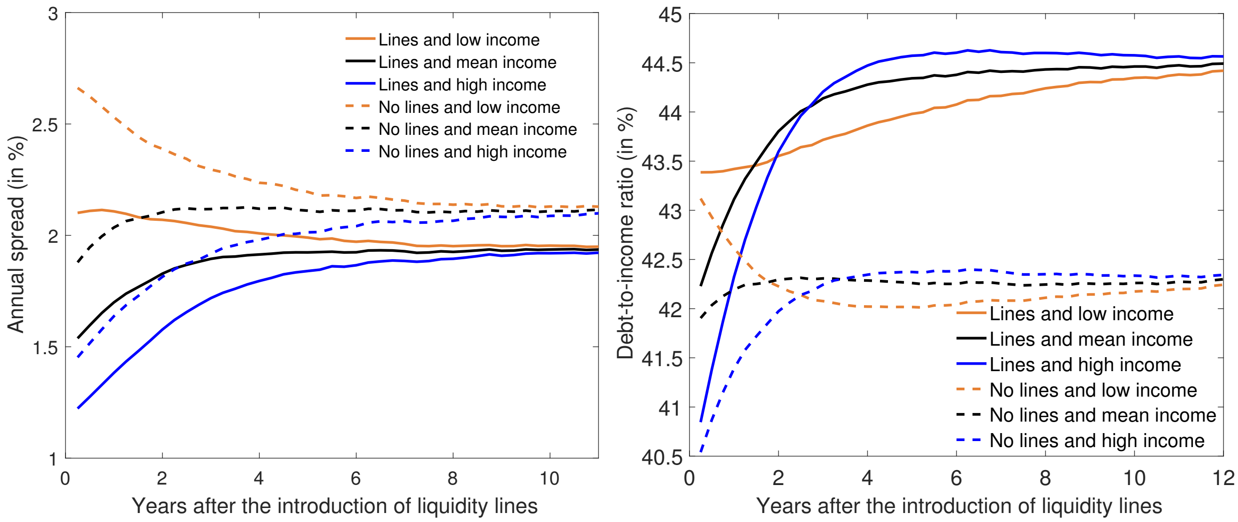


Figure 8: The left panel shows the sovereign spread with and without liquidity lines for samples without defaults. The right panel represents the mean debt-to-income ratio ($\frac{b'/(\delta+r)}{4y}$) during transitions following the introduction of liquidity lines. Solid lines represent the means of transition paths for an economy with liquidity lines, and dashed lines represent the means of transition paths for economies without liquidity lines.

where v_S and v_N denote the value functions with and without liquidity lines, respectively. Thus, a positive welfare gain means that agents prefer the economy with liquidity lines.

Figure 9 is plotted when the economy is in a low liquidity period, while Figure 10 is plotted when the economy is free from a liquidity shock. The initial stock of defaultable debt holdings is set to the ergodic level of the defaultable debt stock in the baseline economy. The figures show that the gains can be substantial. The right panels of the figures highlight the source of the welfare gains for the mean output level. These charts are in line with Figure 1. In particular, thick dots on these price charts show that the economy with liquidity lines issues higher amounts of debt at a lower price, which generates a consumption hike,, and thus, an increase in welfare gains.

We observe lower welfare gains for increasing income levels as the likelihood of defaulting declines, which reduces the price differentials between the liquidity lines and baseline economies. These price differentials are visually noticeable in the left panel of Figure 1 as well. The discontinuity in welfare gains arises because the government defaults below a certain income threshold for a given portfolio allocation and a liquidity condition.

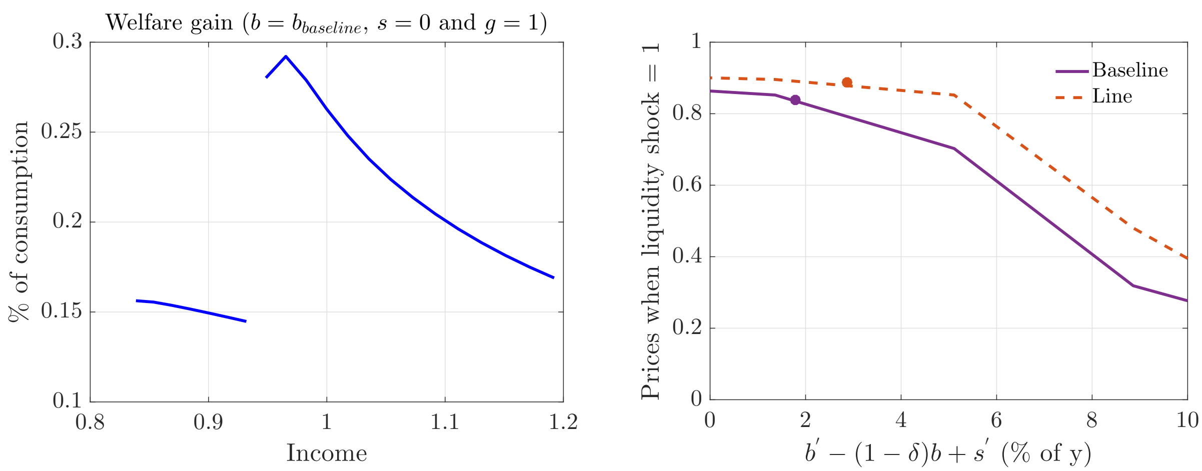


Figure 9: The left panel plots welfare gains measured in consumption-equivalent terms from the inception of liquidity lines. The initial debt portfolio at the time of inception entails no liquidity lines ($s = 0$) and a stock of defaultable debt that equals the long-run average debt-to-mean-annual-income ratio of the baseline economy. The right panel shows the impact of policy on the bond pricing schedule as a function of the ratio of net debt issuance ($b' - (1-\delta)b + s'$ to mean annual income (normalized to one). Thick dots denote equilibrium realizations.

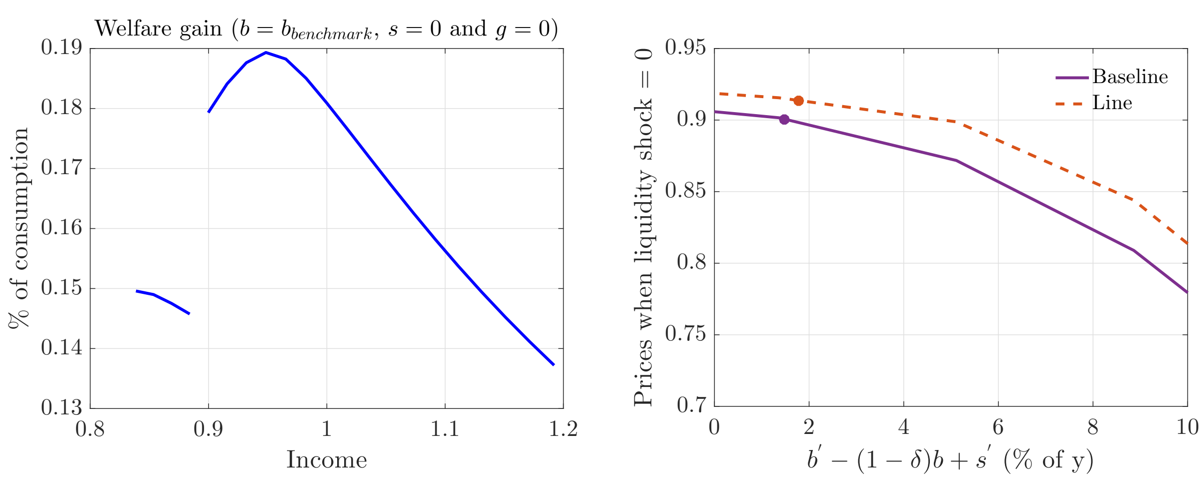


Figure 10: The left panel plots welfare gains measured in consumption-equivalent terms from the inception of liquidity lines. The initial debt portfolio at the time of inception entails no liquidity lines ($s = 0$) and a stock of defaultable debt that equals the long-run average debt-to-mean-annual-income ratio of the baseline economy. The right panel shows the impact of policy on the bond pricing schedule as a function of the ratio of net debt issuance ($b' - (1-\delta)b$ to mean annual income (normalized to one). Thick dots denote equilibrium realizations, and the economy is not in a low liquidity period.

