Shifts in balanced growth and public capital
- an empirical analysis for Belgium -

Gerdie Everaert

Social Economics Research Group

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Correspondence to: Gerdie Everaert, University of Ghent, Hoveniersberg 4, 9000 Ghent, Belgium
Telephone: +32-(0)9-264.34.87
Telefax: +32-(0)9-264.35.92
e-mail gerdie.everaert@rug.ac.be

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Abstract

In the large literature analysing the contribution of public capital formation to productivity growth, the stance of technology has been unremittingly ignored. However, technological progress is one of the most important determinants of long-run growth in many macroeconomic variables. Output elasticities calculated from long-run - cointegrating - estimates omitting technology are therefore very likely to be subject to an important omitted variable bias. In a recent paper, Crowder and Himarios (1997) propose to identify technological progress in the data based on the neo-classical growth model's result that the long-run behaviour of the economy is uniquely determined by technology. Once this common long-run growth component is extracted from the data, the production function can be estimated as a 'period-by-period' constraint. In this paper, this approach is extended to allow for a permanent shift in the balanced growth path, implied by the structural reduction of public capital investment. The methodology is applied to Belgian data for the period 1953-96.

Key words: Balanced growth, public capital, cointegration, common trends, short-run production technology.

I INTRODUCTION

In a series of influential papers Aschauer (1989a, 1989b, 1989c) shows that the decline in public capital outlays observed in a major part of the OECD-countries since the early 1970s may be, to a large extent, responsible for the slowdown of productivity growth, which set in at about the same time. Expanding the conventional aggregate Cobb-Douglas production function with the public capital stock, Aschauer provides empirical evidence that a 1% decrease in the ratio of public to private capital stocks decreases multifactor productivity in the US by 0.39%.

In the large literature that sprung from Aschauer's work, a lot of possible defects in the initial methodology have been identified. One major problem emerges from the fact that all variables included in the production function show stochastic non-stationary behaviour. The finding of a unit root makes Aschauer's results, derived using level data, suspicious due to possible *spurious correlation* (see e.g. Tatom, 1991).

In trying to deal with this kind of non-stationarity, some authors have proceeded to check for cointegration using the residual-based ADF method in the sense of Engle and Granger (1987) or the maximum likelihood estimation procedure developed by Johansen (1988). Unfortunately, the results point in opposite directions. On the one hand, Tatom (1991) finds no evidence of cointegration in the US. Sturm and de Haan (1995) come to the same conclusion for both the US and the Netherlands. On the other hand, Bajo-Rubio and Sosvilla-Rivero (1993) find a clear cointegrating relationship in Spain. Applying the Johansen technique on data for the US, the UK, France and Germany, Clarida (1993) also finds a strong impact of public capital on multifactor productivity.

One problem that has been unremittingly ignored in all these studies is the treatment of the underlying rate of technological progress, which is an important determinant of economic growth. In fact, from the neo-classical growth model we learn that the per capita growth rates of output, investment and capital are exclusively determined by the rate of technological progress once the economy has reached its balanced growth path. Therefore, omitting technology in a cointegration framework will almost certainly yield coefficients that cannot be interpreted as output elasticities, i.e. the coefficients on capital stocks will tend to one for in the long-run capital and output grow at more or less the same rate[1]

Note that this remark is closely related to the critique of reverse causation, which states that public investment is a normal good, i.e. the public capital stock grows with increasing output. To the extent that technological shocks are first reflected in higher economic growth which in its turn affects the accumulation of factors of production, there may indeed be an important reverse causation problem, inducing an upward bias on the coefficient of public capital.

The reason why technological progress is easily omitted is that it is not directly measurable. In an attempt to deal with this problem, Everaert and Heylen (1998) use patent statistics as an approximation. Including this measure, they find a cointegrating relationship with an output elasticity of public capital around 0.29 and causality running from public capital to output. However, an important defect of this approach is the sensitivity of the results to the choice - to some extent arbitrary - of patent lifetime that one has to make to accumulate patent stock data.

In a recent paper Crowder and Himarios (1997) propose an alternative approach that relies on the ideas that (i) technological progress determines the long-run growth of the economy and (ii) the production function is a 'period-by-period' constraint that describes the short- to medium-term behaviour of the variables. Empirically, the approach boils down to analysing whether output and capital stocks cointegrate subject to the balanced growth restrictions from the neo-classical model. If these restrictions are valid, the common stochastic trend - i.e. the growth component - can be identified as technological progress. After filtering out this common trend from output and capital stock series, the production function can be estimated from the stationary data as a short-run constraint. Applying this methodology to US data, the authors find a strong confirmation of Aschauer's result.

One potential problem in Crowder and Himarios' approach is the assumption that the economy is on - or moves to - a balanced growth path which is held fixed over the whole estimation period. The observation that public investment has been structurally reduced

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[1] Note that some authors, e.g. Sturm and de Haan (1995), try to capture technological progress by including a linear trend in the production function. However, this implies looking for cointegration in linearly detrended data, which is clearly in contradiction with the results from unit root tests, indicating a stochastic rather than a linear trend.

implies a shift in the steady-state of the economy, though. A second problem lies in the fact that running unit root tests on capital stock series frequently reveals I(2) behaviour in small samples. Ho and S∅rensen (1994) show that the presence of I(2) components makes the Johansen maximum likelihood estimator, needed in the estimation of the common trends model, unreliable.

The purpose of this paper is to extend the methodology proposed by Crowder and Himarios by allowing for structural shifts in the steady-state growth path resulting from shifts in investment behaviour. In order to deal with the I(2) behaviour of capital stock data, we use investment data to extract the common trend. The analysis concentrates on Belgian data for 1953-96.

