International Comparison of Failure Prediction Models From Different Countries : An Empirical Analysis

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ABSTRACT

This study compares eight international failure prediction models on one data set of Belgian company accounts, using performance indicators based on the inequality principle and performance measures based on a classification rule. After a brief theoretical review of the two basic modelling techniques in failure prediction research and the performance measures used to evaluate them, we report type I and type II error rates corresponding with the original cut-off point and calculate new optimal cut-off points, as well as Gini-coefficients. A wide range of performances was observed for the different models. However the models estimated on a sample of Continental European companies are found to be better performing when validated on a sample of Continental European, i.e. Belgian companies, than the Anglo-Saxon models. A remarkable finding is also that the Greek Gloubos-Grammaticos models show better predictive ability when validated on samples of Belgian failing and non-failing companies than on their own (Greek) validation samples. Another important finding is the robustness of the older discriminant models and the models that were estimated on bigger companies. The validation shows that very simple models can have great predictive ability.

1. INTRODUCTION

Failure prediction or financial distress models are much-discussed in accounting and credit management literature. A lot of studies have been dedicated to the search for the most effective empirical method for failure prediction. Recently a lot of papers are published comparing different scoring techniques on the same data set. Examples are Altman et al. (1993), Bell et al. (1990), Curram et al. (1994), Joos, Ooghe and Sierens (1998), Laitinen and Kankaanpää (1998)...

In this paper, we validate several international failure prediction models on one data set. As our goal was not the re-estimation, but a large and global validation of international models, we worked with populations and samples as large as possible. It is also our objective to suggest possible explanations for differences in performance between the investigated failure predicion models from different countries.

This paper is organised as follows. In the next section, two modelling techniques are explained: linear discriminant analysis and logistic regression. These modelling techniques are the ones used in the models we compare in this study. In order to evaluate international scoring models, different performance measures were used. We discuss type I and type II errors based on old and new cut-off points and we compare the models in a more global way with Gini-coefficients. The theoretical elaboration of these performance indicators is described in section 3. Section 4 discusses and investigates the failure prediction models that were used in this study. For each model a summary is presented containing the variables and coefficients of the model and in appendix 1 an overview of the characteristics of the different models is presented. Section 5 describes the population and the methodology followed to draw up the samples. Section 6 discusses the results of our empirical research and the final section concludes with an overview of the most important findings.

2. MODELLING TECHNIQUES

Modelling techniques for two-group classification in general and failure prediction in particular can roughly be classified in four different groups: classical statistical techniques, recursive partitioning analysis (or tree classification), neural networks and genetic algorithms[1]. The latter three classification methods are sometimes classed under the general denominator of ‘inductive learning’, i.e. learning processes based on examples. It is therefore more difficult to validate this kind of models as an outsider. As a consequence, this study only considers failure prediction models estimated with classical statistical techniques such as linear discriminant analysis and logistic regression. A second reason why we only consider these techniques is because they are mostly used in failure prediction research, both in the earlier versions as in the most recent ones. In this section the following modelling techniques are explained: discriminant analysis and logit analysis.

[1] For a comprehensive summary of methodological issues on estimation and evalution of credit scoring models, see JOOS, OOGHE and SIERENS (1998)

2.1. Discriminant analysis

Discriminant analysis (DA) is a multivariate extension of the univariate variance analysis. DA compares the distribution of one or more variables for different groups / populations which are known and identified, and mutually exclusive. DA is a parametrical technique for it is based on assumptions about differences between variable means (vectors) and covariance structures between groups; furthermore it is important that the independent variables have a multivariate normal distribution (Altman et al.,1981).
Multiple linear DA has following discriminant function with an output in [-∞, +∞]:

$$D_i = b_0 + b_1 X_{i1} + b_2 X_{i2} + ... + b_k X_{ik} \tag{1}$$

with D_i = discriminant score of firm i,
$b_0, ..., b_k$ = estimated coefficients,
$X_{i1}, ..., X_{ik}$ = variables/features of firm i.

The estimation process of the coefficients is aimed at getting the best possible discrimination between both groups. A firm is then classified into the failing or non-failing group by comparing its discriminant score D_i with a cut-off score between the failing and the non-failing firms.

2.2. Logit Analysis

In logit analysis, the conditional probabilities or logit scores lying between 0 and 1 (on a sigmoidal curve) are determined with the next formula (Hosmer & Lemeshow, 1989):

$$P(y=1|\boldsymbol{X}) = P_1(\boldsymbol{X}) = \frac{1}{1+e^{-(b_0+b_1 X_1+...+b_k X_k)}} \tag{2}$$

The exponent in formula (2) expresses the so-called 'logit'. The coefficients are estimated with the *maximum likelihood* method. Therefore, the likelihood function in formula (3) is maximised:

$$L(\mathrm{b}) = \prod_{i=1}^{n} P_1(\mathrm{X_i})^{y_i} \; [1 - P_1(\mathrm{X_i})]^{1-y_i} \tag{3}$$

with $P_1(X_i)$ = probability of failure of *i*th firm,
b = vector with k estimable parameters $b_1, b_2, \ldots, b_k$,
X_i = vector with characteristics of *i*th firm,
Y_i = 1 if *i*th firm fails, 0 if it doesn't fail.

Logit analysis is often used in classification studies because this method has some favourable qualities, e.g. it is not necessary to adapt the method for disproportional samples[2] for only the constant term b_0 is distorted (Maddala, 1992).

[2] In classification research, state-based samples (the probability of being selected depends on the ‘state’ of the firm i.e. non-failing or failing) are often used instead of pure random samples. Since the number of failing units is smaller than the number of non-failing units in most databases, random sampling would lead to very small samples of failing firms, and to inaccurate models.

3. PERFORMANCE CRITERIA

The performance of a classification model indicates how well the model performs and is called 'goodness-of-fit' in econometric literature. Evaluation of the performance is possible in two different contexts: the original dataset that was used to estimate the model or a new validation dataset. It is not our intention to present an exhaustive overview of the various performance measures. In this section two different performance measures will be discussed: measures based on a classification rule and measures based on the inequality principle (Joos, Ooghe and Sierens, 1998). Furthermore, we mention other performance criteria and motivate why these measures were not used in this study.

3.1. Measures based on a classification rule

Since ‘classification’ is the principal goal of the failure prediction models, it is obvious that measures based on a classification rule are frequently applied. A firm is categorised as ‘failing’ or ‘non-failing’, on the basis of the following *classification rules.*

For a continuous score model, the classification rule can be formulated as follows:

$$y_i^* = \begin{cases} 1 \text{ if the logit score } \hat{y}_i \text{ of firm } i > y^* \\ 0 \text{ if the logit score } \hat{y}_i \text{ of firm } i \leq y^* \end{cases} \quad (4)$$

with y_i^* = estimated class of firm i,
y^* = threshold or cut-off point.

The classification rule divides the logit scores into two subdivisions, which causes two types of misclassification costs:

1. *Type I error*: credit risk: if a failing firm is classified as a non-failing one.
2. *Type II error*: commercial risk: if a non-failing firm is classified as a failing one.

The threshold can be determined for which the average of both types of errors is minimised. This is the so-called optimal threshold or cut-off point. In addition and following Koh (1992) the population proportions and misclassification costs can be involved in the identification of the threshold as well.

The *population proportions* show the frequency of failing and non-failing firms in the population.

The *misclassification costs* can be very different for both errors in the context of credit granting. The classification of a failing company as a non-failing one, can have more severe consequences than the classification of a non-failing as a failing one. If these cost factors are integrated, it is obvious that the classification process is dependent of the risk behaviour of the decision-maker and his attitude towards the proportion of cost factors. To identify the threshold, the global cost function must be minimised (Koh, 1992):

$$expected\ cost = EC = \pi_{failing} C_{TypeI}\, TypeI + \pi_{non-failing} C_{TypeII}\, TypeII \quad (5)$$

with $\pi_{failing}, \pi_{non-failing}$ = population proportion of failing and non-failing firms,
C_{TypeI}, C_{TypeII} = cost of type I and type II error,
Type I, Type II = type I and type II misclassifications resulting from resp. type I and type II errors.

Minimising a cost function is only one way to evaluate the performance of a classification model. It is also possible to evaluate the performance statistically; without taking the population proportions and the misclassification costs into account. Significance can be tested by using the Kolmogorov-Smirnov test (Siegel & Castellan, 1988). This allows to test whether the scores of the failing firms are significantly higher than the scores of the non-failing firms. The Kolmogorov-Smirnov test is based on the cumulative distribution functions of the scores of the non-failing ($F_{non\text{-}failing}$) and failing ($F_{failing}$) firms. The greatest cumulative difference between both functions reveals whether the samples originate from the same population or not.

$$D_{non-failing,\,failing} = \max[F_{non-failing}(y) - F_{failing}(y)] \quad (6)$$

with $D_{non-failing,\,failing}$ = maximum difference between the cumulative scoring distributions of non-failing and failing firms,
$F_{non-failing}(y)$ = cumulative distribution of the scores of non-failing firms,
$F_{failing}(y)$ = cumulative distribution of the scores of failing firms,
y = discriminant or logit score.

