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CAPITALIST-WORKER WEALTH DISTRIBUTION IN A TASK-BASED MODEL OF AUTOMATION

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Abstract: I integrate the notion of task-based automation into a two-class economy with capitalists and workers, and subsequently study the interaction between the degree of automation and the wealth distribution between capitalists and workers. I find that, as the economy becomes more automated, the possibility of a sustained capitalist class naturally arises. In a laissez-faire scenario, a stable steady state with partial automation can occur both in a dual context (where workers own all wealth) or Pasinetti context (where both capitalists and workers have a strictly positive wealth share). In contrast, the fully automated scenario only occurs in case of an anti-dual economy where capitalists own all wealth. In the case of capital income taxation, the stable fully automated case can also occur in the dual or Pasinetti context. I show that Piketty's citation of a rising capital-output ratio as an explanation for an increasing capital income share and rising wealth inequality does not require the contested 'sigma $>$ 1' assumption.

Keywords: automation; two-class model; task-based production; factor shares; Pasinetti theorem; wealth inequality

JEL classification codes: E25, O33, O40

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Statements and declarations

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1 Introduction

Some recent studies have been preoccupied with using macroeconomic models in the tradition of Pasinetti (1962) to study evolutions in wealth inequality. However, they do so from a Pikettian (2014) point of view, and thus focus on the standard neoclassical production function where capital-labour substitutability is fully governed by the elasticity of substitution (Mattauch et al., 2017; Taylor, 2014; Zamparelli, 2017). In particular, this research is strongly inspired by the work of Zamparelli (2017). He shows that, in a general two-class neoclassical growth model, capitalists' share of total wealth tends to one if the elasticity of substitution between capital and labour is high enough to ensure endogenous growth. In this work, I move away from this condition on the elasticity of substitution by endogenizing technology through a general task-based model in which the output elasticities are determined by the optimal automation choice of firms (in the style of Acemoglu and Restrepo (2018)). Introducing endogenous automation and allowing firms to rationally consider whether they want to produce a task using capital or labour creates the potential of endogenous growth without having to impose large values for the elasticity of substitution, which are implausible according to most empirical estimates (e.g., Chirinko, 2008; Gechert et al., 2022; Knoblach et al., 2020; Oberfield & Raval, 2021). By replacing the contested assumption of a high elasticity of substitution with the incorporation of automation into the model, I can study the relationship between technology and capitalist-worker wealth dynamics in a more credible Pasinetti-style framework.

This study is closely related to two very distinct strands in the economic literature. On the one hand, the recently burgeoning literature on the theoretical modelling of automation. Automation in the task-based framework has negative implications for the labour share of income (Acemoglu & Restrepo, 2018), and this seems to be in line with the bulk of the empirical evidence (Acemoglu et al., 2020; Acemoglu & Restrepo, 2020; Autor & Salomons, 2018; Dauth et al., 2017; Eden & Gaggl, 2018; Graetz & Michaels, 2018; Guimarães & Gil, 2022). This literature has also studied the distributional effects of automation via the heterogeneous labour market effects of automation (e.g., Acemoglu & Loebbing, 2022; Jacobs & Heylen, 2021; Lankisch et al., 2017; Prettner & Strulik, 2020), but there have been fewer studies focussing on the implications of automation on wealth inequality. The recent paper by Moll et al. (2022) is a notable exception: they find that task-based automation leads to rising wealth inequality by raising returns to wealth.

On the other hand, this paper also relates closely to the long-standing literature on two-class economy models, which is exactly preoccupied with studying these distributional issues. In the words of Pasinetti (1962), who kickstarted this line of research, it has tried to "clear up the confusion which has been made of two different concepts of distribution of income: distribution of income between profits and wages, and distribution of income between capitalists and workers" (p. 270). Pasinetti (1962) focussed on (i) the eponymous steady state in which capitalists and workers both hold a positive share of the total capital stock, and in which worker savings are surprisingly irrelevant for the macroeconomy. In response, Samuelson and Modigliani proved that, if capitalists were insufficiently thrifty, they would not emerge and workers would own all wealth. This is (ii) the so-called dual steady state. Finally, Darity (1981) showed that the opposite, (iii) anti-dual case could also arise. In his own words, this is a "case of perpetual dynamic disequilibrium where the capitalists acquire an ever-increasing share of the capital stock" (p. 980). In my two-class model, all three possible outcomes will occur, depending on the degree of automation of the economy.

Most closely related to this study is the work of Mattauch et al. (2017). Mattauch et al. (2017) use a two-class model to study how capital tax-financed public investment affects the wealth distribution and relate this to the notion of automation. However, automation is operationalized by varying the value of the elasticity of substitution (like Zamparelli (2017)) and not through a task-based model.

Our main findings are as follows. Rising productivity of capital will incite firms to expand the set of automated tasks, and this implies a fall in the labour share of income. In terms of the wealth distribution between capitalists and workers, two phases of rising capital productivity can be distinguished. At first, when the share of automated tasks is still sufficiently low, rising capital productivity does not generate the possibility of a durable capitalist class. If the share of automated tasks is pushed above a certain threshold, however, a durable capitalist class emerges. From then onwards, rising capital productivity will also increase the wealth share of the capitalist class. Eventually, if capital productivity becomes large enough, the economy is fully automated (AK-technology) and capitalists own the totality of the capital stock. I also consider the implications of capital income taxation of which the revenue is transferred to workers. I find that, even under full automation, capitalists will then not fully own the total capital stock. For very large values for the capital income tax rate, capitalists will not even exist under full automation. I also illustrate how the integration of endogenous automation into the Pasinetti framework can lead to a coherent, wealth-based explanation of income inequality. Finally, I discuss what my findings imply for the credibility of Piketty's (2014) joint explanation of the fall in the labour share of income and rising wealth inequality through the increase in the capital-output ratio. I find that, in my framework where the distribution of income between capital and labour is determined by the rational automation choice of firms, this mechanism no longer requires an elasticity of substitution larger than one. As a result, one major criticism against his account of the story seems to collapse.