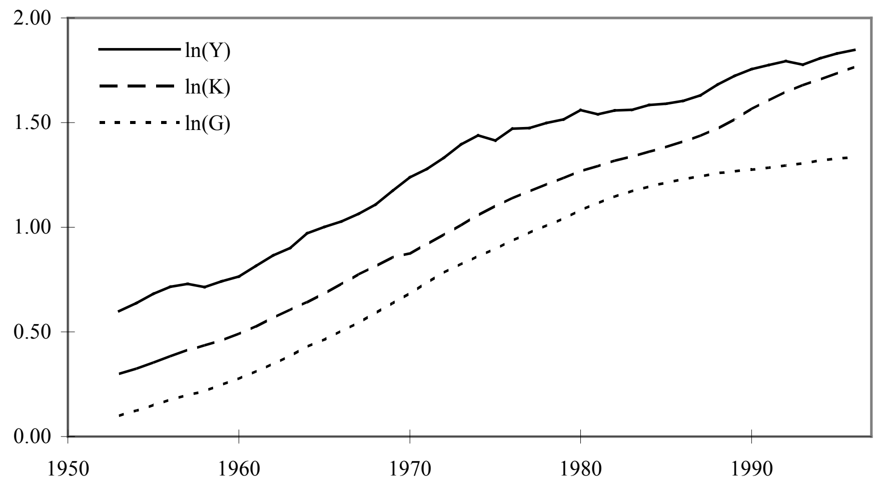
The remainder of this paper is organised as follows. The next section discusses the long-run growth properties of the neo-classical model and briefly confronts them with the data. Section three examines more thoroughly whether the data satisfy the balanced growth restrictions applying the Johansen maximum likelihood estimation procedure. The fourth section uses the results from the Johansen methodology to estimate the common trend in the data. Estimates of output elasticities are presented in the fifth section. The final section summarises and outlines some directions for future research.

II LONG-RUN PROPERTIES OF THE NEO-CLASSICAL GROWTH MODEL

One of the nice features of neo-classical growth models is that they all have strong implications concerning the long-run behaviour of the economy. It is a well-known result that once the economy has converged to its steady-state growth path, the growth rates of per capita output, investment, consumption and capital stocks should all be equal to the exogenous rate of technological progress. This common deterministic trend makes the ‘great ratios’ of consumption, investment and capital to output constant along the balanced growth path. King *et al*. (1991) point out that when uncertainty is added to the long-run behaviour of the economy - i.e. technological progress has a stochastic rather than a deterministic data generating process - output, consumption, investment and capital stocks exhibit common stochastic trends, implying the ‘great ratios’ to become stationary stochastic processes.

Figure 2.1 plots the logarithms of output (*Y*), gross private capital (*K*) and gross public capital (*G*). At first sight, the three variables display a broadly similar upward trend. In figure 2.2, we graph the ratios of private and public capital stocks to output. Unfortunately, both graphs do not show strong mean reverting behaviour. This raises doubts about the validity of the prediction that capital to output ratios are stationary stochastic processes. These doubts are confirmed by the results from Augmented Dickey-Fuller (ADF) tests, which clearly reveal the presence of a unit root in both processes[2].

Figure 2.1 Real output, gross private capital and gross public capital in Belgium (1953-96)[a]

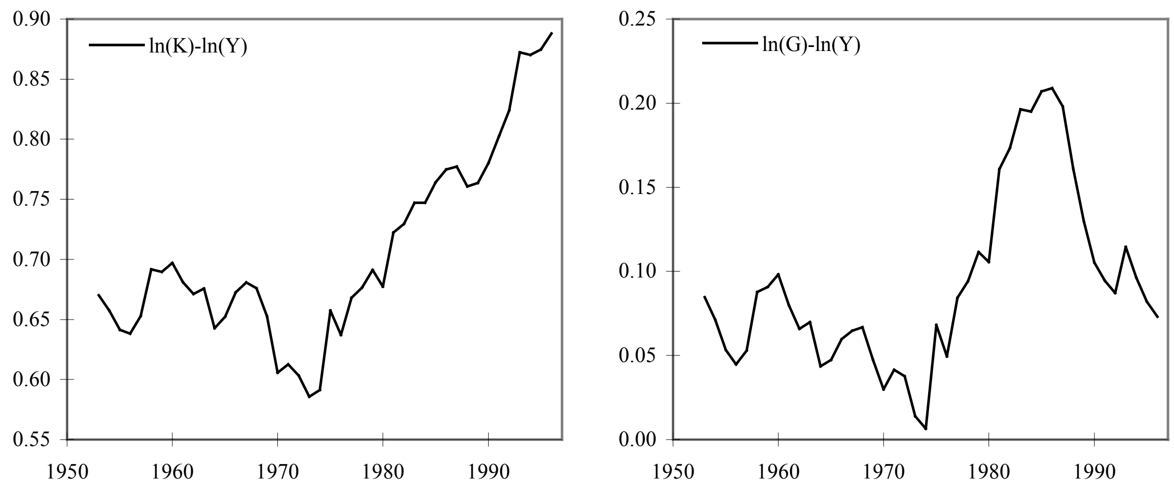


Sources: OECD statistical compendium 1998/1 and Belgian Federal Planning Bureau.
Notes: [a] Constants are added to the logarithms of the variables in order to enhance visual comparability.

Since the neo-classical model only predicts stationarity when the economy is on its steady-state growth path, one possible explanation for the apparent non-stationarity of capital to output ratios is that the economy has been hit by shocks causing permanent shifts in the steady-state. In order to see which variables may cause the steady-state to move, we analyse a simple neo-classical growth model with stochastic technological growth.