The score for which the greatest difference ($D_{non\text{-}failing,\ failing}$) between the cumulative distribution function of non-failing and failing firms exists, is also the ‘optimal’ cut-off point with minimal classification errors. In this context, abstraction is made of population proportions and misclassification costs (Koh, 1992).

3.2. Measures based on the inequality principle

The performance of a model can be demonstrated graphically with the construction of a *trade-off function*. The cumulative frequency distributions for ‘non-failing’ and ‘failing’ firms, are then located in a co-ordinate system with the type I error ($= F_{failing}(y)$) on the X-axis and the type II error ($=1- F_{non-failing}(y)$) on the Y-axis (Steele, 1995).

A model has a better performance as the curve is situated closer to the axis’s. The best performing (i.e. most discriminating) model has a trade-off function that coincides with the axis’s. After all, a perfect model categorises each failing firm as a failing one (the type I error is always 0) and a non-failing firm as a non-failing one (the type II error is also 0 for every value). The worst model (i.e. a model that can not make a difference between non-failing and failing firms) has a linear descending trade-off function from 100% type II until 100% type I. In this case $F_{failing}(y)$ and $F_{non-failing}(y)$ coincide (for each score, there are just as much non-failing as failing firms), with complementary type I and type II errors for each score as a result.

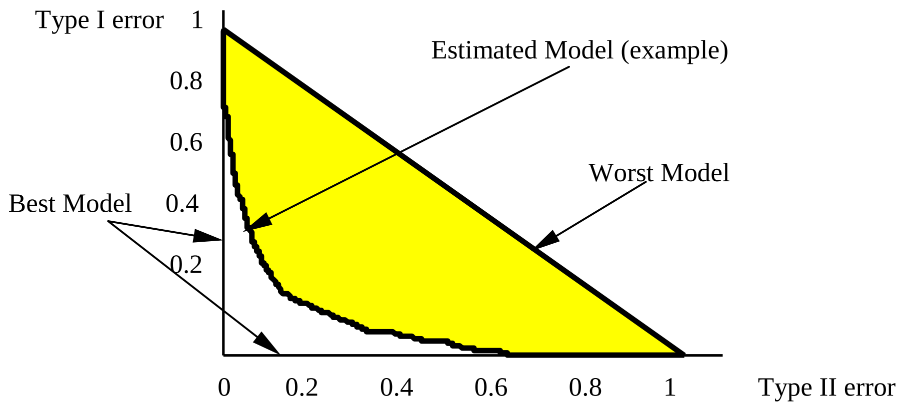

Figure 1: Trade-off function: best, worst and estimated classification models

Each element of this trade-off function represents an optimal threshold for given classification costs ($C_{Type\,I}$ *and* $C_{Type\,II}$) and population proportions ($\pi_{failing}$ *and* $\pi_{non-failing}$).
The difference between the estimated model (trade-off function) and the worst model is an aggregated performance measure and is presented by the Gini-coefficient. This coefficient lies in a normal situation between 0 and 1 and is equal to the proportion of the area between the estimated model and the worst model (grey area in figure 1), and the area between the worst and the best model (i.e. the triangle with the axis's as sides). Therefore: a higher Gini-coefficient corresponds with a curve that is situated closer to the axis's and thus with a better performing model. A negative Gini-coefficient implies that a model classifies more companies falsely than correctly.

An empirical appropriation of the Gini-coefficient is presented in the formula below (Joos, Ooghe and Sierens, 1998):

$$\hat{GINI} = \frac{\dfrac{x_{\max} y_{\max}}{2} - \sum_{i=1}^{n} (x_i - x_{i-1}) \dfrac{y_{i-1} + y_i}{2}}{\dfrac{x_{\max} y_{\max}}{2}} \tag{7}$$

$$= 1 - \sum_{i=1}^{n} (x_i - x_{i-1})(y_{i-1} + y_i)$$

with x_i, y_i = type I and type II error with threshold i,

$x_{\max}, y_{\max}$ = maximum type I and type II, i.e. each 100%.

3.3. Other measures

Two performance criteria for evaluating failure prediction models that were not used in this study are R^2-type measures and measures based on entropy. R^2-type measures indicate the percentage of the variance that is explained by the model, through a comparison of the predicted values with the real values of the dependent variable. As discriminant models generate an output in [-∞, +∞] and there is no variance to be explained, this measure can not be used for evaluating discriminant models (Joos, Ooghe and Sierens, 1998). However it is possible to use the count R^2

measure to evaluate the performance of discriminant models. As this measure indicates the number of correctly and the number of falsely classified firms and this is already measured by other measures based on the inequality principle, especially the Gini-coefficient, we decided not to use these measures.

A second type of performance criteria that were not used in this study are the measures based on entropy. This concept originates from the information theory of Shannon (1948) and was originally introduced in econometrics by Theil (1971). Measures based on entropy were used as performance measure in failure prediction research by Zavgren (1985) and Keasy and McGuinness (1990). Both studies compare the information value of five estimated logit and probit models (5 year, 4 year,…, 1 year prior to failure). A possible disadvantage of the entropy concept however is that it only evaluates the discriminating ability of the model. Furthermore it is impossible to take misclassification costs and population proportions into account a posteriori.

4. FAILURE PREDICTION MODELS UNDER INVESTIGATION

Only failure prediction models estimated with linear discriminant analysis and logistic regression were selected to conduct this study. The reason for this is twofold. First, these techniques are mostly used in failure prediction research, both in the earlier versions as in the most recent ones. Altman started in 1968 with his ‘Z-score’ discriminant model, and the same risk analysis tool is still applied for the scoring models developed by the Central Banks of Austria, France, Germany, Italy, United Kingdom a.o.(‘International Conference of the European Committee of Central Balance Sheet Data Offices’, October 1997, Paris). Logit scores have the advantage that they are easier to interpret because of their binary character. Logit was introduced in a later stage and is at this moment applied in both academic papers, as in research from Central Banks.

The second reason why we chose those models is that they are easier to validate as an outsider than e.g. neural networks. Models were also selected depending on the availability of variables and coefficients: as many recent models are licensed to commercial companies, these models are not fully described in academic publications. The Taffler (1984) model for example was excluded because of the unavailability of its coefficients. Other selection criteria were the availability of performance indicators as type I and type II errors and optimal cut-off points. We also opted for general models and not for models investigating e.g. the probability of failure of new or small firms. Accordingly, the failure prediction model of Laitinen (1992), which was estimated in order to predict failure of newly founded firms, was excluded from our study.

At the end, eight models were withheld: Altman (1968), Bilderbeek (1979), Ooghe-Verbaere (1982) (see Ooghe et al. 1982 and 1998), Zavgren (1985), Gloubos-Grammaticos (discriminant analysis and logistic regression) (1988), Keasy-McGuiness (1990) and Ooghe-Joos-De Vos (1991) (see Ooghe et al. 1991 and 1995). In table 1 an overview is presented of the characteristics of each model under consideration (country, population, period, definition of failure, sampling method, estimation technique, number of variables, model and classification rule). Tables 2 to 8 present the variables and coefficients of each model.

Insert table 1

Insert tables 2 -8

5. POPULATION AND SAMPLES

Before describing the population and method followed to draw the samples, we give some important definitions used in this study.

5.1. Definitions of failing and non-failing firms

Failed firm: a firm in the situation of bankruptcy or official approval of a legal composition.

Non- failing firm: As non-failing firm we included all of the following juridical situations:
- Termination of activity
- Early dissolution- liquidation
- Merger with another company to form a third one
- Absorption by another company
- Closing of a liquidation
- Scission into several companies
- Dissolution by legal ending
- Request for legal composition
- Without any particular legal status

We thus include in the group of non-failing firms also firms which cause doubt about the economic reason of their juridical situation. It is our aim to validate failure prediction models; so it is necessary to do this in a realistic situation and to consider these firms as non-failing ones.

Account 1 year before failure: accounts of a failed firm of which the closing date falls within the period: [date of failure, date of failure - 365d.]

Account 2 year before failure: accounts of a failed firm of which the closing date falls within the period: [date of failure - 365d., date of failure — (2 * 365d.)]

Account 3 year before failure: accounts of a failed firm of which the closing date falls within the period: [date of failure — (2 * 365d.), date of failure — (3 * 365d.)]

5.2. Population

For the validation, Belgian accounting data from the period 1992-1996 were used. It concerns published annual accounts of non-financial companies subject to the Royal Decree of October 8, 1976 on the annual accounts of companies. These data were obtained from the CD-ROM's of the National Bank of Belgium.