4.3 Permanent Liquidity Lines

This section presents the quantitative analysis for an economy in which liquidity lines are always available to establish the importance of a trigger mechanism in the contract. This section highlights the contribution of this paper to the existing literature, which typically assumes that such financial instruments are always granted (see Hatchondo et al. (2017)) and thus finds that the effects of such instruments are only short-lived.

I initially confirm the findings of Hatchondo et al. (2017). For that, as in Hatchondo et al. (2017), liquidity lines are always available, including during default episodes regardless of the state of the world. Formally, the value of repayment and the value of default are expressed as

$$v_R(b,s,y,g) = \max_{b'\geq 0, s'\geq 0} \left\{ u\left(c\right) + \beta \mathbb{E}_{y',g'|y,g} v(b',s',y',g') \right\}, \tag{4.1}$$

subject to

$$c = y - g\phi^g(y) - s + \frac{1}{1+r}s' - b + q(b',s',y,g)\left[b' - (1-\delta)b\right],$$

$$s' \leq \bar{s}.$$

$$v_D(s,y,g) = \max_{s'\geq 0} u\left(c\right) + \beta \mathbb{E}_{y',g'|y,g}\left[(1-\psi)v_D(s',y',g') + \psi v(0,s',y',g')\right], \tag{4.2}$$

subject to

$$c = y - \phi^d(y) - s + \frac{1}{1+r}s',$$

$$s' \leq \bar{s}.$$

Notice that liquidity lines are now always available to the government as a choice, not only during low liquidity periods. Figure 11 displays the transition dynamics of the spread and debt-to-income ratio for the unanticipated introduction of liquidity lines. The figure reveals that the effects of liquidity lines are only short-lived without any restrictions limiting access to these instruments. At the time of introduction, liquidity lines mitigate default risk and thus reduce spreads (left part of the figure). However, spreads soar back to their pre-liquidity line level over the medium run. Similarly, the debt-to-income ratio initially drops and then rises back to the pre-liquidity line level. Additionally, I investigate the evolution of debt and spread dynamics 500 quarters after the initiation of liquidity in Figure 12. The figure also reveals that the economy with permanent liquidity lines displays debt, spread, and default paths that are almost identical to that of the baseline economy. Thus, liquidity lines do not have any discernible long-run effects on debt balances, spreads, consumption, and default dynamics. It now remains to explore why a permanent arrangement fails to feature long-run effects.

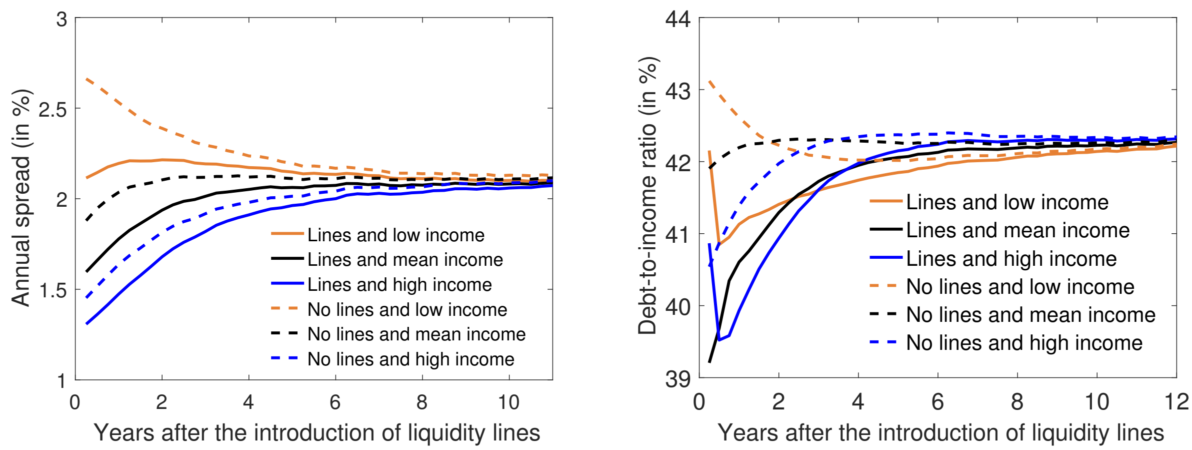


Figure 11: The left panel shows the sovereign spread with and without liquidity lines for samples without defaults. The right panel represents the debt-to-income ratio ($\frac{b'/(\delta+r)}{4y}$) during transitions following the introduction of liquidity lines. Solid lines represent the means of transition paths for an economy with liquidity lines, and dashed lines represent the means of transition paths for economies without liquidity lines.

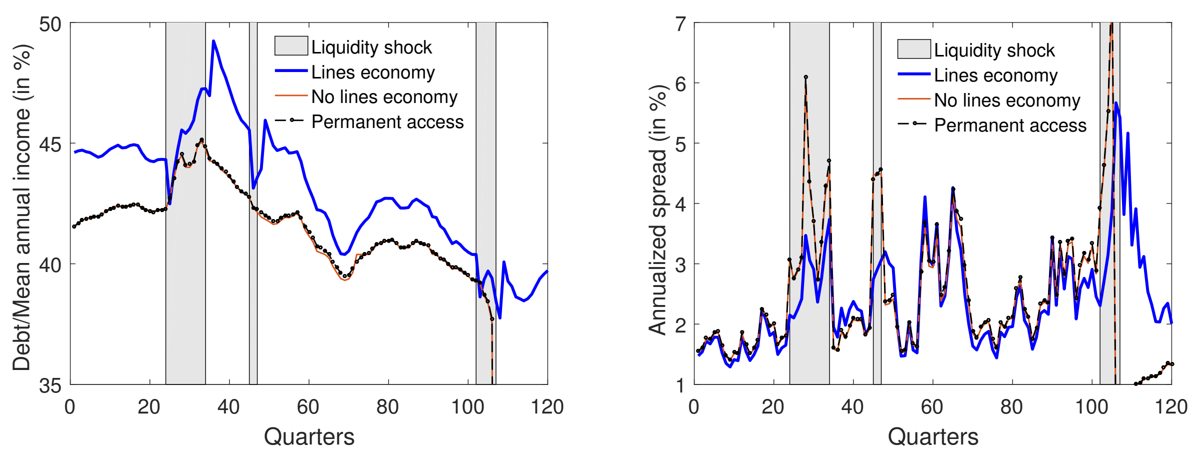


Figure 12: Evolution of debt and spread dynamics in the baseline economy, in the liquidity line economy with a temporal arrangement, and in the liquidity line economy with a permanent arrangement. The liquidity line limit in both economies is 3 percent. The first period in the figure corresponds to the 501st quarter after the initiation of liquidity lines. Mean annual income is normalized to one.

Introducing a state-contingent asset that pays only during a crisis episode completes the markets, though not perfectly. The effect of this state-contingent asset on the price of non-contingent (defaultable) debt mainly works through the increased cost of defaulting when liquidity lines are arranged to be temporal.

