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PERMANENT EXEMPTION FROM PAYROLL TAXES: THE ROLE OF HIRING FRICTIONS

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Permanent exemption from payroll taxes: The role of hiring frictions☆

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Abstract

Belgium's 2016 payroll tax exemption for first-time employers triggered a sharp increase in firms hiring their first worker but little growth among larger firms. To account for this pattern, we develop and estimate a directed search model—with discrete hiring, firm heterogeneity, and endogenous entry—using Belgian microdata. The exemption reduces the high marginal cost of the first hire, enabling many previously non-hiring entrepreneurs to become employers, but most lack the productivity needed to expand beyond one worker. The model matches the post-reform size distribution and identifies the conditions under which size-dependent hiring subsidies can foster sustained firm growth.

Keywords: payroll taxes; size-dependent policies; hiring frictions; wage subsidies; competitive search theory.
JEL Codes: H25, J08, J23, J38, L25

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1 Introduction

In the aftermath of the Great Recession, the persistence of high unemployment rates across many advanced economies prompted policymakers to consider measures aimed at stimulating job creation (Criscuolo et al., 2014). These debates often centered on the use of hiring credits and exemptions from employer payroll taxes as instruments to encourage firm-level employment growth (Neumark and Grijalva, 2017). A well-known example is the Hiring Incentives to Restore Employment (HIRE) Act, introduced in the United States in 2010, which provided temporary payroll tax relief to firms that expanded employment among previously unemployed workers.

A crucial consideration for policymakers designing employment subsidy programs is that firms should benefit differentially depending on their size. While many industrial policies historically target large, established firms (Acemoglu et al., 2018), there is recognition that young firms play a critical role in job creation (Bijnens and Konings, 2020; Decker et al., 2014; Criscuolo et al., 2014; Haltiwanger et al., 2013; Wong et al., 2005). Consequently, government aid and subsidy programs should also be tailored to support these younger firms. Although many new firms display low productivity and face a considerable risk of exiting the market, a significant share experience rapid growth, contributing disproportionately to the economy's overall job creation (Decker et al., 2014).

To evaluate the effectiveness of government policies aimed at stimulating job creation, it is essential to draw on evidence from episodes in which governments have intervened in labor markets through subsidy or tax measures. In this paper, we contribute to this literature by analyzing a reform implemented in Belgium in 2016, under which the government permanently exempted new employers from paying payroll taxes on the gross salary of one employee.

The objective of the reform was to encourage employment growth primarily among newly established firms. Since 2009—the starting year of our sample—Belgium has witnessed a substantial increase in the number of firms with zero employees (see (Cockx and Desiere, 2024) and Figure A.1a in the Appendix). At the same time, the numbers of multi-worker firms have remained largely unchanged. This pattern suggested that newly created firms had not expanded their employment, pointing to the potential importance of labor market frictions and vacancy creation costs. A policy that could induce these firms to transition from being owner-run to employing workers could, in principle, also enable them to continue growing and hire a second, third, or even more employees.

Empirical evidence suggests that this was not the case. Up to three years after the reform, there has been some growth in the number of firms employing one employee, but very little growth in firms with more than one employee (Deng et al. (2024) and our section 2).

In this paper, we construct and estimate a structural search model to interpret these empirical patterns, and to consider more broadly the implications of policies that subsidise job creation. Our model is a directed search equilibrium, broadly similar to that of Kaas and Kircher (2015), in which firms, differing in productivity and workforce size, can hire workers in a frictional labour market by posting multiple vacancies. Heterogeneous firms post vacancies in different segments of the market, offering applicants a wage–retention probability tuple. Workers are assumed to be homogeneous. As in the baseline directed search model, applying for a job offering a higher wage is compensated by a lower job-finding probability and a longer duration of unemployment.

To address the specific reform at hand, we make the following additions to this well-known framework. First, we assume that firm hires are natural numbers. Thus, firms can hire 0, 1, 2 (or more) workers, and do not employ a continuum of workers, as is typically assumed in the literature. We discuss the implications of this assumption in relation to various well-known theoretical results, but most importantly, we highlight that assuming a discrete workforce introduces a non-convexity in firm hiring. Consequently, a firm with no employees may face a steep marginal cost in hiring its first worker, making it less competitive than multi-worker firms in attracting applicants.

Second, following models of occupational choice and entrepreneurship (e.g., Lucas (1978); Cagetti and De Nardi (2006)), we endogenize firm creation within our directed search framework by allowing agents to choose at any time whether to start their own business or search for a job with an established firm. We introduce this assumption in light of extensive empirical evidence showing that many small businesses are founded by individuals who could otherwise have joined existing firms as employees (see, for instance, Fairlie (2013); Hamilton (2000)). Modeling entrepreneurship as an endogenous choice thus captures an important margin of labor market behavior that is particularly relevant for small firms.

Solving this directed search model, while allowing multi-worker firms to post vacancies sequentially, is not an easy task. Since firms that grow over time and experience productivity shocks are likely to create vacancies in different segments of the labour market, the wages offered to their workers may vary depending on when the vacancies were filled. In theory, one would need to track the entire distribution of 'promised values' offered to workers as a state variable in the firm's program in order to solve for optimal hiring and firing policies. This makes solving the firm's value function essentially intractable.

Unfortunately, our assumption that firm employment is a natural number, together with the presence of distortionary taxes in the model, does not allow us to leverage the key property of Kaas and Kircher (2015), which shows that competitive equilibria in this class of models can be obtained as a solution to a social planning program. We instead leverage an analogous result in Schaal (2017), establishing that the firm's value function can be represented as a surplus-maximisation problem in which only firm productivity and employment serve as sufficient state variables for characterising optimal hiring and firing policies. The distribution of promised values need not be included in the state vector. A variant of this result applies in our setting, which features discrete hiring and distortionary taxes.

To ensure that our model provides a suitable laboratory for analyzing the effects of the tax exemption, we estimate its parameters using firm-level microdata from the Belgian economy. Specifically, we identify the deep structural parameters by minimizing the distance between model-implied and empirical moments. The estimation targets a broad set of moments, including firm entry and exit rates, employment growth dynamics, the firm size distribution, the relative revenues of firms with zero and one employee, and aggregate labor market indicators such as the average job-finding rate and the unemployment rate.

Our estimates indicate that firms face convex vacancy creation costs that decline with employment, as well as non-wage operating costs that also decrease with firm size. Examining the resulting optimal policy functions, we find that the model generates sizable inaction regions in which firms do not adjust employment, including a wide range of productivity levels for which zero-employee firms optimally refrain from hiring. The model also implies that productive firms exploit different margins of adjustment to grow. Smaller firms expand primarily by offering more attractive compensation to applicants while maintaining a relatively low number of vacancies, whereas larger and equally productive firms compete with smaller firms by posting more vacancies.

We then simulate the 2016 reform within our estimated model. Consistent with the data, the post-reform equilibrium is characterised by a sharp increase in the number of firms with one employee, while the prevalence of multi-worker firms remains largely unchanged. We examine the mechanisms driving this outcome. According to our structurally estimated model, firms face substantial costs when hiring their first employee. However, most small firms are relatively unproductive and thus have little incentive to expand employment further. Although the combination of government subsidies and declining vacancy creation or operating costs could, in principle, encourage additional hiring, these firms are constrained by their low productivity.

We discuss the conditions that should prevail in the labour market so that these conclusions are reversed, enabling firms that hire one worker to proceed to hire more employees. Notably, our model assigns an important role to the initial productivity distribution. In a counterfactual economy in which a substantial fraction of young firms have high growth potential, alleviating hiring frictions through subsidies can serve the goal of initiating a virtuous cycle of firm growth. We also discuss the role of alternative assumptions regarding vacancy creation costs and their impact on policy outcomes.

One of the advantages of estimating a structural model is that it allows us to evaluate the policy in a broader context — in particular, to consider alternative forms of employment subsidies. In the early 2000s, the Belgian government introduced limited and temporary subsidies for the second to fifth employees, with the aim of stimulating employment growth among small firms. However, these subsidies were largely ineffective in achieving their intended objective. This outcome is perhaps not surprising in a labour market characterised by tight frictions and heterogeneous firm productivity.

We use the model to study the impact of these subsidies when they are *permanent*. According to our findings, subsidies for the third employee may still be ineffective, as they continue to encourage the growth of relatively unproductive firms. In contrast, subsidies that apply to higher marginal hires—beyond the first few employees (for example, the fifth employee)—can reverse these predictions, leading to substantial growth among highly productive, multi-worker firms. Thus, our conclusion is that subsidies targeted at hiring additional employees at higher employment levels generate the desirable outcome of redistributing employment toward more productive establishments—an effect consistent with the impact of mitigating labour market frictions and closer to the neoclassical equilibrium benchmark. Finally, we find that an across-the-board subsidy (a uniform subsidy for all workers and firms) is also quite effective in fostering growth among multi-worker firms.

Our work complements two recent empirical papers that have analysed the 2016 reform in Belgium, Cockx and Desiere (2024) and Deng et al. (2024). In Section 2, we revisit the findings of the first paper, which essentially estimates a sharp increase in the number of new employers following the reform's implementation. Our structural model is consistent with this empirical pattern. Moreover, comparing the cohort of new employers before and after the reform, Deng et al. (2024) conclude that the second group of firms was, on average, less productive than the first. This finding also aligns with the predictions of our model. Compared with these papers, which assess empirical moments, the advantage of estimating a structural model is that it allows for a deeper examination of the mechanisms at work, while also considering a broader spectrum of policies.

Our paper also contributes to an important strand of the literature that studies the effects of size-dependent policies in models with heterogeneous firms (e.g., Guner et al., 2008; Braguinsky et al., 2011; Gourio and Roys, 2014; Garicano et al., 2016, among many others). These papers primarily estimate the impact of various government policies and regulations on firm-level and aggregate productivity, as well as on the allocation of labor across firms.[1] Like these studies, we examine policies that distort the firm size distribution and the allocation of labor across firms. However, in contrast to these papers—which are grounded in neoclassical frameworks—the interaction between labor market frictions and policy distortions is central to our analysis.

Our work is closely related to studies that quantitatively evaluate the effects of labour market policies—such as hiring credits and wage subsidies—using equilibrium search models (Shephard, 2017; Bíró et al., 2022; Breda et al., 2024; Cahuc et al., 2019).[2] Specifically, Shephard (2017) examines a tax reform in the United Kingdom within a framework that incorporates worker heterogeneity, while Breda et al. (2024) develop a search model to quantify the spillover and congestion effects of reductions in labour taxes. Bíró et al. (2022) focus on the effects of a payroll tax cut for older workers; Cahuc et al. (2019) analyse a temporary hiring credit introduced by the French government in 2009 for firms with fewer than ten employees. Using their search model, they derive sufficient statistics that allow them to measure the fiscal cost per job created.

[1] Guner et al. (2008) develop a growth model with an endogenous size distribution of production units and show that policies reducing the average size of establishments by 20% lead to sizable declines in aggregate output and in the number of firms, alongside large increases in the number of establishments (by about 23.5%). Braguinsky et al. (2011) examine the effects of employment protection in Portugal on the firm size distribution, finding that eliminating such distortions can yield large first-order productivity gains. Gourio and Roys (2014) and Garicano et al. (2016) analyze regulations applying to medium and large firms in France–specifically those with more than 50 employees—and show that removing these regulations can improve the allocation of labor across firms (Gourio and Roys, 2014) and generate substantial welfare gains (Garicano et al., 2016).

[2] An older body of this literature includes two papers which employed search-and-matching models to evaluate the effects of permanent payroll tax cuts targeted at low-paid workers in Belgium, (Cardullo and Van der Linden, 2007; Batyra and Sneessens, 2010).

Our work has the most in common with Cahuc et al. (2019), who study a similar size-dependent tax exemption. Methodologically, however, we differ by employing a large-scale quantitative model estimated with microdata to simulate the policy's impact. Moreover, our modelling framework departs from Shephard (2017); Bíró et al. (2022); Breda et al. (2024); Cahuc et al. (2019), who rely on the more standard search-and-matching model. A distinctive feature of our framework is the assumption that employment levels take integer values, which distinguishes it from conventional continuous-employment formulations.

The remainder of the paper is organized as follows: Section 2 briefly discusses the 2016 reform in Belgium presenting empirical evidence on its impact on the size distribution of firms. Section 3 presents the model. Section 4 describes the data and the model estimation output. Section 5 contains our main analysis of the baseline tax exemption; it also uses the structural model to investigate alternative policies. A final section concludes the paper.

2 The policy and some descriptive evidence

Institutional Setting. In October 2015, Belgium implemented an unforeseen and permanent exemption from payroll taxes for private-sector firms that recruited their first employee after January 1, 2016. From that date onward, any firm that hired workers and had no employees in the preceding four quarters became permanently exempt from paying employers' payroll taxes on the gross salary of one employee. The stated objective of this reform was to encourage entrepreneurs to hire their first employee, and foster a virtuous cycle of firm growth.[3] The exemption did not apply to existing employers or to new companies formed by those employers.[4]

Importantly, eligible firms could retain the exemption even if they expanded their workforce or if a subsidized employee left the company. In cases of separation, the firm could transfer the exemption to another employee. Consequently, for eligible firms, the exemption could remain in effect indefinitely, as long as they employed at least one person. Up until 2022, the exemption had no cap.

The savings in payroll tax payments resulting from this policy can be calculated by determining the difference between the average payroll tax rate that a new employer would have paid on gross wages before the reform and the rate applied to their subsidized employee after the reform. After 2016, new employers paid a 2.9% payroll tax rate for their subsidized employees. Before the reform, calculating the expected payroll tax rate for the first employee is more complex. The 2016 reform replaced previous, less generous temporary hiring subsidies for the first employee, which had been in place—with some modifications—since 2004.[5]

[3]According to the press release from the Council of Ministers at the time of the announcement, this measure was intended *"to help as many self-employed workers as possible take the first step toward hiring"* with this first step then paving the way for additional hires (Dejemeppe and Van der Linden (2015)).

[4]In Belgium, business owners are classified as self-employed. Therefore, they are subject to a different payroll tax regime than employees and are ineligible for the exemption.

[5]In 2015, new employers hiring their first employee could claim a €1,550 quarterly payroll tax reduction for the first five quarters, €1,050 for the next four quarters, and €450 for the last four quarters.

In this context, Cockx and Desiere (2024) estimate that the expected payroll tax rate for hiring the first employee was approximately 18.3% before the reform. This figure is calculated as a weighted average of the payroll taxes paid each quarter after hiring. We adopt this estimate in our paper and direct readers to Cockx and Desiere (2024) for the methodology used to derive it. This suggests that the exemption reduced the payroll tax rate by about 15 percentage points. Given the significance of this change, it is not surprising that around 72% of new employers in 2016 applied for the exemption within two quarters of hiring their first employee (López-Novella, 2021)

Impact. Figure 1 illustrates the impact of the policy on the stock of firms by plotting the evolution of the number of firms with 1, 2, and between 3 and 8 employees over time.[6] Firm size is defined as the number of employees on the last day of the quarter, regardless of whether they are part-time or full-time. Each quarter, we compute the ratio of firms employing one worker (and analogously for firms with 2 to 8 workers) to the number of firms with one employee in 2015Q3. We use 2015Q3 as the pre-reform reference quarter because, as shown by Cockx and Desiere (2024), following the announcement of the reform in 2015Q4, some firms postponed hirings from 2015Q4 to 2016Q1.

Prior to the reform, firms with one or more employees exhibited similar trajectories, following parallel negative trends since 2011. The 2016 reform interrupted these negative trends; however, it generated a strong positive trend only for firms with one employee. By 2019, the stock of firms with one employee had increased by 10% (relative to 2015Q3). In contrast, firms with more than one employee did not exhibit a strong positive trend.

It is important to note that our goal in studying these patterns is not to claim that the reform alone explains the entire trajectory of firms after 2016. We acknowledge that these descriptive moments are not informative about other shocks that may have occurred between 2017 and 2019 and could have affected firm growth either positively or negatively. Nevertheless, we do know from the empirical evidence provided by Cockx and Desiere (2024)—who focus on a much narrower window around the policy change—that the reform led to a sizable increase in the number of first-time employers in Belgium.[7]

Based on these observations, we will not require our model to match these numbers exactly, but rather to replicate the qualitative pattern whereby most post-reform employment growth occurred among firms transitioning from zero to one employee.

[6] We focus on firms with eight employees or fewer because these firms followed the same pre-reform trend as those with 1 or 2 employees, whereas this was not the case with larger firms.

Similarly, the number of firms with zero employees has been increasing throughout the entire observation period—see Figure A.1a. We will not attempt to explain this trend. In the model estimation, we will assume that the number of firms with 0 employees in the pre-reform period equals their average over the two years preceding the reform.

[7] This conclusion is reinforced by the empirical section of Desiere et al. (2025), who, using the same data as we do, estimate the impact of the reform over four years with a DiD specification controlling for time fixed effects. Since their point estimates essentially coincide with what can already be inferred from our descriptive evidence, we do not include them in our empirical section.

Moreover, in a technical note, Cockx et al. (2025) extend the sample to include the COVID period; they continue to find a substantially higher number of firms with one employee.

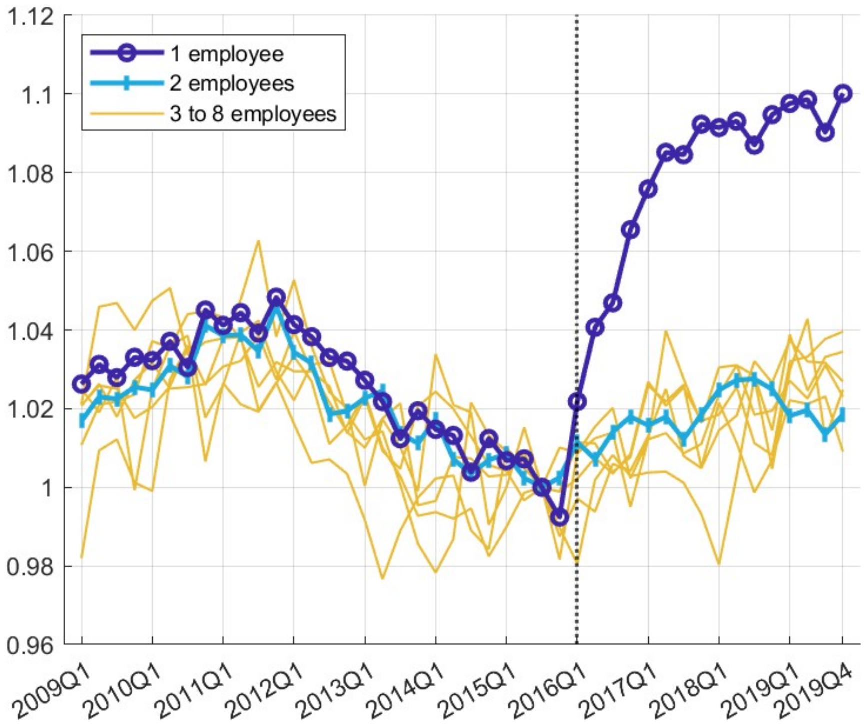

Figure 1: Evolution of the stock of firms. The figure illustrates the percentage change in the stock of firms with 1 to 8 employees relative to 2015Q3. The vertical dotted line marks the beginning of the policy in 2016.

Other Subsidies. The exemption was part of broader reforms aimed at increasing hiring by reducing employers' payroll taxes. Temporary subsidies for hiring the 2nd to the 5th employee have been in place since 2004. In 2016, the government further increased these subsidies and introduced an additional temporary subsidy for hiring the 6th employee.

We will abstract from these policy changes for three main reasons; First, the permanent payroll tax exemption for the first employee is far more generous than the temporary payroll tax reductions for the subsequent five employees.[8] Second, as discussed, temporary subsidies already existed since 2004 and became only slightly more generous in 2016; therefore, it is unlikely that these subsidies substantially altered the firm size distribution after 2016. Finally, the introduction of the temporary subsidy for the sixth employee in 2016 is unlikely to have affected firm decisions to hire their first or second employee and, as can be inferred from Figure 1 it probably did not exert a considerable impact on the distribution of firms.[9]

[8]For example, firms hiring their second employee could claim a payroll tax reduction over thirteen quarters, amounting to a maximum of €8,850 before the 2016 reform and €13,750 afterwards. The subsidies for subsequent hires are less generous than for the second hire.

[9]To avoid clutter, the lines corresponding to firms with 5, 6, and 7 employees are not individually labeled. Nonetheless, the graphs for all firm sizes between 3 and 8 employees display no discernible trend after 2016.