In Belgium companies are bound to deposit their annual accounts in a prescribed form dependent on their size. A distinction can be made between bigger companies that have to prepare their annual accounts in a complete form and smaller companies that prepare their annual accounts in an abbreviated form. The first group of bigger companies is characterized by a number of employees of more than 100 or at least two of the following criteria have to be exceeded:
- Number of employees (yearly average): 50;
- Turnover (V.A.T. excluded) (yearly average): 200 million Belgian francs;
- Total assets: 100 million Belgian francs.

Companies that don't meet these criteria are allowed to prepare their annual accounts in an abbreviated form.

The population of which the failed sample is drawn, consists of all firms that failed in 1995 or 1996. Only firms that failed in 1995 having annual accounts in 1992 or later, and firms that failed in 1996 having annual accounts in 1993 or later, and of which at least one account is available on the CD-ROM of the National Bank of Belgium, are included.
The population of which the non-failing sample is drawn, consists of all firms that where non-failing on January 1, 1998. Only firms with annual accounts in 1992 or later, and of which at least one account is available on the CD-ROM of the National Bank of Belgium are included.

In both populations the following companies were not included because of their special situation:
- Financial intermediation, insurance companies and pension funds;
- Management activities of holding companies and co-ordination centers;
- Public administration;
- Education;
- Health and social work.

Both populations contain complete and abbreviated form annual accounts. Table 9 shows the number of companies used in this study.

Table 9: Population of failing and non-failing firms

Form	Category	Number
Complete form	Failing in 1995 Failing in 1996 Non-failing	167 180 14.747
Complete + abbreviated form	Failing in 1995 Failing in 1996 Non-failing	2.773 3.048 150.952

5.3. Sample construction

For each company that failed in 1995 or 1996, the annual accounts 3, 2 and 1 year prior to failure, if available and if not concerning an extended financial year, are used and compared with annual accounts of non-failing companies in the same period.

Non-failing companies are split in 4 equal groups: group A, B, C and D. For each group of companies, the annual account of one specific year in the period 1992-1995, if available and if not concerning an extended financial year, is taken. This means that for the non-failing companies in group A, the annual account of 1992 was taken; for the firms in group B the annual account of 1993 was taken and so on.

Non-failing firms in group A: annual account of 1992
Non-failing firms in group B: annual account of 1993
Non-failing firms in group C: annual account of 1994
Non-failing firms in group D: annual account of 1995

For the comparison with failing companies, accounts of the two relevant years are taken together. The procedure is explained in table 10.

Table 10: Procedure sample construction

		Failing group	Non-failing group	
	Failing	**Year annual accounts**	**Year annual accounts**	**Non-failing firms**
1 ypf	Failing in 95 Failing in 96	1994 1995	1994 & 1995	Group C & group D
2 ypf	Failing in 95 Failing in 96	1993 1994	1993 & 1994	Group B & group C
3 ypf	Failing in 95 Failing in 96	1992 1993	1992 & 1993	Group A & group B

The validation of the failure prediction models was conducted for two types of samples of failing and non-failing firms. The first type of samples was taken from failing and non-failing firms with annual accounts in a complete form. The second type of samples was taken from firms with annual accounts in the complete form or in the abbreviated form.

In the study for the complete form annual accounts only, all the accounts available in a specific year were applied. For the abbreviated and complete form annual accounts together, we were forced to reduce our sample size because of practical reasons. About one third of the failed annual accounts available and about 10% of the non-failing annual accounts available, were drawn ad random. Table 11 gives the number of annual accounts used in the one - two - three years prior to failure samples.

Table 11: Samples of failing and non-failing firms

	Bigger companies with complete form annual accounts		All companies with complete and abbreviated form annual accounts	
	Failing	**Non-failing**	**Failing**	**Non-failing**
1 ypf	111	6323	613	16312
2 ypf	258	6621	1542	16312
3 ypf	294	6916	1611	16312

The validation of each model started with the same sample. One (or more) ratios could not be calculated for some companies because their denominator contains variables with a zero. This is especially the case if the denominator contains turnover or stocks for it is not obliged to publish turnover for the smaller companies with an abbreviated form of annual accounts, or because they don't have stocks. Table 12 shows the percentage of cases in our samples for which the following variables had value 0.

Table 12: Samples of failing and non-failing firms with zero-values for some variables

Variables	Form of annual accounts	Percentage
Turnover	Complete form Complete + abbreviated form	6,96% 48,96%
Stocks	Complete form Complete + abbreviated form	35,59% 42,73%
Supplier credit	Complete form Complete + abbreviated form	5,29% 10,69%
Short term operational assets	Complete form Complete + abbreviated form	1,06% 3,19%
Short term debts	Complete form Complete + abbreviated form	0,67% 1,31%

Table 13 gives the percentage of the original sample that could be used for the validation of the different models.

Table 13: Samples of failing and non-failing firms that can be used for the different models

Authors	Form of annual account	Year before failure	Percentage
Altman	Complete form	Weighted average	99,4%
	Complete + abbreviated form	w.a.	51,2%
Bilderbeek	Complete form	w.a.	92,8%
	Complete + abbreviated form	w.a.	50,9%
Ooghe –Verbaere	Complete form	1 ypf	99,1%
		2 ypf	92,7%
		3 ypf	98,2%
	Complete + abbreviated form	1ypf	98,4%
		2 ypf	49,2%
		3 ypf	95,5%
Zavgren	Complete form	w.a.	62,7%
	Complete + abbreviated form	w.a.	30,8%
Gloubos & Grammaticos	Complete form	Logit	99,9%
		Discriminant	99,3%
	Complete + abbreviated form	Logit	99,9%
		Discriminant	98,7%
Keasy & McGuiness	Complete form	1 ypf	89,9%
		2 ypf	92,8%
		3 ypf	91,1%
	Complete + abbreviated form	1 ypf	45,1%
		2 ypf	52,1%
		3 ypf	49,2%
Ooghe — Joos – De Vos	Complete form	1 ypf	98,5%
		3 ypf	100,0%
	Complete + abbreviated form	1 ypf	97,9%
		3 ypf	99,9%

6. RESULTS AND INTERPRETATION

This section discusses the results of our validation of the different international failure prediction models on our data set of bigger and smaller Belgian companies. First, we report the classification results obtained by the authors themselves on their own samples. Second, we discuss the general results of the comparison of the failure prediction models. We report type I, type II and unweighted error rates corresponding with the original cut-off point as well as new error rates corresponding with a newly calculated cut-off point. We discuss these validation results for the bigger companies with complete form annual accounts (section 6.3.) and for all companies (bigger and smaller) with complete and abbreviated form annual accounts (section 6.4.). In section 6.5 the research results are presented graphically with the construction of trade-off functions for the eight failure prediction models, for the two types of samples (bigger companies and all companies, including the smaller ones). Finally, we emphasise the differences in performance between the various models and suggest possible explanations for these dispersed performances.

6.1. Performance results in the original studies

Table 14 contains the classification results obtained by the authors themselves, both on their original and validation sample.

Table 14: Classification results in original publications

	Cut-off	Type I error		Type II error		Unweighted error rate	
		Original sample	Validation sample	Original sample	Validation sample	Original sample	Validation sample
1YPF							
- Altman	2,6750	6%	n.a.	3%	n.a.	5%	n.a.
- Bilderbeek	0,0250	n.a.	n.a.	n.a.	n.a.	32%	n.a.
- Ooghe-Verbaere	3,1492	n.a.	13,6%	n.a.	21,7%	n.a.	17,6%
- Zavgren	n.a.	11%	n.a.	24%	n.a.	18%	n.a.
- Gloubos-Grammaticos logit	0,5000	16,6%	33,3%	10%	12,5%	13,3%	22,9%
- Gloubos-Grammaticos discriminant	0,0000	3,3%	33,3%	13,3%	33,3%	8,3%	33,3%
- Keasy-McGuiness	n.a.	14%	44%	14%	30%	14%	37%
- Ooghe-Joos-De Vos	0,3117	n.a.	14,7%	n.a.	22,4%	n.a.	18,5%
2YPF							
- Altman	2,6750	28%	n.a.	6%	n.a.	17%	n.a.
- Bilderbeek	0,0250	n.a.	n.a.	n.a.	n.a.	27%	n.a.
- Ooghe-Verbaere	0,1663	n.a.	27,9%	n.a.	22,8%	n.a.	29,6%
- Zavgren	n.a.	11%		22%	n.a.	17%	n.a.
- Gloubos-Grammaticos logit	0,5000	n.a.	39,1%	n.a.	17,4%	n.a.	28,3%
- Gloubos-Grammaticos discriminant	0,0000	n.a.	39,1%	n.a.	17,4%	n.a.	28,3%
- Keasy-McGuiness	n.a.	16%	22%	21%	29%	18,5%	25,5%
3YPF							
- Altman	2,6750	52%	n.a.	n.a.	n.a.	n.a.	n.a.
- Bilderbeek	0,0250	n.a.	n.a.	n.a.	n.a.	29%	n.a.
- Ooghe-Verbaere	0,3355	n.a.	26,2%	n.a.	32,9%	n.a.	29,6%
- Zavgren	n.a.	31%	n.a.	24%	n.a.	28%	n.a.
- Gloubos-Grammaticos logit	0,5000	n.a.	50%	n.a.	21,4%	n.a.	35,7%
- Gloubos-Grammaticos discriminant	0,0000	n.a.	35,7%	n.a.	14,3%	n.a.	25%
- Keasy-McGuiness	n.a.	28%	27%	19%	44%	23,5%	35,5%
- Ooghe-Joos-De Vos	0,2137	n.a.	18,3%	n.a.	34,1%	n.a.	27,7%

6.2. General results of the comparison of the failure prediction models

In tables 15 and 16, the results of our validation, on the samples of bigger companies with complete form and on the samples of all companies with complete and abbreviated form annual accounts, are shown. Firstly, the type I, type II and unweighted error rate corresponding with the original cut-off point, are given. Secondly, we calculate a new cut-off point and report the corresponding error rates. In the last column, the Gini-coefficient that is independent of changing cut-off points, is reported. In both tables, the authors, the unweighted error rate with the original and the new cut-off point and the Gini-coefficient of the best performing models are printed in bold letters.