To expand on this point, let's assume that lines are permanent, as in Hatchondo et al. (2017). That is, these lines are available all the time including during default episodes (recall that there is no default on liquidity lines as they can always be rolled over). In simulations, the government always chooses the highest amount of possible liquidity lines; that is, both s and s' are equal to the liquidity line limit $\bar{s}$. Once liquidity lines are made available to the government, the sovereign exercises them to the limit and then simply keeps rolling them over. This scenario is plausible as the price of the liquidity asset is always cheaper than the price of the defaultable debt since it is risk-free. If a government defaults as in the world of Hatchondo et al. (2017), the budget constraint during defaults becomes $c = y - \phi^d(y) - s + \frac{1}{1+r}s'$ where the last two terms simply become $\bar{s} \times (1 - \frac{1}{1+r})$, which is essentially a negligible cost increase on top of the income cost of defaulting. This is mainly why Hatchondo et al. (2017) cannot generate long-run effects. When liquidity lines are arranged to be temporal instead, the rise in the cost of defaulting is not negligible. For instance, consider that the government is hit by a liquidity shock during a repayment state. The government uses liquidity lines to the limit during those low liquidity periods. Now if the government defaults in the period after utilizing the liquidity lines, the amount of payments during defaults becomes $\bar{s}$, which is not negligible. Thus, it is costlier to default when these lines are arranged to be temporal rather than permanent.

The exit clauses with temporary lines improve the market discipline because the government now has to repay both non-defaultable debt accrued from the liquidity line (without being able to roll over) and defaultable debt. The government worries about rollover risk at the time of exit as the government has fewer resources available to pay for defaultable debt compared to the baseline economy. In a nutshell, temporal arrangements partially complete the markets, and the government's commitment to not default on liquidity lines works as a disciplining device on its budget balances.

4.4 Model Dynamics Without Income Cost of Global Liquidity Shock

In this subsection, I evaluate the simulation results of the baseline economy by comparing them to economies with liquidity lines with 3 percent and 10 percent limits when we remove the income cost of global liquidity shocks from the model. Table 6 reports the long-run moments and highlights that liquidity lines still yield long-run declines in spreads while the increase in debt levels is very limited. The baseline economy is recalibrated to be able to match the debt-to-income ratio, mean spread, and increase in mean spread during the liquidity shock moments observed in the data. To this end d_0, d_1 and α are set to -0.59, 0.925, and 75, respectively.

Figure 13 is provided to better illustrate how debt and spread dynamics evolve over time. Notice that the dynamics of the debt-to-mean-annual-income (normalized to one) ratio during a liquidity shock in Figure 13 are different than those in Figure 5 where a liquidity shock entails an income cost. The intuition rests on the market discipline imposed on the government's debt balances during a liquidity shock from a higher aversion premium on lenders. The higher discipline arises because lenders ask for a significantly higher spread for each unit of default risk. Thus, the government optimally lowers its debt balances. Both liquidity line economies use their established lines to further deleverage their defaultable debt to lower default risk (see Aguiar et al. (2016) for a fuller discussion on how lenders' high aversion premium affects the government's debt dynamics). This explains why spreads are significantly smaller in economies with liquidity lines during a liquidity shock. The spread dynamics at the time of exit from a liquidity shock are also different. Recall that spreads remain elevated in the liquidity line economy relative to the baseline economy for a couple of quarters following an exit from a liquidity shock in the model in which a liquidity shock entails an income cost. This time, however, Figure 13 displays that spreads with liquidity lines always remain lower relative to those of the baseline economy. This can be attributed to the greater debt reduction achieved by lines during a low liquidity period. Even though the government's available balances decline significantly as it has to pay for liquidity lines without being able to roll over at the time of exit, the government still has a higher fiscal capacity achieved by the debt reduction.

One important takeaway from this analysis is that one would not be able to generate defaults triggered by a liquidity shock in the baseline economy while the number of defaults in 100 years turns out to be 0.39. Thus, using the income cost of a liquidity shock is a shortcut to generating defaults driven by a liquidity shock in the baseline economy.

4.5 Capital Gains

This section discusses the impact of liquidity line arrangements for the holders of previously issued defaultable debt b at the time of the unanticipated announcement.

Figure 14 presents that with the introduction of liquidity lines; holders of previously issued defaultable debt enjoy capital gains. The left panel shows the market value of the debt

Table 6: Long-run statistics: Effects of introducing liquidity lines

	(1) Lines 10 percent	(2) Lines 3 percent	(3) Baseline Recalibrated	(4) Data
Debt Statistics				
Mean debt-to-GDP ratio (in %)	42.6	42.8	42.3	42.2
Mean r_s during a global liq. shock	2.4	3.1	3.6	4.0
Mean r_s	2.1	2.3	2.4	2.3
Mean liq. lines to GDP during a liq. shock (in %)	10	3		

The model is re-calibrated by setting d_0, d_1 and α to -0.59, 0.925 and 75, respectively. Details on how the moments are obtained are provided under the numerical solution algorithm in Appendix B.

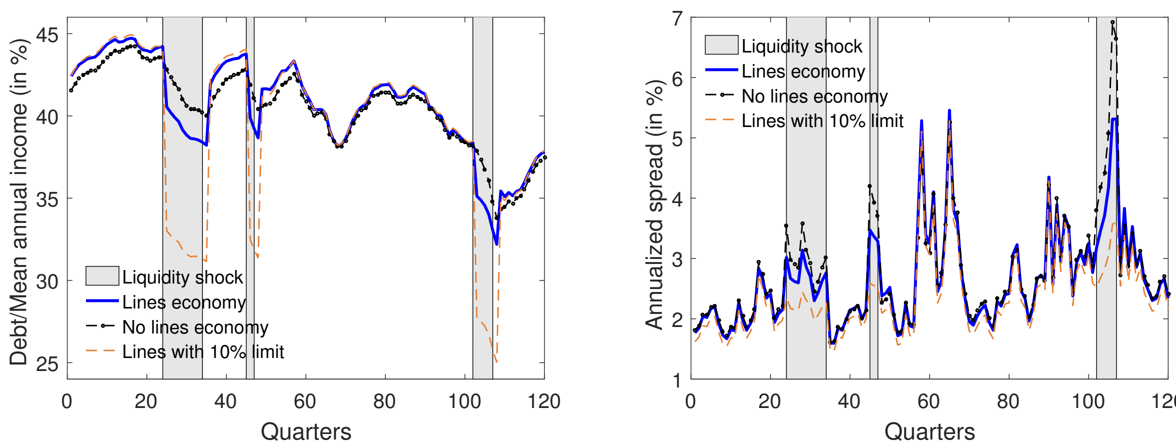


Figure 13: Evolution of debt and spread dynamics when the income cost of a global liquidity shock is removed, $\omega = 0$, in the baseline economy and in economies with liquidity lines with 3 percent and 10 percent limits. The first period in the figure corresponds to the 501^{st} quarter after the initiation of liquidity lines.

with the mean level of income when the unanticipated announcement of a liquidity line arrangement takes place during a global liquidity shock ($g = 1$). Thus, non-defaultable debt through liquidity arrangements is zero ($s = 0$). The right panel shows the same plot for $g = 0$, that is, the economy is not hit by a liquidity shock when liquidity lines are established. Both panels present that the market value of the debt shifts up following the arrangement of liquidity lines. This occurs because liquidity lines reduce default risk, which raises the price of the defaultable debt. Thus, the market value of the debt surges, and holders of previously issued defaultable debt enjoy capital gains. In the Appendix, I show that the gains of arranging liquidity lines can actually be higher if some of these gains can be transferred to the government through debt exchange prior to the inception of liquidity lines.