3 The model

We now present our baseline model, a directed search equilibrium with heterogeneous firms. The environment is broadly similar to that in Kaas and Kircher (2015), but we depart from their framework in three important ways. First, given our focus on government policy, we introduce payroll taxation into the model. We assume that, prior to the reform, employers pay a constant tax rate $\tau > 0$ for each individual they employ, whereas after the reform, new employers pay a reduced tax rate $0 \leq \tau_1 < \tau$ on the gross earnings of a single employee.

Second, we do not assume that firms hire or employ a continuum of workers. Instead, we focus on small and medium-sized firms that are exempt from taxation for one of their employees. The number of workers in the generic firm, L, and the number of vacancies it posts, V, are defined over the set of natural numbers. Under this assumption, firms in our model face uncertainty regarding the number of workers they ultimately employ. As we discuss below, the resulting equilibrium is generally inefficient.[10]

Finally, in contrast to Kaas and Kircher (2015), as well as numerous other models of directed search, we do not assume that the economy is divided into distinct groups of workers and entrepreneurs, with only entrepreneurs able to create jobs. Instead, every individual in the economy can choose either to work as an employee or to become a business owner, so firm creation is a decision open to all. This assumption, which introduces an occupational choice element into our model (similar to the entrepreneurship framework of Cagetti and De Nardi (2006)), is motivated by the observation that a large fraction of small firms are operated by self-employed individuals who were previously employees in other firms and who, at any point, may decide to close their business and return to paid employment.

3.1 Description

Consider an economy with a constant mass of individuals who are infinitely lived, risk-neutral, and discount future income with a factor of $\beta < 1$. Time is discrete. At the beginning of each period, an agent can be employed, searching for a job, or running a firm. Firms produce a homogeneous good using labour as the sole input, and the goods market is perfectly competitive[11]. We further assume no financial market frictions.[12]

[10] In contrast to Kaas and Kircher (2015), a competitive equilibrium cannot be derived from a social planner's problem. Instead, the firm's optimal control problem must be solved explicitly, and the resulting decision rules aggregated to compute the equilibrium.

A further reason why the efficiency result does not apply here is that the introduction of payroll taxation distorts the allocation.

[11] In our quantitative section, we will estimate the model targeting moments for firms with few (less than 15) employees (our dataset). For these small firms, assuming perfectly competitive product markets is likely a good approximation.

[12] The assumption of frictionless financial markets naturally complements our assumptions of linear worker utility and full commitment in firm–worker contracts. Under commitment, firms can effectively relax any financial constraints by borrowing from their workers through backloaded wage payments.

Revenues and costs. Firm revenues are represented with the function $R(L, x)$. $L \in \mathbb{N}_0$ denotes the number of workers at the firm (all employed agents provide one unit of labour per period) and $x \in X = [x_1, x_2, ..., x_n]$ is the firm's idiosyncratic productivity, a random variable drawing from a discrete distribution. We assume that R is strictly increasing and concave in L. Moreover, $R(L = 0, x) > 0$ so that the firm's production is not zero even if there are no employees. In this case, the firm is run by a solo-entrepreneur. We thus implicitly assume that business owners earn revenue supplying their own labour.

Every period, an active firm incurs three types of expenditures. First, the total wage bill for its L workers. Second, additional costs $c(L)$, which are per-period non-pay expenditures associated with managing employees. These costs increase weakly with L at a decreasing rate. We further impose that $c(L = 0) = 0$ (managing costs are zero for solo entrepreneurs).

Third, costs must be incurred to hire new employees. Specifically, following Kaas and Kircher (2015), we postulate a cost function $C(V, L)$ for vacancies $V \in \mathbb{N}$ posted by the firm. This function is strictly convex.[13] Moreover, for any V, $C(V, L)$ decreases with L. Posting a given number of vacancies is less expensive for large firms than it is for smaller firms.

Job Creation. As in the baseline directed search model, firms create jobs by posting vacancies and by advertising the path of wages that the workers will receive. Each vacancy is a long-term contract that specifies for each future period a state-contingent path of wages and a state-contingent retention probability. Firms are perfectly committed to the contract they offer, meaning that they cannot change the proposed state-dependent path of wages and retention probabilities after hiring.

We will represent the optimal contracts recursively in the next paragraph utilizing the promised utility approach (Spear and Srivastava, 1987): a contract promises to deliver utility W to a worker. The per period wage represents the utility flow of the worker.

Analogously, at the vacancy creation stage, the firm posts a vacancy advertizing the value W that the worker will expect to receive if the match is formed. We index the posted contract by W.

As in the standard search model, the number of applicants for the job is an increasing function of W. Let $\lambda_W \in \mathbb{N}_0$ denote the 'queue length', and $m(\lambda_W)$ the probability that the vacancy will be filled given λ_W. We assume that $m(\lambda)$ is increasing and concave in λ, satisfying $m(0) = 0$ and $m(\lambda = +\infty) = 1$.[14] The worker's probability of being hired then

Because workers have linear utility, they are indifferent to the timing of income and thus do not require compensation for deferred payments. In contrast, if workers had non-linear (e.g., concave) utility functions that reflect a preference for consumption smoothing, financial frictions would become relevant: firms could no longer costlessly borrow from workers. Incorporating such preferences however, significantly complicates the analysis, particularly when firms employ multiple workers.

[13] Convexity is imposed to rule out increasing returns to vacancy creation, which would allow firms to grow extremely rapidly by posting large numbers of vacancies at nearly fixed costs.

[14] Combined with the assumption that the cost of posting vacancies is convex, the concavity of m implies that firms that want to rapidly expand their workforce will both post more vacancies and attract more jobseekers per vacancy. This property is supported by empirical evidence (e.g. Davis et al. (2010)

is $\frac{m(\lambda)}{\lambda}$, and it is strictly decreasing in λ.

In the directed search equilibrium, firms that post a higher promised value W (offer higher wages) will be compensated with higher job filling rates. In turn, workers will form longer queues for these vacancies but will face lower odds of being hired. In equilibrium, the expected payoff for workers derived from applying to any available vacant job (the tuple (λ, W)) should be the same. This makes λ strictly increasing in W, as we will formalize later on.

Job Destruction. Filled jobs can be destroyed both due to the arrival of exogenous destruction shocks and due to the firm endogenously deciding to fire one or more of its workers. Exogenous shocks are of two types. First, each individual worker faces at the beginning of every period the possibility that her contract will be ended. This occurs at probability s_0. Second, the firm may receive a sufficiently negative shock that forces the owner to fire all workers and shut down the firm. The likelihood of this event is δ_0.

In addition, endogenous job destruction can occur when the firm has hired too many workers and, given the current level of productivity, desires to close some of the filled positions. Moreover, the entrepreneur may decide at the start of the period, to shut down the firm.

Endogenous firm exits in the model occur when the entrepreneur prefers to look for a job (to become a worker). Hence, they are not driven by the presence of a fixed cost of running the firm (as in e.g. Hopenhayn (1992)). The outside option for the entrepreneur is the value of unemployment, U, and the firm will remain active insofar as the present value of profits exceeds U.

Finally, note that we do not consider an environment with on the job search and so job contracts do not terminate because workers change firms.[15]

Unemployed jobseekers/ new entrepreneurs. At the beginning of each period, unemployed individuals can either apply for a vacancy or start their own business. We define S the expected utility of a jobseeker, the agent that applies to a vacancy. Moreover, denote the present value of a new firm with initial idiosyncratic productivity draw x by $\mathcal{J}(L = 0, x, \{.\})$.

We assume that jobseekers can observe all available vacancies at zero cost. Thus, when

for US evidence and He et al. (2023) for a field experiment conducted with a Chinese job board).

[15] Eurostat experimental statistics measure the quarterly rate of workers who transit from a job to another. Compared to other countries in the EU, this rate is, in all age groups, relatively low in Belgium. This even more true if one compares Belgium to the US.

The low job-to-job transition rates likely reflect Belgium's distinctive employment institutions. The vast majority of paid employment is under *Contrats à durée indéterminée, CDI*, which are standard open-ended contracts providing employment protection (notice periods and severance pay etc). According to the National Bank of Belgium, roughly 90% of employees hold such contracts. Combined with Belgium's relatively low wage inequality, these features are likely to reduce the incentives for workers to change jobs, compared to less regulated labour markets.

Finally, since our model assumes that workers have linear utility, we will not consider employment protection.

applying for a vacancy, they select from the entire set of available vacancies, one of those offering the highest expected utility.

$$\max_W S_W = \max_W b + \beta(1-\delta_0)\frac{m(\lambda_W)}{\lambda_W}W + \beta\left(1-(1-\delta_0)\frac{m(\lambda_W)}{\lambda_W}\right)U'. \tag{1}$$

where U' denotes the value of unemployment in the next period. From this point forward, we impose $U' = U$ focusing on a steady state environment.

Jobseekers receive a flow utility equal to b.[16] Moreover, we assume that when a contact with an active vacancy is made (at rate $\frac{m(\lambda_W)}{\lambda_W}$) at the start of the next period, the firm may still exit due to the arrival of an exogenous δ_0 shock. However, the firm cannot instantaneously endogenously fire a newly matched worker. We assume that the firm will pay (in expectation) W for a new match, and the worker will remain with the firm at least for one period. Extending full commitment to vacancies simplifies the computation of the equilibrium quite a bit.[17]

The unemployed individual faces a binary choice between becoming a jobseeker or a business owner. We assume that business creation yields expected profit value $\sum_{x\in X}\sigma_x\mathcal{J}(L=0,x,\{.\})$—where σ_x is the initial productivity distribution with support X— net of a sunk cost s_b. The sunk cost captures in reduced form the costs of initial investments in resources, time etc, needed to start a business. The value U is a solution to

$$U = \max\left\{\sum_{x\in X}\sigma_x\mathcal{J}(L=0,x,\{.\}) - s_b, \max_W S_W\right\}. \tag{2}$$

3.2 The firm's program

We now state formally the program of an active firm. Consider a firm that has current productivity x and a workforce of L workers. These workers have been hired at different points in time and offered (upon hiring) possibly different contracts that are contingent on the history of firm productivity, and the firm's hiring and firing policies and outcomes. We express the optimal program recursively utilizing the 'promised utility approach'. Let $\mathbf{W} \equiv \left\{W_i\right\}_{i\in\mathbf{L}}$ denote the vector of promised values attributed to the firm's workers, and the expected profit function by $\mathcal{J}(x,L,\mathbf{W})$. $\mathbf{L}$ is the set $\{1,2,...,L\}$, when $L>0$. In the case where the firm employs zero workers, we have $\mathbf{W}=\{.\}$ (the empty set) and $L=0$.

[16]Following standard practice, we interpret b as the total flow utility of unemployment, comprising both income (or consumption) during unemployment and the utility derived from leisure, net of any disutility from the unemployment state.

[17]Otherwise W is not sufficient for the job search program. Worker search choices would need to also be conditioned on the implicit retention probabilities, which (as we will next show) depend endogenously on firm characteristics, employment and productivity.

The firm's profit is a solution to the following functional (Bellman) equation:

$$\mathcal{J}(x,L,\mathbf{W}) = \max_{\widetilde{\phi}^i(.),d(.),V\in\mathbf{N}\cup\{0\},W_V,\widetilde{\mathbf{W}}'} \overbrace{R(L,x)}^{\text{Revenue}} \overbrace{-c(L)-\sum_{i\in\mathbf{L}} w_i(1+\tau)}^{\text{Operating cost}}$$
$$- \underbrace{C(V,L)}_{\text{Vacancy Cost}} +\beta(1-\delta_0)\mathbf{E}\Omega(x',L',\widetilde{\mathbf{W}}')+\beta\delta_0 U \tag{3}$$

subject to

$$L' = \sum_{i=1}^{L}\widetilde{\phi}^i(\widetilde{s}',L,\mathbf{W})I^i_{(\widetilde{s}')}+L_V(\widetilde{s}') \qquad (\text{Employment in } \widetilde{s}') \tag{4}$$
$$I^i_{(\widetilde{s}')}=1 \quad \text{if } i \text{ does not separate exogenously} \quad (Prob=1-s_0) \tag{5}$$
$$L_V(\widetilde{s}') \equiv \sum_{v\in\{1,2,,\ldots,V\}} I^v_V(\widetilde{s}') \tag{6}$$
$$I^v_V(\widetilde{s}')=1 \quad \text{if vacancy } v \text{ is filled in } \widetilde{s}' \quad (Prob=m(\lambda_{W_v})) \tag{7}$$
$$W_i = w_i+\beta(1-\delta_0)\mathbf{E}\widetilde{\phi}^i(\widetilde{s}',L,\mathbf{W})I^i_{(\widetilde{s}')}d_{(\widetilde{s}',L,\mathbf{W})}W'_{\widetilde{s}',i}+$$
$$+\mathbf{E}\left[\delta_0+(1-\delta_0)\left(d_{(\widetilde{s}',L,\mathbf{W})}\left((1-I^i_{(\widetilde{s})})+(1-\widetilde{\phi}^i(\widetilde{s}',L,\mathbf{W}))I^i_{(\widetilde{s}')}\right)+(1-d_{(\widetilde{s}',L,\mathbf{W})})\right)\right]U \tag{8}$$

A few lines are needed to explain the above objects.

The firm chooses the number of vacancies V it posts in the set $\{0,1,2...\}$ and the contracts W_V, expressed in terms of the promised utility of its potential hires. Note that this notation is inclusive of hiring policies that focus on a specific "market segment' for all vacancies (multiple vacancies with the same contract), or the firm may 'randomize', posting vacancies in different segments of the labour market. These vacancy posting policies map into filling probabilities m_{W_v} for any vacancy $v\in V$ as discussed previously.

The expectation operator $\mathbf{E}$, is a sum over all contingencies for vacancy filling rates, exogenous separations and future productivity x'. We introduced the variable $\widetilde{s}'$ to denote the joint (next period) state over all events. Thus, from equation (4) L' -the firm's employment level in the next period- is a random variable which is dependent on the state $\widetilde{s}'$.

L' is the sum of two components: the total number of new employees of the company L_V — $L_V(\widetilde{s}')\equiv\sum_{v\in\{1,2,,\ldots,V\}}I^v_V(\widetilde{s}')$— and the total number of workers who remained with the firm $\sum_{i=1}^{L}\widetilde{\phi}^i(\widetilde{s}',L,\mathbf{W})I^i_{(\widetilde{s}')}$. In our notation, I^v_V and I^i are indicator variables. I^v_V equals 1 with probability $m(\lambda_{W_v})$ (if the vacancy v has been filled) and 0 otherwise. Analogously, $I^i=1$ if worker i did not separate exogenously from the firm.

The map $\widetilde{\phi}^i(\widetilde{s}',L,\mathbf{W})$ is the firm's retention policy function for worker i contingent on $\widetilde{s}'$.[18] Effectively, a firm can decide to lay off worker i when $\widetilde{s}'$ realizes, if -for example-

[18]Notice that the dependence of $\widetilde{\phi}^i$ on $(\widetilde{s}',L,\mathbf{W})$ (rather than explicitly on future productivity and

productivity is low or a large fraction of the firm's posted vacancies were filled and the new workers have been promised value less than W_i.[19]

Equation (8) is the 'promise keeping constraint' for worker i. The firm must deliver an expected utility W_i to this worker by offering wage w_i in this period and the continuation utility that can be written as the sum of the expected payoff if the worker remains with the firm in the next period, and the value of unemployment if the worker separates (due to firm shut down, an exogenous or an endogenous separation). $d_{(\widetilde{s}',L,\mathbf{W})}$ is the map that indicates the firm's endogenous exit decision: $d_{(\widetilde{s}',L,\mathbf{W})} = 1$ if the firm remains active and 0 if an exit is optimal.

The decision to exit is implicit in the definition of the firm's continuation profit function

$$\Omega(x', L', \widetilde{\mathbf{W}}') = \max\Big\{\mathcal{J}(x', L', \widetilde{\mathbf{W}}'), U - \text{Cost}(W_V(\widetilde{s}'))\Big\} \tag{9}$$

We define $\widetilde{\mathbf{W}}'$ in the firm's continuation profit $\mathcal{J}(x', L', \widetilde{\mathbf{W}}')$ as the vector obtained from the union of the set of promised values to new hires in state $\widetilde{s}'$, $W_V(\widetilde{s}')$, and the set $f(\mathbf{W}) \subseteq \mathbf{W}$ where f is a partial map which selects the workers that the firm retains in $\widetilde{s}'$ and assigns the continuation utilities to form the next period's state vector. When the firm exits, the entrepreneurs payoff is U minus the wage costs implied by the promised utilities $W_V(\widetilde{s}')$.

Simplifying the firm's program. Solving the dynamic program defined in (3) is a formidable task. It involves solving the Bellman equation with $L+2$ state variables where L can potentially be a very large number. We now show that the firm's problem can be considerably simplified, optimal policies can be recovered as a solution to a dynamic programming problem where the state vector does not include $\mathbf{W}$, keeping track of promised utilities is not necessary. The following proposition summarizes the result:

Lemma 1. *The optimal policies for firing, hiring and firm exit, can be obtained as the solution to a dynamic program in which the firm maximizes a function $K(x, L)$, with productivity and employment as the only state variables. It is not necessary to keep track of the vector $\mathbf{W}$ of promised values.*

The online appendix provides the formal proof of this property, following an analogous result in Schaal (2017). As in that paper, we show that solving a surplus maximization problem is sufficient, leveraging the linearity of the promise-keeping constraints along

employment) is meant to capture that retention is a function of the beginning of period employment level. Hence, the state $\widetilde{s}'$ reveals how many of the L workers of the firm did not separate exogenously and the number of vacancies filled, as well as the productivity level x'. Together with the promised utility vector and the firms wage posting policy, these arguments are sufficient to determine the total workers available for production at the start of the period and the total wage cost.

We use the same arguments in the function $d(.)$ which determines the optimal exit policy.

[19] In contrast to continuous settings where firms are certain that the number of vacancies filled will be equal to postings times the average vacancy filling rate, here filled vacancies are random. Thus, a firm may have targeted to hire $V \times m$ workers, but it may end up with all of the V vacancies filled.

with the property that continuation utilities do not distort the incentives of firms and workers.[20] It is thus possible to first recover the optimal values of $\widetilde{\phi}^i$, W_V, and V by maximizing $K(x, L)$, and subsequently construct an optimal contract that implements the promised utilities.

Lemma 1 extends the result of Schaal (2017) in several important respects. First, because we do not assume a continuum of workers, firms face uncertainty regarding the size of their workforce in the following period. Second, the government imposes distortionary taxation. Third, the decision to exit the market depends on the entrepreneur's outside option U, rather than on the presence of a fixed cost of production, as in Schaal (2017).

The inclusion of the first two elements implies that surplus maximization does not necessarily lead to an efficient allocation. More specifically, the allocation may be inefficient because firms facing uncertainty about their future workforce may create too many vacancies (as a precaution) or hoard labor (firing less than in the efficient allocation).[21] Moreover, though under commitment and linearity in taxes and utility the worker and the firm are indifferent with regard to the timing of wages (taxes do not distort the allocation along this margin), distortionary taxes do impinge a loss in the sense that job creation in the decentralized equilibrium is generally below the efficient level.

Thus, in our setting, the competitive equilibrium cannot be characterized as the solution to a social planner's problem (see Kaas and Kircher (2015)). To find the competitive equilibrium we first need to solve the individual firm's program and aggregate up all of the individual decision rules.

We will now rewrite the program of the firm -leveraging Lemma 1- to demonstrate an efficient procedure to approximate numerically the competitive equilibrium. The appendix establishes the following additional result that helps us reduce the complexity of the task:

Lemma 2. *Given the total number of workers $\sum_{i=1}^{L} I^i_{\widetilde{s}'} - F'$ that the firm wants to retain in $\widetilde{s}'$ (where F' denotes workers fired in $\widetilde{s}'$), the optimal policy $\widetilde{\phi}^i(\widetilde{s}', L, \mathbf{W})$ (retain worker i) is a lottery in which worker i is retained with probability $\frac{\sum_{i=1}^{L} I^i_{\widetilde{s}'} - F'}{\sum_{i=1}^{L} I^i_{\widetilde{s}'}}$.*

The result in Lemma 2 can be explained by the assumption that firms display full commitment. It is not profitable to replace workers with a high promised value W with less costly workers or with new workers, since contracts are contingent on vacancies and hiring policies. Furthermore, a firm that wants to fire part of its workforce is essentially indifferent between firing worker i with a promised value W_i and worker j with a promised value W_j, even if $W_i \neq W_j$. Under commitment the cost of firing either one of these workers is the same.