The remainder of this paper is structured as follows. In section 2, the model is set out. In section 3, I build a simple system of two differential equations and use it to study the interaction between the degree of automation and the wealth distribution between capitalists and workers in a laissez faire scenario. In section 4, I introduce capital income taxation into the economy and re-evaluate the dynamic system. In section 5, I study whether the relationship between wealth inequality and income inequality implied by the model is empirically plausible. In section 6, I examine what my findings mean for the credibility of Piketty's (2014) explanation of the fall in the labour share of income and rising wealth inequality. Section 7 concludes.

2 Model

2.1 Production structure

In the model economy, output is produced through the execution of tasks. The range of tasks is normalized to the zero-one interval. The elasticity of substitution between tasks is 1 such that we obtain a Cobb-Douglas representation of the task-based production function. In equation 1, $\prod_a^b$ denotes the product integral, which is the product-type counterpart of the sum-type integral.

$$Y = \prod_0^1 t_i^{d_i} \qquad (1)$$

Firms can produce a task by assigning labour or capital to the execution of that task. In equation 2, the two factor inputs act as perfect substitutes for the production of tasks, as they do in Acemoglu and Restrepo (2018).

$$\forall i \in [0,1]\colon t_i = \delta(i)k_i + \ \gamma(i)l_i \qquad (2)$$

I impose more structure on the productivity schedule by imposing a uniform productivity of capital over different tasks and an exponentially increasing productivity of labour over different tasks (equation 3).

$$\forall i \in [0,1]\colon\ \delta(i) = A\ \wedge\ \gamma(i) = e^{Bi} \text{ with } B > 0 \quad (3)$$

This results in labour having a competitive advantage in the higher-indexed tasks. Firms will only use the input that is most effective. The marginally automated task $\tilde{I}$ is the task for which capital and labour are equally cost-effective. Given the competitive advantage schedule, tasks with an index i larger than $\tilde{I}$ are more cost-effectively executed using labour, while the opposite holds for tasks with an index i smaller than $\tilde{I}$. In equation (4a), w denotes the wage rate and r denotes the interest rate. For simplicity, I assume no depreciation of the capital stock.

$$\exists \tilde{I} \in [0,1]\colon\ \frac{A}{r} = \frac{e^{B\tilde{I}}}{w} \Longleftrightarrow \tilde{I} = \frac{ln(A\frac{w}{r})}{B} \qquad (4a)$$

$$\forall i \in \left[0,\tilde{I}\right]\colon\ t_i =\ Ak_i\ \wedge\ \forall i \in\ ]\tilde{I};1]\colon\ t_i = e^{Bi}l_i \qquad (4b)$$

Using equation (4b) and inserting it into equation (1) delivers equation (5a).

$$(4b)\ \&\ (1) \Rightarrow\ Y = \prod_0^{\tilde{I}}\ (Ak_i)^{di} \prod_{\tilde{I}}^1\ (e^{Bi}l_i)^{di} \qquad (5a)$$

Total capital supply is denoted by K and total labour supply by L. Factor markets clear (equation 6a).

$K = \int_0^{\tilde{I}} k_i di \ \wedge \ L = \int_{\tilde{I}}^1 l_i di$ (6a)

Rational firms assign capital (labour) to tasks such that the marginal product of capital (labour) does not vary over tasks. Given the Cobb-Douglas production function, firms will spread out the total capital stock K (labour supply L) over all automated tasks (non-automated tasks) on an equal basis.[1] This results in equations (5b) and (6b).

$$(5a) \Rightarrow \ Y = \prod_0^{\tilde{I}} (Ak)^{di} \prod_{\tilde{I}}^1 (e^{Bi} l)^{di} \qquad (5b)$$

$$(6a) \Rightarrow \ K = \int_0^{\tilde{I}} k \ di \ \Leftrightarrow \ k = \frac{K}{\tilde{I}} \ \wedge \ L = \int_{\tilde{I}}^1 l \ di \ \Leftrightarrow \ l = \frac{L}{1-\tilde{I}} \qquad (6b)$$

By combining equations (5b) and (6b), we obtain a production function in equation (7).

$$Y = \prod_0^{\tilde{I}} \left(A \frac{K}{\tilde{I}}\right)^{di} \prod_{\tilde{I}}^1 \left(e^{Bi} \frac{L}{1-\tilde{I}}\right)^{di} \qquad (7)$$

Using the definition of the product integral, namely that $\prod_a^b f(i)^{di} = \exp(\int_a^b ln\, f(i)\, di)$, equation (7) can be simplified to equation (8).

$$Y = \left(\frac{AK}{\tilde{I}}\right)^{\tilde{I}} \left(\frac{L}{1-\tilde{I}}\right)^{1-\tilde{I}} e^{\frac{B}{2}\left(1-\tilde{I}^2\right)} = \left(\frac{AK}{\tilde{I}}\right)^{\tilde{I}} \left(\frac{e^{\frac{B}{2}(1+\tilde{I})} L}{1-\tilde{I}}\right)^{1-\tilde{I}} \qquad (8)$$

The production function in equation (8) can also be expressed in per capita terms, where small variable names denote the per capita variables (equation 9).