Table 15: Performance results, bigger companies with complete form of annual accounts

	Cut-off point, original	Type I-error, original	Type II-error, original	Unweighted error rate	Cut-off point, new	Type I error, new	Type II error, new	Unweighted error rate	Gini
1YPF									
- Altman	2,6750	20,9%	82,0%	51,4%	0,0000	94,5%	2,1%	48,3%	-8,3%
- Bilderbeek	0,0250	13,9%	60,3%	37,1%	0,9641	24,1%	33,9%	29,1%	47,3%
- **Ooghe-Verbaere**	3,1492	13,6%	27,8%	**20,7%**	3,2351	9,1%	29,4%	**19,2%**	**74,2%**
- Zavgren	n.a.	n.a.	n.a.	n.a.	0,4946	41,0%	37,6%	39,3%	7,2%
- Gloubos-Grammaticos Logit	0,5000	6,3%	50,2%	28,3%	0,0627	19,8%	23,7%	21,7%	66,7%
- Gloubos-Grammaticos Discriminant	0,0000	4,5%	57,3%	30,9%	-1,1517	19,1%	31,2%	25,1%	54,7%
- Keasy-McGuiness	n.a.	n.a.	n.a.	n.a.	-1,0035	44,9%	32,3%	38,6%	23,4%
- Ooghe-Joos-De Vos	0,3117	24,5%	34,5%	29,5%	0,3496	26,4%	29,9%	28,1%	53,6%
2YPF									
- Altman	2,6750	18,0%	82,0%	50,0%	1,3695	44,5%	48,1%	46,3%	2,4%
- Bilderbeek	0,0250	12,3%	60,5%	36,4%	0,7615	26,0%	39,2%	32,6%	41,1%
- Ooghe-Verbaere	0,1663	26,9%	39,8%	33,3%	-0,4425	43,8%	21,6%	32,7%	46,0%
- Zavgren	n.a.	n.a.	n.a.	n.a.	0,0808	31,3%	50,4%	40,8%	13,1%
- **Gloubos-Grammaticos Logit**	0,5000	11,6%	49,6%	**30,6%**	0,1773	22,1%	35,2%	**28,7%**	**52,0%**
- Gloubos-Grammaticos Discriminant	0,0000	9,8%	56,8%	33,3%	-0,4659	16,0%	46,4%	31,2%	40,9%
- Keasy-McGuiness	n.a.	n.a.	n.a.	n.a.	4,0478	39,5%	32,9%	36,2%	37,6%
3YPF									
- Altman	2,6750	13,7%	81,3%	47,5%	1,5023	41,0%	51,9%	46,4%	3,6%
- Bilderbeek	0,0250	12,5%	60,8%	36,6%	0,3056	17,8%	53,2%	35,5%	34,6%
- **Ooghe-Verbaere**	0,3355	26,8%	36,4%	**31,6%**	0,2357	31,3%	29,9%	**30,6%**	**46,6%**
- Zavgren	n.a.	n.a.	n.a.	n.a.	0,1941	31,1%	63,5%	47,3%	3,9%
- Gloubos-Grammaticos Logit	0,5000	17,4%	49,4%	33,4%	0,3709	20,8%	44,7%	32,7%	42,4%
- Gloubos-Grammaticos Discriminant	0,0000	14,7%	56,0%	35,3%	-0,4348	22,9%	46,1%	34,5%	32,6%
- Keasy-McGuiness	n.a.	n.a.	n.a.	n.a.	1,4494	28,2%	45,5%	36,8%	32,5%
- Ooghe-Joos-De Vos	0,2137	45,9%	24,8%	35,4%	0,1792	36,1%	32,3%	34,2%	37,3%

Table 16: Performance results, all companies with complete + abbreviated form of annual accounts

	Cut-off point, original	Type I-error, original	Type II-error, original	Unweighted error rate	Cut-off point, new	Type I error, new	Type II error, new	Unweighted error rate	Gini
1YPF									
- Altman	2,6750	18,9%	81,5%	50,2%	2,8159	15,3%	83,5%	49,4%	-7,9%
- Bilderbeek	0,0250	8,3%	54,4%	31,4%	1,1900	23,0%	28,5%	25,8%	59,7%
- **Ooghe-Verbaere**	3,1492	11,8%	34,4%	**23,1%**	2,4750	16,1%	27,6%	**21,8%**	**68,7%**
- Zavgren	n.a.	n.a.	n.a.	n.a.	0,5692	79,4%	18,1%	48,8%	17,4%
- Gloubos-Grammaticos Logit	0,5000	7,1%	52,5%	29,8%	0,0102	25,4%	18,6%	22,0%	68,6%
- Gloubos-Grammaticos Discriminant	0,0000	5,9%	62,1%	34,0%	-1,6668	23,0%	30,8%	26,9%	53,5%
- Keasy-McGuiness	n.a.	n.a.	n.a.	n.a.	-0,9810	41,0%	30,9%	36,1%	33,5%
- Ooghe-Joos-De Vos	0,3117	17,6%	36,0%	26,8%	0,4052	23,9%	28,0%	25,9%	61,3%
2YPF									
- Altman	2,6750	19,8%	82,2%	51,0%	-0,0007	98,9%	0,6%	49,8%	-7,3%
- Bilderbeek	0,0250	13,7%	54,4%	34,0%	0,6388	24,3%	38,9%	31,6%	46,8%
- **Ooghe-Verbaere**	0,1663	18,3%	42,6%	**30,5%**	-0,0125	21,9%	38,2%	30,1%	51,7%
- Zavgren	n.a.	n.a.	n.a.	n.a.	0,0896	54,1%	31,7%	42,9%	6,0%
- **Gloubos-Grammaticos logit**	0,5000	9,5%	53,3%	31,4%	0,0342	27,5%	27,8%	**27,6%**	**56,3%**
- Gloubos Grammaticos Discriminant	0,0000	6,6%	62,0%	34,2%	-1,1417	21,6%	40,1%	30,8%	43,6%
- Keasy-McGuiness	n.a.	n.a.	n.a.	n.a.	4,1524	36,2%	33,4%	34,8%	37,2%
3YPF									
- Altman	2,6750	20,7%	81,2%	50,9%	0,0084	98,0%	0,9%	49,5%	-8,3%
- Bilderbeek	0,0250	17,4%	55,3%	36,3%	0,7225	33,7%	36,5%	35,1%	38,3%
- Ooghe-Verbaere	0,3355	19,3%	49,2%	34,2%	0,0086	27,3%	39,0%	33,1%	44,0%
- Zavgren	n.a.	n.a.	n.a.	n.a.	0,2321	50,2%	21,7%	36,0%	24,5%
- Gloubos-Grammaticos Logit	0,5000	16,8%	52,9%	34,8%	0,1359	24,9%	39,4%	**32,1%**	43,9%
- Gloubos-Grammaticos Discriminant	0,0000	12,0%	61,5%	36,7%	-0,8372	23,5%	46,2%	34,8%	33,3%
- Keasy-McGuiness	n.a.	n.a.	n.a.	n.a.	1,2512	32,0%	44,2%	38,1%	28,0%
- Ooghe-Joos De Vos	0,2137	23,5%	42,9%	**33,2%**	0,2430	28,1%	37,4%	32,8%	**44,9%**

In our study, we define a ‘new’ optimal cut-off point as the failure prediction score for which the unweighted average of type I and type II errors reaches a minimum. This is the most objective way of working for the comparison of different models. Allocation of weights to different types of errors, is subjective and depends on the risk aversion of the risk analyst. We also do not take population proportions into account either, because of the unbalanced proportion of sample sizes. The overrepresentation of non-failing companies would cause a too tolerant decision process.