Note that Lemma 2 is essentially subsumed by Lemma 1. If the optimal policies that derive from solving (3) are exactly those obtained from maximizing function $K(x, L)$,

[20] In the presence of distortionary taxes, the sum of the surplus accruing to the firm and the worker is strictly less than the total surplus generated by the match (i.e., the total value the match creates relative to outside options). Our notion of surplus is the total surplus including revenue from distortionary taxes.

[21] By contrast, if all firms of a given type could trade applications and pool workers, efficiency could be restored.

it follows readily that firms will not post vacancies to replace workers and they will randomize to determine which workers to fire. We can thus dispense with the policy functions $\widetilde{\phi}^i$, determining only the total number of workers that a firm will fire in state $\widetilde{s}'$.

We proceed by making the following assumptions regarding the wage profiles that firms offer to their workers and the firms hiring/firing policies.

Assumption 1. An optimal contract that promises W to a newly hired worker offers the following (frontloaded) wage profile: The worker gets $\bar{w}$ in the initial period (signing bonus) and a continuation wage, $\underline{w} = (1-\beta)U$, for each period the employee produces output.

Assumption 2. (No replacement hiring.) *The firm will not want to simultaneously hire and fire workers.*

The wage profile defined in Assumption 1 is not the only one that we could employ. Lemma 1 instructs us to solve for the optimal firing and hiring decisions through solving a surplus maximization problem; thus any wage profile that satisfies promise keeping can be used to decentralize the solution. However, as it will be evident shortly, Assumption 1 offers tractability as it enables us to express the firm's exit condition solely as a function of U.

Assumption 2 is made only for convenience (to state Lemma 3 and the definition of the equilibrium below). However, it will turn out to describe a property of optimal firm behavior once we solve the model numerically in Section 4. Given sufficiently high vacancy creation costs and our assumed stochastic process for firm productivity, the model admits a large inaction region in which firms neither fire nor hire.

Building on these assumptions, the next Lemma describes the efficient computation of the firm's dynamic program when (x, L) is the state vector.

Lemma 3. Efficient computation of value functions. *Let $e(L,x), f(L,x), h(L,x)$ denote the profit functions of a firm that exits the market, fires workers and hires new workers (respectively) given the state vector (x, L). Moreover, define*

$$J(L,x) = \max\Big\{e(L,x), f(L,x), h(L,x)]\Big\} \tag{10}$$

the upper envelope function. Also $Pr(L', L-F)$ denotes the probability that a firm that laid off F workers this period, will have $L' \in [0, ..., L-F]$ workers next period (accounting for the realizations of the exogenous job destruction shocks). Finally, let $Pr(H, m(\overline{w}), V)$ denote the probability that a firm posting vacancies V and contracts $m(\overline{w})$ will hire H workers. $e(L,x), f(L,x), h(L,x)$ *obey the following functional equations:*

$$e(L,x) = U, \tag{11}$$

$$f(L,x) = \max_{F\in\{0,1,..,L\}} \Bigg[R(L-F,x) - (1+\tau)\cdot \underline{w} \cdot (L-F) - c(L-F)$$
$$+ \beta \cdot (1-\delta_0) \cdot E_{x'}[\sum_{L'\in\{0,...,L-F\}} Pr(L', L-F)\cdot J(L',x')] + \beta\cdot\delta_0\cdot U\Bigg]. \tag{12}$$

and

$$h(L,x) = \max_{V\in\mathbb{N}, \bar{w}\in\overline{W}} \Bigg[R(L,x) - (1+\tau)\cdot \underline{w}\cdot L - c(L) - C(V,L)$$
$$+ \beta\cdot(1-\delta_0)\cdot \sum_{H\in\{0,...,V\}} Pr\big(H, m(\bar{w}), V\big)\Big[\sum_{L'\in\{0,...,L\}} Pr\big(L', L\big)\cdot \big[E_{x'} J(L'+H, x') \tag{13}$$
$$- H\cdot(1+\tau)\cdot\bar{w}\big]\Big] + \beta\cdot\delta_0\cdot U\Bigg].$$

Finally, $Pr\big(H, m(\bar{w}), V\big)$ is such that the following indifference condition is satisfied:

$$\bar{w} = \frac{U(1-\beta) - b}{\beta(1-\delta_0)\frac{m(\lambda)}{\lambda}}. \tag{14}$$

for $\lambda = 1, 2, 3, ...$

Competitive Equilibrium. We now define the competitive equilibrium in the pre-reform steady state.

Let $V^*(x,L)$ denote the optimal policy function for the number of vacancies and $\overline{w}^*(x,L)$ the optimal correspondence for promised utility (hiring bonus) of newly hired workers. Let $F^*(x,L)$ denote the optimal firing policy rule and $d^*(x,L)$ the optimal exit rule. Moreover, let $\psi(x,L)$ denote the invariant measure of firms across productivity and L in steady state. The following conditions hold:

1. The optimal policy rules are derived from the solution of the Bellman equations stated in Lemma 3.

2. The measure $\psi(x,L)$ is consistent. Let $\psi(\mathcal{X},\mathcal{L})$ denote the measure of firms that have productivity in the set $\mathcal{X}$ and employment in set $\mathcal{L}$. Define $\widetilde{\psi}(x,L)$ as the analogous measure before endogenous exits of firms take place. Then

$$\widetilde{\psi}(\mathcal{X},\mathcal{L}) = (1-\delta_0)\sum_{x,x'\in\mathcal{X}}\sum_{\substack{L'\in\mathcal{L}\\ L\in\{0,1,2,...\}}} Pr(x',x)Pr(L', L-F^*)\psi(x,L) +$$
$$(1-\delta_0)\sum_{x,x'\in\mathcal{X}}\sum_{\substack{L'+H\in\mathcal{L}\\ L\in\{0,1,2,...\}}} Pr(x',x)Pr(L',L)Pr(H,m(\overline{w}),V)\psi(x,L) + B_N I_{0\in\mathcal{L}}\sum_{x,x'\in\mathcal{X}}\sigma_x Pr(x',x)$$

for any sets $\mathcal{X}$ and $\mathcal{L}$. $Pr(x', x)$ denotes the transition probability from x to x'. B_N is the number of new firms (over total population), while $I_{0\in\mathcal{L}}$ is an indicator variable that takes the value 1 if $L = 0$ (zero workers) is an element of the set $\mathcal{L}$. [22]

We then have $\psi(\mathcal{X}, \mathcal{L}) = Q\widetilde{\psi}(\mathcal{X}, \mathcal{L})$ where Q maps $(\mathcal{X}, \mathcal{L})$ onto itself setting $\psi = \widetilde{\psi}$ wherever $d^* = 1$ and $\psi = 0$ otherwise.

3. The steady state unemployment rate is constant (inflow into unemployment equals the outflow from unemployment).

$$\text{Inflow} = \underbrace{\delta_0 \sum_x \sum_{L\in\{0,1,2,\ldots\}} (L+1)\psi(x, L)}_{\text{Exogenous exits}} + \underbrace{(1-\delta_0)\sum_x \sum_{L\in\{0,1,2,\ldots\}} (L-L')Pr(L', L-F^*)\psi(x, L)}_{\text{Job destruction in shrinking firms}} +$$
$$(1-\delta_0)\underbrace{\sum_x \sum_{L\in\{0,1,2,\ldots\}} (L-H-L')^{-}Pr(L', L)Pr(H, m(\overline{w}), V)\psi(x, L)}_{\text{Net job destruction of firms that posted vacancies}} + \underbrace{\sum_{x,L}(L+1)I_{Q_{x,L}=0}\widetilde{\psi}(x, L)}_{\text{Endogenous exits}}$$

$$\text{Outflow} = (1-\delta_0)\underbrace{\sum_x \sum_{L\in\{0,1,2,\ldots\}} (L-L'-H)^{+}Pr(L', L)Pr(H, m(\overline{w}), V)\psi(x, L)}_{\text{Net job creation of expanding firms}} + \underbrace{B_N}_{\text{New firms}}$$

where $(L-L'-H)^{-}$ symbolizes the negative part function (i.e. $-\min\{L-L'-H, 0\}$) and $(L-L'-H)^{+}$ is the positive part function.

4. $U = \sum_x \sigma_x J(0, x) - s_b = S_W$ for any contract $W(\overline{w})$.

A couple of comments are useful to clarify the above conditions. Note that in the definition of the flows out of unemployment in point 3 we include new firm creation. This is needed because in our model new firms are created by previously unemployed workers and hence represent an outflow from unemployment. Moreover, the inflow into unemployment includes net job destruction for firms that expand employment. This inclusion does not contradict Assumption 2. Whereas a firm that currently creates vacancies will not fire workers, a firm that created vacancies in the previous period may end up having fewer workers next period due to exogenous job destruction.

Finally, as in Kaas and Kircher (2015) B_N is determined so that the equality in 4 holds.

3.3 Introducing the exemption for the first hire

We now describe how the introduction of the tax subsidy to firms affects the equilibrium. We assume that initially the economy is in steady state and unexpectedly the government

[22] Hence $B_N I_{0\in\mathcal{L}} \sum_{x,x'\in\mathcal{X}} \sigma_x Pr(x', x)$ is the fraction of new firms that will have productivity in $\mathcal{X}$ in the next period.

implements a change in the tax code whereby for a single employee firms pay taxes $0 \leq \tau_1 < \tau$ and pay τ for all other employees.[23] Not all firms are eligible for the exemption. Eligible firms are those with zero employees at the time of the reform, and firms established after the announcement of the reform. Therefore, in the new competitive equilibrium a subset of the firms continue solving the program described in the previous subsections, in which a tax rate τ applies to all of their employees.[24]

Obviously, in the long run, after the reform, the economy will settle in a new steady state in which the measure of non-eligible firms is zero. However, we are particularly interested in tracing the behavior of the economy in the short term, along the transition path, since this trajectory is pertinent for evaluating the empirical patterns we documented in Section 2.

It turns out that the task of computing the equilibrium along the transition, is not much more complicated than solving for the steady state equilibrium. A particularly appealing feature of this computation is that, the new value of unemployment is sufficient to solve the firms and the workers programs. This value is constant throughout the transition, independently of the distribution of agents across the two labour market states, and the distribution of firms over (x, L).[25] Hence, given a guess for the new (post reform) value U, we can solve the firms and workers value functions, aggregate firm decisions and, given the initial steady state measures of firms, it is simple to construct the sequence of distributions $\psi_t(x, L)$ and $\widetilde{\psi}_t(x, L)$ from $t = 0$ (when the reform is introduced) to $t = \infty$ (when the economy reaches the long run steady state). The equilibrium value of unemployment needs to satisfy equation (2) and the requirement that in every period along the transition, a positive measure of firms enter into the market.[26]

To solve the firm's program post reform we make the following additional assumptions.

[23] We abstract from government budget balance requirements. In the new equilibrum, the total social security contributions collected by the government will decrease (unless of course enough more jobs are created or the distribution of employment changes sufficiently across firms to compensate for the lower tax rates). This fall in revenue is not compensated by adjusting benefits.

[24] Note that we assume that non-eligible firms never become eligible for the exemption, even if at some point they lose all of their workers. This assumption simplifies our computations and helps to avoid implausible predictions whereby a firm strategically fires its employees to benefit from the new tax code. In practice, by law, a firm with zero employees, which was previously an employer, needs to remain without employees for at least one year in order to become eligible for the exemption. This provision has obviously been made to eliminate strategic firing: the opportunity cost of having no employees when the optimal firm size is not zero is considered high enough. Accounting explicitly for this feature in the model is, however, complicated. We conjecture that making a subset of the firms permanently ineligible is roughly equivalent.

[25] Obviously, this result follows from the well-known property that equilibria in models of directed search are block recursive; value functions depend only on exogenous aggregate state variables, not on the endogenous distributions and, therefore, transitional dynamics are absent (Menzio and Shi, 2011, 2010). Notice that in our model this property can be verified to hold given the assumed homogeneity on the worker side. This assumption enables to pin down (though the usual indifference argument) the ratio λ_W in every submarket where firms may post vacancies. With heterogeneous workers (for example in a model with on the job search) block recursivity may not hold. In Schaal (2017) it does hold because firms hire a continuum of workers, which is not the case here.

[26] See Kaas and Kircher (2015); Schaal (2017).

Assumptions 3. *i) After an exemption is assigned to an employee, it cannot be reassigned until that employee leaves the firm.*

ii) If the subsidized employee is fired (exogenously or endogenously) the exemption is randomly reassigned to one employee from the combined pool of remaining employees and newly hired workers.

iii) Firms that create multiple vacancies and may end up hiring multiple workers, randomly assign the exemption to one of their new hires.

Assumptions i) to iii) guarantee that firms will not act strategically to maximize the benefits they can extract from the reform. For example if i) does not hold then the firm could backload the wages of all employees that have not yet been subsidized and then sequentially assign the subsidy to them. This will reduce the total social security costs. If ii) fails then the firm will assign the subsidy to the most expensive employee. Finally, unless iii) holds a firm may find profitable to assign the exemption to a single vacancy promising a very high W ($\overline{w}$) and promise a lower value for its other vacancies.[27]

With these assumptions we can continue expressing the firms program as in Lemma 3. Eligible firms with positive employment (assigned exemption to one of their existing employees) will pay $\tau \cdot \underline{w} \cdot \max[0, L-1] + (\tau_1) \cdot \underline{w} \cdot \min[1, L]$ as total contributions for their active workers and $\tau(\overline{w} + \beta\underline{w})$ for each new hire. Firms that have zero employees (positive number employees when the subsidized worker separated) and hire will pay $\tau_1(\overline{w} + \beta\underline{w})$ for the worker to whom the exemption is (randomly) assigned.

[27]Given the convexity of the cost function firms in our model will optimally create multiple vacancies with the same $\overline{w}$, rather than 'randomize', even though we do not preclude them from doing so. iii) basically requires that incentives for vacancy creation are not distorted along this dimension, because of the subsidy.

Assumptions (i) and (ii) are required in order to restrict the firm's value function to (x, L) as the only state variables. To see this, consider a firm that hires two workers with promised values $\overline{W}$. Abstracting from job destruction and productivity shocks, $\overline{W}$ is the present value of wages in a match of infinite duration.

If the subsidy is randomly assigned to worker 1, the firm is in principle indifferent between paying $(\tau_1 + \tau)\overline{W}$ in contributions immediately (by fully frontloading wages) or paying contributions based on any wage schedule that delivers $\overline{W}$. As in the benchmark model, the timing of wages is therefore irrelevant.

Without i), however, timing affects the firm's tax bill. For example, the firm could frontload the wages of worker 1 and pay worker 2 zero wages in the first period, while promising continuation utility $\overline{\overline{W}} = \frac{\overline{W}}{\beta}$) in the next. The subsidy would then be reassigned to worker 2 in period 2. In this case the present value of contributions is

$$(\tau_1\overline{W} + \tau_1\beta\frac{\overline{W}}{\beta}) < (\tau_1 + \tau)\overline{W}$$

Hence, the firm minimizes tax liabilities by distorting the wage profile, and the timing of wages is no longer innocuous. Intuitively, the subsidy introduces a nonlinearity into the tax schedule that creates such distortions.

If assumption (ii) does not hold, further distortions arise. In particular, the firm's incentives over endogenous separations may be affected. Even when separations are purely exogenous, the optimal wage schedule may be altered—for instance, the firm may backload the wages of high-cost workers in anticipation that the subsidized worker will separate.

4 Fitting the model to Belgian Data

In this section, we fit the model to the data. We begin by briefly describing the dataset employed and outlining the procedure used to match the empirical moments estimated from the data with those generated by the model. The estimation proceeds in two steps: first, we externally calibrate a subset of the model parameters; second, we estimate the remaining parameters by minimizing the distance between the model-implied and data moments. We evaluate the performance of the model in terms of matching the data moments.

4.1 Data

We draw on a firm-level panel dataset comprising the population of firms with no more than fifteen employees.[28] The dataset, obtained from the National Bank of Belgium (NBB), provides quarterly observations spanning the period 2009Q1–2019Q4. It includes both newly established firms and firms that employed at most fifteen workers in at least one quarter of the sample period. By contrast, firms that were founded prior to 2009Q1 and consistently employed more than fifteen workers throughout the entire period are excluded from the sample.

The NBB compiles these data from various administrative sources. For the purposes of this paper, the most relevant source is the National Social Security Office (NSSO), which administers payroll taxes and covers all employers in Belgium. The dataset at our disposal reports the total number of full-time and part-time employees on the last day of each quarter for every employer. The dataset does not provide information on the exact number of days or hours worked over the course of a quarter. The NSSO dataset is complemented by a dataset from the Crossroads Bank for Enterprises (CBE), a register that contains all entities engaged in business activities, including those without salaried employees. This dataset allows us to compute the share of firms without employees, as well as the share of firms without employees that hire their first employee in a given year. Finally, the VAT database allows us to compute revenues for firms both with and without employees.

To align the data more closely with our model, we measure firm size by the total number of employees, irrespective of whether they are employed on a full-time or part-time basis. For the estimation, we supplement the NBB data with publicly available aggregate statistics from the National Employment Office (ONEM) and the Statistical Office of the European Union (EUROSTAT). A detailed description of this data can be found in Appendix A.

[28] Our focus in this paper is on small businesses that are likely to be impacted by the 2016 reform. We will thus not include moments for large firms as targets for our model, since firms with many employees may have an altogether different productivity process and / or be subject to different hiring frictions.

4.2 Functional forms

We first discuss our assumptions for the model's functional forms. We specify the matching technology as a Cobb–Douglas function following the most common approach in the literature (e.g. Bilal et al. (2022) and Cahuc et al. (2023)). Letting $\gamma_m \in (0,1)$ be the elasticity of the vacancy filling rate with respect to the number of jobseekers, and μ_m the matching efficiency parameter, the probability of filling a vacancy is given by $q(\lambda) = \mu_m \cdot (\frac{1}{\lambda})^{-\gamma_m}$ and the analogous probability of a jobseeker being contacted by a firm is $p(\lambda) = \mu_m \cdot (\frac{1}{\lambda})^{1-\gamma_m}$.

As discussed previously, firm productivity is a discrete random variable that takes values in the set X. We set $X = \{x_1, x_2, ..., x_n\}$ to be a uniform grid, that is $x_i = x_1 + (i-1)\Delta x$ for $i = 1, 2, .., n$ and where $\Delta x = \frac{x_n - x_1}{n-1}$ denotes the increment. The probability that the new entrant firm draws x is $\sigma_x \in \Sigma = \{\sigma_{x_1}, \sigma_{x_2}, ..., \sigma_{x_n}\}$. As in Bilal et al. (2022), the values of σ_x derive from a bounded Pareto distribution with shape parameter σ_P. The probability density function for a bounded Pareto random variable is given by $\Pr(x = z) = \frac{\sigma_P \cdot x_1 \cdot^{\sigma_P} \cdot z^{-\sigma_P - 1}}{1 - (\frac{x_n}{x_1})^{\sigma_P}}$.[29] In terms of our discretized process this implies that the elements of Σ are set according to $\sigma_{x_1} = Pr(x \leq x_1 + \frac{1}{2} \cdot \Delta x)$, $\sigma_{x_2} = Pr(x \leq x_2 + \frac{1}{2} \cdot \Delta x) - Pr(x \leq x_2 - \frac{1}{2} \cdot \Delta x)$, where the cumulative probabilities can be easily computed using the (numerical) integrals.

Moreover, once firm productivity has been assigned at entry, we assume that it follows a random walk process whereby $x' = x + \epsilon$, with $\epsilon \sim \mathcal{N}(0, \sigma_{\mathcal{N}})$. Based on this specification, we can compute the transition probabilities over X, that is discretize the random walk to have $x' \in X$. [30]

Finally, our specifications for the revenue and cost functions are the following. First, the firm's revenue function is set as $R(L, x) = x \cdot (L+1)^\alpha$, $L \geq 0$, consistent with our assumption that firms that are run by solo-entrepreneurs produce output and generate revenues. Second, the per-period non-pay expenditures are specified as $c(L) = \mu_o \cdot L^{\gamma_o}$, $L \geq 0$, $\mu_o \geq 0$ and $\gamma_o \geq 0$. These costs are 0 for solo-entrepreneurs and are weakly increasing in L. The cost of posting vacancies function is $C(V, L) = \mu_C \frac{V^{(1+\gamma_C)}}{(L+1)^{\gamma_C}}$, $\mu_C > 0$ and $\gamma_C > 0$. As in Kaas and Kircher (2015), $C(V, L)$ is convex in V and decreasing L.

[29] This choice of distribution is quite common in the literature, mainly due to its simplicity and flexibility. See Dewitte et al. (2022) for a review.