$$(8) \Rightarrow y = \frac{Y}{L} = \left(\frac{Ak}{\tilde{I}}\right)^{\tilde{I}} \left(\frac{e^{\frac{B}{2}(1+\tilde{I})}}{1-\tilde{I}}\right)^{1-\tilde{I}} \qquad (9)$$

Based on equation (9) and the assumption of perfect competition, the remuneration (or factor cost) related to both capital and labour can be computed as the marginal product (equations 10a and 10b).

$$(9) \Rightarrow r = \frac{\partial y}{\partial k} = \tilde{I} \frac{y}{k} \qquad (10a)$$

$$(9) \Rightarrow w = y - \frac{\partial y}{\partial k} k = y\left(1 - \tilde{I}\right) \qquad (10b)$$

[1]For capital, this is rather logical given its constant productivity over the different tasks. For labour, this property is only present because of the elasticity of substitution of 1. If the elasticity of substitution were below 1, firms would compensate for the relative "over-execution" of tasks where labour is very productive (the highest-indexed tasks) by assigning more labour to the tasks where labour is less productive (the lower-indexed tasks). The opposite would happen in case tasks are gross substitutes (elasticity of substitution larger than 1): firms would exploit the substitutability by assigning more labour to the tasks where labour is most productive. Only in the Cobb-Douglas case is the distribution of the total labour supply over all non-automated tasks uniform.

The share of automated tasks in the economy is denoted by $\tilde{I}$. Note that, since the share of automated tasks is the output elasticity with regard to capital and the production function is of the Cobb-Douglas type, the capital share of income is also equal to $\tilde{I}$ (and the labour share is equal to $1-\tilde{I}$). This follows from the simple rearrangement of equation (10a).

2.2 The capital-labour ratio and automation

In this production set-up, firms determine the level of automation $\tilde{I}$ optimally based on the relative supply of the production factors k and the two technology parameters A and B. Equation (11) indicates how firms choose the level of automation $\tilde{I}$ as the capital-labour ratio k varies. Equations (12), (13) and (14) then respectively show how the output per capita level y, the wage rate w, and the interest rate r arise from this optimal automation choice.

Based on equations (4a), (10a) and (10b), $\tilde{I}$ can be defined as an implicit function of the capital-labour ratio k in equation (11). For strictly positive values for A and B, it can be shown that $\tilde{I}(k)$ is a continuously differentiable, monotonically increasing and concave function of k for which $\tilde{I}(0)=0$ and $\underset{k\to\infty}{lim}\ \tilde{I}(k)=1$.

$$(4a)\ \&\ (10\text{a})\ \&\ (10\text{b})\ \Rightarrow k=\frac{e^{B\tilde{I}}}{A}\frac{\tilde{I}}{1-\tilde{I}} \quad (11)$$

Combining equations (9) and (11), we can express the output per capita level y as a convex, monotonically increasing function of the share of automated tasks $\tilde{I}$.

$$y=\left(\frac{Ak}{\tilde{I}}\right)^{\tilde{I}}\left(\frac{e^{\frac{B}{2}(1+\tilde{I})}}{1-\tilde{I}}\right)^{1-\tilde{I}}=\frac{e^{\frac{B}{2}(\tilde{I}^2+1)}}{1-\tilde{I}} \quad (12)$$

Based on equation (12), we find that $\underset{\substack{\tilde{I}\to 1\\ <}}{lim}\ y\left(\tilde{I}\right)=+\infty$. This is logical because (1) the production technology in equation (9) becomes of the AK-type if all tasks are automated ($\tilde{I}=1$) and (2) such complete automation requires that $k\to+\infty$.

Combining equation (12) and (10b), we find a simple expression for the wage rate w in equation (13). Note that the wage rate coincides with the marginal product of labour because of the assumption of perfect competition.

$$w=y\left(1-\tilde{I}\right)=\frac{e^{\frac{B}{2}(\tilde{I}^2+1)}}{1-\tilde{I}}\left(1-\tilde{I}\right)=e^{\frac{B}{2}(\tilde{I}^2+1)} \quad (13)$$

We find that the wage rate w (and thus the marginal product of labour) is a monotonically increasing function of the share of automated tasks $\tilde{I}$.

Taking into account that $w = y\left(1-\tilde{I}\right)$, this result can be phrased in terms of the typical counteracting labour demand effects of automation identified by the literature. Acemoglu and Restrepo (2018) use the term 'displacement effect' to characterize the loss in labour demand coming from the contraction in the set of tasks executed by labour $(1-\tilde{I})$ and the term 'productivity effect' to characterize the rise in labour demand coming from the rise in average labour productivity y. We find that the productivity effect dominates the displacement effect since the rise in $\tilde{I}$ generates a more than proportional rise in y.

Combining equation (12) and (10B), we find a simple expression for the interest rate in equation (14).

$$r = \tilde{I}\frac{y}{k} = Ae^{\frac{B}{2}\left(1-\tilde{I}\right)^2} \qquad (14)$$

We find that the interest rate r (and thus the marginal product of capital) is a monotonically decreasing function of the share of automated tasks $\tilde{I}$. As $k \to +\infty$ and $\tilde{I} \underset{<}{\to} 1$, the production technology turns AK and thus the marginal product of capital converges to A from above.

2.3 Capitalists and workers

In the previous sub-section, I showed how the capital-labour ratio k determines the optimal choice firms make on $\tilde{I}$ and how the output per capita level, the wage rate, the interest rate and the factor shares all follow from this choice. In this section, I detail how the supply of both production factors are established in a two-class economy.