Figure 2.2 Private and public capital to output ratios in Belgium (1953-96)

2 The ADF(1) *t*-statistic for ln(*K*)-ln(*Y*) - with trend and intercept - has a value of -0.91 and a value of -1.30 for ln(*G*)-ln(*Y*). Both test-statisitcs are clearly not significant, suggesting a unit root in both processes. These results were found to be insensitive to alternative specifications of the ADF test.



Sources: see figure 2.1

Consider the following Cobb-Douglas production function, characterised by constant returns to scale over all inputs:

$$Y_t = K_t^{\alpha} G_t^{\beta} \left(A_t L_t \right)^{1-\alpha-\beta}, \tag{2.1}$$

with Y, K and G as defined above and L and A denoting labour input and technology respectively. Uncertainty is introduced by assuming that technology A is generated by the following logarithmic random walk with drift:

$$\ln(A_t) = g + \ln(A_{t-1}) + \varepsilon_t \,. \tag{2.2}$$

The drift term g determines the average rate of growth in A. Temporary deviations from this average are captured by the error term ε_t.

If K and G are generated as:

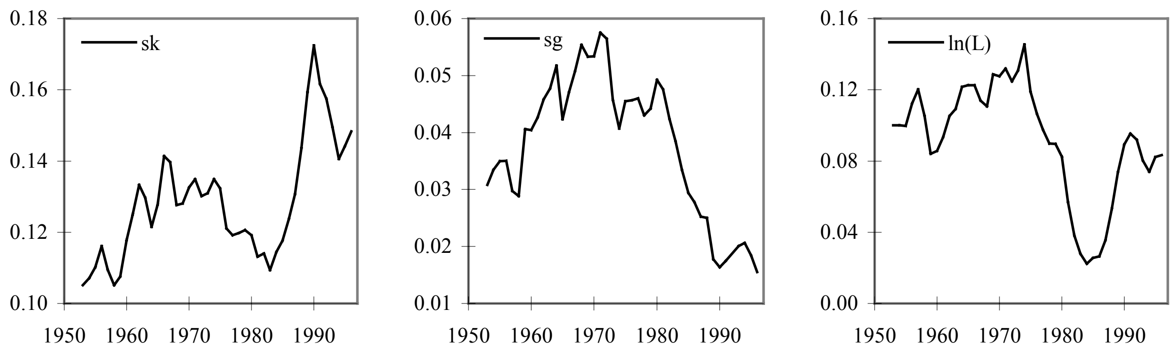
$$\frac{dK_t}{dt} = s^k Y_t - \delta^k K_t \qquad \text{and} \qquad \frac{dG_t}{dt} = s^g Y_t - \delta^g G_t , \tag{2.3}$$

with s^k and s^g denoting the shares of output invested in private and public capital respectively and δ^k and δ^g denoting depreciation rates, it is straightforward to show that in steady-state the economy should satisfy:

$$\left(\frac{K_t}{Y_t} \right)^* = \frac{s^k}{n+g+\delta^k} \qquad \text{and} \qquad \left(\frac{G_t}{Y_t} \right)^* = \frac{s^g}{n+g+\delta^g} , \tag{2.4}$$

with n denoting the rate of growth of labour input L. Equation (2.4) implies that - for constant s and n - the ratios of capital to output are stationary along the steady-state growth path. Permanent shocks to technology (ε_t) induce only temporary deviations around this steady-state, i.e. the ratios change as capital and output move toward their new steady-state values. Once the economy has converged, the ratios return to their initial levels.

Figure 2.3 Determinants of steady-state capital to output ratios in Belgium (1953-96)



Sources: see figure 2.1

Equation (2.4) suggests two important sources of permanent shifts in steady-state capital to output ratios. Both a higher fraction of output invested in capital and a slower growth or decrease in the labour force raise the capital to output ratio[3]. Figure 2.3 shows the evolution of the labour force and the shares of output invested in private and public capital. Till the early 1970s, weak upward trends in investment rates as well as in employment keep the capital to output ratios more or less constant. From the mid 1970s though, some significant shifts in the steady-state occur. Combined with more or less stable investment rates, the decline in labour input over the period 1974-84 has induced a strong increase in the capital to output ratios. After 1984, employment recovers, putting downward pressure on the capital to output ratios. The reason why ln(K)-ln(Y) does not actually decline results from a very strong increase of investment in private capital over the period 1983-90. The downward pressure on ln(G)-ln(Y) in contrast is strengthened by a significant reduction in public investment from the early 1980s onward, caused mainly by the drastic fiscal consolidation programs of the 1980s and the 1990s.

3 Note that the positive effect of an increase in investment on the capital to output ratio derives from the model's assumption of diminishing marginal products to private and public capital. In this case, higher investment raises output less than it raises the capital stock. Similarly, a slower growth or decrease of the labour force makes the production process more capital intensive for output is reduced more than the capital stock (see equations (2.1) and (2.3)).

III EMPIRICAL LONG-RUN BALANCED GROWTH RESTRICTIONS

In this section, we formally test whether the data for Belgium are consistent with the steady-state properties of the neo-classical growth model. The outline is as follows. The first subsection looks more closely at the characteristics of the data, including tests for structural breaks in investment behaviour. In 3.2, we briefly discuss the popular Johansen approach to long-run structural modelling in a multivariate environment with I(1) variables and specify a model allowing for structural shifts in investment behaviour. The third subsection presents results from estimating the unrestricted model. Section 3.4 moves to an over-identified model and reports estimates of the resulting error correction model.

3.1 The data

Consistent with the assumed data generating process of technology, augmented Dickey-Fuller tests (see table 3.1) show that there is strong evidence of a unit root in output, private capital and public capital. The growth rate of labour input is found to be stationary. Therefore, the long-run properties of the neo-classical model have a straightforward interpretation in terms of cointegration. After allowing for possible shifts in investment behaviour and fluctuations in the rate of growth of employment, there should be two cointegration vectors that are reasonable representations of the balanced growth restrictions identified in the previous section.