[30] Specifically, let $\Pr(x' = x_i | x = x_j)$ denote the transition probability from 'node' j to node i in the discretized process. Then

$$\Pr(x' = x_i | x = x_j) = \Phi\left(\frac{x_i + \frac{\Delta x}{2} - x_j}{\sigma_{\mathcal{N}}}\right) - \Phi\left(\frac{x_i - \frac{\Delta x}{2} - x_j}{\sigma_{\mathcal{N}}}\right)$$

is the ji element of the $n \times n$ transition probability matrix and Φ is the CDF of the standard Normal random variable.

4.3 Externally set and internally estimated parameters

Calibrated parameters. A model period is one quarter, corresponding to the frequency of the NBB data. We set the quarterly discount rate ($\beta^{-1} - 1$) equal to 1.3%—which is equivalent to a yearly discount rate of about 5%. In the pre-reform period, we set the rate of employers' payroll taxes, τ, equal to 18.3% for all workers. For the post-reform period, we set the rate of employers' payroll taxes to 2.9% for one employee for eligible firms.

We discretize idiosyncratic productivity in the model using a large number of nodes, $n = 100$, to ensure that our model can accurately capture firm size/productivity distributions. We normalize the minimum productivity level x_1 to 1. Furthermore, following the most common approach in the literature, we set the elasticity of the matching function with respect to the number of jobseekers, γ_m, to 0.5.[31]

Our next two choices concern the exogenous firm exit and job destruction parameters. In our baseline, we set $s_0 = 0$, so that all job separations in active firms occur only when they endogenously fire workers. As we will show later, under this assumption the model can readily generate employment outflows and unemployment rates that align with their empirical counterparts.[32]

Finally, as in Kaas and Kircher (2015), we assume that firm exits occur primarily due to δ_0 shocks and calibrate exogenous exits according to the following function of L $\delta_0 \in \{0.0230, 0.0120, 0.0113, 0.0109, 0.0106\}$ for $L \in \{0, 1, 2, 3, 4\}$ and $\delta_0 = 0.0103$ when $L \geq 5$. These exit rates have been estimated from the NBB data. Further details on this estimation are provided in Appendix A.1, where we also discuss why focusing on exogenous firm destruction shocks is preferable in our setting. In particular, we suspect that a fraction of closed firms in our sample remain legally active for a few periods, reporting zero employees instead of formally exiting. As a result, our sample likely overestimates exits of firms with zero employees and underestimates exits from positive employment. To avoid biasing other parameters of the model we attribute firm exits to exogenous shocks.[33]

Table 2 reports the externally set or standardized parameters.

Estimated parameters. The 11 parameters left to be estimated are the highest value of the idiosyncratic productivity x_{100}; the shape parameter of the bounded Pareto distribution σ_P; the standard deviation of the shocks of the random walk process $\sigma_{\mathcal{N}}$; the

[31]Our data does not allow us to estimate this parameter. 0.5 is the estimate for γ_m reported by Petrongolo and Pissarides (2001) and it has been widely used in calibrated models.

[32]In contrast, setting $s_0 = 0.01$ or 0.02 (common values for US calibrations) our model does worse in terms of matching targeted moments (for brevity we do not report these experiments). Including the parameter s_0 in the estimation, would thus yield an estimate approximately 0, but dealing with a corner solution would complicate the estimation of the model.

[33]On the technical side, the model imparts a tradeoff between matching the number of firms with 0 workers and matching the exit rate. If exits are endogenous and driven by productivity, then matching the large exit rates of small firms would make the model unable to match the large data fraction of firms with 0 employees. Thus, the 'exogenous exits only' scenario, would be consistent with estimating the parameters δ_0 (along with the rest of the model parameters) if the weight assigned to matching the firm distribution is high.

production function elasticity α; the scale parameters μ_C and the elasticity parameter γ_C of the cost of posting vacancy function; the scale parameter of the matching function μ_m; the scale parameter of the per-period non-pay expenditures μ_o and the elasticity of the per-period non-pay expenditures with respect to the number of employees γ_o; the flow utility of unemployment b and the one-time sunk cost to open a firm s_b.

4.4 Empirical and simulated moments

We estimate the parameters to minimise the weighted sum of the squared distances between certain empirical moments and their respective simulated counterparts. The empirical moments are computed using NBB data and publicly available statistics from the national employment agency (ONEM) and EUROSTAT, focusing on observations from two pre-reform years, 2013 and 2014. We excluded 2015 from our sample, to eliminate biases due to possible anticipatory effects (see Cockx and Desiere (2024) for a discussion of this possibility). Similarly, we do not include information from years or quarters prior to 2013, as we believe that relatively old statistics may no longer reflect the economic conditions or firm behaviour at the time of the reform's implementation in 2016.

Let θ denote the vector of empirical moments, and $\hat{\theta}$ the model simulated counterparts. Our approach is to find the vector of model parameters $\upsilon = \{x_{100}, \sigma_P, \sigma_{\mathcal{N}}, \alpha, ...\}$ to minimize the following objective function

$$(\hat{\theta}(\upsilon) - \theta)' W^{-1} (\hat{\theta}(\upsilon) - \theta) \tag{15}$$

The matrix W contains the squares of the elements of θ on the main diagonal, with zeros elsewhere. In other words, our distance criterion is based on the percentage difference between the observed and simulated moments. This weighting matrix is commonly used in cases where it is not feasible to calculate the variance and covariances of certain moments employed in the estimation, while ensuring that the units of measurement do not influence the weight assigned to each moment (Low and Meghir, 2017; Bilal et al., 2022). This issue applies here, as the standard deviations for the moments sourced from ONEM and EUROSTAT are unknown and their correlations with the moments derived from our NBB panel data cannot be computed.

To reduce the computational time required to solve the model, we impose a limit of 50 employees per firm.[34] Moreover, most of the moments we target pertain to the distribution and dynamics of small firms, primarily those with between 0 and 5 employees. We adopt this approach for several reasons, including that the policy change has been found to mainly impact small firms and thus we want our model to be as good a laboratory as possible to analyze the behavior of these firms.[35] Also, firms with up to 5 employees form

[34]We implement this limit by assuming that if a firm exceeds 50 employees at the end of a period due to hiring, its workforce is immediately reduced to 50. Consequently, firms have no incentive to post vacancies once they reach the cap, and those approaching it will reduce vacancy postings to avoid unproductive hiring costs.

[35]Nonetheless, the behaviour of large firms is also implicitly targeted since our model will match aggregate statistics.

the bulk of our sample, as we will later show.[36]

We now outline the moments targeted in our estimation.[37] We also explain the parameters of the model that are informed by the selected moments. We discuss the moments we target and the corresponding parameters, in two separate subgroups. First, we discuss moments that primarily inform us about parameters that govern the size distribution of firms in steady state and which would be important even in a frictionless environment (in a neoclassical version of our model). These are parameters that relate to the productivity process, the production technology and the non-pay cost function $c(L)$. Subsequently, we discuss moments that primarily inform us about the magnitude of the frictions, and which relate to parameters that impact the dynamics of employment growth for firms of given size. To be clear, we do not claim that the first set of moments is not determined also by the frictions and the second set by the productivity and technology parameters. We simply consider some of the moments as more informative for certain parameters than for others.

In the first group of moments we include the following objects: Using data from the NBB and statistics from Statbel, we calculate the proportion of firms with 0, 1, 2, 3, 4, and 5 employees relative to the total number of firms in the economy. Moreover, we target the ratio of average revenues between firms with zero employees and firms with one employee. Finally, we add the proportion of new firms in a given quarter that have 0, 1, 2, or 3 employees, relative to the total number of new firms.[38]

These moments primarily provide information on the productivity process (the maximum value x_n and the initial productivity parameter σ_P), the curvature of the production function (α), the non-pay cost function $c(L)$ (mainly through its scale parameter μ_0), and the sunk cost of firm creation, s_b.

The link between each of these parameters and the steady-state distribution is straightforward. For example, a higher value of σ_P (or x_n) implies a more dispersed productivity distribution and, consequently, smaller employment shares for small firms. Analogously, a steeper non-pay cost function $c(L)$ leads to a larger fraction of firms with small size. The same applies when the value of α is low: decreasing returns to scale reduce marginal productivity more rapidly in multi-worker firms. Finally, by matching the output of firms with zero employees to that of firms with one employee, we identify the model using information on the relative productivity of solo entrepreneurs, which is primarily determined by the parameter σ_P and the sunk cost s_b.

The second set of moments comprises the following objects: First, for small firms (with 0, 1, or 2 employees), we compute partial transition probabilities across different future employment levels. Specifically, we take firms with zero employees in a given quarter and

[36]Since we cap firm size at 50 employees, we effectively assume that very large firms are unaffected by any policy changes considered in this paper. Although this assumption is made for computational reasons, we believe it is also pragmatic. In practice, very large firms tend to be so productive that a government subsidy for one or a few employees would have a negligible effect on their overall performance. A further reason is that these firms may face different hiring frictions than smaller firms, may be able to screen applicants more effectively, may have unionized workforces, etc.

[37]A more detailed description of each moment than we provide here is contained in the Appendix A.

[38]New firms are defined as those that did not exist in the previous quarter.

calculate the proportions that (i) remain at zero employees, (ii) grow to one employee, and (iii) grow to two employees after four quarters. Similarly, we compute annual transition probabilities for firms with one and two employees across the same employment states (0, 1, and 2 employees). By construction, matching these transitions also captures the proportion of firms that grow beyond two employees. Finally, we ask our model to target the economy-wide unemployment rate,[39] the quarterly job-finding rate, and the relative job vacancy rate for firms with fewer than 10 employees relative to firms with more than 10 employees (data moment released by Eurostat).

These moments are particularly informative about the impact of search frictions in our model. The stronger the frictions (for a given productivity process), the more difficult it becomes for firms to grow, and transitions from 0 to 1, 2, or more employees become less frequent. Similarly, more severe frictions imply a lower job-finding rate and a higher unemployment rate in the economy. By matching the moments described above, we capture with high accuracy the frictions faced by small and newly established firms, which is crucial for our analysis. Matching the aggregate, economy-wide moments allows the model to also account for the frictions affecting larger firms.

Before closing this paragraph, we comment on the target that we specify for the last estimated parameter of our model, the flow value of utility/income in unemployment b. In the macro-labour literature it is typical to set the value of this parameter to have a ratio of b to average wage income between 0.5 and 0.7 (see for example Hall and Milgrom (2008), Pissarides (2009), Kaas and Kircher (2015) and discussion therein). In contrast, in many estimated micro search models the value of b is lower (in some cases even negative, Bunzel et al. (2001); Hornstein et al. (2011)). Basically, setting b to a high value allows macro models to match the cyclical volatility of unemployment and vacancies (Hagedorn and Manovskii (2008)); and setting b to a low value allows micro models to match measures of cross sectional dispersion (in wages, productivity etc).[40] For our model's purposes, it is perhaps preferable to prioritize cross sectional inequality over macroeconomic volatility. However, as a compromise between the two approaches, we set the target for the ratio of b to average wage equal to 0.5, that is at the lower range considered in macro models.[41]

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[39]We target the unemployment rate provided by ONEM. Details on how this statistic is constructed by ONEM, and the advantages of using it in our setup, are provided in Appendix A.

[40]Intuitively, b determines the reservation wage/productivity and higher values of b imply more compressed cross sectional distributions.

[41]With this choice, b is lower than the value consistent with the replacement rate of income during unemployment relative to salary income in Belgium. This target would typically lie between 0.7 and 0.8, although it varies across workers (see `https://www.oecd.org/en/data/indicators/benefits-in-unemployment-share-of-previous-income.html`). A lower value of b would, however, be consistent with a broader definition of unemployment—one that includes not only individuals eligible for benefits but also new entrants and passive jobseekers.

Alternatively, b can be interpreted as the value of leisure, which may be lower than the replacement rate if unemployment entails a disutility for individuals (for example, due to a stigma effect).

4.5 Results and model fit

Table 1 compares the actual data with the simulated moments. Overall, the match between the observed and simulated moments is very satisfactory. The model performs very well in terms of matching the proportions of firms with 1, 2,..., 5 employees. It performs slightly worse in matching firms with 0 employees; however, as noted previously, this discrepancy is not concerning, as in the data a fraction of these firms may be inactive firms that have not yet de-registered from the official statistics.[42]

The performance of the model is also very good in terms of matching the distribution of new firms with 0, 1, 2, 3 employees. In the data, new firms that have not yet hired any worker are 95%, and they are 96% in the model. The fractions with 1, 2 and 3 employees are 2.3% .9% and .4%, respectively, in the data, v.s. 2.0% .9% and .4% in the model.

Where the model performs slightly worse is in matching the transition rates across employment levels. More specifically, the model understates the fraction of firms with one employee that transition to zero employees in the next year, and it also understates the probability that a firm with two employees will have zero or one employee in the following year. In other words, the model slightly overstates the persistence of firms with one and two employees in these states.

The weaker performance in terms of matching these moments is, however, not concerning. Since some of the firms that transition to 0 employees in the data are likely to be firms which are about to exit the market (and have not yet de-registered from official statistics) we are likely to observe larger flows to 0 employees. Thus, not matching these data numbers very accurately is not necessarily problematic for the overall fit of the model.[43]

Finally, note that the model can accurately predict the transitions of firms with 0 employees (which is a particularly relevant target for our quantitative exercise), the unemployment rate and, the job finding rate as in the data.[44]

[42]We experimented with alternative assumptions about the number of inactive firms in our sample, assuming for example that the true statistic for the number of active firms with 0 workers, is up to 50 percent lower than what we find in the data. In each case we re-estimated the model parameters and simulated the effects of the reform. Our results did not change relative to the baseline model. For brevity we will not show the output from these exercises here.

[43]Note that the motivation to adopt a yearly horizon to compute these flows has been to attempt to mitigate the potential bias. Consider the following simple example: Let A be the set of firms with 0 employees and B the set with positive number of employees. Let δ_0 denote the exit rate for firms in B, and let p denote the true transition probability to A. Each period a fraction q of the exiting firms will be assigned to A and stay there for one period.

The quarterly transition to A is $p+\delta_0 q$. The transition over two quarters is $p+(1-p)(1-\delta_0)(p+\delta_0 q)$. We can estimate $\hat{p}_{1q} = p+\delta_0 q$ and $\hat{p}_{2q} = \frac{1-\sqrt{1-4\left[p+(1-p)(1-\delta_0)(p+\delta_0 q)\right]}}{2}$. For relevant numerical values we can show that $p < \hat{p}_{2q} < \hat{p}_{1q}$. Estimating the transitions with longer-horizon data is preferable in terms of reducing the measurement error induced bias.

[44]In the model's steady state, the job-finding rate plus the firm-entry rate equals the separation rate. The model implies a quarterly separation rate of 1.4%, which is comparable to the empirical moment: average layoffs / involuntary separations are typically 4–5% annually, i.e. roughly 1.0–1.25% per quarter. Thus, although we impose zero exogenous separations, the model still generates a plausible

Table 1: Comparison between observed and simulated moments

Moment	Description	Data	Simulated
Share of firms with 0–5 employees	N_0/N	79.09%	71.43%
	N_1/N	6.50%	5.82%
	N_2/N	3.51%	2.72%
	N_3/N	2.21%	1.78%
	N_4/N	1.52%	1.37%
	N_5/N	1.08%	1.45%
Ratio of average revenues (0 vs. 1 employee)		25.74%	25.58%
New firms by initial size	$N_{\text{new},0}$	95.55%	96.40%
	$N_{\text{new},1}$	2.30%	1.99%
	$N_{\text{new},2}$	0.93%	0.91%
	$N_{\text{new},3}$	0.44%	0.44%
Firms with 0 employees: transitions (next year)	$0 \to 0$	88.88%	88.97%
	$0 \to 1$	1.62%	1.80%
	$0 \to 2$	0.31%	0.24%
Firms with 1 employee: transitions (next year)	$1 \to 0$	17.91%	7.94%
	$1 \to 1$	64.41%	71.75%
	$1 \to 2$	9.98%	10.80%
Firms with 2 employees: transitions (next year)	$2 \to 0$	6.12%	3.12%
	$2 \to 1$	17.02%	4.74%
	$2 \to 2$	54.67%	63.25%
Unemployment rate		10.75%	11.74%
Quarterly job finding rate		8.90%	10.03%
Ratio of JVR for firms with ≤ 9 vs. > 9 employees		1.60	2.05
b over average gross wage		0.50	0.50

Notes: EES = employees; JVR = job vacancy rate. N is the total number of firms, N_0 the number with zero employees, and $N_{\text{new},0}$ the number of new firms with zero employees.

In Table 2 we summarize the values of the estimated parameters. We estimate a value of the production elasticity $\alpha = 0.76$. This value is close to (but slightly lower than) the estimates provided in Bilal et al. (2022) (0.82) and Schaal (2017) (0.85) based on matching data from the United States. Moreover, different from these papers, our estimation focuses on the sample of smaller firms, to better capture the employment

overall separation rate.

dynamics in this segment of the distribution.

For the same reason, our estimate for the scale parameter of the matching function, 0.12, is lower than the value 0.195 reported Bilal et al. (2022) (matching efficiency is likely to be higher in the United States than it is in Belgium, as job finding rates are also quite higher.) Given the elasticity of the matching function in our model, 0.5, for $\lambda = [1, 2, 3, 4, 5, \dots]$ we obtain vacancy filling rates $q = [0.12, 0.17, 0.20, 0.24, 0.26, \dots]$ and job finding probabilities $p(\lambda) = [0.12, 0.08, 0.07, 0.06, 0.05, \dots]$. These numbers seem reasonable for the Belgian economy.

The estimated sunk cost parameter ($s_b = 90$) implies that the entry cost corresponds to approximately 37% of the expected value of opening a firm, $\sum_{x \in X} \sigma_x J(L = 0, x)$. An empirical counterpart for this parameter is difficult to identify in the existing literature; however, calibrating large entry costs is common in theoretical models (see, e.g., Kaas and Kircher (2015)). Moreover, we estimate the elasticity parameter of the vacancy posting cost function to be $\gamma_C = 0.66$, implying that that vacancy costs are convex (consistent with Kaas and Kircher (2015), whose baseline calibration assumes $\gamma_C = 1$).

In contrast, the non-pay cost function is concave, with an estimated elasticity parameter of $\gamma_o = 0.32$. Thus, as in many models of firm dynamics that include a fixed operating cost, our results imply that firms may exhibit some degree of increasing returns to scale as employment expands (average costs, $\frac{c(L)}{L}$, decline with firm size). We find however that a (slightly) positive derivative of the $c(L)$ function is needed to match the firm data.

Finally, the parameter of the Pareto distribution function, $\sigma_P = 1.22$, implies that for $\left\{x_1, x_2, x_3, ..., x_n\right\}$ the corresponding initial probabilities are $\sigma_x \in \left\{0.18, 0.13, 0.10, ..., 0.0003\right\}$. In our model, most firms start with low idiosyncratic productivity and subsequently grow when they experience positive shocks. Because productivity follows a random walk, the initial conditions continue to influence firm employment even in the long run.

Firm exit and entry rates. We evaluate the model's predictions in terms of two additional (untargeted) moments, the firm exit and entry rates.

As was discussed previously, our calibration of the δ_0 function was based on the observed data exit rates by firm size. The model matches well the firm distribution. Therefore, the average quarterly exit rate predicted by the model is close to the data counterpart (1.9% v.s. 2.05%).

Moreover, in the stationary equilibrium, total new firm creation equals firm exits. We compare the flow of new entrants over the total labour force in the data and in the model. The model ratio is 0.32%. In the NBB data, the number of firms created each quarter between 2013 and 2014 was approximately 13,000. From the Labour Force Survey, we know that the number of active individuals in 2013 and 2014 was about 4.9 million.[45] The corresponding ratio is 0.27%, which aligns with the model's prediction.

To conclude, the model performs well in matching the targeted moments, and the estimated parameter values are broadly consistent with those found in the existing liter-

[45] Statistic computed by Statbel from LFS data https://bestat.statbel.fgov.be/bestat/crosstable.xhtml?datasource=577a64da-ea7b-4018-aab2-196693cf5dce

Table 2: Externally calibrated and internally estimated parameters

Parameter	Description	Value
Externally set parameters		
β	Quarterly discount rate	0.987
τ	Payroll tax rate (pre-reform)	0.183
τ_1	Payroll tax rate (post-reform)	0.029
n	Number of idiosyncratic productivity levels	100
x_1	Lowest idiosyncratic productivity	1
δ_0	Quarterly exogenous exit rate	[0.023, ...]
s_0	Quarterly exogenous separation rate	0
γ_m	Elasticity of matching function w.r.t. λ	0.5
Estimated parameters		
x_n	Highest idiosyncratic productivity X	17.7
σ_P	Shape parameter, Pareto distribution	1.22
α	Production function elasticity	0.76
$\sigma_{\mathcal{N}}$	Std. dev. of shocks in random walk	0.55
μ_C	Scale parameter, cost of posting vacancies	1.76
γ_C	Elasticity of cost of posting vacancies	0.66
μ_m	Scale parameter, matching function	0.12
μ_o	Scale parameter, non-pay expenditures	1.98
γ_o	Elasticity of non-pay expenditures w.r.t. L	0.32
b	Quarterly out-of-work income	1.69
s_b	Sunk cost to open a firm	90

ature. We will later provide additional evidence on the model's good fit, most notably showing that it can accurately replicate the empirical responses of firms to the tax reform discussed in Section 2.