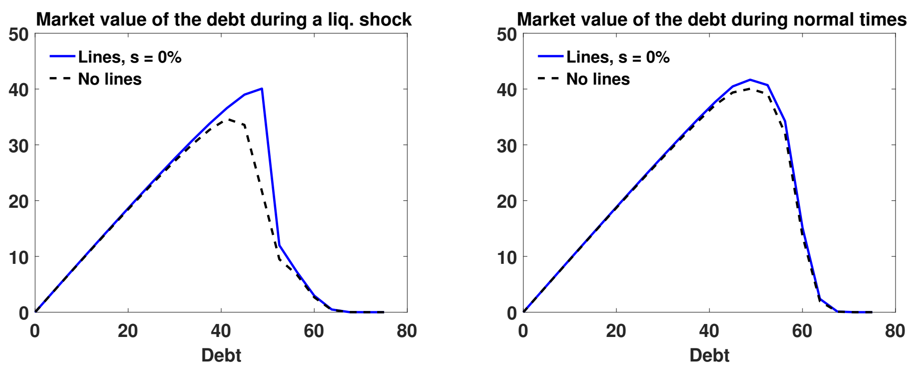


Figure 14: The panels depict the market value of debt at time t computed by $b \times q(b', s' = 0, y = \bar{y}, g)$ where $\bar{y}$ denotes the mean trend income. The left panel plots the market value of debt for the case in which liquidity lines are introduced when the economy is hit by a liquidity shock ($g = 1$), and the right panel plots the case in which liquidity lines are introduced during normal times ($g = 0$). Dashed lines represent the market value of the debt before liquidity lines, and the solid lines represent the market value of the debt after liquidity lines.

4.6 Investigating Moral Hazard

To discipline the potential moral hazard problem that may arise following the introduction of liquidity lines, I initially undertake the following analysis. I compare two scenarios: one with an unexpected (e.g. surprise) credit line added afterward in a crisis, and a second with an ex-ante expected credit line in a crisis.

I investigated this in two alternative ways. First, the government knows for sure that credit lines will be implemented today or 20 quarters after today. Second, I added another state variable with which I introduce a shock which follows a Markov process such that credit lines can be implemented with a probability π_L each period and once it is implemented it can remain in effect with a probability χ. For simplicity I set both parameters to 50 percent. With this way, the government anticipates the implementation of credit lines with a positive probability. I show the results of this analysis in Figure 15.

The left panel of the figure shows that the economy with "Line probability", which anticipates liquidity lines to be implemented with 50 percent probability in each period, does not raise its borrowing as much as the economy that has full access, depicted with "Lines economy". The economy with "Line probability" shows sharp movements during a global liquidity shock. This is because the government loses its access to the liquidity lines in the 14^{th} quarter and regains it in the 15^{th} quarter. Not surprisingly, as I increase (decrease) this 50 percent probability of having access to the credit lines, the economy gets closer to the economy with full (no) access. I do not include these results for brevity.

The right panel of Figure 15 shows that the economy which anticipates liquidity lines to be implemented 20 quarters after today ("20q after" line in the chart) follows identical paths of the economy which was fully granted in the first period ("Lines economy" in the chart) until the realization of a liquidity shock in the 11^{th} quarter. The economy which anticipates credit lines to be implemented in the 20^{th} quarter follows a similar dynamics to the baseline economy, which is depicted as "No lines economy" in the chart, during the liquidity shock and converges to the path of lines economy around the 32^{nd} quarter.

As ex-ante expected credit line results differ from the one with an unexpected credit line, it may signal a moral hazard problem. To dig deeper, that is to determine if a policy generates a moral hazard problem, the policy should incentivize the government to take more risk as the government does not bear the full cost of that risk. One way to analyze is to investigate whether the policy exacerbates the inefficiency available in the model. The first order inefficiency in quantitative sovereign default models is default. Yet, it appears that default likelihoods are not changing as presented in Table 3. The second order inefficiency arises in quantitative sovereign default models with long-term debt: the debt dilution problem which refers to the decline in the value of existing defaultable debt because of the issuance of new debt. As the government increases its indebtedness with the policy, debt dilution problem could have been exacerbated. The analysis in Section 4.5 shows that market value of previously issued debt in fact goes up at the time of policy arrangement. For the long-run effect, the equilibrium response of consumption can be investigated to assess the toll of debt dilution. Figure 6 shows that consumption allocations under long-term debt are slightly lower during normal times for low levels of income realizations. Thus, liquidity lines do not seem to exacerbate the debt dilution problem under long-term debt but rather work as a tool to transfer resources from normal periods to low liquidity periods.

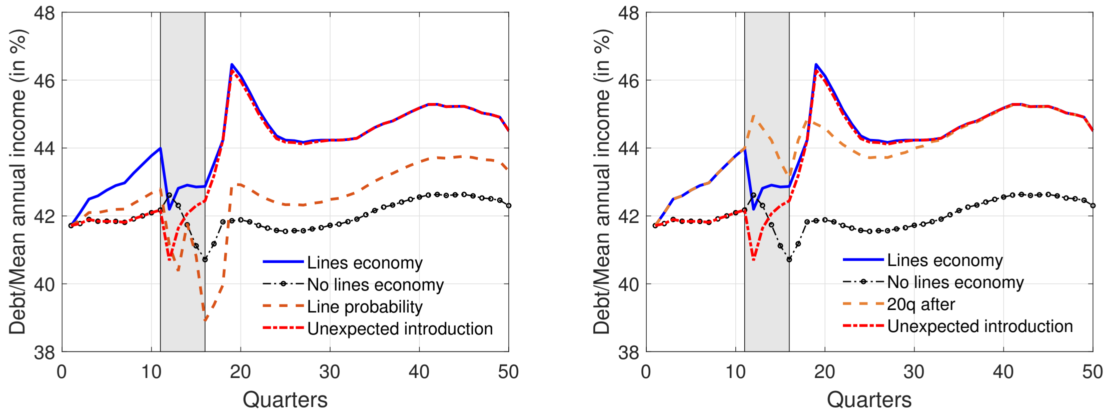


Figure 15: The economy with "Lines economy" is introduced with credit lines in the first period to the "No lines economy", whereas the economy with "Line probability" anticipates to access these lines with 50 percent probability during a liquidity shock. The economy with "Unexpected introduction" is introduced with lines unanticipatedly right prior to the observation of liquidity shock. The economy with "20q after" in the right panel anticipates credit lines to be available at the 20^{th} quarter but not before.

Lastly, recall that the baseline analysis relies on a 3 percent liquidity line limit. With the inception of liquidity lines, the government starts borrowing more in the long run as liquidity lines reduce default risks for a given level of debt. Now, to determine whether this overborrowing is partly a result of moral hazard behavior, or the government is just borrowing more because it becomes cheaper with the availability of liquidity lines, I increase the liquidity line borrowing limit to 10 percent. This is done to see whether the government borrows more with a higher limit, as one would expect moral hazard incentives, if there is any, to heighten. I document in Table 7 that this is not the case. In fact, the government's total debt balances decline slightly with 10 percent limit compared to a 3 percent limit. This is also consistent with Table 6 where we remove the income cost of liquidity lines from the model. The reason for the decline is that, as discussed before, the government worries about rollover risk when the low liquidity period is over. The government is required to repay the 10 percent liquidity line without being able to roll over that debt as well as the defaultable debt when exiting the low liquidity period. In a nutshell, even if a moral hazard channel exists, it is likely to be small with such a low limit compared to the gains obtained from the inception of liquidity lines.