The limitation of these decisions, is that some strange outcomes can be noticed. In the Altman model 1ypf for the bigger companies (complete form) e.g., a lower unweighted error rate is reached if the original cut-off point 2,6750 is updated to 0,0000. At the same time however, a strong shift from type II errors to type I errors occurs. In other words, with the new cut-off point more failing companies are misclassified and less good companies, which of course cannot be the aim of a good failure prediction model (see table 15).

Besides the unweighted error rate, we also concentrate on the trade-off function, as measured by the Gini-coefficient, to discuss the ‘fit’ of the models. This measure gives a more global judgement than the discussion of type I and type II errors separately, and is suited for the comparison of models validated on different samples.

It is remarkable that in almost all of the cases the performance results based on the unweighted error rate with the original cut-off point and with the new cut-off point and on the Gini-coefficient are indicating the same model as the best performing one, especially with the bigger companies with complete + abbreviated form 1ypf. In the 2 and 3ypf cases, the differences between the two best performing models (Ooghe-Verbaere and Gloubos-Grammaticos logit for 2ypf and Gloubos-Grammaticos logit and Ooghe-Joos-De Vos for 3ypf) are rather small.

6.3. Results for the bigger companies with complete form annual accounts (table 15)

Using the original cut-off point for each model under consideration, the best-performing model 1ypf is Ooghe-Verbaere, with an unweighted error rate of 20,7%. When a ‘new’ optimal cut-off point is calculated, the Ooghe-Verbaere model still has the lowest unweighted error rate (19,2%). The same conclusion can be made when looking at the Gini-coefficient of the different models. With a Gini-ratio of 74,2%, the Ooghe-Verbaere model is clearly the best suited model for short-term failure prediction of bigger Belgian companies with complete form annual accounts, on the basis of our validation sample.

The unweighted error rates range from 20,7% to 51,4% using the original cut-off point of the different models and from 19,2% to 48,3% using the newly calculated cut-off point. The Altman-model is clearly the worst-performing model. Of the eight failure prediction models under consideration, it has the lowest unweighted error rates, both for the original as for the new cut-off point. The validation on a sample of failing and non-failing Belgian companies with complete form annual accounts leaves the Altman-model with a negative Gini-coefficient. This means that more companies are falsely classified than correctly.

The results for the 2ypf validation show a somewhat different view. The Gloubos-Grammaticos (logit and discriminant) models have the lowest unweighted error rates and the highest Gini-coefficients and the Altman-model is still the worst performing model.

The 3ypf research reveals similar results as the 1ypf validation. The Ooghe-Verbaere model is the best-performing model. Once again the worst performing model is Altman, based on the lowest Gini-coefficients, and the Zavgren-model based on the unweighted error rate when a new cut-off point is used.

6.4. Results for all (bigger and smaller) companies with complete and abbreviated form annual accounts (table 16)

The results for all companies (complete and abbreviated form annual accounts) are similar to the results for the bigger companies (complete form). Ooghe-Verbaere is the best performing model 1ypf and Altman is the worst performing model.

Two years prior to failure Gloubos Grammaticos logit is the best performing model with an unweighted error rate of 27,6% when using the new cut-off point. Once again Altman is the worst performing model with the heighest unweighted error rates and the lowest Gini-coefficient.

The same goes for the 3ypf validation. In this case the Ooghe-Joos-De Vos model seems the best model to discriminate between failing and non-failing companies, based on the highest Gini-coefficient and the logit model Gloubos-Grammaticos based on the lowest unweighted error rate with a new cut-off point.

6.5. Graphical presentation of the research results

In section 3.2. we showed that the performance of a failure prediction model can be demonstrated graphically with the construction of a trade-off function. In figures 2 to 7, the trade-off functions for the eight failure prediction models are plotted for the two types of samples: bigger firms with complete form annual accounts (figure 2 to 4) and all companies (bigger and smaller) with complete + abbreviated form annual accounts (figure 5 to 7). This was done for the three years prior to failure.

Insert figures 2 to 7

Figures 2 to 7 show that the relative performance order of the investigated models is not very different for **the number of years prior to failure**, or **the type of account form (complete or abbreviated)**. Roughly 3 categories can be distinguished: Ooghe-Verbaere, Ooghe-Joos-De Vos and Gloubos-Grammaticos as the best performing models, Bilderbeek, Keasy-McGuiness as mid-category and Altman and Zavgren as the least performing.

When measuring performance, not only the fit of models is important, but also the ease of use and the capacity to judge as many companies as possible. We mentioned earlier that it was not possible to validate all models on the same sample, because some models contain ratios having a denominator with a zero value. This is especially the case for the Zavgren-model that contains ratios with respectively turnover, stocks, capital and current assets in the denominator. The exact percentages of the samples that could be used in our research are given in section 5 about population and samples. Models with a ratio having ‘turnover’ in the denominator cannot be applied for almost 50% of the companies in our research because of the Belgian accounting law. This law makes a difference between the regulations for complete form annual accounts of bigger companies and abbreviated form annual accounts of smaller companies. One of the differences is that companies publishing their results in an abbreviated form only have to mention the ‘gross margin’ and not the turnover and related costs.

6.6. Differences in performance and possible explanations

Tables 15 and 16 clearly show that the range of performances differs for the models 1, 2 and 3 years prior to failure. For the validation 1 year prior to failure, the Gini-coefficients have values from -8.3% to 74,2% (complete form) and from -7,9 % to 68,7% (complete + abbreviated form). In this range, the performances are ‘dispersed’.

For the validation 3 years prior to failure, the Gini-coefficients have values from 3,6% to 46,6% (complete form) and –8,3% to 44,9% (complete + abbreviated form). Not only is the range of Gini-coefficients smaller, but there is also practically no distance between the six ‘best’ models.

The global performance of the ‘best models’ in the 3ypf research is also less than those in the 1ypf research. One of the reasons for this is the fact that specific features of failing companies are less pronounced three years than one year before failure. Therefore it is more difficult to distinguish both categories and to define discriminating variables and models.

How could these differences in performance between the eight models under consideration be explained? Possible explanations of this phenomenon include:

- Age of the model;
- Company size: small versus big enterprises;
- Modelling technique: logit versus discriminant analysis;
- Number of variables;
- Complexity of variables.
- Estimation on Anglo-Saxon or European companies;

First of all, it is stated sometimes that the performance of a failure prediction model is inversely related with the age of the model. This would mean that the classification results of recent models are better than those of older models. The models in our study do not support this view entirely. The oldest model in our study, the Altman-model (1968) indeed shows the worst overall performance. More recent models like the Zavgren-model (1985) and Keasy-McGuiness (1990) however, are the second and third worst performers. The Ooghe-Verbaere model (1982) on the other hand is, together with Gloubos-Grammaticos Logit (1988), the best performer and both models have overall better classification results than the Ooghe-Joos-De Vos model (1991). Based on our comparison of eight failure prediction models, the age of the models does not explain the differences classification results.

A second possible explanation for the differences in performance could be the size of the failing and non-failing firm for which the models were designed to discriminate between. Some models were designed only to discriminate between bigger (failing and non-failing) companies. The estimation sample for the Ooghe-Verbaere model for example only consists of Belgian enterprises publishing their results in a complete form. However there seems to be no correlation with performance. Even when validated on abbreviated form annual accounts, this model is among the best performing models. For the validation 1ypf (see table 16) this model has better performance than the Ooghe-Joos-De Vos model which was originally designed to discriminate between failing and non-failing, bigger and smaller companies with both complete and abbreviated form of annual accounts.

Third, it is possible that the used estimation technique determines the classification results. More recent techniques (like logistic regression) are usually believed to be more discriminating between failing and non-failing companies than older techniques like discriminant analysis. Numerous empirical studies have tried to determine the most effective empirical method for prediction (e.g. Zavgren, 1983, Jones, 1987). In a recent study Laitinen and Kankaanpää (1998) study the six most popular failure prediction techniques (linear discriminant analysis, logit analysis, recursive partitioning, survival analysis, neural networks and human information processing) to test whether the failure prediction accuracy can be increased by using alternative methods. They find no superior method: "even one of the latest applications, neural networks, is in its present form only as effective as discriminant analysis was as early as thirty years ago". Our validation confirms this. Although the logit model by Gloubos and Grammaticos is better performing than their discriminant model, a similar relation is not true for the Ooghe-Joos-De Vos model and the Ooghe-Verbaere model. The former is a logit model that generally doesn't perform better than the discriminant model by Ooghe and Verbaere. Only in the 3ypf validation for all companies with complete + abbreviated annual accounts, the logit model performs better than the discriminant model.