Before turning to this experiment, we will first characterize -in the next paragraph- the firm's optimal policy functions for any pair (L, x), in the pre-reform period, given the values of the estimated parameters of the model. This allows us to unveil the model's mechanics, which will serve as a benchmark for understanding how the exemption alters firms' behavior.

4.6 Optimal policy functions in the pre-reform steady state

Figure 2 shows the firm's firing and hiring policies as a function of the number of employees and current productivity. The dark blue shaded area in the graph identifies the region over which the firm reduces employment. The light blue area is the inactivity region: the firm neither hires nor fires workers in that part of the state space. Finally, the yellow shaded area corresponds to firms that seek to expand their workforce, posting vacancies and engaging in active hiring of workers.

The dynamic adjustments of firm employment are easy to read off this graph. For example, consider a firm whose current employment level is 15 employees and the current productivity x_{50}. This firm is in the *Wait* region. If negative shocks drive productivity down to, say, x_{30}, the new employment level can be found along the vertical line at x_{30}, where the *Wait* region ends and the *Fire* region begins. This corresponds to 5 employees. After the arrival of the shocks, the firm decides to lay off 10 of its employees.

It is worth noting that, since the area covered in light blue is quite wide, an implication of the graph is that firms that have just fired employees will need to experience a considerable increase in productivity before being induced to hire again. In the previous example, for this firm with five employees, it would take a very large shock (or several positive shocks) that raise productivity to at least x_{49} before the firm starts hiring.

Analogously, for firms that have just entered the market and currently have 0 employees, it is optimal not to hire unless productivity is above x_{34}. Given the estimated distribution of the initial idiosyncratic productivity, it is clear that many new firms will initially have productivity lower than x_{34} and will hire only if they experience productivity increases.

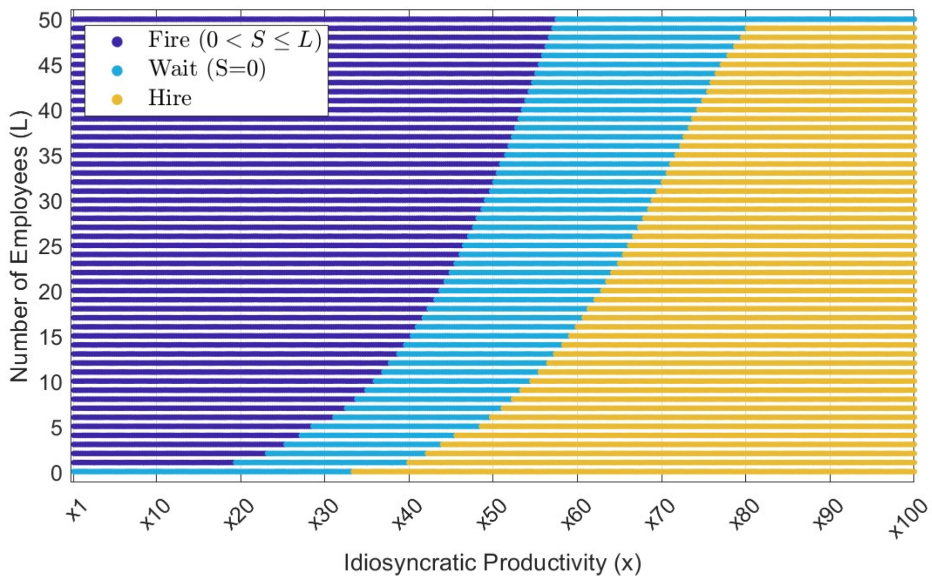


Figure 2: Firm optimal policy functions. The figure shows combinations of idiosyncratic productivity (x-axis) and number of employees (y-axis) at which firms choose different employment adjustment policies. The dark blue-shaded region indicates where firms reduce employment (*fire region*); the blue-shaded region corresponds to no employment adjustment (*wait region*); and the yellow-shaded region represents employment expansion (*hire region*).

Hiring policies. Within the hiring region shown in Figure 2, firms with different pairs of (L, x) hire at varying rates. When hiring, these firms will make two decisions: first, how many vacancies V to post and second, how quickly to fill each vacancy. To fill a vacancy faster, a firm needs to attract more jobseekers for that vacancy, by posting higher wages. We now turn towards these decisions of expanding firms.

Figure 3 shows the number of vacancies posted by a firm with a given pair (x, L). Unsurprisingly, firms post more vacancies when they are more productive. In contrast, the relation between employment and vacancies is non-monotonic; the number of posted vacancies first increases and then decreases in L. We can attribute this property to the joint impact of the firm revenue function R and the cost of posting vacancies. Revenues increase at a decreasing rate in L; the cost of posting vacancies, $C(V, L)$ decreases in L. Consequently, over a certain range, the vacancy yield rises as firm employment expands, but eventually declines as L becomes larger.

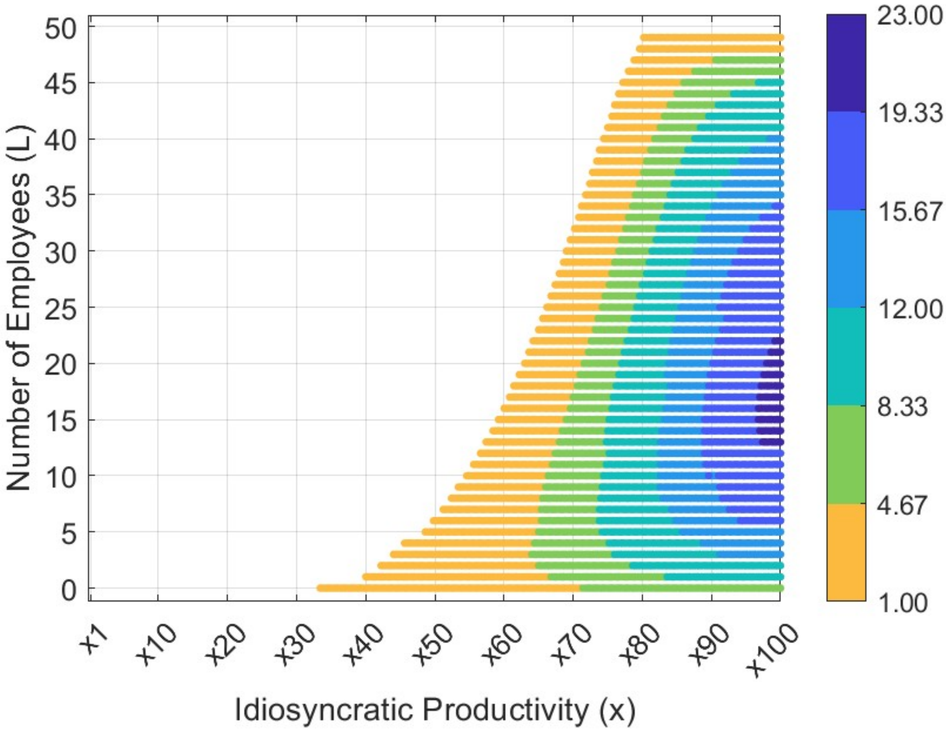


Figure 3: Number of vacancies by firm productivity and employment level. The figure shows the number of vacancies posted by expanding firms for each (L, x) pair. Different colours denote the number of vacancies posted (contour lines). The mapping between colours and vacancy postings is shown on the right axis. The uncoloured region corresponds to (L, x) combinations in which firms post 0 vacancies, as denoted in Figure 2.

Figure 4 shows the vacancy-filling rate as a function of (L, x). In general, more productive firms offer more generous contracts and achieve higher job-filling rates, thereby attracting a larger pool of applicants.[46] This effect is particularly pronounced for highly productive firms that have not yet reached their desired employment level. Very small but highly productive firms tend to hire more aggressively by offering higher wages.

Thus, highly productive firms initially expand employment by offering higher wages

[46] See Figure C.1a in the appendix for the number of jobseekers per vacancy.

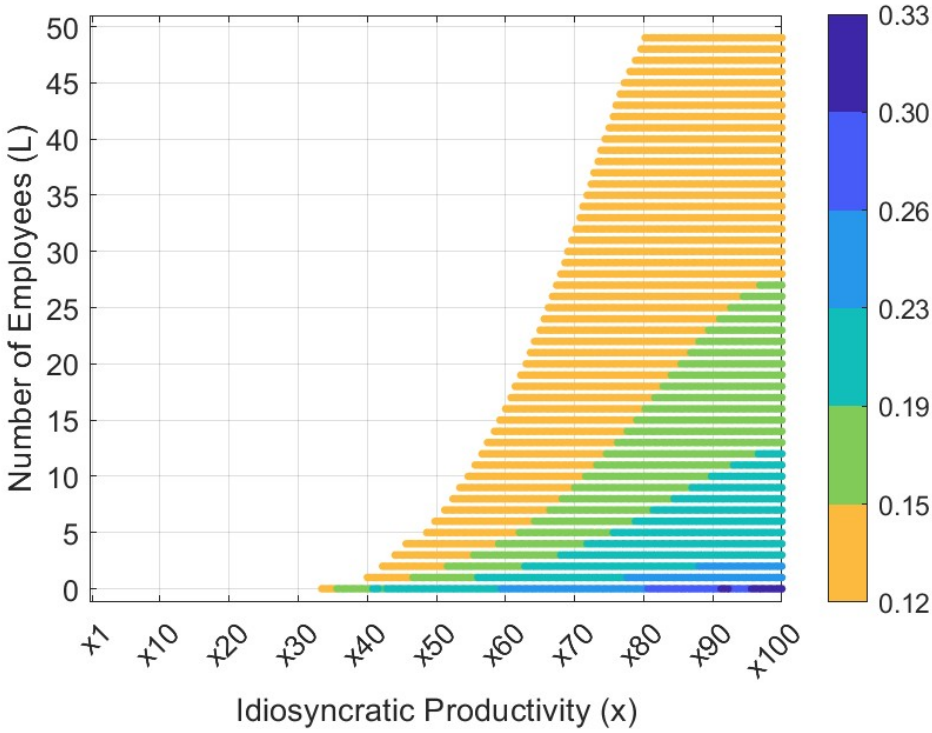


Figure 4: Vacancy filling rates. The figure shows the vacancy filling rate (the probability that a posted vacancy receives an application as a function of (L, x)). Different colours denote the vacancy filling rates (contour lines). The mapping between colours and rates is shown on the right axis. The uncoloured region corresponds to (L, x) combinations in which firms post 0 vacancies, as shown in Figure 2.

to attract applicants. Subsequently, they continue growing by increasing the number of posted vacancies, while gradually lowering the wages offered.

Figure 5 presents the expected number of hires —calculated as the product of the number of posted vacancies and the vacancy-filling rate— for expanding firms. Interestingly, expected hires remain roughly constant for productive small and medium-sized firms. This suggests that both types of firms are similarly efficient in expanding employment, although they adopt different strategies to do so.

5 The impact of employee subsidies

5.1 Firm Responses to the tax shock

We now introduce the tax exemption and study the dynamic adjustment of the economy. We consider that initially the economy is in steady state, and an unanticipated permanent change in the tax code occurs. Following this event, the economy gradually converges to the new steady state.

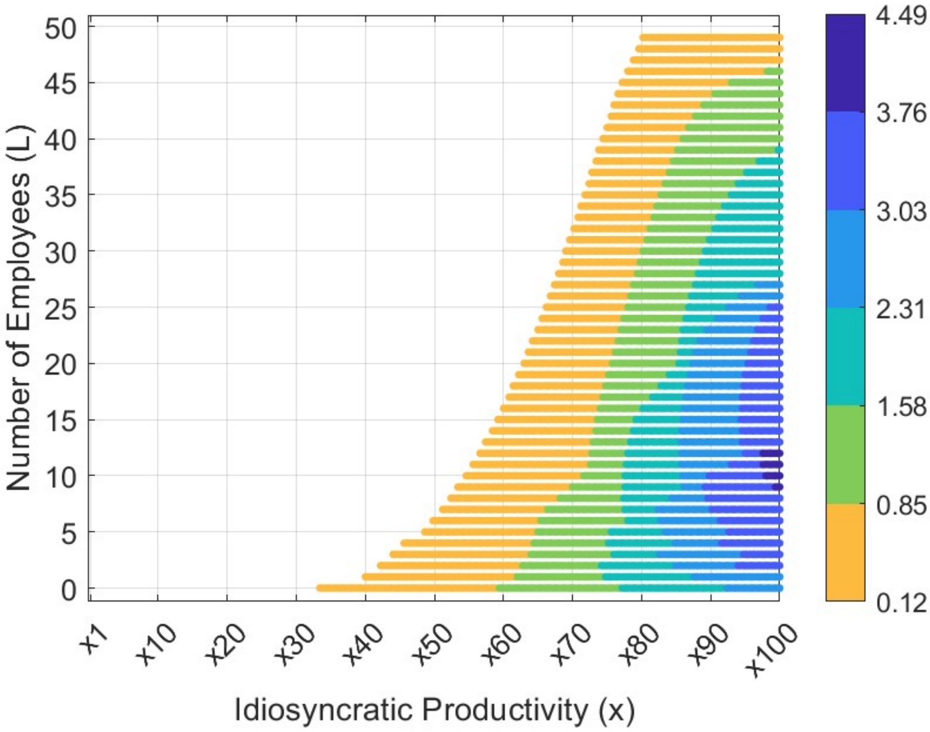


Figure 5: Expected number of hires. The figure shows the expected number of hires for each pair (L, x). Different colours denote contour lines corresponding to different numbers of hires. The mapping between colours and values is shown on the right axis. The uncoloured region corresponds to (L, x) combinations in which firms post 0 vacancies, as denoted in Figure 2.

Figure 6 shows the model's predictions (dotted lines) for the change in the number of firms with 1 employee, 2 employees, and more than 2 employees. The solid lines denote the corresponding data points as reported in Section 2.

Qualitatively, the model reproduces the patterns observed in the data. Following the change in the tax code, the number of firms with 1 employee increases, while the number of firms with 2 or more employees remains roughly constant. As in the data, the change in the tax code primarily affects firms at the margin of hiring their first employee. For additional hires, the policy has little to no effect.

Though, as explained previously, our goal is not to match the absolute magnitudes of the changes (particularly those after 2017), we note that the model performs reasonably well in quantitative terms. While the growth of single-employee firms after 2017 is faster in the model than in the data, the data clearly show a slight downward trend beginning in 2012 that is reversed— following the tax change. Hence, detrended growth in the data may actually be larger and more consistent with the model's prediction, which assumes the steady state as the initial condition.

Changes in the firms' optimal policy. What explains the response of firms to the tax reform? By fitting a structural model to the data we can inspect the different channels through which firm behavior is impacted.

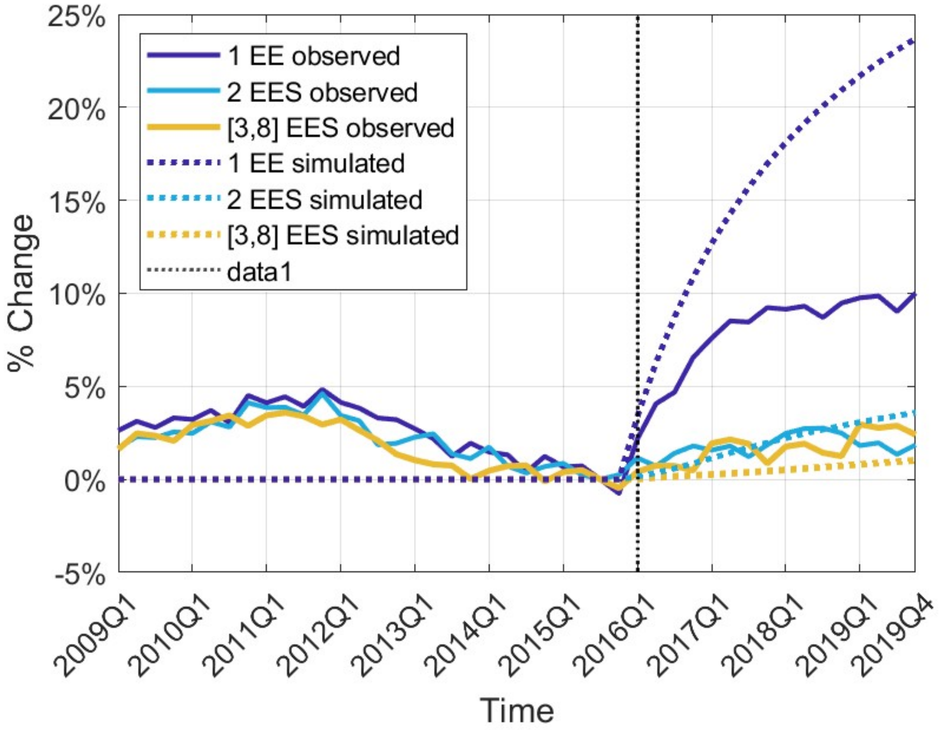


Figure 6: Impact of policy: model and data. The figure illustrates the changes in the stock of firms with 1, 2, and [3–8] employees in both the data and the model simulation. For each firm group, the number of firms is normalised to 1 in 2015Q3. The vertical dotted line marks the beginning of the policy in 2016.

We begin by considering the impact of the tax code on the firms hiring policies in a partial equilibrium setting, abstracting from the general equilibrium effects which (as we discuss below) increase the wage costs of firms through a rise in the value of unemployment, U.

In Figure 7 we summarize the change in the firms hiring policy function due to the tax exemption. We consider only firms that are eligible for the subsidy. We represent with rectangles values of the (L, x) vector for which firms change their hiring policy. We measure these changes in terms of the expected number of hires per firm (the colour code on the right indicates the percentage changes). If no rectangle is shown, it means that firms do not adjust their vacancy posting behavior. Finally, the black vertical dashed line denotes the minimum productivity level at which firms with zero employees hired pre-reform.

Notice that the only firms that experience an increase in expected hires are firms with initial employment level $L = 0$. Firms with positive employment rates do not adjust their hiring policy. The theoretical results established in Section 3 of the paper are useful for interpreting this model prediction. Most notably, we showed that the timing of worker wages is irrelevant for the optimal allocation (and hence also for future hiring policies). A firm could frontload the wages of its workers, and thus also frontload its social security contributions. In light of this property, it is therefore not surprising that optimal hiring decisions for firms with $L > 0$ are unaffected by the reform: for these firms that have already benefited from the new tax code, the benefit is already sunk.[47]

Consider now eligible firms with 0 employees. The change in the tax code expands the hiring region (to the left of the vertical line) and also induces additional hiring among higher-productivity firms, that would have hired even in absence of the reform. We find that the threshold productivity level (above which firms hire) falls by 5.7% relative to the pre-reform equilibrium. These marginal firms are most likely to begin hiring by posting a single vacancy. Consequently, we expect an increase in the number of firms with $L = 1$.

Firms that are further from the hiring margin, however, tend to post multiple vacancies. In this case, an increase in expected hires implies that such firms receive more job applications per vacancy —through higher offered wages— post more vacancies, or both. According to our model, both channels are operative, implying a faster growth in the number of firms with 1, 2, or more employees.[48]

It is important to note, however, that not all firms on the $L = 0$ grid experience an increase in expected hires. Many firms do not adjust their hiring policies after the reform, and in some cases, we even observe a slight reduction in expected hires. These outcomes arise entirely from the discrete (non-convex) nature of hiring in our model. For some firms, posting an additional vacancy or increasing (discretely) the promised wage to attract one or two additional applicants is too costly. In addition, some firms may even find it profitable to reduce the number of posted vacancies while targeting a higher number of jobseekers per vacancy. This substitution can lead to a reduction in expected hires.

Accounting for general equilibrium effects. We now switch on the general equilibrium impact of the reform and continue investigating the firm hiring policies. As discussed previously, the change in the tax code induces a one off permanent increase in the value of unemployment U. Conveniently, this is the only equilibrium effect that we need to account for; the firm policy functions will not display any transitional dynamics after the introduction of the reform. Under the new tax code the value U increases by 0.84% relative to the pre reform steady state.