Table 7: Long Run Statistics: Effects of introducing liquidity lines

	(1)	(2)	(3)
	Lines 10%	Lines 3%	Baseline
Panel A: Debt Statistics			
Mean Debt-to-GDP	44.1	44.4	42.6
Mean r_s during a global liq. shock	1.9	2.7	3.
Mean r_s excluding a global liq. shock	1.8	1.9	1.9
Mean liq. lines to GDP during a liq. shock (in %)	10	3	

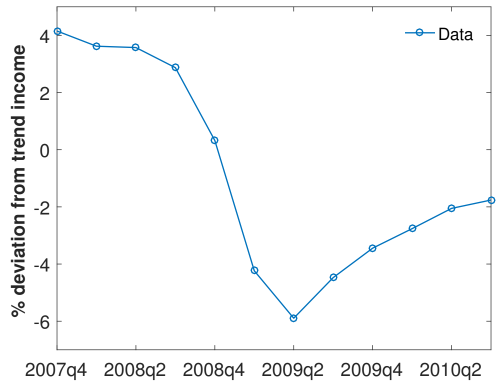
Details on how the moments are obtained are provided under Computation Algorithm in Appendix B.

4.7 Event Analysis

I now explore my model implications in an event study analysis during the liquidity crisis in 2008 and 2009. The top left panel of Figure 16 plots deviations of Mexico's real GDP from a linear trend for the 1980Q1-2014Q3 period, and the right panel presents the event analysis. Notice that the economy faced a global liquidity shock in 2008Q4 ,and Mexico subsequently arranged a swap line facility with the FED for the following three quarters. Thus, starting from 2009Q1, the economy switched to the liquidity line economy, and up to that point, the economy had operated under the dynamics of the baseline economy. In both the baseline and liquidity line economies, I ensure that the model economies face identical income shocks to their empirical counterparts.[10]

The right panel of Figure 16 plots the time series of Mexico's EMBI spreads (the dashed line with circles), which are more than doubled following concerns over global liquidity conditions. The light-shaded area denotes the period in which VIX was two standard deviations above its historical average, while the darker-shaded region denotes the period in which Mexico's swap line arrangement with the FED and VIX was one and a half standard deviations above its historical average. Even though income was still deeply in negative territory in the data, spreads subsided with the inception of swap lines. The chart also plots the implications of the event study analysis using identical income shocks. The dashed line presents the outcome of the baseline economy, and the solid line denotes the model economy with liquidity lines. The initial debt level for the baseline economy is set to the ergodic level of the debt stock. The last data point of the baseline economy stands for a hypothetical situation that attempts to predict what would have happened had no

[10]I follow the methodology provided in Önder and Sunel (2020) for this event analysis. Please refer to it for the technical details.



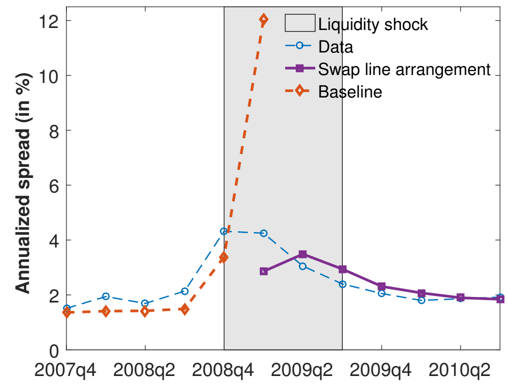


Figure 16: The left panel shows the evolution of cyclical deviations of income from a linear trend, and the right panel presents the event study analysis before and after Mexico's swap line arrangement with the FED.

liquidity lines been established. It shows that the spreads could have jumped to 12 percent. The case of Mexico suggests that the inception of liquidity lines can play a pivotal role in avoiding a severe debt crisis.

5 Conclusion

This paper proposes an endogenous sovereign default model quantifying the gains of arranging liquidity lines as a hedge against a liquidity crisis. I show that a temporary arrangement of liquidity lines with a 3 percent limit during a liquidity crisis would yield a significant reduction in spreads and generate substantial welfare gains for the sovereign as well as capital gains for holders of previously issued defaultable debt. However, if these lines are permanently available to the government, the benefits are short-lived, mainly because temporal arrangements induce market discipline as the government is worried about rollover risk at the time of exit. The amount of resources available to the government to pay for defaultable debt declines compared to the baseline economy at the time of exit since the government is obliged to repay for liquidity lines without being able to roll over them.

I also explore a potential moral hazard channel of larger debt accumulation and find that, if it ever exists, it is likely to be small compared to the benefits coming from the liquidity lines. Lastly, I show in an event analysis that Mexico's arrangement of swap lines with the FED amid the global financial crisis helped avoid a substantial debt crisis.

References

Aguiar, Mark and Gita Gopinath, "Defaultable debt, interest rates and the current account," *Journal of International Economics*, 2006, *69*, 64–83.

_ **, Satyajit Chatterjee, Harold Cole, and Zachary Stangebye**, "Quantitative Models of Sovereign Debt Crises," *Handbook of Macroeconomics*, 2016, *2*, 1697–1755.

Aizenman, Joshua and Gurnain K. Pasricha, "Selective Swap Arrangements and the Global Financial Crisis," *International Review of Economics and Finance*, 2010, *19(3)*, 353–365.

Alfaro, Laura and Fabio Kanczuk, "Optimal Reserve Management and Sovereign Debt," *Journal of International Economics*, 2009, *77(1)*, 23–36.

Arellano, Cristina, "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 2008, *98(3)*, 690–712.

_ **and Ananth Ramanarayanan**, "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 2012, *120* (2), 187–232.

Bianchi, Javier, Juan Carlos Hatchondo, and Leonardo Martinez, "International Reserves and Rollover Risk," *American Economic Review*, 2018, *108(9)*, 2629–2670.

Boz, Emine, "Sovereign Default, Private Sector Creditors, and the IFIs," *Journal of International Economics*, 2011, *83*, 70–82.

Caballero, Ricardo and Stavros Panageas, "A Global Equilibrium Model of Sudden Stops and External Liquidity Management," 2007. MIT Department of Economics Working Paper No. 08-05.

Calvo, Guillermo A., Alejandro Izquierdo, and Luis-Fernando Mejia, "Systemic Sudden Stops: The Relevance Of Balance-Sheet Effects And Financial Integration," 2008. NBER Working Paper No. 14026.

_ **and Carmen M. Reinhart**, "When Capital Inflows Come to a Sudden Stop: Consequences and Policy Options," 2000. MPRA Paper 6982 (Munich: University of Munich).

_ **and** _ , "Systemic and Idiosyncratic Sovereign Debt Crises," 2014. NBER Working Paper 20042.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan, "Sudden Stops and Output Drops," *American Economic Review*, 2005, *95(2)*, 381–387.

Chatterjee, Satyajit and Burcu Eyigungor, "Maturity, Indebtedness and Default Risk," *American Economic Review*, 2012, *102(6)*, 2674–99.

Cruces, Juan Jose, Marcos Buscaglia, and Joaquin Alonso, "The Term Structure of Country Risk and Valuation in Emerging Markets," 2002. manuscript, Universidad Nacional de La Plata.

Eaton, Jonathan and Mark Gersovitz, "Debt with potential repudiation: theoretical and empirical analysis," *Review of Economic Studies*, 1981, *48*, 289–309.

Eichengreen, Barry and Poonam Gupta, "Managing Sudden Stops," 2016. World Bank Policy Research Working Paper 7639.

Fink, Fabian and Almuth Scholl, "A Quantitative Model of Sovereign Debt, Bailouts and Conditionality," *Journal of International Economics*, 2016, *98*, 176–190.

Fleming, Michael J. and Nicholas J. Klagge, "The Federal Reserve's foreign exchange swap lines," *Current Issues in Economics and Finance, Federal Reserve Bank of New York*, 2010, *16.*

G20, "G20 Cannes summit communique," https://g20.org/wp-content/uploads/2014/12/Declaration_eng_Cannes.pdf 2011.