Two types of individuals exist in the model economy. First, there is a class of workers whose income sources are twofold. On the one hand, they inelastically provide one unit of labour and earn a wage w in return. On the other hand, they save a certain share s_w of their total income. They thus build up the workers' capital stock K_w on which they earn the interest rate r. Note that since every worker inelastically provides one unit of labour, the capital-labour ratio k coincides with the capital stock per worker. The growth rate of the worker class over time is denoted by n.[2] This results in differential equation (15) for the evolution of the worker capital stock per worker.

$$\dot{k}_w = (s_w r + \frac{s_w w}{k_w} - n)k_w \qquad (15)$$

[2] Note that, more generally, L can also be thought to denote labour in efficiency units. In that case, small letters (like y or k) do not denote the variable in per capita terms. Instead, y then denotes the output per efficiency unit of labour. In this case, n does not just denote the population growth rate, but the sum of the population growth rate and the aggregate growth rate of labour productivity. This will be important for section 5.

Second, there is a class of capitalists whose only source of income is the return on their capital stock K_c. They save a share s_c of their income and they are imposed to be thriftier than the worker class ($s_c > s_w$). The class of capitalists is assumed to be of constant number over time. This results in differential equation (16) for the evolution of the capitalists' capital stock per capita.

$$\dot{k}_c = (s_c r - n)k_c \quad (16)$$

Jointly, capitalists and workers own the entire capital stock K. (equation 17)

$$K_c + K_w = K \quad (17)$$

I use z to denote the share of total wealth owned by the group of capitalists (equation 18).

$$z = \frac{K_c}{K_c + K_w} = \frac{K_c}{K} = \frac{k_c}{k} \quad (18)$$

3 Dynamic system in the laissez faire scenario

In continuous time, the interaction between the capitalist-worker wealth distribution z and the share of automated tasks $\tilde{I}$ can be described by a system of two differential equations.

3.1 Differential equation for z

First, I construct a simple differential equation for the evolution of the wealth share of capitalists z. Differentiating z in equation (18), we find equation 19.

$$\frac{\dot{z}}{z} = \frac{\dot{k}_c}{k_c} - \frac{\dot{k}}{k} = \frac{\dot{k}_c}{k_c} - \frac{\dot{k}_c + \dot{k}_w}{k} \quad (19)$$

Based on equations (15) and (16) for $\dot{k}_w$ and $\dot{k}_c$ respectively, we obtain equation (20).

$$\Leftrightarrow \; \frac{\dot{z}}{z} = \frac{(s_c r - n)k_c}{k_c} - \frac{(s_c r - n)k_c + (s_w r + \frac{s_w w}{k_w} - n)k_w}{k} \quad (20)$$

We can use the expressions (10a) and (10b) for the factor costs r and w respectively to concretize equation (20) into (21).

$$\Leftrightarrow \; \frac{\dot{z}}{z} = s_c \tilde{I} \frac{y}{k} - n - \frac{(s_c \tilde{I} \frac{y}{k} - n)k_c + (s_w \tilde{I} \frac{y}{k} + \frac{s_w y (1-\tilde{I})}{k_w} - n)k_w}{k} \quad (21)$$

Based on equations (17) and (18), we can eliminate k_c and k_w from equation (21). Subsequently, we rearrange the expression to find equation (22).

$$\Leftrightarrow \; \frac{\dot{z}}{z} = s_c \tilde{I} \frac{y}{k} - n - \frac{(s_c \tilde{I} y - nk)z + (s_w \tilde{I} y + \frac{s_w y (1-\tilde{I})}{1-z} - nk)(1-z)}{k}$$

$$\Leftrightarrow \; \frac{\dot{z}}{z} = \frac{1}{k}\left[s_c \tilde{I} y(1-z) - s_w \tilde{I} y(1-z) - s_w y\left(1-\tilde{I}\right)\right]$$

$$\Leftrightarrow \; \frac{\dot{z}}{z} = \frac{1}{k}\left[(s_c - s_w)\tilde{I} y(1-z) - s_w y\left(1-\tilde{I}\right)\right] \quad (22)$$

The level of output per capita y and the capital-labour ratio k can be replaced by their respective expressions in equations (9) and (11). After some rearrangements, this results in our final differential equation (23) for the evolution of the wealth share of capitalists z.

$$\Leftrightarrow \; \dot{z} = zAe^{\frac{B}{2}(1-\tilde{I})^2}\left[(s_c - s_w)(1-z) - s_w\left(\frac{1-\tilde{I}}{\tilde{I}}\right)\right] \quad (23)$$

3.2 Differential equation for $\tilde{I}$

Second, I construct a differential equation for the evolution of the share of automated tasks $\tilde{I}$.

Equation (11) expresses the capital-labour ratio k as a function of the share of automated tasks $\tilde{I}$. We take logarithms on both sides of equation (11) and differentiate with respect to time to obtain equation (24).

$$\Longrightarrow \ln(k) = -\ln(A) + B\tilde{I} + \ln\left(\tilde{I}\right) - \ln\,(1-\tilde{I})$$

$$\Longleftrightarrow \frac{\dot{k}}{k} = B\dot{\tilde{I}} + \frac{\dot{\tilde{I}}}{\tilde{I}} + \frac{\dot{\tilde{I}}}{1-\tilde{I}}$$

$$\Longleftrightarrow \dot{\tilde{I}} = \frac{\tilde{I}(1-\tilde{I})}{B\tilde{I}\left(1-\tilde{I}\right)+1}\frac{\dot{k}}{k} \quad (24)$$

To find a differential equation for $\frac{\dot{k}}{k}$, we proceed in similar fashion as in sub-section 3.1 and obtain equation 25.