A fourth possible explanation could be the number and complexity of the variables used in the various models. One could assume that the number of variables used in the model is positively correlated with the model's ability to discriminate between failing and non-failing companies. However, there seems to be no clear correlation between the number and complexity of variables and the fit of the models. The most obvious example is the comparison of Gloubos-Grammaticos (logit) and Ooghe-Joos-De Vos, which belong to the best performers. The former contains 3 'basic' ratios with the same coefficients for 1-3 years prior to failure whereas Ooghe-Joos-De Vos contains 12 more 'sophisticated' ratios, some of them derived from a specific failure prediction theory, in a different combination and with other coefficients for the model 1 and 3 years prior to failure. The reason for this is probably that the more sophisticated and specific a model, the better the internal validity, but not necessarily the external validity. The same phenomenon is described in publications in which basic statistical methods are compared with advanced neural networks: complexity is no guarantee for robustness in time[3].

The previously suggested possibilities all have failed to explain the wide range of failure prediction performances in our validation. If we take a look at the nationality of the estimation sample of the investigated models, there seems to be a clear difference between the performance of models estimated on Anglo-Saxon companies and models estimated on (continental) European companies. The models with a population of USA or UK companies (Altman, Zavgren and Keasy-McGuiness) are the least performing when validated on a sample of Belgian companies. On the other hand models estimated on a population of European companies (Bilderbeek, Gloubos-Grammaticos logit and discriminant, Ooghe-Verbaere and Ooghe-Joos-De Vos) are better able to discriminate between failing and non-failing Belgian companies. Intuitively, this outcome is quite logical as we may expect that models that were estimated on a Belgian sample have greater predictive ability when validated on a sample on Belgian companies than, let us say, a sample of American companies. However a strange outcome can be noticed when looking at the performance results of the Greek Gloubos-Grammaticos logit model. When new cut-off points are calculated, this model performs in several cases even better on our sample of Belgian companies, than on their own validation sample.

[3] Laitinen and Kankaanpää (1999) rank six estimation techniques according to classification accuracy, both for their estimation sample (ex post) and for a validation sample (ex ante) of Finnish data one, two and three years prior to failure. The ranking of the six estimation techniques based on their ex post classification results differs significantly from the ranking based on their ex ante classification results, indicating that better internal validity does not guarantee better prediction ability or external validity.

7. SUMMARY AND CONCLUSIONS

In this study we have validated eight international failure prediction models on one data set of Belgian company accounts. All of these eight models use one of two basic modelling techniques in failure prediction research, i.e. linear discriminant analysis or logistic regression. The performance indicators we used to evaluate the predictive ability of the scoring models were of two different types. We discussed type I and type II errors based on old and new cut-off points. On the other hand we compared the models in a more global way with Gini-coefficients.

As our goal was not the re-estimation, but a large and global validation of international models, we worked with populations and samples as large as possible. Eight failure prediction models were validated on two data sets: one data set of complete form annual accounts of failing and non-failing companies and one data set of complete + abbreviated form annual accounts.
The results from the validation on complete form annual accounts indicate that the Ooghe-Verbaere model which was estimated on complete form annual accounts is the best performing model, one and three years prior to failure. Surprisingly, the Greek Gloubos-Grammaticos (logit and discriminant) models have the lowest unweighted error rates and the highest Gini-coefficients, two years prior to failure. The results from the complete and abbreviated form annual accounts research are similar to the results from the complete form research. The logit model that was estimated on Belgian complete and abbreviated form annual accounts (Ooghe-Joos-De Vos) was the best-performing model three years prior to failure. However, one year prior to failure the older Ooghe-Verbaere discriminant model shows the lowest unweighted error rate. Once again the Gloubos-Grammaticos (logit) model is the best performing model two years prior to failure.

The graphical presentation of the research results shows a wide range of performances of the failure prediction models. Roughly three categories could be distinguished: Ooghe-Verbaere, Ooghe-Joos-De Vos and Gloubos-Grammaticos as the best performing models, Bilderbeek, Keasy-McGuiness as mid-category and Altman and Zavgren as the least performing.

Several explanations were suggested for these differences in performance: nationality of the estimation sample, age of the model, company size, modelling technique, number and complexity of variables. Our validation suggests that the nationality of the estimation sample is most relevant to explain the differences in performance. Models estimated on a population of European companies are better able to discriminate between failing and non-failing Belgian companies than models estimated on Anglo-Saxon companies.

A remarkable finding however, is that the Greek Gloubos-Grammaticos models seem to have better predictive ability when validated on samples of Belgian failing and non-failing companies than on their own (Greek) validation sample. Another important finding is the robustness of the older discriminant models (e.g. Ooghe-Verbaere). The size of the companies for which the models were estimated does not seem to have a strong impact on the predictive ability of these models. Models that were estimated on big companies are found to be better failure predictors than some models that were designed for both small and big firm failure prediction. Finally, there seems to be no clear correlation between the number and complexity of the variables included in the model and the fit of the models. The validation shows that very simple models can have great predictive ability.

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APPENDIX 1: Tables

Table 1: Characteristics of the models under investigation

	Altman 1968	Bilderbeek 1972	Ooghe-Verbaere 1982	Zavgren 1985	Gloubos-Grammatikos 1988	Keasy-McGuiness 1990	Ooghe-Joos-Devos 1991
Country	United States	The Netherlands	Belgium	United States	Greece	United Kingdom	Belgium
Population	American Industrial companies	Dutch industrial and trade companies	Belgian enterprises publishing their accounts in a complete form	American companies listed on the Stock Exchange with annual accounts available on Compustat tapes	Greek enterprises	UK companies with data available on Datastream	Belgian enterprises publishing their results in a complete or abbreviated form
Period	1946-1965	1950-1975	1977-1980	1972-1978	1977-1985	1976-1984	1985-1990
Definition of failure	Declaration of bankruptcy by court	Declaration of bankruptcy by court	Declaration of bankruptcy by court or request of legal composition	Request of chapter 10 or 11 from the bankruptcy law	Declaration of bankruptcy by court Failed companies sustained in operation by the Greek government are excluded from the non-failed sample	Declaration of bankruptcy by court	Declaration of bankruptcy by court or request of legal composition
Sample							
Sound	33 annual accounts	43 (original) 220 (validation)	753 (original) 347 (validation)	45 annual accounts	30 (original) 24 (validation)	43 (original) 15 (validation)	347 (original) 170 (validation)
Failed	33 annual accounts	40 (original) 127 (validation)	395 (original) 268 (validation)	45 annual accounts	30 (original) 24 (validation)	43 (original) 15 (validation)	268 (original) 218 (validation
Method	matched on industry and size of total assets	matched on industry turnover, size of total assets and numbers of employees	random selection	matched on industry and size of total assets	matched on industry and total assets	matched on industry and size of net assets	systematic selection
Estimation technique	Linear discriminant analysis	Linear discriminant analysis	Linear discriminant analysis	Logistic regression	Mutiple discriminant, logit & probit analysis and linear probability models	Logistic regression	Logistic regression
Number of variables	5	5	5 (for each model)	10	5 (discriminant analysis) 3 (logit)	10	11
Model	one model applicable to data 1-5 ypf	one model (and two derived classification functions) applicable to data 1-5 ypf	three models, 1 - 3 ypf and one general model each with different variables & coefficients	models, 1 - 5 ypf each with different variables & coefficients	one model, applicable to data 1-3 ypf (based on data 1 ypf)	models, 1 - 5 ypf, each with different variables and coefficients	models, 1 - 3 ypf, each with different variables and coefficients
Classification rule	cut-off score	cut-off score	cut-off score	cut-off point and entropy	cut-off score	cut-off score and entropy	cut-off score

Table 2: Altman, 1968

<table>
<tr><th></th><th>Variables</th><th>Codes complete form</th><th>Codes abbreviated form</th><th>Coeff 1ypf</th><th>Coeff 2ypf</th><th>Coeff 3ypf</th></tr>
<tr><td>X1</td><td>Working capital/Total assets</td><td>(|29/58| - |29| - |42/48| - |492/3|) / |20/58|</td><td>id.</td><td>+0.012</td><td>id.</td><td>id.</td></tr>
<tr><td>X2</td><td>Retained earnings/Total assets</td><td>(|13| + |140| - |141|) / |20/58|</td><td>id.</td><td>+0.014</td><td>id.</td><td>id.</td></tr>
<tr><td>X3</td><td>Earnings before interest and taxes/Total assets</td><td>(|70/67| - |67/70| + |9134| + |650| + |653| - |9126|) / |20/58|</td><td>(|70/66| - |66/70| + |780| - |680| - <65> - |9126| - <656>)</td><td>+0.033</td><td>id.</td><td>id.</td></tr>
<tr><td>X4</td><td>Market value equity/Book value of total debt</td><td><10/15> / (|16| + |17/49|)</td><td>id.</td><td>+0.006</td><td>id.</td><td>id.</td></tr>
<tr><td>X5</td><td>Sales/Total assets</td><td>(|70| + |74| - |740|) / |20/58|</td><td>|70| / |20/58|</td><td>+0.999</td><td>id.</td><td>id.</td></tr>
<tr><td>CO</td><td>Cut-off point</td><td></td><td></td><td>2.675</td><td>2.675</td><td>2.675</td></tr>
</table>