Table 3.1 reveals an important problem, though. Taking first-differences of the public capital stock is not sufficient to remove all non-stationary components.[4] Although a more thorough analysis of the underlying data generating process reveals that public capital should be I(1) (see Everaert and Heylen, 1998), the I(2) behaviour in the available small sample may create important problems in estimating the vector error-correction model (VECM) underlying the cointegration analysis. In fact, explorative results show that the VECM is unstable, making it useless for the identification of common trends in the next section. Consistent with the conclusion of Ho and S∅rensen (1994), this suggests that the maximum likelihood estimator is not able to deal with capital stock series in small samples. Fortunately, the neo-classical

[4] Note that this problem does not depend on the type of test used. Tests based on the work of Phillips and Perron (1988), Kwiatkowski et al. (1992) and Leybourne and McCabe (1994) all point to I(2)-behaviour.

model offers a valuable alternative. Investment in private and public capital should be characterised by the same stochastic trend as the capital stocks for the steady-state growth rate of investment is predicted to be equal to technological progress too. Table 3.1 shows that both investment by the private sector (*Ik*) and the public sector (*Ig*) have one unit root in their data generating process. Therefore, instead of capital stocks we will use investment data in order to estimate the common trend.

Table 3.1 Augmented Dickey-Fuller unit root test (1953-96)[a]

Series		τ_τ^b	τ_μ^c	Series		τ_τ^b	τ_μ^c
ln(*Y*)	*p*=0	-0.51	-1.91	Δ(ln(*Y*))	*p*=0	-5.50**	-5.21**
ln(*K*)	*p*=1	-2.05	-0.73	Δ(ln(*K*))	*p*=0	-2.95	-2.97**
ln(*G*)	*p*=1	-1.18	-2.63	Δ(ln(*G*))	*p*=0	-1.63	-0.33
n	*p*=0	-3.88**	-3.93**				
ln(*Ik*)	*p*=1	-2.78	-1.11	Δ(ln(*Ik*))	*p*=0	-4.18**	-4.20**
ln(*Ig*)	*p*=0	-1.03	-2.07	Δ(ln(*Ig*))	*p*=0	-5.89**	-5.22**

Notes: [a] The lag length is denoted by *p*. The maximum value for *p* was set equal to 4.
[b] Based on regression $\Delta x_t = \mu + \alpha x_{t-1} + \beta_1 \Delta x_{t-1} + \ldots + \beta_p \Delta x_{t-p} + \gamma t + \varepsilon_t$. The MacKinnon critical values for the rejection of a unit root equal -3.52 and -3.19 at the 5% and 10% levels of significance respectively.
[c] Based on regression $\Delta x_t = \mu + \alpha x_{t-1} + \beta_1 \Delta x_{t-1} + \ldots + \beta_p \Delta x_{t-p} + \varepsilon_t$. The MacKinnon critical values equal -2.93 and -2.60 at the 5% and 10% levels respectively.
* Significant at the 10% level, ** significant at the 5% level.

As in the case of capital to output ratios, there is a clear unit root in investment to output ratios (see the first line of table 3.2). Figure 2.3 suggests that this non-stationary behaviour might be due to structural breaks in investment behaviour occurring in the early 1980s. As a first check whether a regime shift is indeed responsible for the apparent non-stationarity, we run augmented Dickey-Fuller tests allowing for a permanent change in the mean of the series. For this purpose, two alternative models, proposed by Perron and Vogelsang (1992), are used. The first is the '*Innovational Outlier Model*' (IOM), which tests for a unit root (α=1) in the regression:

$$x_t = \mu + \delta DU_t + \theta D(T_b)_t + \alpha x_{t-1} + \sum_{i=1}^{p} \beta_i \Delta x_{t-i} + \varepsilon_t \qquad (3.1)$$

with DU_t=1 if $t>T_b$ and $D(T_b)_t$=1 if $t=b+1$, both being 0 otherwise. The second model is the '*Qdditive Outlier Model*' (AOM), which tests for a unit root (α=1) in the two-step procedure:

$$\begin{aligned} x_t &= \mu + \delta DU_t + \tilde{x}_t \\ \tilde{x}_t &= \sum_{i=0}^{p} \omega_i D(T_b)_{t-i} + \alpha \tilde{x}_{t-1} + \sum_{i=1}^{p} \beta_i \Delta \tilde{x}_{t-i} + \varepsilon_t \end{aligned} \tag{3.2}$$

In both models, the timing of the breakpoint, T_b, is determined endogenously at (i) the minimum of the t-statistic for testing α=1 (t_α) or at (ii) the extremum of the t-statistic for testing δ=0 (t_δ).