$$\Longleftrightarrow \frac{\dot{k}}{k} = Ae^{\frac{B}{2}\left(1-\tilde{I}\right)^2}\left[(s_c - s_w)z + \frac{s_w}{\tilde{I}})\right] - n \qquad (25)$$

Integrating equation 25 into equation 24 results in our final differential equation (26) for the evolution of the share of automated tasks $\tilde{I}$.

$$\Longleftrightarrow \dot{\tilde{I}} = \frac{\tilde{I}(1-\tilde{I})}{B\tilde{I}\left(1-\tilde{I}\right)+1}\left\{Ae^{\frac{B}{2}\left(1-\tilde{I}\right)^2}\left[(s_c - s_w)z + \frac{s_w}{\tilde{I}})\right] - n\right\} \qquad (26)$$

3.3 Discussion

3.3.1 The dual and Pasinetti steady states

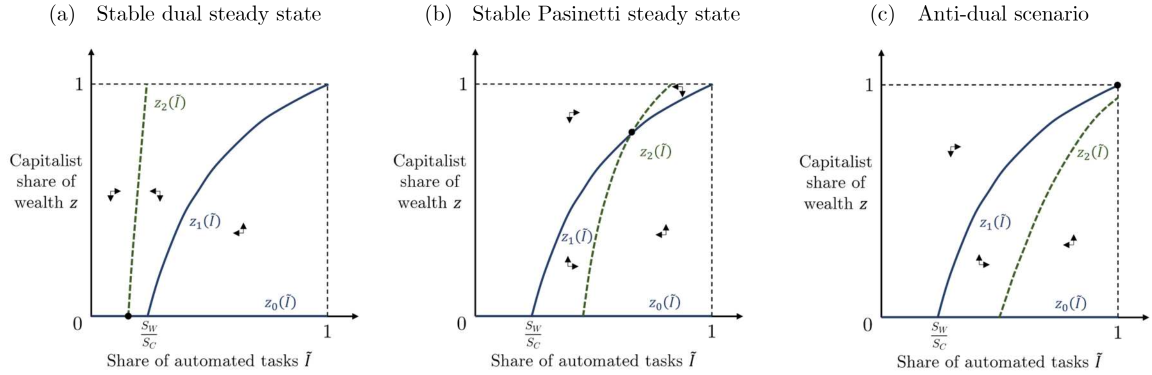
In Figure 1, I illustrate what the dynamic system looks like in a phase diagram. The '$\dot{z} = 0$'-nullcline (dark blue and solid) and the '$\dot{\tilde{I}} = 0$'-nullcline (dark green and dotted) are drawn in the $\tilde{I}z$-plane. Based on equation (23), we can establish that the nullcline '$\dot{z} = 0$' can hold for either $z_0\left(\tilde{I}\right) = 0$ or $z_1\left(\tilde{I}\right) = 1 - \frac{s_w}{s_c - s_w}\left(\frac{1-\tilde{I}}{\tilde{I}}\right)$. The logic behind the $z_0\left(\tilde{I}\right)$-function is that, if capitalists own no wealth ($z = 0$), this wealth distribution is an equilibrium for any level of automation $\tilde{I}$. Since their sole source of income comes out of their capital stock, once capitalists' wealth is gone, they can never (re-)emerge. The $z_1\left(\tilde{I}\right)$-function has a positive slope $z_1'\left(\tilde{I}\right) = \frac{s_w}{(s_c - s_w)\tilde{I}^2}$. The logic behind this positive slope is that a higher wealth share of the capitalists z can only be sustained if the labour share of income is lowered through a rise in $\tilde{I}$. Based on equation (26), the '$\dot{\tilde{I}} = 0$'-nullcline is given by the function $z_2\left(\tilde{I}\right) = \frac{1}{s_c - s_w}\left(\frac{n}{Ae^{\frac{B}{2}\left(1-\tilde{I}\right)^2}} - \frac{s_w}{\tilde{I}}\right)$. The $z_2\left(\tilde{I}\right)$-function has a positive slope $z_2'\left(\tilde{I}\right) = \frac{1}{(s_c - s_w)}\left(\frac{Bn\left(1-\tilde{I}\right)}{Ae^{\frac{B}{2}\left(1-\tilde{I}\right)^2}} + \frac{s_w}{\tilde{I}^2}\right)$. The logic behind this is that a higher level of automation $\tilde{I}$ necessitates a higher capital-labour ratio (see equation 11). This can only be sustained if savings are higher, and thus if the thriftier capitalists own a larger share of the wealth. Note that $z_2'\left(\tilde{I}\right)$-$z_1'\left(\tilde{I}\right) = \frac{Bn\left(1-\tilde{I}\right)}{(s_c-s_w)Ae^{\frac{B}{2}\left(1-\tilde{I}\right)^2}}$ such that the $z_2\left(\tilde{I}\right)$-function is strictly steeper than the $z_1\left(\tilde{I}\right)$-function for $0 \leq \tilde{I} < 1$.

When $A < \frac{n}{s_c}$, capital accumulation does not indefinitely outweigh the growth of the labour force. As a result, the capital-labour ratio k is bounded and will converge to a certain steady-state value until that steady state is reached. Two distinct types of steady states may occur.

First, even if capitalists start out with positive initial wealth, they might end up holding a negligible share of the total capital stock ($z = 0$). This is the so-called 'dual' steady state, in which the presence of a capitalist class cannot be sustained. It is defined by the intersection of the '$z_0\left(\tilde{I}\right) = 0$'-function and the '$z_2\left(\tilde{I}\right)$'-function in Figure 1. The dual equilibrium $(\tilde{I}^*, z^*)$ has the coordinates $\left(\frac{s_w}{n}r\left(\tilde{I}^*\right), 0\right)$.