Goldberg, Linda S., Craig Kennedy, and Jason Miu, "Central Bank Dollar Swap Lines and Overseas Dollar Funding Costs," 2010. NBER Working Paper No. 15763.

Hatchondo, Juan Carlos and Leonardo Martinez, "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 2009, *79*, 117 – 125.

_ , _ , and Cesar Sosa-Padilla, "Debt Dilution and Sovereign Default Risk," *Journal of Political Economy*, 2016, *124*, 1383–1422.

_ , _ , and Horacio Sapriza, "Quantitative properties of sovereign default models: solution methods matter," *Review of Economic Dynamics*, 2010, *13* (4), 919–933.

_ , _ , and Yasin Kürşat Önder, "Non-Defaultable Debt and Sovereign Risk," *Journal of International Economics*, 2017, *105*, 217–229.

_ , _ , and _ , "Default risk and sovereign bond price guarantees," 2019. Unpublished manuscript.

_ , _ , _ , and Francisco Roch, "Sovereign cocos," 2020. Unpublished Manuscript.

Hollo, Daniel, Manfred Kremer, and Marco Lo Duca, "Ciss - a composite indicator of systemic stress in the financial system," in "Working Paper Series 1426, European Central" 2012.

IMF, "Consideration of a New Liquidity Instrument for Market Access Countries," 2006. available via the Internet at http://www.imf.org/external/np/sec/pn/2006/pn06104.htm.

_, "Mexico Seeks $47 Billion Credit Line from IMF," 2009. IMF Survey Magazine: Countries & Regions.

_, "Assessing Reserve Adequacy," 2013. International Monetary Fund Policy Paper.

_, "Mexico: Arrangement Under the Flexible Credit Line and Cancellation of the Current Arrangement - Staff Report and Press Release," 2014. International Monetary Fund Country Report No:14/323.

_, "The Review of Flexible Credit Line, the Precautionary and Liquidity Line, and the Rapid Financing Instrument," 2014. International Monetary Fund Policy Paper.

Jeanne, Olivier and Romain Ranciere, "The Optimal Level of Reserves for Emerging Market Countries: a New Formula and Some Applications," *Economic Journal*, 2011, *121(555)*, 905–930.

Krishnamurthy, Arvind, Stefan Nagel, and Annette Vissing-Jorgensen, "ECB Policies Involving Government Bond Purchases: Impact and Channels," *Review of Finance*, 2018, *1*, 1–44.

Mendoza, E. G., "Sudden stops, financial crises, and leverage," *American Economic Review*, 2010, *100* (5), 1941–1966.

Mendoza, Enrique and Vivian Yue, "A General Equilibrium Model of Sovereign Default and Business Cycles," *The Quarterly Journal of Economics*, 2009, *127 (2)*, 889–946.

Obstfeld, Maurice, Jay C. Shambaugh, and Alan M. Taylor, "Financial Instability, Reserves, and Central Bank Swap Lines in the Panic of 2008," *American Economic Review*, 2009, *99(2)*, 480–86.

Önder, Yasin Kürşat, "Does multiplicity of equilibrium arise in the Eaton-Gersovitz model of sovereign default?," 2016. Central Bank of Turkey, Working Paper No: 16/10.

_, "Commodity Prices, Sovereign Default and Sovereign Wealth Funds (SWFs)," 2017. Unpublished Manuscript.

— **and Enes Sunel**, "Inflation-default trade-off without a nominal anchor: The case of Greece," 2020. forthcoming, https://doi.org/10.1016/j.red.2020.05.002.

Rose, Andrew K. and Mark M. Spiegel, "Dollar illiquidity and central bank swap arrangements during the global financial crisis," *Journal of International Economics*, 2012, *88(2)*, 326–340.

Verdelhan, Adrien and Nicola Borri, "Sovereign Risk Premia," 2010. 2010 Meeting Papers 1122, Society for Economic Dynamics.

A Appendix

Figure A.1 shows that during the global financial crisis Mexico's credit default swaps (CDS) - an indicator being widely used as an assessment of default risk - had started soaring up until it established $30bn worth of liquidity swap lines with the US Federal Reserve. Shortly after, Mexico extended its liquidity lines by making an arrangement with the IMF. Mexico announced its interest in arranging FCL with the IMF on 31 March 2009 (see IMF (2009)). For details on the utilization, renewal and repayment terms of FCL Mexico had with the IMF, see IMF (2014a). Mexico was not the only country that benefited from liquidity swap lines. The US Federal Reserve also extended swap lines with a number of countries: Australia, Canada, United Kingdom, Japan, European Central Bank, Switzerland, Brazil, Korea, Denmark, New Zealand, Norway, Singapore and Sweden. The official Fed release said that "these facilities (swap lines), like those already established with other central banks, are designed to help improve liquidity conditions in global financial markets and to mitigate the spread of difficulties in obtaining U.S. dollar funding in fundamentally sound and well managed economies". The announcement is in line with IMF's FCL and PLL programmes which aim to provide liquidity for emerging economies that are financially sound and solvent but experiencing liquidity problems. Furthermore, IMF (2013) reports that swap lines can also be utilized as an alternative for accumulating reserves to stem capital outflows and these lines can be used as liquidity buffers to mitigate the crisis.

Table A.1 documents the countries that have arranged FCL and PLL with the IMF. Mexico, Poland and Colombia have never drawn funds from the FCL and mainly extended its arrangements to insure itself against global downside risks in case it might run into a liquidity problem. The cost of arranging these lines are deemed to be significantly below the market rate and according to IMF (2014b) the cost of these lines can be considered to be lower than the opportunity cost of accumulating reserves.

Table A.1: FCL and PLL arrangements with the IMF

Country	Type	Tenor	Trade date	Amount agreed	Amount withdrawn
Mexico	FCL	1 yr	Apr 17, 2009	SDR 31.5 bn - US$47 bn	0
Mexico	FCL	1 yr	Mar 25, 2010	SDR 31.5 bn - US$48 bn	0
Mexico	FCL	1 yr	Jan 10, 2011	SDR 47.3 bn - US$72 bn	0
Mexico	FCL	2 yrs	Nov 30, 2012	SDR 47.3 bn - US$73 bn	0
Mexico	FCL	2 yrs	Nov 26, 2014	SDR 47.3 bn - US$70 bn	0
Poland	FCL	1 yr	May 6, 2009	SDR 13.7 bn - US$20.6 bn	0
Poland	FCL	1 yr	Jul 2, 2010	SDR 13.7 bn - US$20.4 bn	0
Poland	FCL	2 yrs	Jan 21, 2011	SDR 19.2 bn - US$30 bn	0
Poland	FCL	2 yrs	Jan 18, 2013	SDR 22 bn - US$33.8 bn	0
Poland	FCL	2 yrs	Jan 14, 2015	SDR 15.5 bn - US$23 bn	0
Poland	FCL	2 yrs	Jan 13, 2017	SDR 6.5 bn - US$8.2 bn	0
Colombia	FCL	1 yr	May 11, 2009	SDR 7 bn - US$10.5 bn	0
Colombia	FCL	1 yr	May 7, 2010	SDR 2.3 bn - US$3.5 bn	0
Colombia	FCL	2 yrs	May 6, 2011	SDR 3.9 bn - US$6.2 bn	0
Colombia	FCL	2 yrs	Jun 24, 2013	SDR 3.9 bn - US$5.8 bn	0
Colombia[a]	FCL	2 yrs	Jun 17, 2015	SDR 3.9 bn - US$5.5bn	0
Colombia	FCL	2 yrs	Jun 13, 2016	SDR 8.2 bn - US$11.5 bn	0
Morocco	PLL	2 yrs	Aug 3, 2012	SDR 4.1 bn - US$6.2 bn	0
Morocco	PLL	2 yrs	Jul 28, 2014	SDR 3.2 bn - US$5 bn	0
Morocco	PLL	2 yrs	Jul 22, 2016	SDR 2.5 bn - US$3.5 bn	0
Macedonia	PLL	2 yrs	Jan 19, 2011	SDR 0.4 bn - US$0.6 bn	SDR 0.2 bn

Table shows the FCL and PLL arrangements between IMF and member countries. Tenor denotes the duration of the arrangement, trade date is the time in which IMF has approved the arrangement and Amount agreed denotes the amount of extension that was granted. Source: IMF.