Table 3.2 Unit root and level-shift hypotheses in investment to output ratios (1953-96) [a]

model[b]			ln(*Ik*)-ln(*Y*)					ln(*Ig*)-ln(*Y*)		
ADF			τ_τ^c	τ_μ^c				τ_τ^c	τ_μ^c	
		p=1	-2.73	-2.44			p=0	-2.16	0.23	
IOM[d]			α	t_α	t_δ			α	t_α	t_δ
(i)	T_b=1986	p=1	0.64	-4.14	3.19**	T_b=1982	p=0	0.74	-3.27	-4.05**
(ii)	T_b=1986	p=1	0.64	-4.14*	3.19**	T_b=1982	p=0	0.74	-3.27	-4.05**
AOM[e]			α	t_α	t_δ			α	t_α	t_δ
(i)	T_b=1985	p=1	0.61	-4.16	6.01**	T_b=1982	p=0	0.67	-3.75	-10.72**
(ii)	T_b=1987	p=1	0.61	-3.80**	7.42**	T_b=1984	p=0	0.64	-3.35*	-13.06**

Notes: [a] The lag length is denoted by p. The maximum value for p was set equal to 4.
[b] (i) T_b determined at the minimum of the t-statistic for testing α=1 (t_α).
(ii) T_b determined at the extremum of the t-statistic for testing δ=0 (t_δ).
[c] See table 3.1 for notes.
[d] Critical values for specification (i) and (ii) in the IOM are respectively equal to -4.76 and -4.26 at the 5% level and -4.42 and -3.82 at the 10% level.
[e] Critical values for specification (i) and (ii) in the AOM are respectively equal to -4.67 and -3.68 at the 5% level and -4.33 and -3.35 at the 10% level.
* significant at the 10% level, ** significant at the 5% level.

The results in table 3.2 point at a significant shift in the mean of investment to output ratios, i.e. δ is highly significant in all cases. The breakpoint for the private investment ratio seems to be situated around 1986-87, while for public investment 1984 seems optimal. The evidence that this structural break is able to render investment to output ratios stationary is less strong, though. The IOM is only able to provide weak evidence against a unit root in the private investment to output ratio while the AOM maximising $|t_\delta|$ is able to reject the null hypothesis at the 5% level. For the public investment to output ratio, the AOM rejects the null hypothesis only at the 10% level while the IOM cannot reject the null of a unit root at any reasonable level of significance. A potential problem is that we do not correct for fluctuations in employment, suggested to be an important source of short-term fluctuations (see section 2). In the multivariate model specified in section 3.2, these fluctuations will be corrected for.

3.2 Empirical specification

Consider the following reduced-form vector autoregression (VAR) with three endogenous I(1) variables ($\ln(Y_t)$,$\ln(Ik_t)$,$\ln(Ig_t)$) and one exogenous I(0) variable (n_t) - explaining only short-term disturbances to the variables - as our basic statistical model:

$$x_t = \mu + \sum_{i=1}^{k} \Pi_i x_{t-i} + \sum_{i=0}^{q} B_i n_{t-i} + \varepsilon_t, \tag{3.3}$$

with x_t=($\ln(Y_t)$,$\ln(Ik_t)$,$\ln(Ig_t)$)' and μ a vector of deterministic terms.

A particularly useful methodology for analysing long-run relationships in multi-dimensional models has been suggested by Johansen (1988) and extended in Johansen and Juselius (1990). They propose a reparameterization of the VAR under (3.3) in a VECM:

$$\Delta x_t = \mu + \Pi\, x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \sum_{i=0}^{q} B_i n_{t-i} + \varepsilon_t, \tag{3.4}$$

Since x_{t-1} is the only level term in equation (3.4), Π is the only matrix that contains information about the long-run relationships. If this matrix Π has reduced rank r, there are r independent linear combinations that are stationary, i.e. there are r cointegrating relationships. In this case, Π can be written as the product of a ($3\times r$) matrix α and a ($r\times 3$) matrix β', both having rank r:

$$\Pi = \alpha\beta' \tag{3.5}$$

with $\beta' x_{t-1}$ representing r cointegrating relationships and α measuring the speed of adjustment towards the long-run equilibrium.

In order to allow for structural shifts in investment behaviour, the drift term μ is decomposed as:

$$\mu = \mu_1 + \mu_2\, k(t) \quad \text{with} \quad k(t) = 1_{[T_b,1]} \tag{3.6}$$

which implies a break in the drift term at $T_b \in [0,1]$, with time defined on the interval [0,1] instead of $\{0,1,...,T\}$. In view of the analysis in the next section, it is useful to further decompose the drift function as (see Johansen and Nielsen, 1993):

$$\mu = \left(\alpha\beta'_1 + \alpha_\perp \gamma'_1\right) + \left(\alpha\beta'_2 + \alpha_\perp \gamma'_2\right) k(t) \tag{3.7}$$

with β_1 and β_2 denoting ($r\times1$) vectors containing the intercepts of the cointegrating relationships, $\alpha_\perp$ being a ($3\times(3\text{-}r)$) matrix chosen orthogonal to the columns of α and γ_1 and γ_2 denoting the ($(3\text{-}r)\times1$) vectors of linear slope coefficients (see Johansen and Juselius, 1992). The following hypotheses are of particular interest:

$$\begin{aligned} &H_{2,0}(r): && \beta_1, \beta_2 \text{ unrestricted}, && \gamma_1, \gamma_2 \text{ unrestricted.} \\ &H_{1,1}(r): && \beta_1, \beta_2 \text{ unrestricted}, && \gamma_1 \text{ unrestricted}, \ \gamma_2 = 0. \\ &H_{0,2}(r): && \beta_1, \beta_2 \text{ unrestricted}, && \gamma_1 = \gamma_2 = 0. \end{aligned} \tag{3.8}$$

The first hypothesis, $H_{2,0}(r)$, is the unrestricted case, allowing for a break in the drift. In the second hypothesis, $H_{1,1}(r)$, the break is restricted to occur only in the intercept of the cointegrating relationship while maintaining a constant drift in the model. The last hypothesis, $H_{0,2}(r)$, excludes a drift from the model. The hypotheses can be tested using likelihood ratio tests which are χ^2 distributed with (n-r) degrees of freedom (see e.g. Johansen and Juselius, 1990).