Note that, if A rises, the '$z_2\left(\tilde{I}\right)$'-function shifts to the right (and the other curves stay in place). If A is sufficiently small, the situation of panel A in Figure 1 will arise. The dual equilibrium is then the only steady state which exists.

Figure 1: Illustration of the phase diagram in the $\tilde{I}z$-plane.



The '$\dot{z} = 0$'-nullcline is marked in dark blue and is solid. The '$\dot{\tilde{I}} = 0$'-nullcline is marked in dark green and is dotted. The laws of motion are denoted by the arrows.

When A is sufficiently high such that $\frac{n}{s_c} > A > \frac{n}{s_c} e^{-\frac{B}{2}\left(1-\frac{s_w}{s_c}\right)^2}$, an additional steady state exists, which is commonly referred to as the 'Pasinetti' steady state. This is the situation of panel B in Figure 1. The Pasinetti steady state is defined by the intersection of the '$z_1\left(\tilde{I}\right) = 0$'-function and the '$z_2\left(\tilde{I}\right)$'-function. In this steady state, both capitalists and workers own a strictly positive share of the capital stock ($z > 0$).[3] Note that the condition that $A > \frac{n}{s_c} e^{-\frac{B}{2}\left(1-\frac{s_w}{s_c}\right)^2}$ boils down to stating that the share of automated tasks $\tilde{I}$ generated by the Pasinetti steady state has to be larger than $\frac{s_w}{s_c}$, which is the intersection of the $z_1\left(\tilde{I}\right)$-function with the $z = 0$ axis. That is to say, for there to exist a Pasinetti equilibrium, capitalists must be sufficiently thrifty relative to workers (low $\frac{s_w}{s_c}$) or the economy has to be sufficiently automated (high $\tilde{I}$) to sustain a positive capitalist wealth share z. In essence, this result is not new. Since the share of automated tasks $\tilde{I}$ is also the capital share of income, my findings simply echo the Samuelson and Modigliani (1966) result that the Pasinetti steady state requires the profit share to be larger than the ratio of the saving rates $\frac{s_w}{s_c}$. Note that the capital share of income is determined within the model by the optimal automation choice of firms.

The Pasinetti equilibrium $(\tilde{I}^*, z^*)$ has the coordinates $\left(1 - \sqrt{\frac{2}{B} ln\left(\frac{n}{As_C}\right)}, \frac{s_c - s_w/\tilde{I}^*}{s_c - s_w}\right)$ with the interest rate r equal to $\frac{n}{s_c}$. Note that workers' saving rate s_w has no impact on the level of automation (and thus not on the factor shares) in the Pasinetti steady state. Moreover, s_w has no impact on the interest rate r. This is all intuitive given the well-known property of the irrelevance of worker savings for the level of the steady state capital stock in the Pasinetti

[3] Only $B > 0$ is required for there to exist a region of values for A for which the Pasinetti steady state exists.

steady state (Pasinetti, 1962) and the direct linkage between the capital-labour ratio and the level of automation in equation (11). Based on the laws of motion, it is clear that the Pasinetti steady state is globally stable if it exists. On the other hand, the dual steady state is stable if the Pasinetti steady state does not exist and it is unstable if the Pasinetti steady state exists.[4]

Note that, in a laissez faire scenario, it is impossible to have a dual-type economy (where capitalists have a zero wealth share) that is fully automated ($\tilde{I} = 1$). Intuitively, this is very clear. In a fully automated scenario, all income is capital income and no labour is used (AK-technology). As long as capitalists start off with non-zero wealth, their share in the total capital stock will tend to 1 asymptotically, since they save a larger fraction of their capital income than the workers. Mathematically, stability of the dual steady state requires that the Pasinetti steady state does not exist and thus that $\tilde{I} < \frac{s_w}{s_c}$. This can only coincide with full automation ($\tilde{I} = 1$) in the improper case where workers are thriftier than capitalists ($s_c < s_w$).

3.3.2 The anti-dual case of unbounded capital accumulation

If A rises even further, we ultimately end up in the situation of panel c in Figure 1. If $A \geq \frac{n}{s_c}$ (and capitalist start off with non-zero initial wealth), there is no longer any steady state: we then enter an anti-dual 'equilibrium' setting ($z = 1$) with full automation ($\tilde{I} = 1$) and with eternal capital accumulation $\frac{\dot{k}}{k} > 0$. To see this, it suffices to combine the expression for the interest rate (equation 14) into the capitalist accumulation rule (equation 16).

$$\frac{\dot{k}_c}{k_c} = s_c r - n = s_c A e^{\frac{B}{2}\left(1-\tilde{I}\right)^2} - n \quad (27)$$

Clearly, capitalist capital accumulation will never stop if $A \geq \frac{n}{s_c}$. As indicated in sub-section 2.2, the lower bound to which the interest rate converges as $\tilde{I} \underset{<}{\to} 1$ is simply A. If capitalists are sufficiently thrifty, capitalist capital accumulation will always overcompensate for the growth of labour such that the capital-labour ratio k rises indefinitely. In equation (23), one can observe that if $\tilde{I} = 1, \dot{z} > 0$ as long as $z < 1$ such that the system must go to an anti-dual 'equilibrium' where workers' share of the wealth is zero. This is logical: in case of an AK-

[4] So the capital share of income $\tilde{I}$ is positively affected by a rise in the capitalist savings rate. This is the opposite of what happens in the standard Pasinetti (1962) model where the profit share falls if capitalists become thriftier (eq. 15 in the 1962 paper). In contrast to the standard model, (1) we assume a neoclassical production function with perfectly competitive factor markets such that factor costs and marginal productivity coincide, and (2) firms here can shift the production technology in response to the increased availability of capital by executing more tasks using capital.

technology, the individual (class) with the highest savings rate will ultimately own the entirety of the capital stock.[5]

[5] Note that if capitalists start out with zero initial wealth (and thus never emerge), a fully automated scenario with eternal capital accumulation will occur if $A \geq \frac{n}{s_w}$. Since capitalists do not emerge, $z = 0$ in this case.