[a] This arrangement is canceled and renewed on Jun 13, 2016.

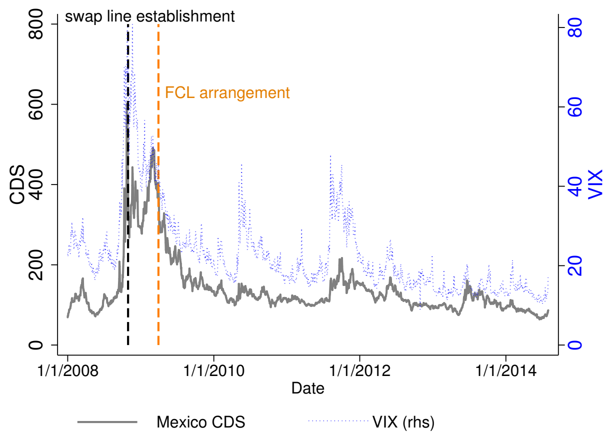


Figure A.1: Mexico's CDS displays a dramatic fall after the introduction of swap lines with the Federal Reserve Bank of the US. Following Mexico's intention of establishing FCL to meet its liquidity needs further dropped the CDS. It can also be seen that CDS of Mexico closely follows VIX. Source: Bloomberg

Table A.2 shows each swap transaction that took place between the U.S. Federal Reserve and Mexico since the establishment. Those arrangements were not extended after its termination on February 1, 2010 up until COVID-19 pandemic. Mexico rolled-over its obligations for two consecutive periods where each period consists of 88 days (almost a quarter) in 2009. Mexico, Korea and Brazil so far have been the emerging market nations that have arranged swap lines with the US Federal Reserve.

Table A.2: Swap line utilization with the US Federal Reserve

Counterparty	Tenor	Trade date	Settlement date	Amount extended	Interest rate
Mexico	88	04.21.2009	04.23.2009	3,221.0	0.74
Mexico	88	07.16.2009	07.20.2009	3,221.0	0.74
Mexico	88	10.14.2009	10.16.2009	3,221.0	0.70
Mexico	84	04.01.2020	04.03.2020	5,000.0	0.91
Mexico	84	04.06.2020	04.08.2020	1,590.0	0.36

Table presents the swap line transactions between the U.S. Federal Reserve and the emerging market countries in which an arrangement has taken place. Brazil had an arrangement as well but has never exercised it. Tenor denotes the duration that the liquidity swap is outstanding, trade date is the time in which both parties agreed upon the terms and conditions, settlement date is the date on which U.S. dollars are extended to the counterparty upon the receipt of the foreign currency, amount extended is the amount of money in millions USD that is provided to the counterparty on the settlement date and the interest rate is the interest payable by the counterparty accrued by swap transaction. Source: U.S. Federal Reserve.

A.1 Liquidity lines with debt ceilings

When countries have access to liquidity channels through FCL or PLL, they might also be subject to implementation of binding fiscal rules as a prerequisite. IMF (2014b) is willing to mitigate the risks that are borne by illiquidity through providing funds to the governments that are financially sound. IMF (2014b) specifically lists debt rules as one of the indicators to assess the eligibility of a country's access to these lines. I show that it is also for the benefit of the government to implement a limit on the amount of debt issuances.

This subsection presents that if an access to a liquidity line follows with a committed debt rule (fiscal rule), then the gains from these lines are bigger. In particular, I assume that access to liquidity lines for up to 3 percent of mean annual income during a global liquidity shock is complemented with a defaultable debt limit of 42.6 percent of mean annual income which is the government's ergodic level of defaultable debt stock. Since this policy analysis combines debt limits and liquidity lines at the same time, the effects of these two policies are confounded. To decompose the impact of each policy, I introduce each policy one at a time in Figure A.2. Dashed lines denote the baseline economy; dashed-lines with circles denote the economy with debt limits only and solid lines denote the economy with two policies. It is visible that each policy has its long run effects. Debt limits alone reduce the spreads by around 50 basis points (bps) while liquidity lines with debt limits mitigate the spreads by 100 bps. Similarly, the number of defaults slightly reduces from 0.92 to 0.87 under the debt limit policy while it falls to 0.66 when the debt limit policy is vested with liquidity lines.

Gains from arranging debt ceilings mainly arise following the elimination of debt dilution problem which refers to the reduction in the value of existing debt by issuing new debt. This problem is eliminated with one-period debt contracts; however, issuing only one-period debt contracts is not practical as the government becomes subject to a greater rollover risk. In an alternative complementary analysis, one may want to think of the arrangement of liquidity lines as the source of financing the government needs to be able to transition to a debt limit which would eliminate default risks entirely. Without providing necessary financing, this enforcement implies a steep consumption fall in the short run which may trigger a default. The amount of debt that can be purchased back from the secondary markets by using the proceeds of liquidity lines can be computed as follows. The sovereign offers the bondholders the level of sovereign debt b_v that would not make them worse off as lenders enjoy capital gains following the inception of lines that is shown in Section 4.5. The post exchange amount of debt b_v satisfies

$$\frac{\bar{s}}{1+r} + b_v(b,s,y,g)\left[1 + (1-\delta)\, q\left((1-\delta)\, b_v\left(b,s,y,g\right), s, y, g\right)\right] = b(1 + (1-\delta) q_b(\hat{b}_b(b,y,g), y, g)$$

where q_b and $\hat{b}_b$ denote the price of the asset and borrowing policy of the economy without liquidity lines, respectively. The amount of debt exchange that can be supported with liquidity lines would be $b - b_v$. In this voluntary debt exchange, all the gains are reaped by the government because the price q_b without liquidity lines is smaller, as presented in Figure 1. Thus, b_v is a lower bound post exchange debt level that does not make bond holders of previously issued debt worse off. I show that with such a voluntary debt exchange, the government can attain to a 36 percent debt to mean annual income ratio with which default risks are eradicated almost entirely. I also conduct a debt exchange analysis in which the exchange is arranged at the time of liquidity lines inception before they are utilized. Thus, capital gains of previously issued debt are transferred to the government. Figure A.3 presents voluntary debt exchange ratio and corresponding welfare gains of this analysis.