3.3 Rank determination and unrestricted cointegration space

The rank of the Π-matrix in equation (3.4) equals the number of its characteristic roots or eigenvalues λ that differ from zero. After ranking the characteristic roots in descending order, the rank can be tested for using two different likelihood ratio tests, i.e. the λ_{trace} and the λ_{max} statistic (Johansen and Juselius, 1990). The latter statistic tests the null hypothesis of r cointegrating vectors against the alternative of r+1 cointegrating vectors. The former is more general in that it tests whether the number of cointegrating vectors is less than or equal to r against the alternative that it is greater.

The asymptotic distribution of both test statistics is not given by the standard χ^2-distribution. Correct critical values are simulated by Johansen and Juselius (1990) and extended by Osterwald-Lenum (1992). Since both studies do not allow for structural breaks in the deterministic term, Johansen and Nielsen (1993) further extend the analysis for the presence of intervention dummies.

Results of rank tests are reported in table 3.3.[5] In line with the idea behind the IOM and the AOM discussed in section 3.1, the specific timing of the structural break, T_b, is chosen to maximise the evidence in favour of two stationary long-run (balanced growth) relationships contained in the Π-matrix. In this respect, setting T_b=1982 was found to be optimal.[6] With T_b=1982, the λ_{trace}-test statistics clearly point to 2 cointegrating vectors[7]. The χ^2-tests at the bottom of the table are in favour of $H_{2,0}(r)$, implying a break in the drift of the model.

Table 3.3 Rank determination and unrestricted cointegrating space, T_b=1982.

A. Likelihood ratio tests for reduced rank of Π^a							**B. Unrestricted cointegrating space**			
λ	λ_{trace}-test							Normalised coefficients		
				95% c.v.	90% c.v.			ln(Y)	ln(Ik)	ln(Ig)
0.53	r=0	r≥1	50.03**	24.52	22.14		β_1	-1.394	1.000^n	0.278
0.35	r≤1	r≥2	18.76**	9.57	7.75		β_2	1.000	-1.522	1.000^n
0.02	r≤2	r=3	0.73	3.84	2.71					
$H_{1,1}(r)$: $\chi^2(1)$=6.22 [0.01]* $H_{0,2}(r)$: $\chi^2(2)$=7.53 [0.02]*										

Notes: [a] Critical values are simulated with DisCo, a program written by Johansen and Nielsen (1993).
[n] Normalised
* significant at the 10% level, ** significant at the 5% level.

The coefficient estimates of the (normalised) cointegrating vectors are reported in the second part of table 3.3. Note however that direct economic interpretation of the estimates is generally not interesting for any linear combination of the two reported long-run relationships is also stationary, i.e. only the space spanned by these vectors is identified[8]. The unrestricted cointegrating vectors are therefore very unlikely to coincide with the true structural economic relationships.

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5 Prior to the econometric analysis, it is very important to pin down the appropriate order of the VAR. To do so, we have estimated the unrestricted VAR under (3.3) starting with a relatively long lag-length and then applied system specification tests to assess whether lags can be eliminated. The results of these tests point to k=2 and q=0.

6 Note that we have opted for a common breakpoint in both private and public investment ratios for tentative explorations with separate breakpoints did only yield minor improvements.

7 Results of the λ_{max} test are not reported since no critical values are available.

3.4 (Over-)identifying restrictions on the cointegrating space

In order to identify the individual cointegrating vectors, some economic theory has to be imposed. Note that since one can always take linear combinations of the unrestricted vectors, one normalisation and *r*-1 additional restrictions can be imposed on each cointegrating vector without changing the log-likelihood function. As such, these so-called exactly identifying restrictions cannot be tested for.

Table 3.4 Exact identification and over-identifying restrictions on the cointegrating space

	A. Exact identification				**B. Over-identifying restrictions**		
	ln(Y)	ln(Ik)	ln(Ig)		ln(Y)	ln(Ik)	ln(Ig)
β_1	-1.175	1.000^n	0.000^r	β_1	-1.000^r	1.000^n	0.000^r
	(0.079)						
β_2	-0.788	0.000^r	1.000^n	β_2	-1.000^r	0.000^r	1.000^n
	(0.248)						
LR-test of balanced growth restrictions $\chi^2(2) = 4.10$ [0.13]							

Notes [a] Standard errors of non-restricted parameters are reported in parentheses.
[n] Normalised, [r] restricted.

If we assume that the first relationship relates to the private investment to output ratio and the second to the public investment to output ratio, exact identification is obtained by setting the coefficient on public investment in the first equation and on private investment in the second equal to zero. Since the system is now fully identified, the reported vectors are interpretable as long-run relationships.[9] The results are reported in the first part of table 3.4. Both vectors are fairly well in line with the neo-classical long-run growth properties outlined in section 2. Consistency with the theory can formally be tested for by imposing additional, overidentifying restrictions[10]. The second half of table 3.4 reports the restricted cointegration vectors. In contrast to the conclusion of Crowder and Himarios (1997), the likelihood ratio

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8 This can easily be seen from the fact that different combinations of α and β can be contained by the same matrix Π. Any invertible ($r \times r$) matrix R can for instance be used to produce $\alpha\beta' = \alpha RR^{-1}\beta'$, with $R^{-1}\beta'$ being an equally possible set of cointegrating vectors.

9 Exact identification must not be confused with unique identifaction: different normalization and exactly identifying restrictions may lead to different relationships.

10 The validity of these restrictions can be tested using log likelihood ratio tests, comparing the eigenvalues obtained under the restricted model with those from the unrestricted model. Since all tests are conditional on

statistic testing the validity of these overidentifying restrictions shows that the data are consistent with the balanced growth restrictions from the neo-classical model.