4 Dynamic system in the case of capital income taxation

I follow Zamparelli (2017) in considering how the dynamic system evolves if capital income is taxed at a rate τ and the tax revenue is transferred to the workers in the form of lump-sum transfers. As a result, the new differential equation for the evolution of the worker capital stock per capita is given by equation (28).

$$\dot{k}_w = (s_w r + \frac{s_w w}{k_w} + s_w \tau r \frac{k_c}{k_w} - n) k_w \quad (28)$$

The new differential equation for the evolution of the capitalists' capital stock per capita is given by equation (29).

$$\dot{k}_c = (s_c(1-\tau)r - n)k_c \ (29)$$

Using differential equations (28) and (29), we obtain differential equations (30) and (31) for z and $\tilde{I}$ respectively. To do this, the respective approaches are parallel to those of section 3.1 and 3.2.

$$\dot{z} = \ zAe^{\frac{B}{2}(1-\tilde{I})^2}\left[(s_c(1-\tau) - s_w)(1-z) - s_w(\tau z + \frac{1-\tilde{I}}{\tilde{I}})\right] \quad (30)$$

$$\dot{\tilde{I}} = \frac{\tilde{I}(1-\tilde{I})}{B\tilde{I}\left(1-\tilde{I}\right)+1}\left\{Ae^{\frac{B}{2}\left(1-\tilde{I}\right)^2}\left[(s_c(1-\tau) - s_w)z + s_w(\tau z + \frac{1}{\tilde{I}}))\right] - n\right\} \quad (31)$$

The '$\dot{z} = 0$'-nullcline and the '$\dot{\tilde{I}} = 0$'-nullcline can, again, be drawn in the $\tilde{I}z$-plane. Based on equation (23), we can establish that the '$\dot{z} = 0$'-nullcline holds for either $z_0\left(\tilde{I}\right) = 0$ or $z_1\left(\tilde{I}\right) = 1 - \frac{s_w}{(1-\tau)(s_c - s_w)}\left(\frac{1-(1-\tau)\tilde{I}}{\tilde{I}}\right)$. Based on equation (26), the '$\dot{\tilde{I}} = 0$'-nullcline is given by the function $z_2\left(\tilde{I}\right) = \frac{1}{(1-\tau)(s_c - s_w)}\left(\frac{n}{Ae^{\frac{B}{2}\left(1-\tilde{I}\right)^2}} - \frac{s_w}{\tilde{I}}\right)$. In Figure 1, an increase in the capital income tax rate τ will cause an inward rotation of the '$\dot{\tilde{I}} = 0$'-nullcline (in dotted dark green) around its intersection with the $z = 0$ axis. Moreover, it will also generate a shift of the $z_1\left(\tilde{I}\right)$-arm of the '$\dot{z} = 0$'-nullcline (in solid dark blue) towards the right. This implies that for values of τ different than zero, the (1,1)-point no longer lies on the '$\dot{z} = 0$'-nullcline.

Both $z_1(\tilde{I})$ and $z_2\left(\tilde{I}\right)$ are positively-sloped, and again it is easy to see that $z_2'\left(\tilde{I}\right) - z_1'\left(\tilde{I}\right) = \frac{Bn\left(1-\tilde{I}\right)}{(1-\tau)(s_c - s_w)Ae^{\frac{B}{2}\left(1-\tilde{I}\right)^2}} > 0$. There are again two types of equilibria. The intersection of $z_0\left(\tilde{I}\right)$ and $z_2\left(\tilde{I}\right)$ results in the dual steady state where $\left(\tilde{I}^*, z^*\right) = \left(\frac{s_w}{n} r\left(\tilde{I}^*\right), 0\right)$. The dual steady state is the same as for the laissez faire scenario, because a redistributive policy intervention from capitalists to workers has no effect if capitalists hold a negligible wealth share ($z = 0$). The intersection of $z_1\left(\tilde{I}\right)$ and $z_2\left(\tilde{I}\right)$ results in the Pasinetti steady state where $\left(\tilde{I}^*, z^*\right) =$

$\left(1-\sqrt{\frac{2}{B}\ln\left(\frac{n}{(1-\tau)s_c A}\right)}, \frac{1}{s_c-s_w}\left(s_c-\frac{s_w}{(1-\tau)\tilde{I}}\right)\right)$ with the interest rate $r=\frac{n}{(1-\tau)s_c}$. Again, the Pasinetti steady state exists if it can generate a positive capitalist wealth share z, which requires that $\tilde{I}^* > \frac{s_w}{(1-\tau)s_c}$. This implies that the productivity of capital A must be sufficiently large: $A > \frac{n}{(1-\tau)s_c} e^{-\frac{B}{2}\left(1-\frac{s_w}{(1-\tau)s_c}\right)^2}$. Note that a larger capital income tax rate τ unambiguously lowers the level of automation $\tilde{I}^*$ and the capitalist wealth share z^*.