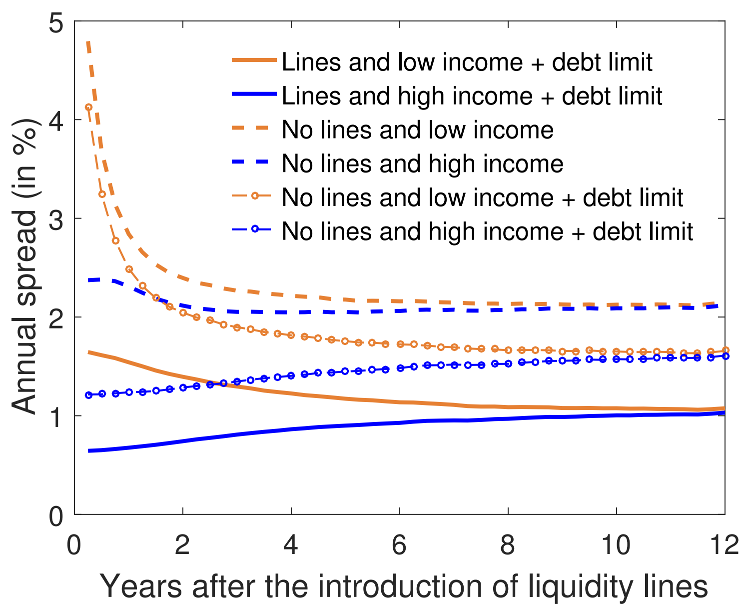


Figure A.2: Solid lines represent the mean of transition paths for an economy with liquidity lines vested with a debt limit, dashed lines represent the transition paths for the baseline economy and dashed-line with circles represent the transition paths for the baseline economy vested with debt limit. The panel represents the transitional dynamics when the economy is hit by a global liquidity shock.

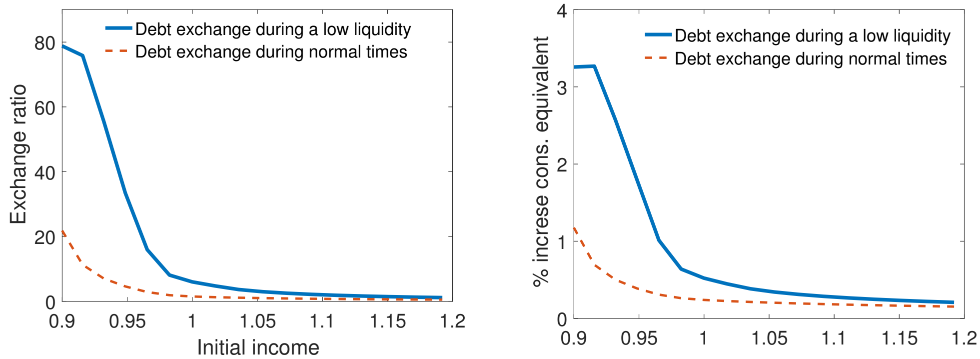


Figure A.3: Voluntary debt exchange occurs at the time of liquidity lines inception and before their utilization. The left panel presents voluntary debt exchange ratio and the right panel denotes the welfare gain from introducing liquidity lines with a voluntary debt exchange.

B Numerical solution algorithm

The numerical algorithm, as briefly explained in Section 3.1, relies on value and price function iterations[11].

An approximation scheme is required for the evaluation of the equilibrium value functions v_R and v_D and price function q that lie outside the grid points. For that purpose this paper engineers a two-dimensional tensor-product spline interpolation over asset space (b, s) and linear interpolation over endowment levels. In particular, routines DBS2IN and DBS2VL are employed using the IMSL library for Fortran.

The numerical algorithm that is used to solve the model works as follows:

1. Guesses of v_R, v_D and q are provided as the last-period of the finite-horizon economy:
   - $v_R(b, s, y, g) = u(y - g\phi^g(y) - s - b)$,
   - $v_D(s, y, g) = u(y - \phi^d(y) - s)$,
   - and $q = 0$.
2. Optimization problem defined in equation (2.4) is solved for each grid point. To do that, I first take 10 points for s' and for each of these points I search over 40 points for optimal bonds policy b' in order to reach the maximum point of the objective function. So for a fixed s', I now have the optimal b' which was attained by using the one-dimensional DUVMIF routine of the IMSL library for Fortran. So, for a given long term debt, liquidity line debt and income grid points and for a fixed s', I now

[11] Computational algorithm follows Hatchondo et al. (2010).

have the optimal bonds policy b'. I then use this fixed s' and the corresponding optimal b' as the initial guesses of optimal portfolio allocation and feed into the bi-dimensional optimization Powell routine to find the optimal pair of b' and s' for each grid points defined above maximizing equation (2.4).

3. Iterate the procedure defined in 2 for equations (2.4) to (2.6) until criteria for convergence is attained.

To simulate the model I use the converged optimal decision rules, specifically do the following steps:

- Set the number of samples N and the number of periods T in which T_0 will be dropped. In particular, set $N = 300$, $T = 1501$ and $T_0 = 1000$.
- Choose y_0 to be mean y, b_0 to be zero, s_0 to be zero and draw sequences of y_t and g_t for $t = 1, 2, ..., T$, using a random number generator. It is suggested to keep the draws so that the same draws can be used again for each $n \in N$. It is also assumed that there is no default and no global liquidity shocks at time zero.
- Trim the first T_0 periods to remove the influence of arbitrarily chosen initial states.

I report the results from simulated sample paths such that the moments presented in the paper calculated from 300 samples such that each sample includes 100 periods (25 years) with no defaults and the sample period begins at least 5 years following a default. Moments are reported using the detrended series and trends are computed using a linear trend.

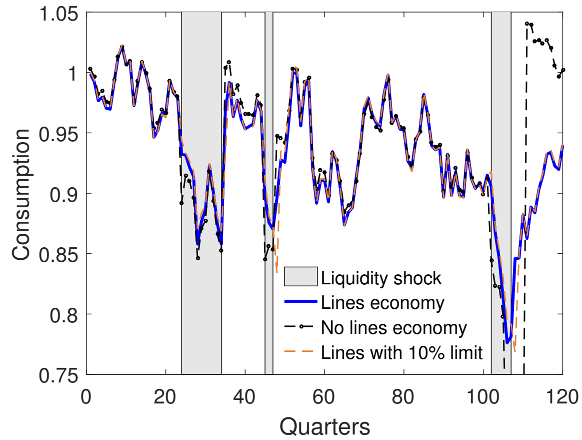


Figure B.1: Evolution of consumption dynamics in the baseline economy, and in liquidity line economies with 3 percent and 10 percent limits. The first period in the figure corresponds to the 501^{th} quarter after the initiation of liquidity lines.

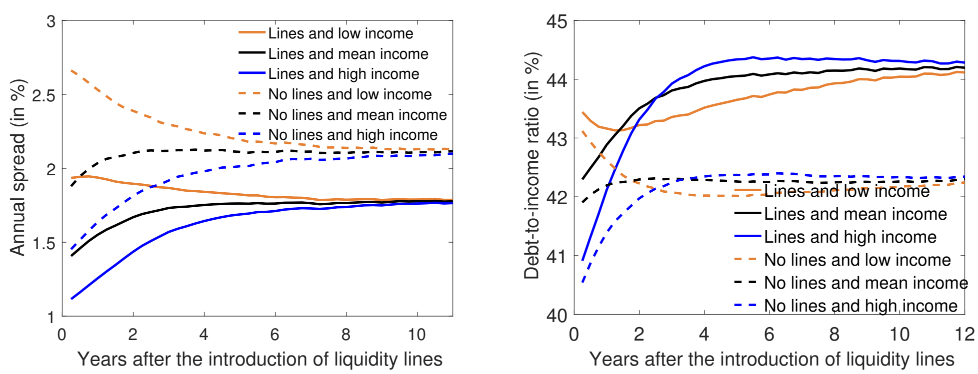


Figure B.2: The left panel shows the sovereign spread with and without liquidity lines for samples without defaults. The right panel represents the mean debt-to-income ratio ($\frac{b'/(\delta+r)}{4y}$) during transitions following the introduction of liquidity lines. Solid lines represent the means of transition paths for an economy with liquidity lines, and dashed lines represent the means of transition paths for economies without liquidity lines. The limit on liquidity lines is set to be 10 percent in liquidity line economies.