Table 3: Bilderbeek, 1979

<table>
<tr><th></th><th>Variables</th><th>Codes complete form</th><th>Codes abbreviated form</th><th>Coeff</th></tr>
<tr><td></td><td>Intercept</td><td></td><td></td><td>+0.45</td></tr>
<tr><td>X1</td><td>Net Profit/Shareholder’s Equity</td><td>(|70/67| - |67/70|)/<10/15></td><td>id.</td><td>+0.15</td></tr>
<tr><td>X2</td><td>Accounts Payable/Turnover</td><td>(|44|)/(|70| +|74| - |740|)</td><td>(|44|)/(|70|)</td><td>+4.55</td></tr>
<tr><td>X3</td><td>Turnover/Total assets</td><td>(|70| +|74| - |740|) / (|20/58|)</td><td>(|70|) / (|20/58|)</td><td>+0.17</td></tr>
<tr><td>X4</td><td>Added Value/Total assets</td><td>(|70/74| - |740| - |60| - |61|) / |20/58|</td><td>(|70/61| - |61/70|) / (|20/58|)</td><td>-1.57</td></tr>
<tr><td>X5</td><td>Accumulated Profits + Reserves/Total Assets</td><td>(|13| + |140| - |141|) / (|20/58|)</td><td>id.</td><td>-5.03</td></tr>
<tr><td>CO</td><td>Cut-off Point</td><td></td><td></td><td>0.025</td></tr>
</table>

Table 4: Ooghe-Verbaere, 1982

<table>
<tr><th></th><th>Variables</th><th>Codes complete form</th><th>Codes abbreviated form</th><th>Coeff</th></tr>
<tr><td colspan="5">1 year prior to failure</td></tr>
<tr><td></td><td>Intercept</td><td></td><td></td><td>+2.6803</td></tr>
<tr><td>X1</td><td>Overdue short-term priority debts /Short-term liabilities</td><td>(|9072| + |9076|) / (|42/48| + |492/3|)</td><td>id.</td><td>-51.3394</td></tr>
<tr><td>X2</td><td>Accumulated profits /Total liabilities</td><td>(|140| - |141|)/ (|10/49|)</td><td>id.</td><td>+10.0870</td></tr>
<tr><td>X3</td><td>Gross earnings before Interests and taxes/Total Assets</td><td>(|70/74| + <60/64> + |630| + <631/4>+ <635/7> + |75| - |9125| - |9126| - <652/9> + |653| + |6560| - |6561| + |763| + |764/9| + |77| - |664/8| + |669| - |9138|) /(|20/58|)</td><td>(|70/66| - |66/70| - <65> - |9126| + |8079| - |8089|+ |8279| - |8289| + |8475| -|8485| - <631/4> - <635/7> - |9125|) / |20/58|</td><td>+4.4145</td></tr>
<tr><td>X4</td><td>Equity Capital/Total Liabilities</td><td>(<10/15>) / (|10/49|)</td><td>id.</td><td>+2.0318</td></tr>
<tr><td>X5</td><td>Cash/Current Assets</td><td>(|54/58|) / (|29/58| - |29|)</td><td>id.</td><td>+2.6314</td></tr>
<tr><td>CO</td><td>Cut-off Point</td><td colspan="2"></td><td>3.1492</td></tr>
<tr><td colspan="5">2 years prior to failure</td></tr>
<tr><td></td><td>Intercept</td><td></td><td></td><td>+0.1837</td></tr>
<tr><td>X1</td><td>Accumulated Profits + Reserves /Total Liabilities</td><td>(|140| - |141|)/ (|10/49|)</td><td>id.</td><td>+4.6524</td></tr>
<tr><td>X2</td><td>Overdue Short-Term Priority Debts /Short-Term Liabilities</td><td>(|9072| + |9076|) / (|42/48| + |492/3|)</td><td>id.</td><td>-16.5456</td></tr>
<tr><td>X3</td><td>Cash/Current Assets</td><td>(|54/58|) / (|29/58| - |29|)</td><td>id.</td><td>+3.2732</td></tr>
<tr><td>X4</td><td>Stock goods in course of production, waste products, finished products/Current working assets</td><td>(|32| + |33| + |37|) / (|3| + |40/41| + |490/1|)</td><td>|3| / (|3| + |40/41| + |490/1|)</td><td>-1.7381</td></tr>
<tr><td>X5</td><td>Cash Flow / Sales</td><td>(|70/67| – |67/70| +| 630| + <631/4> + <635/7> + |6501| + <651> + |6560| – |6561| + |660| + |661| + <662> + |663| + |680| – |760| – |761| – |762| – |780| – |9125|) / (|70| + |74| –|740|)</td><td>(|70/67| - |67/70|+<656> -|780| + |680| +|8079| -|8089| + |8279| - |8289| + |8475| - |8485| - <631/4> - <635/7> - |9125|) / |70|</td><td>+0.0738</td></tr>
<tr><td>CO</td><td>Cut-off Point</td><td></td><td></td><td>0.1663</td></tr>
<tr><td colspan="5">3 years prior to failure</td></tr>
<tr><td></td><td>Intercept</td><td></td><td></td><td>+0.2153</td></tr>
<tr><td>X1</td><td>Overdue Short-Term Priority Debts /Short-Term Liabilities</td><td>(|9072| + |9076|) / (|42/48| + |492/3|)</td><td>id.</td><td>-18.3474</td></tr>
<tr><td>X2</td><td>Accumulated Profits + Reserves /Total Liabilities</td><td>(|13|+ |140| - |141|)/ (|10/49|)</td><td>id.</td><td>+3.3847</td></tr>
<tr><td>X3</td><td>Cash/Current Assets</td><td>(|54/58|) / (|29/58| - |29|)</td><td>id.</td><td>+2.3601</td></tr>
<tr><td>X4</td><td>Stock goods in course of production, waste products, finished products/Current working assets</td><td>(|32| + |33| + |37|) / (|3| + |40/41| + |490/1|)</td><td>|3| / (|3| + |40/41| + |490/1|)</td><td>-1.9230</td></tr>
<tr><td>X5</td><td>Net earnings/Equity Capital + Long-Term Liabilities</td><td>(|70/67| – |67/70| + |9134| + |650| + |653| – |9126|) / (<10/15> + |16| + |17|)</td><td>(|70/66| - |66/70| + |780| - |680| - <65> - |9126| - <656>) /(<10/15> + |16| + |17|)</td><td>+0.0617</td></tr>
<tr><td>CO</td><td>Cut-off Point</td><td></td><td></td><td>0.3355</td></tr>
</table>

Table 5: Zavgren, 1985

	Variables	**Codes complete form**	**Codes abbreviated form**	**coeff 1ypf**	**coeff 2ypf**	**coeff 3ypf**
	Intercept			-0.23883	-2.61060	-1.51150
X1	Inventory/Sales	|3| / (|70| + |74| - |740|)	|3| / |70|	+0.00108	+0.04185	+0.06257
X2	Receivables/Inventory	(|29| + |40/41|) / |3|	id.	+0.01583	+0.02215	+0.00829
X3	Cash/Total Assets	(|50/53| + |54/58|) / |20/58|	id.	+0.10780	+0.11231	+0.4248
X4	Quick Assets/Current Assets (Acid test)	(|40/41| + |50/53| + |54/58|) / |42/48|	id.	-0.03074	-0.02690	-0.01549
X5	Total Income/Total Capital	(|70/67| - |67/70|) / <10/15>	id.	-0.00486	-0.01440	+0.00519
X6	Debt/Total Capital	(|16| + |17| + |42/48|) / <10/15>	id.	+0.04350	+0.04464	+0.01822
X7	Sales/Net Plant	(|70| + |74| - |740|) / |20/58|	|70| / |20/58|	-0.00110	+0.00063	+0.00002
CO	Cut-off Point				n.a.	