IV A COMMON TRENDS MODEL

In the recent macroeconomic literature the common trends model has proven to be a very powerful tool in the analysis of long-run growth in a stochastic environment. King *et al.* (1991) show that there is a straightforward duality between cointegration and common trends in a VECM, i.e. in our model with three endogenous variables and two cointegrating relationships only one common stochastic trend explains the long-run growth of the variables. The econometric justification of this claim is that in a p-dimensional system with cointegration rank r, the long-run behaviour of the variables is determined by accumulations of $z=p-r$ independent permanent innovations (see Crowder *et al.*, 1998). The remaining r independent innovations explain only transitory fluctuations. The intuition behind all this is that a limited number of structural disturbances, e.g. shocks to technology or economic policy, drive the long-run growth in a large number of macroeconomic variables.

The neo-classical model outlined in section 2 suggests that the single stochastic trend in our system reflects the growth rate of technology. In this section the common trend is extracted from the data through imposing the cointegrating restrictions tested in the previous section. In order to see how this can be done, the unrestricted VECM under (3.4) must be inverted to yield the *Wold moving average representation*:[11]

$$\Delta x_t = \delta + \mathrm{C(L)}\varepsilon_t + \mathrm{B(L)}n_t \tag{4.1}$$

The matrix polynomial C(L) can now be seperated into a long-run and a short-run component to obtain:

$$\Delta x_t = \delta + \mathrm{C}(1)\varepsilon_t + (1-\mathrm{L})\mathrm{C}_1(\mathrm{L})\varepsilon_t + \mathrm{B(L)}n_t \tag{4.2}$$

with C(1) denoting the matrix of long-run multipliers and $\mathrm{C}_1(\mathrm{L}) = \dfrac{\mathrm{C(L)} - \mathrm{C}(1)}{(1-\mathrm{L})}$ (see Johansen, 1991). Rewriting (4.2) in levels, i.e multiplying by $(1\text{-L})^{-1}$, yields

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the reduced rank of matrix Π, we are working in the I(0) space, implying likelihood ratio test statistics to be asymptotically χ^2 distributed with the degrees of freedom equal the number of over-identifying restrictions.

[11] A simple algorithm for the inversion of a VECM can be found in Mellander *et al.* (1992).

$$x_t = x_0 + \mathrm{C}(1)\sum_{i=1}^{t} \varepsilon_i + \mathrm{C}_1(\mathrm{L})\varepsilon_t + (1-\mathrm{L})^{-1}\mathrm{B}(\mathrm{L})n_t \tag{4.3}$$

This reparameterisation of (4.1) clearly shows that the long-run behaviour of the variables is determined by $\mathrm{C}(1)\sum_{i=1}^{t} \varepsilon_i$.

Note that (4.3) is a *reduced-form model*, which is not suitable for a clear economic interpretation for it does not allow to trace out the time paths of structural shocks to the system[12]. For that purpose, we need to know the parameters of the underlying *structural model*, which takes the form:

$$x_t = x_0 + \Gamma(1)\sum_{i=1}^{t} \eta_i + \Gamma_1(\mathrm{L})\eta_t + (1-\mathrm{L})^{-1}\mathrm{B}(\mathrm{L})n_t \tag{4.4}$$

with η_t a (3×1) vector of serially uncorrelated structural innovations.

From King *et al.* (1991), we know that in a model with three endogenous variables and two cointegrating relationships, there is only one permanent innovations, η_t^p, which explains the long-run behaviour of the variables. The two remaining disturbances, η_t^{t1} and η_t^{t2}, explain only transitory fluctuations. In this simple case, deducing the single permanent innovation from the reduced-form errors ε_t is straightforward,[13] i.e. the balanced growth restrictions suggest that a technological shock has a unit long-run impact on output and investment,

$$\Gamma(1) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \tag{4.5}$$

implying that the permanent structural innovation η_t^p can be calculated directly from:

$$\Gamma(1)\eta_t = \mathrm{C}(1)\varepsilon_t\,. \tag{4.6}$$

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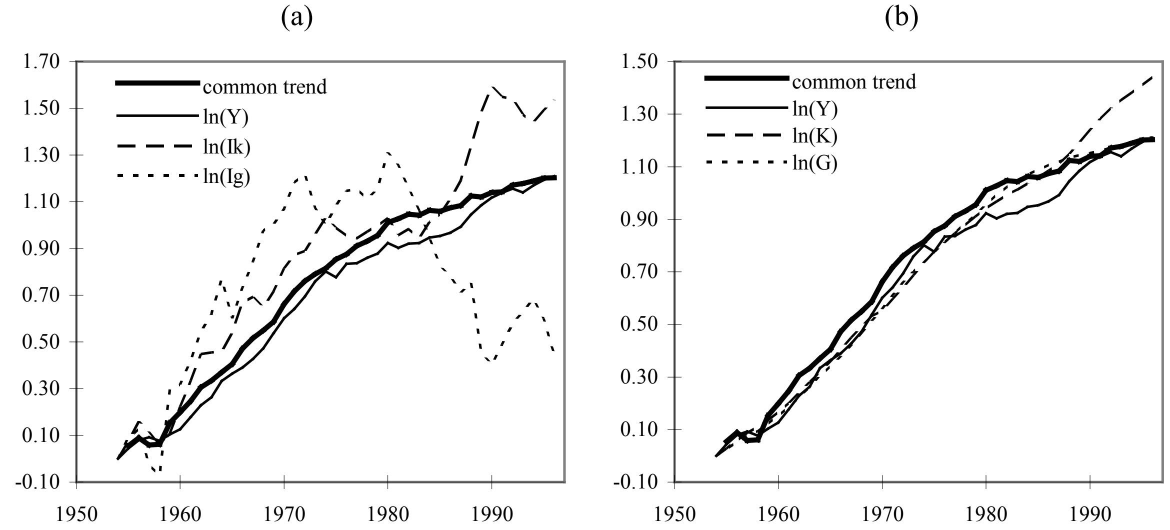
[12] In fact, the disturbances included in the vector ε_t are linear combinations of the true structural innovations in the system.