Figure 8 shows firms' optimal hiring policies. It is essentially the same as Figure 7, but now we account for the higher value of U in the post-reform equilibrium. We again focus on eligible firms (that is, firms with positive employment that have benefited from

[47] Of course, a firm might experience a sequence of negative shocks and be forced to reduce employment to zero. Under this contingency, the tax exemption may affect future profit streams—if negative shocks are followed by positive ones and hiring once again becomes optimal. However, it seems implausible that a firm would hire more today under the prospect that its future employment level will drop to zero.

This property also holds approximately under positive exogenous job destruction shocks. Theoretically, if $s_0 > 0$, a firm with positive employment might benefit from the reform in the future if the subsidized employee separates. Although in our baseline model we set $s_0 = 0$ to match the data—and thus this channel is absent—we experimented with a calibration of $s_0 = 0.02$ and found no difference in the model's predictions. Plausible values of s_0 are too small for the tax code to materially affect future firm profits when current employment $L > 0$.

[48] However, we find that the bulk of the adjustment operates through higher wages.

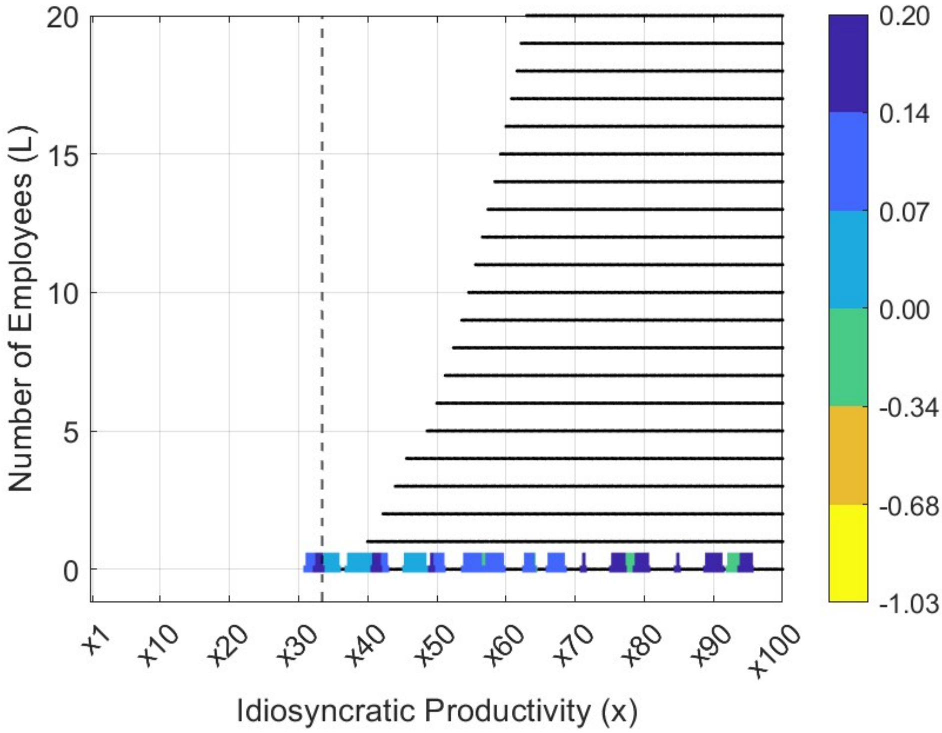


Figure 7: Change in the post-reform expected number of hires for eligible firms (fixed U). The black horizontal dashed lines represent the (L, x) pairs where firms hire the same expected number of individuals both before and after the reform. The coloured rectangles indicate pairs where firms hire a different expected number of employees post-reform, with the colour coding showing the magnitude of the change. The black dashed line denotes the minimum productivity level at which firms with zero employees were hiring prior to the reform's implementation.

the new tax code). The green and ochre areas indicate values of the state vector for which firms reduce expected hires. Specifically, the ochre regions correspond to firms that reduce the number of jobseekers per vacancy (by not increasing promised wages sufficiently to maintain their queue length), whereas the green regions represent firms that reduce the number of vacancies.

Notice that for some of the new firms, the reduction in taxes more than compensates for the increase in wage costs driven by the higher value U. Hence, we continue observing an increase in the expected hires for firms on the $L = 0$ grid. Elsewhere in the domain of the policy functions, higher employment costs lead to a reduction in the desired level of hiring.

In the appendix we show (Figure C.2) the counterpart of Figure 8 for firms that are not eligible for the reform. By and large the changes in the optimal policy functions are as for eligible firms. Job creation drops following the change in taxes, echoing the larger wage costs of employment.

Comparing the general equilibrium impact with the data. How plausible is the

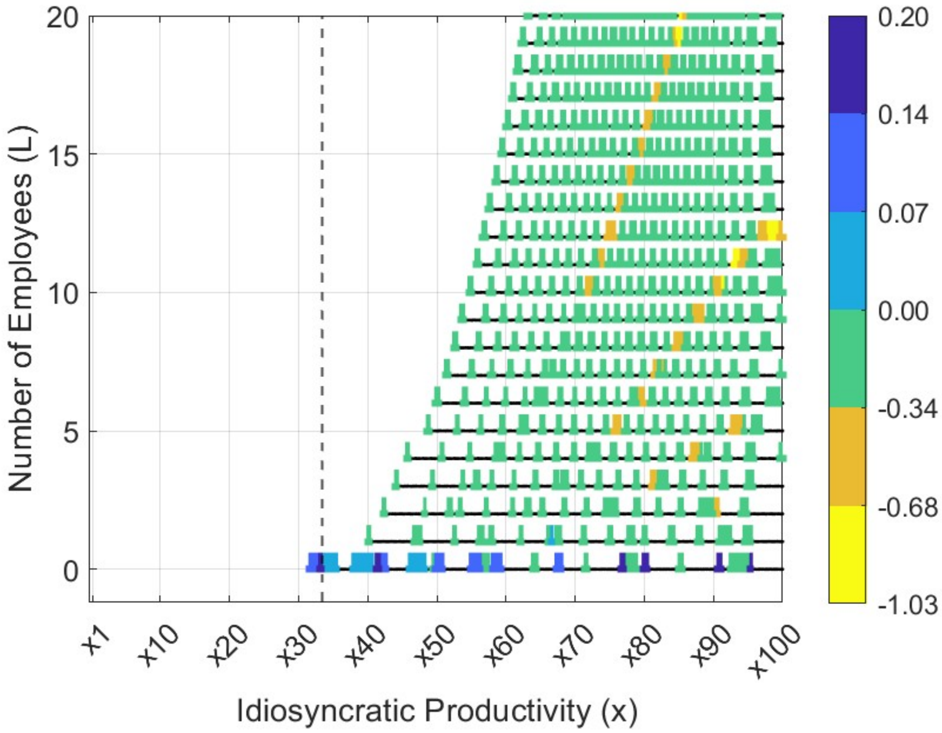


Figure 8: Change in the post-reform expected number of hires for eligible firms (general equilibrium)). See the legend under Figure 7.

magnitude of the general equilibrium effects implied by the model? We find a counterpart in the data by noting that the value of unemployment U maps directly to the wage cost of firms. After the introduction of the exemption, U increased by 0.84%, implying an identical 0.84% increase in the continuation wage. This increase in U, along with the policy-driven rise in the number of applicants that some firms aim to attract, leads to a 1.92% increase in the average signing bonus. Together, these two changes result in an approximate 0.9% increase in flat wage-equivalent costs.

In the appendix, we present a graph (Figure A.1b) illustrating the evolution of real wages in Belgium before and after 2016. Real wages declined by about one percentage point between 2013 and 2016 but subsequently began to rise, increasing by roughly 1.5 percent by 2019. While not all of this growth can be attributed to the reform, the pattern of wage dynamics is broadly consistent with the firm-level employment trends discussed earlier.[49] Taken together, these observations suggest that the model does not imply implausible general equilibrium effects.

[49] The wage data we refer to cover the entire economy, not only firms with up to 15 employees. Wages in Belgium are automatically indexed to inflation and wage growth norms are typically determined through collective bargaining between unions and large firms. Although these norms may not directly bind small businesses, they can influence wage-setting more generally. At the same time, average economy-wide wages will also reflect the outside options of workers employed in small firms, as well as composition effects induced by changes in the distribution of employment across firms—elements that our model explicitly captures.

5.2 Why does the tax reform not lead to much growth in multi-worker firms?

The objective behind the introduction of the exemption by the Belgian government was to enable/encourage firms with 0 workers to hire a first employee, with the expectation that this would then lead sufficiently productive entrepreneurs to continue hiring. Hence, the intended impact of the reform was to foster employment growth for productive multi-worker firms.

We have seen that in the model (much like in the data) following the change in the tax code, the number of firms with more than 1 employee does not grow considerably. In Figure 9a we plot the changes that occur in the firm-size distribution due to the reform. For convenience, we focus only on the steady state of the model and a subset of the employment levels. The number of firms with 1 employee increases by 39% in the post reform equilibrium. The analogous numbers for firms with 2 and between 3 and 8 employees increase much less (5% and 3% respectively). The fraction of firms with more than 8 employees drops by roughly 1 percentage point.

Notice that the most crucial parts of the model to explain these predictions are i) the productivity distribution of firms with 0 employees and ii) the shape of the employment cost function $c(L)$ and that of the vacancy posting costs $C(V, L)$. In principle, if the pool of zero-employee firms (pre-reform) includes high-productivity firms that face a steep cost of hiring their first employee —either because $c(1)$ is high or because posting vacancies at $L = 0$ is expensive, in addition to the non-convexity of hiring in our model— then subsidizing the first employee could substantially increase the fraction of firms with multiple workers.

According to our estimated model, firms are reluctant to hire their first employee before the reform not only because of the steep costs involved, but also because of their low productivity. Therefore, even when the subsidy encourages them to hire one employee, they remain unwilling to hire additional workers, despite the fact that the average cost $c(L)/L$ and the cost per vacancy both decline as employment rises.

5.3 An objective for assessing the effectiveness of policy.

Even though the reform does not lead to significant growth in the number of multi-worker, this outcome may still be desirable. From the perspective of an optimizing government. setting the tax rate as a function of firm size to maximize its objective function subject to a given revenue constraint, subsidizing the first employee may form part of the optimal tax policy.

Verifying whether this is indeed the case is a formidable task. Beyond explicitly specifying the government's objective, it requires solving a complex optimization problem in which taxes can be set for each employee or firm type, and possibly subject to additional constraints that may limit the degrees of freedom in terms of varying taxes across firms and workers.[50] Solving this problem is beyond the scope of our paper.

[50]For practical purposes a policy that entails too much variation of tax rates across workers and

Instead, we adopt a simple metric to discern whether subsidizing the first employee may be a desirable policy. We interpret that the implicit objective of the Belgian government is to alleviate the frictions that inhibit firms to expand their employment. We consider that a desirable reform in the tax code would be one that could imitate an equilibrium allocation in which frictions are less significant. We model 'looser frictions' by considering an increase in the matching efficiency parameter (μ_m).

Figure 9b shows the effects of this change. Since our goal is not set a quantitative target for policy but rather to study the qualitative features of an equilibrium with less search frictions, we simply consider a calibration of the model where matching efficiency increases by 50 percent (leaving all other parameters of the model as in the pre reform steady state), and plot the changes the distribution of firms relative to the pre-reform steady state.[51] The light-colored bars in the figure show the impact of this change. For comparison purposes, the dark-colored bars repeat the first employee exemption baseline shown in Figure 9a.

Assuming higher matching efficiency changes the firm distribution (relative to the original steady state) in two salient ways. First, there is a higher number of firms with 0 and 1 employees. Second, there is a higher number of firms with 3 or more employees. (Interestingly, the number of firms with 2 employees drops).

Subsidizing the first employee, accomplishes partly these 'objectives'. It boosts the number of firms with one employee, however it fails to induce higher firm entry and growth of multi-worker firms. We will next consider the policy in alternative environments and also consider alternative reforms in the tax code. For our subsequent experiments we will maintain that the implicit targets for policy are the ones we identified by relaxing the frictions in the economy.

5.4 When do 1st employee subsidies work?

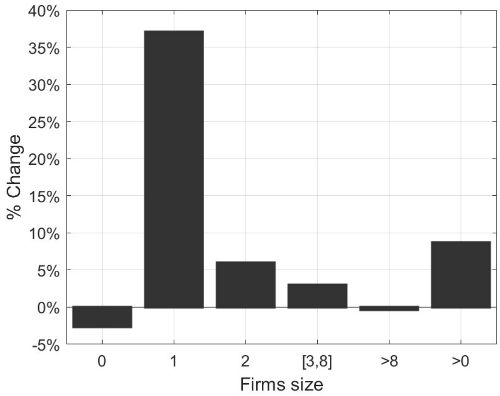
High productivity entrants. We now consider alternative parameterizations of the model and study the impact of the reform. Our first experiment assumes that a larger

firm employment levels is not palatable for several reasons. For example, it may be that managers find difficult to navigate around such policies when they determine the optimal level of hiring/firing. Too much variability in taxes can then impinge distortions. Thus, constraining to tax functions that satisfy 'implementability' is important. Obviously, identifying the set of relevant tax functions is not an easy task either.

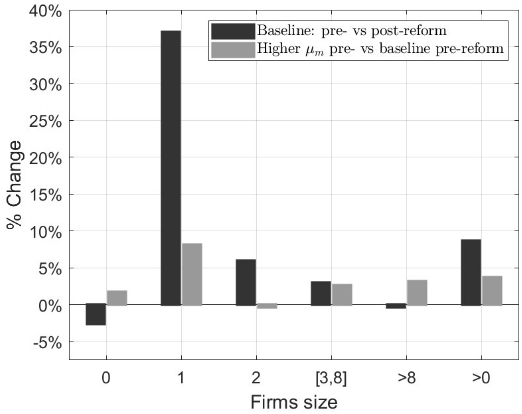
[51] Obviously, if frictions are completely absent, we obtain the neoclassical outcome whereby the distribution of employment is such the marginal product of labour is the same across firms. Implicitly, we are assuming that the objective of the government is to design a tax code that brings the economy closer to this outcome.

Note also that previous papers (Kaas and Kircher, 2015; Schaal, 2017), have characterized constrained efficient allocations whereby the planner maximizes output net of vacancy costs and taking labour market frictions as given. Kaas and Kircher (2015) and Schaal (2017) establish that the competitive equilibria in their settings are constrained efficient (optimal distortionary taxes are then equal to zero).

This is a different argument than we adopt here. We do not necessarily consider that the objective of the government is to maximize net output and/or the government budget could constrain the tax schedule. In this context zero taxes may not be feasible/or even optimal.



(a) Changes in the firm size distribution: Tax exemption



(b) Changes in the firm size distribution: Tax exemption vs higher matching efficiency.

Figure 9: Changes in the firm size distribution. The left panel shows the percentage change in the number of firms with 1, 2, 3–8, >8, and >0 employees induced by the change in the tax code. The percentage changes are relative to the initial (pre reform) steady state. The right panel reproduces the bars from our baseline model and compares them with a scenario in which taxes remain constant but matching efficiency permanently increases (light-colored bars).

fraction of new firms are productive. We recalibrate the shape parameter of the Pareto distribution to be half of the estimated value. Under the new parameterization we have $\sigma_x = [0.11, 0.09, 0.07, \ldots, 0.001]$ (instead of $\sigma_x = [0.18, 0.13, 0.19, \ldots, 0.0003]$ in the baseline). We wish to test whether in economies where small firms are more productive (than they are in Belgium), but face frictions that inhibit them from accomplishing their desired employment levels, a first worker subsidy could produce different outcomes than we observed in our baseline.

The results (together with those of the baseline model) are plotted in Figure 10a (light-colored bars). When the initial firm distribution features a larger fraction of highly productive firms, then subsidizing the first employee entails a drastically different impact than in the baseline economy. Most notably, the number of multi-worker firms now increases considerably in the new steady-state equilibrium.

This experiment confirms our previous intuition; the change in the tax code may not lead in growth of multi-worker firms when the new entrants and firms with zero employees have low productivity. In contrast, if the pool of new entrants contains a large number of productive businesses then the subsidy brings about better outcomes, precisely because it enables productive firms to grow faster by alleviating the frictions.

In the appendix (Figure C.3) we perform an additional check of this hypothesis. Focusing on the baseline model we assume that the employee subsidy can be fully targeted towards productive small firms (firms that will eventually grow to employ multiple workers). We show that if such tagging of subsidies to productivity is possible, then the change in the tax code induces growth in the number of multi-worker firms. It does so by enabling the productive small firms to post higher wages and to compete with large firms in attracting applicants.

The role of vacancy creation costs. We now consider an alternative calibration of the model which eliminates the dependence of vacancy creation costs on firm size. Thus, in the model analyzed in this paragraph, firms continue facing a convex cost of posting vacancies however costs do not drop when the firm expands its employment level.

We wish to test if the nature of vacancy creation costs matters for our baseline results. As we saw previously, in our baseline model, small productive businesses are able to grow by offering higher wages to applicants, whereas larger (medium sized) productive firms have a relative advantage in vacancy creation costs and continue growing by posting more vacancies. Eliminating the dependence of vacancy costs on firm employment means that medium firms will need to compete more intensively with small firms in terms of the wages they offer.

Note that it is not a priori known what an employment subsidy will accomplish in this new environment. If small firms can offer lower wages per vacancy, they will grow faster even in the pre reform steady state. In contrast, if the wages offered by small firms grow due to the more intensive competition, then an employment subsidy could be useful for small firms.

We plot the results in Figure 10a (middle bars). Though the impact of policy does not dramatically change relative to the baseline model, we do observe more growth for firms with 2 employees. This is driven by relatively unproductive firms. These firms would optimally grow to have 2 employees (instead of 1) under the different cost function we assume in this paragraph.

5.5 Alternative reforms

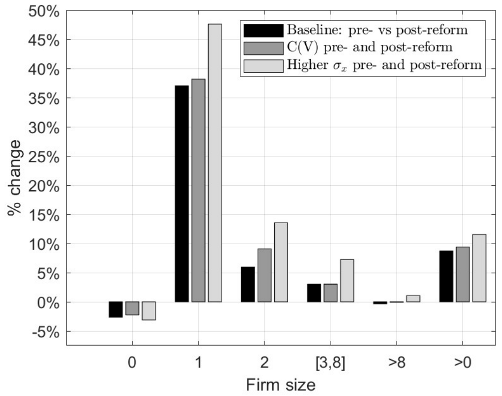
Subsidies beyond the first employee. Thus far, our analysis has examined the effects of the permanent payroll tax cut on the first employee. We utilized the model to investigate why the change in the tax code introduced by the Belgian government in 2016 had only a limited impact on the number of multi-worker firms.

As discussed previously, the permanent exemption for the first hire was not the only policy the government introduced to foster employment growth among small firms. Since 2004, similar subsidies had been offered for the second, third, fourth, and fifth employees. Unlike our baseline tax change, however, these subsidies were temporary.

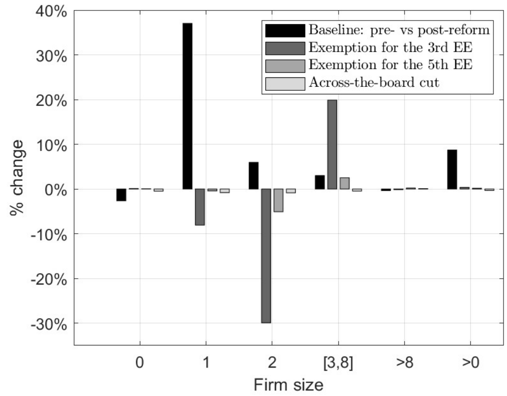
Notice that temporary tax cuts are unlikely to induce considerable changes in firm employment dynamics in our model. Simply put, any short-lived benefit (such as a subsidy lasting only a few quarters) is likely to be too small in present value terms for firms to respond by increasing vacancies, especially given the non-convexities inherent in hiring in our model.[52] We therefore focus on permanent exemptions for employees beyond the first

[52]This property would be consistent with the recent empirical evidence in Belgium. Recall that temporary subsidies had been applied to the sixth employee since 2016, but as we saw in Section 2, they most likely did not result in any significant increase of the share of firms with six or more employees.

It is also worth noting that temporary subsidies can be surprisingly complex to analyze within the



(a) Changes in the firm size distribution: Different initial productivity and vacancy costs.



(b) Changes in the firm size distribution: Subsidizing the 3rd, 5th employees and across-the-board subsidies.