Table 6: Gloubos and Grammatikos, 1988

	Variables	**Codes complete form**	**Codes abbreviated form**	**Coeff**
		Discriminant analysis		
	Intercept			+4.423
X1	Current Assets/Current Liabilities	(|29/58| - |29|) / (|42/48| + |492/3|)	id.	-2.044
X2	Net Working Capital/Total Assets	(|29/58| - |29| - |42/48| + |492/3|) / (|20/58|)	id.	+4.421
X3	Total Debt/Total Assets	(|16|+|17|) / (|20/58|)	id.	-4.404
X4	Gross Income/Total assets	(|70/74| + <60/64> + |630| + <631/4> + <635/7> + |75| – |9125| – |9126| - <652/9> + |653| + |6560| – |6561| + |763| +|764/9| + |77| – |664/8| + |669| – |9138|) / (|20/58|)	(|70/66| – |66/70| – |65|– |9125| – |9126| - <631/4> - <635/7> + |8079| – |8089| + |8279| – |8289| + |8475| – |8485|) / (|20/58|)	-2.778
X5	Gross Income/Current Liabilities	(|70/74| + <60/64> + |630| + <631/4> + <635/7> + |75| – |9125| – |9126| - <652/9> + |653| + |6560| – |6561| + |763| +|764/9| + |77| – |664/8| + |669| – |9138|) / (|42/48| + |492/3|)	(|70/66| – |66/70| – |65|– |9125| – |9126| - <631/4> - <635/7> + |8079| – |8089| + |8279| – |8289| + |8475| – |8485|) / (|42/48| + |492/3|)	+4.423
CO	Cut-off point			**0**
		Logistic regression		
	Intercept			+3.548
X1	Net working Capital/Total Assets	(|29/58| - |29| - |42/48| + |492/3|) / (|20/58|)	id.	+5.585
X2	Total Debt/Total Assets	(|16|+|17|) / (|20/58|)	id.	-8.504
X3	Gross Income/Total assets	(|70/74| + <60/64> + |630| + <631/4> + <635/7> + |75| – |9125| – |9126| - <652/9> + |653| + |6560| – |6561| + |763| +|764/9| + |77| – |664/8| + |669| – |9138|) / (|20/58|)	(|70/66| – |66/70| – |65|– |9125| – |9126| - <631/4> - <635/7> + |8079| – |8089| + |8279| – |8289| + |8475| – |8485|) / (|20/58|)	+13.070
CO	Cut-off point			**0.5**

Table 7: Keasy and McGuiness, 1990

<table>
<tr><th></th><th>Variables</th><th>Codes complete form</th><th>Codes abbreviated form</th><th>Coeff</th></tr>
<tr><td colspan="5">1 year prior to failure</td></tr>
<tr><td></td><td>Intercept</td><td></td><td></td><td>+0.0881</td></tr>
<tr><td>X1</td><td>Capital Gearing</td><td>(|10/15|) / (|16| + |17/49|)</td><td>id.</td><td>+0.0316</td></tr>
<tr><td>X2</td><td>Creditors Turnover</td><td>(|600/8| + |61| + |9145|) /|44|</td><td>|60/61| / |44|</td><td>-0.2710</td></tr>
<tr><td>X3</td><td>Pre tax profit margin</td><td>(|70/67| - |67/70| + |9134|) / (|70| + |74| - |740|)</td><td>(|70/67| - |67/70| - <67/77>)/ |70|</td><td>-0.3227</td></tr>
<tr><td>CO</td><td>Cut-off point</td><td></td><td></td><td>n.a.</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="5">2 years prior to failure</td></tr>
<tr><td></td><td>Intercept</td><td></td><td></td><td>+3.3612</td></tr>
<tr><td>X1</td><td>Inventory/Sales</td><td>|3|/ (|70| + |74| - |740|)</td><td>|3| /|70|</td><td>+8.4286</td></tr>
<tr><td>X2</td><td>Working Capital Ratio</td><td>(|29/58| - |29| - |42/48| - |492/3|) / |20/58|</td><td>id.</td><td>-2.7244</td></tr>
<tr><td>X3</td><td>Return on capital employed</td><td>(|70/74| + <60/64> + |9125|) / (|20| + |21| + |22/27| + |3| + |40/41| + |490/1|)</td><td>(|70/64| - |64/70| + |9125|) / (|20| + |21| + |22/27|+ |3| + |40/41| + |490/1|)</td><td>-0.1081</td></tr>
<tr><td>X4</td><td>Return on shareholders equity</td><td>(|70/67| - |67/70|) / <10/15></td><td>id.</td><td>-0.01947</td></tr>
<tr><td>CO</td><td>Cut-off point</td><td></td><td></td><td>n.a.</td></tr>
<tr><td colspan="5">3 years prior to failure</td></tr>
<tr><td></td><td>Intercept</td><td></td><td></td><td>+6.4202</td></tr>
<tr><td>X1</td><td>Quick Assets Ratio</td><td>(|40/41| + |50/53| +|54/58|) / |42/48|</td><td>id.</td><td>-1.5599</td></tr>
<tr><td>X2</td><td>Creditors Turnover</td><td>(|600/8| + |61| + |9145|) / |44|</td><td>|60/61| / |44|</td><td>-0.3010</td></tr>
<tr><td>X3</td><td>Turnover/Net Plant</td><td>(|70| + |74| - |740|) / |20/58|</td><td>|70| / |20/58|</td><td>-0.8799</td></tr>
<tr><td>X4</td><td>Pre tax profit margin</td><td>(|70/67| - |67/70| + |9134|) / (|70| + |74| - |740|)</td><td>(|70/67| - |67/70| - <67/77>)/ |70|</td><td>-0.4216</td></tr>
<tr><td>CO</td><td>Cut-off point</td><td></td><td></td><td>n.a.</td></tr>
</table>

Table 8: Ooghe – Joos - De Vos, 1991

<table>
<tr><th></th><th>Variables</th><th>Codes complete form</th><th>Codes abbreviated form</th></tr>
<tr><td colspan="4">1 year prior to failure</td></tr>
<tr><td>X1</td><td>Direction of the Financial Leverage (1 if > 0, 0 if <0)</td><td>{(|70/66| - |66/70| + |780| - |680| - <65> -|9126|) / |20/58|} - {(-<65> - |9126| - |6560| + |6561|) / (|17| + |42/48|)}</td><td>{(|70/66| - |66/70| + |780| - |680| - <65> - |9126|) / |20/58|} - {(-<65> - |9126| - <656>) /(|17| + |42/48|)}</td></tr>
<tr><td>X2</td><td>Accumulated Profits + Reserves/Total Liabilities less deferrals and accruals</td><td>(|13| + |140| - |141|) / (|10/49| - |492/3|)</td><td>id.</td></tr>
<tr><td>X3</td><td>Cash / Total Assets</td><td>(|51/53| + |54/58|) / |20/58|</td><td>(|50/53| + |54/58| - |8721|) / |20/58|</td></tr>
<tr><td>X4</td><td>Overdue Short-Term Priority’s Debts (1 if >0, 0 else)</td><td>(|9072| + |9076|)</td><td>id.</td></tr>
<tr><td>X5</td><td>Operational Net Working Capital / Total Assets</td><td>(|3| + |40/41| - |44| - |45| - |46|) / (|20/58|)</td><td>id.</td></tr>
<tr><td>X6</td><td>Net Operating Result / Working Assets</td><td>(|70/64| - |64/70| + |9125|) / (|20| +|21| + |22/7| + |3| + |40/41|)</td><td>id.</td></tr>
<tr><td>X7</td><td>Financial Debts (Short Term)/Short-term Liabilities</td><td>(|430/8|) / (|42/48|)</td><td>id.</td></tr>
<tr><td>X8</td><td>Guaranteed Portion of Amounts Payable by the Firm</td><td>(|9061| + |9062|) / (|17| + |42/48|)</td><td>id.</td></tr>
<tr><td>CO</td><td>Cut-off Point : 0.6883</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="4">3 years prior to failure</td></tr>
<tr><td>X1</td><td>Accumulated Profits + Reserves/Total Liabilities</td><td>(|13| + |140| - |141|) / (|10/49| - |492/3|)</td><td></td></tr>
<tr><td>X2</td><td>Publication lag</td><td></td><td></td></tr>
<tr><td>X3</td><td>Overdue Short-Term Priority’s Debts (1 if >0, 0 else)</td><td>(|9072| + |9076|) 1 if >0, else 0</td><td></td></tr>
<tr><td>X4</td><td>Operational Cash-flow before Taxes – Capital Investments/Total Assets</td><td>{(|70/66|- |66/70| - <65> - |9126| - <631/4> + <635/7> + |807| - |808| + |827 | - |828| + |847| -|848| - |860| - |861| - |9125|) - (|816| - |817| + |822| - |823| - |829| + |830| + |836| - |837| + |842| +| 843| -|849| + |850| - <8545> + |858| - |859|)} / (|20/58|)</td><td>{(|70/66|- |66/70| - <65> - |9126| - <631/4> - <635/7> + |8079| - |8089| + |8279| - |8289| + |8475| - |8485| - |9125|) - (|8169| - |8179| + |8229| - |8239| - |8299| + |8309| + |8365| - |8375| + |8425| - |8435| - |8495| + |8505| - <8545>)} / (20/58)</td></tr>
<tr><td>X5</td><td>Relationships with Affiliated Enterprises</td><td>(|9291| + |9381| + |9401|) / (|20/58|)</td><td>(|9291| + |9294| + |9295|) / (|20/58|)</td></tr>
<tr><td>X6</td><td>Debt/Total Liabilities</td><td>(|17| + |42/48|) / (|10/49| - |492/3|)</td><td>id.</td></tr>
<tr><td>CO</td><td>Cut-off Point: 0.7863</td><td></td><td></td></tr>
</table>

(*) coefficients can not be given

APPENDIX 2: TRADE-OFF FUNCTIONS 1 YPF, 2 YPF, 3YPF; COMPLETE & COMPLETE + ABBREVIATED FORM