[13] With more than one common trend, additional restrictions have to be imposed in order to identifying unique permanent innovations. This implies an ordering of the disturbances (Cholseki decomposition) similar to the identification scheme in standard VAR models (see e.g. King *et al.*, 1991).

Panel (a) of figure 4.1 plots the resulting common trend in relation with actual output and investment data. Based on theoretical considerations, the same common trend should be present in the capital stock data. As shown in panel (b) of the graph, the common trend fits the capital stock data remarkably well.

Figure 4.1 The common stochastic trend in real output, investment and capital stocks



Sources: See figure 2.1.

V SHORT-RUN PRODUCTION FUNCTION CONSTRAINT

Extracting the common trend identified in the previous section from output and capital stock series produces stationary deviations about the balanced growth path, i.e. ln(Y'), ln(K'), ln(G'). Using these stationary series, the production function:

$$\ln(Y') = a_1 + a_2 \ln(K') + a_3 \ln(G') + a_4 \Delta \ln(L') \qquad (5.1)$$

can be estimated as a short-run constraint. Since labour input is found to be non-stationary in levels, L enters the regression in first-differences.

Table 5.1 reports the results from estimating equation (5.1) both unrestricted and restricted, imposing constant returns to scale over all inputs. In all regressions, we corrected for first-order serial correlation in the error term. The first regression is the simple unrestricted case. Public capital enters significantly, but only at the 10% level. The coefficient estimate on public capital is moreover quite high. Imposing the restriction of constant returns to scale (regression 2), raises the significance of public capital to the 5% level and reduces its point estimate to 0.30. Note however that the *F*-test rejects the hypothesis of constant returns to scale at the 1% level of significance in regression (1), which is mainly due to the estimated high output elasticity of labour input. One potential reason for this high estimate is that firms do not always react to business-cycle fluctuations by laying off or hiring labour, i.e. they opt for labour hoarding in recessions and overtime in booms. If this kind of behaviour is important on the macroeconomic level, the economy may not always be operating on its production function, i.e. labour hoarding pushes the economy below the production function while the inverse holds for overtime. This kind of friction on the labour market generates pro-cyclical behaviour of labour productivity, inducing an upward bias on the output elasticity of labour in equation (5.1). In order to correct for this phenomenon, we have added the change in the log of unemployment as an additional variable, capturing business cycle fluctuations (regression (3))[14]. Since the output elasticity of labour input is now significantly lower, the restriction of constant returns to scale cannot be rejected at the 5% level. With constant returns to scale imposed (regression (4)), the coefficient estimate on public capital equals 0.33. The rate of return to public capital implied by this estimate equals about 29%, which is much more plausible than the 150% (see Hurst, 1994) obtained by Aschauer and in line with the results of Everaert and Heylen (1998).

Table 5.1 Production function estimates, Belgium (1953-96)^{a,b}

		c^{st}	$\ln(K')$	$\ln(G')$	$\Delta \ln(L)$	$\Delta \ln(U)$	restriction	Adj. R^2	DW	F-test^c
(1)	$\ln(Y')$	-0.09 (-1.40)	0.40** (2.03)	0.41* (1.82)	0.91** (3.88)		-	0.89	2.01	0.01
(2)	$\ln(Y')$	-0.07 (-3.50)	0.38** (3.18)	0.30** (2.01)	0.32** (4.34)		$a_2+a_3+a_4=1$	0.87	1.98	-
(3)	$\ln(Y')$	-0.10 (-1.13)	0.42** (2.09)	0.40* (1.73)	0.73** (2.80)	-0.02 (-1.47)	-	0.89	1.79	0.06
(4)	$\ln(Y')$	-0.08** (-2.27)	0.42** (3.20)	0.33** (2.18)	0.25** (3.63)	-0.02** (-3.67)	$a_2+a_3+a_4=1$	0.88	1.65	-

Notes: [a] *t*-statistics are reported in parentheses.
[b] The results are corrected for first-order serial correlation in the error terms.
[c] The *F*-test has constant returns to scale over all inputs as null hypothesis. Reported is the *p*-value giving the lowest significance level at which the null hypothesis can be rejected.
* significant at the 10% level, ** significant at the 5% level.

VI CONCLUSION

[14] Ideally, the rate of capacity utilisation should be used. Data running from 1953 were not available though.

In this paper, we apply a three-step procedure to estimate the contribution of public capital to output. The first step starts from a standard neo-classical growth model with stochastic technological growth. The model predicts that in the long-run, capital, investment and output per capita should all grow at the rate of technology, i.e. the ratios of capital and investment over output should be stationary along the balanced growth path. The validity of the model can be tested for by imposing the balanced growth restrictions on the cointegrating vectors in a vector error correction model. After allowing for structural shifts in investment behaviour, likelihood ratio tests show that the simple neo-classical model's predictions are consistent with the Belgian growth experience over the period 1953-1996. In a second step, the balanced growth restrictions serve to identify the common stochastic trend, i.e. technological growth, in the data. Extracting this common trend from the data produces stationary deviations about the balanced growth path, which can be used to estimate the production function as a period-by-period constraint in the third step. Public capital was shown to enter the production function with a statistically significant output elasticity of 0.33.

The major advantages of this approach are that (i) it takes into account the underlying rate of technological growth, which is easily omitted in most applied research, and (ii) it deals with the non-stationarity in the data. One problem is that the choice of the timing of the structural break in investment is, up to some point, arbitrary. An interesting extension would therefore be to endogenise investment behaviour and, even more important, analyse the relationship between public and private investment.

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