Figure 10: Policy Counterfactuals. The left panel shows the percentage change in the number of firms with 1, 2, 3–8 and more than 8 employees, induced by the change in the tax code under the baseline parameters, when the Pareto parameter σ_P is half of the estimated value, and when the cost of posting vacancies function C is independent of L. The right panel reproduces the bars from our baseline model and compares them with alternative policies. Specifically, we consider the case where the government subsidizes the 3rd and 5th employees and when an across-the-board subsidy is introduced, whereby taxes are reduced equally for all workers/firms in the economy.

and, in the interest of parsimony, restrict our analysis to the 3rd and 5th employees.

The results can be seen in Figure 10b. Consider first subsidizing the 3rd employee. According to our model, this policy induces higher firm entry (opposite to the baseline model, it increases the number of firms with 0 employees), it reduces the numbers of firms with 1 and 2 workers, and it leads to substantial growth of firms with 3 or more employees. As a result, the reported number of firms with between 3 and 8 employees expands, though this is mainly driven by firms with 3 or 4 employees. Otherwise, the reform reduces the numbers of medium sized firms including those that have more than 8 employees (second to last set of columns in the Figure).

Next, consider the impact of a permanent subsidy for the 5th employee. This change in policy induces higher entry of firms to the economy, slightly more firms with between 3 and 8 employees and also a higher number of firms with > 8 employees.

Why are the outcomes between the two types of subsidies different? Just like in the baseline policy, a subsidy of the third employee, leads to an expansion of the workforce

model. For instance, a temporary exemption would break the result concerning the neutrality of the timing of wages. Firms would have a clear incentive to frontload wages in order to benefit as much as possible from the lower tax rate. Modeling temporary subsidies would therefore require, for example, restricting attention to flat wage contracts—which would substantially enlarge the state space—or introducing nonlinearities in the tax schedule, such as a maximum level of income eligible for exemption, and so forth.

of relatively unproductive firms. Firms that would otherwise have 1 or 2 employees are now able to hire a third. But for these establishments no further growth is optimal. In contrast a reform that subsidizes the 5th employee enables productive firms that would otherwise grow, to grow faster, mitigating the effect of frictions on their optimal hiring rates. Interestingly, the changes in the firm size distribution resemble qualitatively the changes induced by a higher matching efficiency parameter (Figure 9b).

Across the board subsidies. We close this paragraph by considering the impact of an across the board tax cut —that is, a uniform reduction in labor taxes for all workers in the economy, irrespective of firm size. This type of policy has been studied extensively in the academic literature (see, e.g., Mortensen and Pissarides (2001)) and has received significant attention from policymakers, particularly as a means of stimulating employment growth during periods of slack labor demand.[53]

Figure 10b shows the impact of this reform (along with the experiments of the previous paragraph). We assume that the magnitude of the tax reduction is such that the government's steady-state revenues remain equal to those in the baseline scenario.[54] Consequently, taxes per employee fall by a smaller amount, which explains the more modest changes in the number of firms. Nevertheless, our primary interest lies in evaluating the qualitative effects of this policy on the firm size distribution.

The across the board subsidy clearly favors large firms. According to our simulations it is only firms that have more than 8 workers that experience positive growth in the new steady state, whereas the numbers of all smaller firms shrink.

It is easy to understand which model mechanisms are responsible for this outcome. Since larger firms have an advantage in terms of vacancy posting costs, they can outcompete smaller firms by posting more vacancies. Therefore, a subsidy that increases desired job creation in large firms unavoidably leads to more competition and less jobs filled in smaller firms.

5.6 Aggregate and firm level outcomes

Our findings in the previous sections suggested that tax cuts favoring small firms (specifically, those with 1 and 3 employees) induce changes in the firm-size distribution that reallocate economic activity and workers from large to small firms. In contrast, reforms that implicitly target more productive (and hence larger) firms (e.g., the 5th employee) or that uniformly redistribute the tax burden across the firm-size distribution tend to

[53] See for example Kitao et al. (2010). See also the U.S. Hiring Incentives to Restore Employment Act (HIRE) of 2010 which discusses tax exemptions from employer social security contributions for workers hired from unemployment. The across the board subsidy can be seen as similar to a hiring cost subsidy (e.g. Kaas and Kircher (2015)) in our model. Though both types of subsidies may impact similarly job creation, they may impact differently worker retention. However, under full commitment (when wage paths are irrelevant as in our model) retention will not respond differently to a wage-cost subsidy than to a hiring subsidy.

[54] Specifically, we find that reducing τ from 18% to 16.8% achieves a 5.4% decrease in steady-state payroll tax revenues, matching the revenue change in the baseline reform.

promote the growth of multi-worker firms by raising the relative hiring and employment costs faced by smaller companies.

Moreover, a distinctive feature of our model is the presence of owner-run firms. As we have shown, the policies most impactful on this margin—specifically, in reducing the fraction of firms with zero employees—were the tax cuts for the 1st and 3rd employees.

Before concluding the paper in the next section, we explore the implications of the alternative reforms we have considered for a wide range of aggregate and firm-level outcomes. We use this paragraph to connect our findings with a broad literature on size-dependent policies. While numerous papers grounded in the neoclassical framework find that subsidies to small firms may entail large negative effects on productivity (Guner et al. (2008); Restuccia and Rogerson (2008); Buera et al. (2013); Acs et al. (2016), among others), other related work—emphasizing the importance of frictions and market failures in shaping the firm-size distribution—highlights that policies targeting job creation in small firms may have desirable features (Evans and Jovanovic, 1989; Hurst and Lusardi, 2004; Moll, 2014; Siemer, 2019; Hombert et al., 2020; Ando, 2021).

Workers, Firms. Table 3 reports the percentage changes in average firm size (including solo entrepreneurs), average employer size, as well as in the number of firms, employees, and jobseekers. There are several noteworthy properties. First, all policies that subsidize a single employee lead to a reduction in average firm and employer size. The across-the-board subsidy is the only policy that leads to an increase in these moments. Perhaps unsurprisingly, the baseline reform has the largest negative effect; as we previously noted, this reform induces a significant increase in the number of firms with only one employee.

Second, single-employee reforms increase the total number of firms in the economy, whereas the across-the-board subsidy reduces the total number of firms. However, the number of new entrepreneurs/firms drops under the first-employee subsidy and under the across-the-board subsidy, and rises when the government subsidizes the third and fifth employees.

Finally, the number of job seekers increases when tax cuts are targeted to the first employee and decreases in all other scenarios.

To understand these patterns, notice that a key determinant of equilibrium entry (the number of new firms created) is the size distribution of firms. Recall that firm exit rates in the model are a decreasing function of size, with the gradient particularly steep when size increases from 0 to 1 employee and from 1 to 2 employees. Hence, by replacing owner-run firms with firms that employ one worker, the baseline tax-cut policy reduces firm exits considerably. As a result, firm entry falls in the steady state, but the number of active firms increases.

The third- and fifth-employee reforms predict much smaller changes along these margins; for the most part, they redistribute employment across firms that have similar exit rates (firms with 3–8 employees). What is perhaps more remarkable is the larger impact induced by the across-the-board tax cut. Although a uniform tax cut does not lead to abrupt changes within any particular size category, it redistributes employment across the entire size distribution, reducing firm exit rates substantially.

Job Seekers. The numbers reported in the table reveal another intriguing prediction of the model. The baseline reform leads to an increase in the number of job seekers. This is somewhat counterintuitive, as we have seen that the tax change induces a rise in the value of U. Where, then, does the gain in U come from? The answer lies in the higher number of business owners in the new equilibrium. As firm exits decline, the higher survival rate of active firms increases the fraction of business owners in the population.

Hence, the response of U to the reform is not explained by a reduction in the number of unemployed individuals (if these are conventionally measured as active job searchers). According to our model, accounting for endogenous entrepreneurship choices is crucial for accurately capturing individuals' outside options.

Beyond the baseline, the remaining reforms shown in Table 3 result in a reduction in the number of job seekers. However, this does not always imply that aggregate employment increases (as in the 5th-employee subsidy). Again, changes in the number of business owners must be taken into account to understand these predictions.

Productivity Metrics. The last four rows of the table report measures of average productivity—–both revenues and revenues net of non-pay expenditures $c(L)$ —at the firm level, as well as per worker. In the latter case, productivity is computed as gross revenues divided by the number of workers in each firm, and then averaged across all firms. The across-the-board subsidy generates the largest gains in average revenues per firm, as this reform shifts employment toward larger, and therefore more productive, firms. In contrast, the baseline subsidy for the first employee shifts employment toward smaller firms and reduces average firm revenues.

Interestingly, the pattern reverses when we consider revenues per worker, excluding non-pay costs. According to this productivity measure, the baseline reform produces the largest gains (0.2%) compared with negative percentage changes under all other reforms. Shifting employment from larger to smaller firms unsurprisingly increases the marginal product of labor in large establishments. This effect essentially compensates for the lower average product among small firms—particularly those transitioning from zero to one employee. Accounting for the non-pay expenditures, reverses this prediction of the model.

6 Conclusion

We leveraged micro–firm–level data and an estimated directed search model with heterogeneous multi-worker firms to analyze Belgium's permanent payroll tax exemption for the first hire. According to our structural model, the policy substantially lowers the marginal cost of hiring a first worker, enabling many previously non-hiring entrepreneurs to become employers. However, most of the new employers are relatively unproductive and remain constrained by low productivity, which limits their ability to grow beyond one employee. These mechanisms allow the model to reproduce the empirical evidence.

Table 3: Firm and Labor Market Variables (% change relative to the pre-reform steady state)

Variable	Baseline	3rd employee	5th employee	Across the board
Aggregate outcomes				
Average firm size	−0.78	−0.18	−0.09	0.54
Average employer size	−8.30	−0.37	−0.16	0.43
# of firms	0.62	0.18	0.08	−0.44
# of employees	−0.17	0.01	−0.01	0.10
# of jobseekers	0.19	−0.32	−0.06	−0.05
# of agents creating a firm	−0.71	0.10	0.04	−0.46
Firm-level outcomes				
R/firm	−0.31	−0.09	−0.04	0.35
$(R - c(L))$/firm	−0.47	−0.12	−0.06	0.35
R/worker	0.20	−0.15	−0.08	−0.05
$(R - c(L))$/worker	−0.43	−0.10	−0.05	−0.05

Notes: All entries represent percentage deviations from the pre-reform steady state. "Baseline" refers to the benchmark model with the existing payroll tax cut on the first employee. The "3rd employee" and "5th employee" columns correspond to scenarios where the payroll tax reform applies only to firms hiring at least three or five workers, respectively. "Across the board" applies the reform to all firms.

Using the estimated model as a general-equilibrium laboratory, we evaluated alternative size-dependent subsidies. We found that permanent subsidies for the second or third employee have limited effects on firm growth, as they continue to target firms with low growth potential. In contrast, subsidies that apply to higher marginal hires—such as the fifth employee—or uniform subsidies that reduce labor costs for all workers can meaningfully shift employment toward more productive firms generating growth among multi-worker firms.

Our findings underscore that the effectiveness of size-dependent hiring subsidies depends critically on the joint distribution of firm-level employment and productivity. Our paper contributes to a large literature on size-dependent policies by introducing a framework in which firms face discrete hiring frictions, heterogeneous marginal costs of expanding employment, and in which workers have an occupational choice through endogenous entrepreneurship. Fitting our model to Belgian data, we find that the productivity distribution remains the primary determinant of the impact of size dependent policies. The model also shows that endogenous entrepreneurship plays a central role in shaping workers' outside options and the wage response to the policies.

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A Data Appendix

A.1 Construction of micro-data moments

Firm exits. We explain how we calculate the proportion of firms that exit the market within a year and show relevant data moments related to exits.

Let N_i, where $i \in [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]$, represent the number of firms with i employees in the market during a given quarter. We first count how many of these firms are no longer registered in the NBB dataset four quarters later. The second column of Table A.1 presents the average de-registration rates computed over four quarters: 2013Q1/2014Q1, 2013Q2/2014Q2, 2013Q3/2014Q3, and 2013Q4/2014Q4.[55] The de-registration rates are very low, as firms do not always de-register from the NBB dataset if they are no longer active. Therefore, these rates may not be accurate estimates of the firms' true exit rate.

In the third column of the same table, we calculate the proportion of firms with i employees in a given quarter that have 0 employees one year later. Interestingly, about 3.1-3.9% of firms with 10, 11, 12, 13, 14, and 15 employees see their workforce shrink to 0 employees within a year. We count these transitions as latent firm exits. We thus make the conservative assumption that, in addition to the de-registration rate from the NBB dataset, an additional 3.6% of firms exit the market each period—where 3.6% is the average proportion of firms with 10–15 employees that have 0 employees one year later.

The imputed yearly exit rates are shown in the last column of Table A.1. To derive the quarterly analogues, we apply the following formula (y): $q = 1 - (1 - y)^{1/4}$, where y and q denote the yearly and quarterly rates respectively.[56]

In estimation, we set δ_0 equal to the imputed quarterly exit rates.

Firms and employee distributions Using the NBB data, we calculate the average number of firms with 1 to 9 employees for the years 2013 and 2014. Because the NBB data do not include information on larger firms, we complement these figures with publicly available data from Statbel, the Belgian statistical office. Statbel (2022b) provides annual statistics on the number of active firms and their corresponding number of paid employees. These statistics are limited to for-profit firms in the industry and services sectors—sections B to N of the NACE Rev. 2 classification—excluding holding companies (NACE code K64.2). Further details on the firms covered by the Statbel statistics are available in Statbel (2022a).

Table A.2 presents the distribution of firms and employees obtained by combining our granular NBB data for small firms with the Statbel data for larger firms.

Relative revenues of firms with 0 and 1 employee. We calculate the average

[55] We selected this pre-reform year because, in 2015Q4, some firms altered their behaviour in anticipation of the policy. See (Cockx and Desiere, 2024).

[56] This formula sometimes predicts that the exits rates can be locally increasing in firm size, though the difference are very small. We impose weak monotonicity, setting δ_0 constant after 8 employees.

Table A.1: Yearly transitions

N EES	No longer in the dataset	With 0 EES	Imputed exit rate
0	5.33%	92.45%	8.89%
1	1.16%	21.48%	4.73%
2	0.87%	9.68%	4.43%
3	0.73%	6.54%	4.30%
4	0.62%	5.67%	4.18%
5	0.46%	4.93%	4.09%
6	0.52%	4.48%	4.08%
7	0.47%	4.05%	4.04%
8	0.39%	4.03%	3.96%
9	0.39%	4.16%	3.96%
10	0.42%	3.73%	3.96%
11	0.54%	3.96%	3.96%
12	0.52%	3.62%	3.96%
13	0.48%	3.68%	3.96%
14	0.49%	3.29%	3.96%
15	0.40%	3.11%	3.96%

Notes: This table reports the proportion of firms with 1, 2, etc. employees in a given quarter that have deregistered from the NBB datasets within one year (second column) and those with zero employees after one year (third column). The fourth column combines this information to provide the proportion of firms that have exited the market within one year.

Table A.2: Firms' and employment distribution

Firm size	**N firms**	**N firms (%)**	**N EES**	**N EES (%)**
0	655,083	79.09%		
1	53,829	6.50%	53,829	2.39%
2	29,055	3.51%	58,110	2.58%
3	18,296	2.21%	54,888	2.44%
4	12,567	1.52%	50,268	2.23%
5	8,962	1.08%	44,808	1.99%
6	6,834	0.83%	41,003	1.82%
7	5,311	0.64%	37,177	1.65%
8	4,267	0.52%	34,135	1.52%
9	3,480	0.42%	31,320	1.39%
[6,+∞]	30,557	3.69%	1,846,300	81.99%
total	828,240	100.00%	2,251,837	100.00%

Notes: These statistics are derived from our NBB data and publicly available Statbel statistics. They represent the averages for the period between 2013 and 2014.

quarterly turnover for firms with 0 and 1 employee between 2013Q1 and 2014Q4, amounting to €14,711 and €47,510, respectively. However, turnover is reported only for firms subject to VAT. In Belgium, most firms may opt out of VAT if their annual turnover is below €25,000. As a result, the NBB does not record turnover data for 29.32% (4.20%) of firms with 0 (1) employees. To account for this substantial share of firms with 0 employees whose turnover is unknown, we make the conservative assumption that all such firms have an annual turnover of €25,000, corresponding to a quarterly turnover of €6,250. Under this assumption, the average quarterly turnover of firms with 0 employees decreases to €12,230. Consequently, the ratio of the average turnover of firms with 0 employees to that of firms with 1 employee is 25.74%.

Distribution of new firms. To measure the size distribution of new firms, we proceed as follows. Using the NBB data, we compute for each quarter between 2013Q1 and 2014Q4 the number of firms that did not exist in a previous quarter and report how many of these firms have 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 employees. Notice that when a firm changes its identification number (CBE number), it will appear as a new firm in the NBB data. Therefore, we may mistake firms that change CBE number as new firms. For this reason, as in Bijnens and Konings (2020), we assume that all firms which have 10 employees or more in the quarter they are created are not really new firms and, therefore, are not included in this statistic.[57] In Table A.3, we report the employment distribution of new firms.

Table A.3: Employment distribution of new firms

size	**N firms**	**N firms (%)**
0	12,821	95.55%
1	308	2.30%
2	125	0.93%
3	59	0.44%
4	40	0.30%
5	24	0.18%
6	16	0.12%
7	11	0.08%
8	8	0.06%
9	6	0.04%
Total [0,9]	13,418	100.00%

Notes: These statistics are computed from our NBB data. They present the averages between 2013Q1 and 2014Q4.

A.2 Statistics from ONEM and Statbel

The unemployment rate is derived from ONEM's annual reports. The average unemployment rate for 2013 and 2014 is 10.75% (the average unemployment rate for 2013 is 10.8% and for 2014, 10.7%). ONEM computes the unemployment rate as the ratio between the number of individuals who receive unemployment allowances and are seeking a job, and the number of individuals insured against unemployment.

[57] Geurts and Van Biesebroeck (2016) estimate that already 57% of new entrants between 10–19 employees are spurious, with this percentage increasing with the firm's size at entry

Similarly, using the ONEM public statistics on transitions into and out of unemployment for each quarter between 2013Q1 and 2014Q4, we determine the number of individuals who were unemployed (receiving unemployment allowances and seeking a job) in quarter $t-1$ and who are either in salaried employment (A) or still unemployed (B) in quarter t. Next, we compute the job-finding rate for each quarter as the ratio A/(A+B). Finally, we calculate the average over these eight quarters, which is 8.9%.

The standard definition of the unemployment rate used by many administrative bodies (e.g., the OECD, Eurostat) does not impose any restrictions regarding the receipt of unemployment benefits or insurance against unemployment. Unemployed individuals are typically defined as: a) without work during the reference period, b) currently available for work, and c) actively seeking employment. The unemployment rate is then calculated as the ratio of the number of unemployed persons to the labour force. According to this definition, the Belgian statistical office, Statbel, reports an average unemployment rate of 8.6% for Belgium over 2013 and 2014, which is lower than the value reported by ONEM.

Our model, as it does not include any unemployment allowance and therefore does not account for the fact that some individuals may be unemployed and receiving benefits while others do not, more closely replicates the unemployment rate reported by Statbel.

However, ONEM data provide detailed information on transitions into and out of unemployment, which are not available with the same level of accuracy from other sources. Given the centrality of both statistics to our model, we have chosen to rely on ONEM figures for both moments, despite this leading to an overestimation of the target unemployment rate.

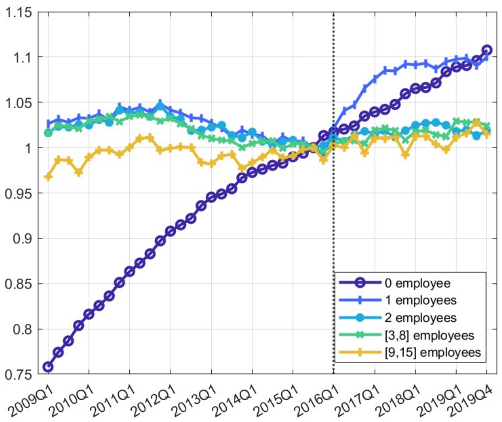
EUROSTAT provides the job vacancy rate (JVR), which is the ratio of the number of vacant jobs to the total number of vacant jobs and occupied posts for firms with 10 or more employees, as well as for firms with fewer than 10 employees. A 'job vacancy' is defined as a paid post that is newly created, unoccupied, or about to become vacant: (a) for which the employer is taking active steps and is prepared to take further steps to find a suitable candidate from outside the enterprise concerned; and (b) which the employer intends to fill either immediately or within a specific period of time; An occupied post means a paid post within the organisation to which an employee has been assigned (Eurostat, 2022). For 2013 and 2014, the JVR for small firms was 3.78% and 3.40%, while for big firms was 2.40% and 2.12% (Statbel, 2022c). Then, the average ratio of the job vacancy rate between small and large firms is 1.60.

A.3 Additional Empirical Output

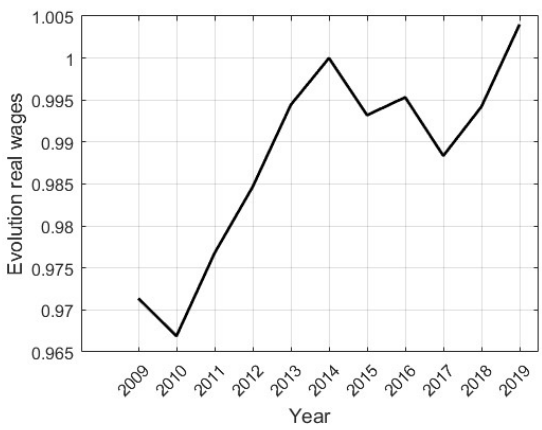
We present two additional empirical moments. First, Figure A.1a shows the evolution of the stock of firms before and after the reform. The legend explains the mapping between the markers/colors and firm types. relative to the figures we presented in the main body of text, Figure A.1a adds firms with 0 employees and firms with more than 8 employees. (The data for firms with between 3 and 8 employees is now depicted using the average).

Clearly, throughout the period considered, solo entrepreneur firms exhibit a strong upward trend. This is an important fact. It can be argued that the Belgian government introduced the tax exemption for the first employee, precisely because of this trend, so as to encourage growth for the newly created firms.

Figure A.1b shows real wages in Belgium since 2009. We normalize the real wage in 2014 to unity. According to the data, real wages have been dropping since 2014, and resumed positive growth in 2017.



(a) Evolution of the stock of firms. This panel shows the evolution in the number of firms with 0, 1, 2, [3–8], and [9–15] employees. For each group of firms, the number of firms is normalized to 1 in 2015Q3. The vertical dotted line marks the introduction of the policy in 2016. Note that the dataset contains information only for firms with up to 15 employees.



(b) Real wages in Belgium. The panel shows the evolution of average annual wages in Belgium at constant prices. Real wages in 2014 are normalized to 1, and values for other years are adjusted accordingly. *Source:* OECD Data Explorer, average annual wages.

Figure A.1: Firm dynamics and wage trends. The left panel illustrates the evolution in the number of firms by size over time. The right panel shows how real wages have evolved over time in Belgium.

B Proofs of Lemmata

Proof of Lemma 1. To prove Lemma 1 we follow closely Schaal (2017). We first write the firms profit function as:

$$\mathcal{J}(x,L,\mathbf{W}) = \max_{w,\widetilde{\phi}^i(.),d_{(.)},V\in\mathbf{N}\cup\{0\},W_V,\widetilde{\mathbf{W}}'} \widetilde{\mathcal{J}}(x,L,\mathbf{W}) = \max_{w,\widetilde{\phi}^i(.),d_{(.)},V\in\mathbf{N}\cup\{0\},W_V,\widetilde{\mathbf{W}}'} \widetilde{R}(L,x,V)$$
$$-\sum_{i\in\mathbf{L}} w_i(1+\tau) + \beta(1-\delta_0)\mathbf{E}\left[d_{(\widetilde{s}',L,\mathbf{W})}\mathcal{J}(x',L',\widetilde{\mathbf{W}}') - (1-d_{(\widetilde{s}',L,\mathbf{W})})\text{Cost}(W_V(\widetilde{s}')\right] + \beta\delta_0 U \tag{16}$$

subject to

$$L' = \sum_{i=1}^{L}\widetilde{\phi}^i(\widetilde{s}',L,\mathbf{W})I^i_{(\widetilde{s}')} + \sum_{v\in\{1,2,,\dots,V\}} I^v_V(\widetilde{s}') \tag{17}$$
$$I^v_V(\widetilde{s}') = 1 \quad \text{if vacancy} \quad v \quad \text{is filled in } \widetilde{s}' \quad (Prob = m(\lambda_{W_v}))$$
$$W_i = w_i + \beta(1-\delta_0)\mathbf{E}\widetilde{\phi}^i(\widetilde{s}',L,\mathbf{W})I^i_{(s')}d_{(\widetilde{s}',L,\mathbf{W})}W'_{s',i}+$$
$$+\mathbf{E}\left[\delta_0 + (1-\delta_0)\left(d_{(\widetilde{s}',L,\mathbf{W})}\left((1-I^i_{(\widetilde{s}')}) + (1-\widetilde{\phi}^i(\widetilde{s}',L,\mathbf{W}))I^i_{(\widetilde{s}')}\right) + (1-d_{(\widetilde{s}',L,\mathbf{W})})\right)\right]U \tag{18}$$

To condense notation we have used $\widetilde{R}(L,x,V) = R(L,x) - c(L) - C(V,L)$.

Next, we define the following function $\widetilde{K}(x,L)$

$$\widetilde{K}(x,L) := \widetilde{\mathcal{J}}(x,L,\mathbf{W}) + \sum_i W_i(1+\tau) = \widetilde{R}(L,x,V)$$
$$+\beta(1-\delta_0)\mathbf{E}\left[d_{(\widetilde{s}',L,\widetilde{\mathbf{W}}')}\left(\mathcal{J}(x',L',\widetilde{\mathbf{W}}') + \sum_i(1+\tau)W'_{\widetilde{s}',i}I^i_{(\widetilde{s}')}\widetilde{\phi}^i(\widetilde{s}',L,\mathbf{W})\right)+\right.$$
$$\left.(1-d_{(\widetilde{s}',L,\mathbf{W})})((1+\tau)U - \text{Cost}(W_V(\widetilde{s}')\right] + 2\beta\delta_0 U \tag{19}$$

Notice that maximizing $\mathcal{J}(x,L,\mathbf{W})$ subject to constraints (17) and (18) is equivalent to maximizing $\widetilde{K}(x,L)$ subject to (17), since $\sum_i W_i(1+\tau)$ is a predetermined constant. In other words, the optimal functions for firm exits, worker retention and hiring that derive from solving program (16) coincide with the optimal policies that solve

$$K(x,L) = \max \widetilde{K}(x,L) \tag{20}$$

subject to (17).

Given this equivalence, it is obvious that we can write

$$K(x,L) = \max_{d_{(x',L')},V,W_V,\widetilde{\phi}^i(x',L')} \widetilde{R}(L,x,V)$$

$$+\beta(1-\delta_0)\mathbf{E}\Bigg[d_{(\widetilde{s}',L)}\Bigg(K(x',\sum_{i=1}^{L}\widetilde{\phi}^i(\widetilde{s}',L)I^i_{(\widetilde{s}')}+\sum_{v\in\{1,2,,\ldots,V\}}I^v_V(\widetilde{s}'))-\sum_v(1+\tau)W^v_V(\widetilde{s}')I^v_V(\widetilde{s}'))\Bigg)+$$

$$(1-d_{(\widetilde{s}',L)})((1+\tau)U-\text{Cost}(W_V(\widetilde{s}'))\Bigg]+2\beta\delta_0 U \tag{21}$$

where it is now made explicit that $d(.)$ and $\widetilde{\phi}^i(.)$ depend only on the vector $(\widetilde{s}',L)$, and hence only on productivity and the beggining of next period employment in state $\widetilde{s}'$.

The optimal policies that solve (21) and any wage profile that is consistent with (18) solve also the original firm dynamic program. ■

Proof of Lemma 2. The proof follows immediately from (21). Since all $\widetilde{\phi}^i$ are functions of $(\widetilde{s}',L)$ only and do not depend on promised utility, there is nothing in the model that pins down the relative separation rates of the firm's workers (Schaal (2017)). It follows readily that we can set $\widetilde{\phi}^i=\widetilde{\phi}^j$ for all worker pairs i,j. Thus, given the total number of workers F' that the firms wants to fire in state $\widetilde{s}'$ and $\sum_i I^i_{\widetilde{s}'}$ is the total number of workers that did not separate exogenously in that state, we have $\widetilde{\phi}^i=\frac{\sum_i I^i_{\widetilde{s}'}-F'}{\sum_i I^i_{\widetilde{s}'}}$. ■

Proof of Lemma 3. We provide a descriptive proof that, under Assumptions 1 and 2 and Lemmata 1 and 2, the firm's program can be expressed as functional equations (10) to (13) in text.

The usefulness of Assumption 2 is clear. It enables us to separate the firm's optimization program into f,h. Since firms that fire workers do not simultaneously want to post vacancies, the choice variable in f is the number of workers that the firm fires, F. and the choice variables in h are V (number of vacancies) and $\overline{w}$ upfront payment for each of the posted vacancies. Allowing $F=0$ (firm does not fire any workers) and $V=0$ (firm does not post vacancies) implies that choosing f or h is compatible with inaction.

The importance of Lemmas 1 and 2 is also obvious. Using Lemma 1 we can be sure that we can recover optimal hiring and firing policies as the solution to a functional equation where (x,L) are the only state variables and use the promise keeping constraint to determine the path of wages/utility. Lemma 2 allows to solve the optimal firing policies by determining only the total number of workers that the firm will fire.

Note also that a key difference in terms of the notation we use in Lemma 3 relative to Lemma 1 is that we treat L as the beginning of period employment. Therefore, a firm that chooses f in state (L,x) is essentially a firm that has committed to fire workers in that state. Since the firm fires F workers (randomly chosen) its employment level will be $L-F$. From Assumption 2 the firm pays $\underline{w}(1+\tau)$ for each of the $L-F$ workers that it retains. Since vacancy creation costs are zero, the net revenue of the firm is $R(L-F,x)-c(L-F)$.

If the firm does not exit exogenously its continuation payoff is $E_{x'}[\sum_{L'\in\{0,\ldots,L-F\}} Pr(L',L-$

$F) \cdot J(L', x')] + \beta \cdot \delta_0 \cdot U\Big]$. $Pr(L', L - F)$ accounts for the uncertainty due to exogenous job destruction shocks. Given this notation we use $E_{x'}$ instead of E to clarify that the expectation is over the residual uncertainty for future productivity.

Consider now state h. Since the firm does not fire any workers, its beginning of period employment equals the total number of retained workers. The profit flow is therefore $R(L, x) - (1 + \tau) \cdot \underline{w} \cdot L - c(L) - C(V, L)$. Conditional on no exogenous exit, the firm will have a continuation payoff

$$\sum_{H \in \{0,\dots,V\}} Pr\big(H, m(\bar{w}), V\big) \Big[\sum_{L' \in \{0,\dots,L\}} Pr\big(L', L\big) \cdot \big[E_{x'} J(L' + H, x') - H \cdot (1 + \tau) \cdot \bar{w}\big]\Big]$$

We use L' to denote the number of workers that will not separate exogenously in the next period and given the realization of H (total hires) the next beginning of period employment for this firm is $L' + H$. Finally, (13) imposes that the firm pays $H \cdot (1 + \tau) \cdot \bar{w}$ to the newly hired workers regardless of the optimal action in next period (fire, hire, exit, implicit in function J).

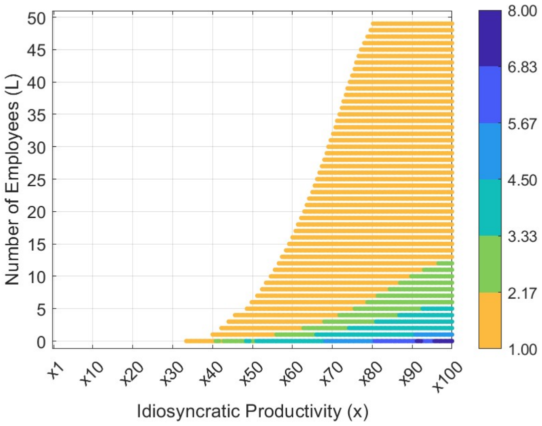
C Additional model output

For completeness, we revisit in this subsection the firm's optimal policy functions. Figure C.1a plots the number of jobseekers per vacancy in the (x, L) plane, whereas Figure C.1b shows the optimal number of fired workers for shrinking businesses, in the pre-reform steady state. Finally, Figure C.2 plots the change in expected hires for non-eligible firms following the introduction of the tax exemption.

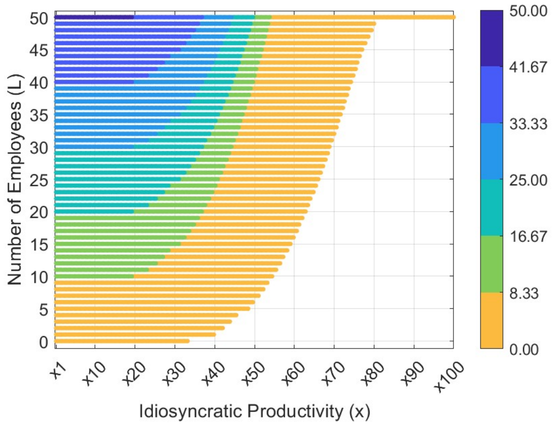
Marginal and inframarginal firms.

We now separately analyse how two groups of firms respond to the policy: those that, in the absence of the reform, would not have been productive enough to hire employees—located to the left of the dashed line in Figures 7 and 8—and those that would have hired workers regardless, located to the right of the dashed line. We refer to the first group as marginal firms and to the second as inframarginal firms.

We begin by assuming that the policymaker can perfectly observe the productivity x of each firm and grants the exemption only to marginal firms. Figure C.3 (dark grey bars) plots the change in the number of firms by employment size when comparing the pre- and post-reform steady states. Relative to the scenario in which all firms are eligible (black bars), we observe that firms that were previously too unproductive to hire account for the bulk of the increase in single-employee firms. However, only a small fraction of these new employers grow beyond one employee. This confirms our intuition: these newly established, low-productivity firms tend to remain within the policy region where hiring additional workers is not profitable. Only a few of them—those receiving a sufficiently positive productivity shock—expand further and hire more employees.



(a) Number of jobseekers per vacancy before the reform. This panel displays the number of jobseekers per vacancy attracted by firms for each combination of idiosyncratic productivity x and employment level L.



(b) Number of fired employees pre-reform. The panel shows the number of fired employees by firms for each combination of idiosyncratic productivity x and employment level L.

Figure C.1: Jobseekers and endogenous separations. The left panel shows the number of jobseekers per vacancy by firm productivity and employment level, while the right panel illustrates the number of fired employees along the same dimensions.

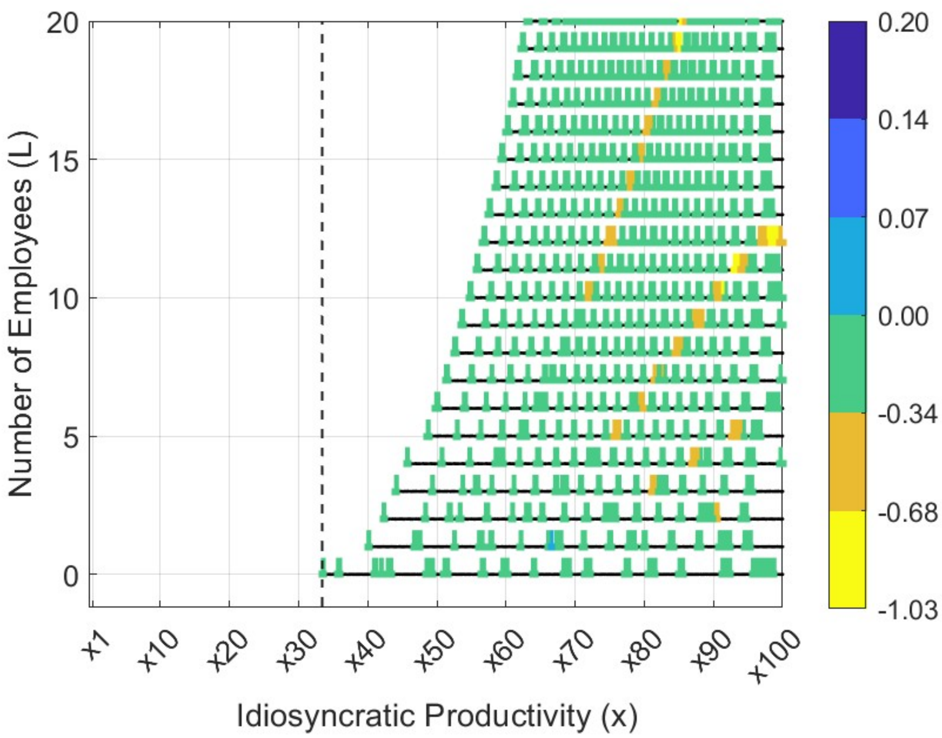


Figure C.2: Change in the post-reform expected number of hires for non-eligible firms (General equilibrium). See the legend under Figure 7.

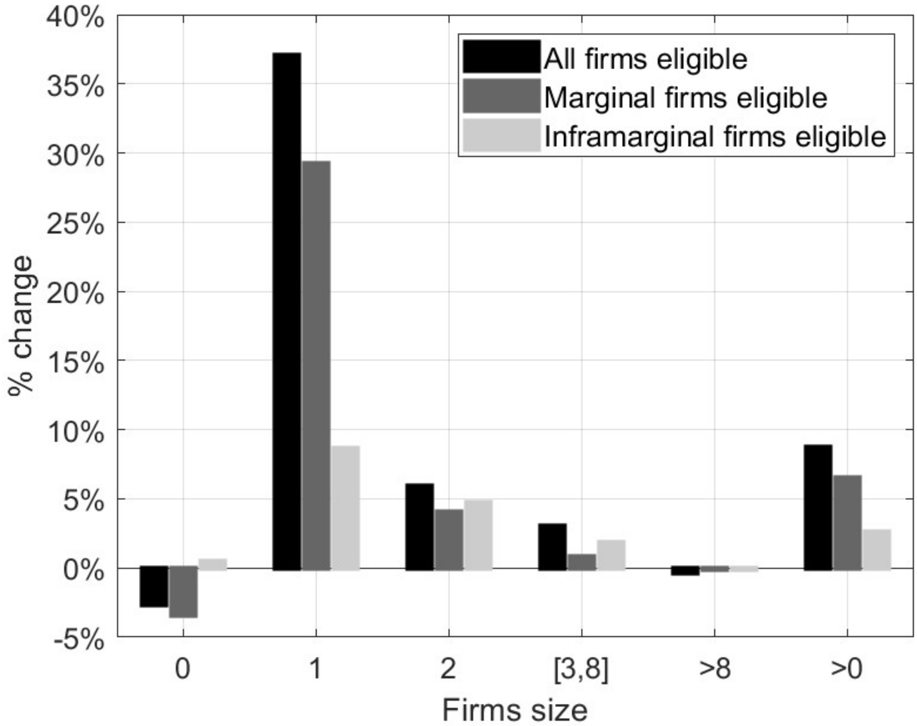


Figure C.3: Change (in %) in the stock of firms in steady state - decomposition. Percentage change in the stock of firms between the pre- and post-reform steady states. The figure shows three scenarios: (i) all firms are eligible for the exemption; (ii) only firms that were not productive enough to hire before the reform (marginal firms) are eligible; and (iii) only firms that were productive enough to hire before the reform (inframarginal firms) are eligible.

Next, we assume that the policymaker grants the exemption only to inframarginal firms. The impact is shown by the light grey bars in Figure C.3. In contrast to both the baseline reform and the reform that subsidizes only marginal firms, we now observe an increase in firm entry. Moreover, we find more growth among multi-worker firms, as the changes induced in the firm-size distribution are clearly more tilted toward larger firms. This experiment confirms that initial firm productivity is an important determinant of the impact of the reform and that, provided tax subsidies can be targeted toward more productive firms, the policy can lead to stronger growth in the number of larger firms.[58]

The finding that most of the increase in firms with one employee is driven by firms that were not productive enough to hire prior to the reform is consistent with Deng et al.

[58]It is also worth noting that the general equilibrium effects triggered by subsidizing marginal versus inframarginal firms differ in magnitude. When only marginal employers are subsidized, U increases by just 0.13% in the steady state. This is because marginal employers represent only a small fraction of firms receiving the exemption in our baseline analysis (roughly 18%). In contrast, when inframarginal firms are subsidized, U increases by 0.71%. Hence, the general equilibrium impact is stronger in the second policy change than in the first.

If we equalize the change in U across the two reforms, we would find even greater growth of multi-worker firms under the second reform. We intentionally allow the general equilibrium impacts to differ, however, so that Figure C.3 provides a clear decomposition of the baseline distributional changes.

(2024). Comparing the cohort of employers created after the reform with the cohort created just before it, Deng et al. (2024) show that firms entering after the reform are, on average, less productive than those entering before.