In the Pasinetti case, full automation ($\tilde{I}^*=1$) occurs if $A \geq \frac{n}{(1-\tau)s_c}$. In contrast to the laissez faire scenario, a fully automated economy (where $\tilde{I}^*=1$ and the production technology is thus of the AK-type) is no longer synonymous with an anti-dual situation in which capitalists own the totality of the capital stock ($z=1$). In the Pasinetti economy, the capitalist share of wealth can be lower than 1 under full automation: $z^*=\frac{1}{s_c-s_w}\left(s_c-\frac{s_w}{(1-\tau)}\right)$ such that any strictly positive capital income tax rate τ implies that the capitalist wealth share will not tend to one under full automation.[6]

Interestingly, it is even possible to have full automation ($\tilde{I}^*=1$) in the dual case where workers own the totality of the capital stock ($z=1$). For a stable dual steady state to exist, it is required that the Pasinetti steady state does not exist, meaning that $\tilde{I}^* < \frac{s_w}{(1-\tau)s_c}$. Clearly, this is consistent with $\tilde{I}^*=1$ if $(1-\tau)s_c < s_w$ which is plausible for large values of τ.

Finally, consider the possibility that the capital income taxation rate τ is negative. This implies that capital income is subsidized by the government and workers are taxed on a lump sum basis to finance this subsidy. Note that such capital income subsidy is similar to imposing that capitalists' rate of return is higher than the rate of return of workers, as first done by Moore (1974) in the Pasinetti Paradox context. In such a case, it is possible that capitalists own the totality of the wealth stock ($z^*=1$) without full automation ($\tilde{I}^*=1$) occurring. Where $z^*=1$ requires that $\tilde{I}^*=1$ in the laissez faire scenario, now it only requires that $\tilde{I}^*=\frac{1}{1-\tau}$.

Three different stories of technical progress in capital productivity A can now be told, depending on whether the capital income tax rate is zero, positive, or negative. If the productivity of capital at the execution of tasks A is equal to zero, all tasks are executed by workers ($\tilde{I}^*=0$ and $z^*=0$) regardless of the policy scenario. Rising capital productivity will, in a first phase, only drive automation without the emergence of a sustainable capitalist class

[6] This can also be obtained by imposing that $\frac{\dot{k}_w}{k_w}=\frac{\dot{k}_c}{k_c}$ in equations (28) and (29) while setting $r=A$ and setting wage income of workers equal to zero.

($0 < \tilde{I}^* < 1$ and $z^* = 0$) under the dual steady state. If capital income taxation is very large ($(1-\tau)s_c < s_w$), it is possible that rising capital productivity continues driving automation without creating the possibility of a sustained capitalist class until the economy is fully automated ($\tilde{I}^* = 1$ and $z^* = 0$). In the absence of very large capital income taxation ($(1-\tau)s_c > s_w$), rising capital productivity will at some point generate the possibility of a sustained capitalist class ($0 < \tilde{I}^* < 1$ and $0 < z^* < 1$). This possibility of a sustained capitalist class will emerge for lower levels of automation if capital income is subsidized ($\tau < 0$) and higher levels of automation if capital income is taxed ($\tau > 0$). From then onwards, rising capital productivity will not only increase the level of automation $\tilde{I}^*$, but also the wealth share of the capitalist class (the Pasinetti steady state). If capital income is taxed ($\tau > 0$), the economy will reach a point of full automation where workers still own a non-zero share of the capital stock ($\tilde{I}^* = 1$ and $0 < z^* < 1$). If capital income is not taxed ($\tau = 0$), rising capital productivity will eventually bring the economy to a state of full automation where capitalists own all wealth ($\tilde{I}^* = 1$ and $z^* = 1$). If capital income is subsidized ($\tau < 0$), rising capital productivity will result in complete ownership of the capital stock by capitalists before the economy is fully automated ($0 < \tilde{I}^* < 1$ and $z^* = 1$). Continued increases in capital productivity will then eventually also lead to complete automation ($\tilde{I}^* = 1$ and $z^* = 1$).

5 The link between wealth and income inequality in the model

Apart from the wealth distribution, we can also monitor the distribution of income in our model economy. The share of national income going to capitalists is easily computed as the product of the capital share of income $\tilde{I}$ and the capitalists' share of wealth holdings z. In other words, the *capitalists'* share of national income is given by $\tilde{I}z$ in the model. Assuming that the number of capitalists is negligible relative to the number of workers, it is easy to compute that the wealth Gini is z and the income Gini is $\tilde{I}z$.

Two things can be noted about the relationship between the wealth Gini and income Gini in the Pasinetti steady state. First, our model comes to the realistic conclusion that the wealth Gini is larger than the income Gini (by a factor of $\frac{1}{\tilde{I}}$). This is a stylized fact in advanced economies. The second prediction comes from the model's connection between the distribution of income between production factors (capital and labour), and the distribution of wealth between classes (capitalists and workers). Given the inherent co-movement[7] between z and $\tilde{I}$, our model namely predicts that the wealth Gini should be a *concave*, monotonically increasing function of the income Gini: $Gini_{Wealth} = \frac{s_c Gini_{Income}}{s_w(1-Gini_{Income})+s_c Gini_{Income}}$.[8]

From a wider theoretical perspective, this is not a surprising prediction. In fact, this result is obtained in most models where those with higher income save more. In an economy with two types of agents (those with a high savings rate s_H and those with a low savings rate s_L), the wealth share of the thrifty group of agents is easily computed as $WealthShare_H = \frac{s_H IncomeShare_H}{s_L(1-IncomeShare_H)+s_H IncomeShare_H}$. Their wealth share is thus a concave function of their income share as long as $s_H > s_L$. If the thrifty group of agents is the wealthier group, and their group size is negligible compared to the total population, we find that the wealth Gini is a concave function of the income Gini under quite general conditions.

In short, our model finds the very same relationship between the wealth Gini and the income Gini as one would obtain from a very general theoretical perspective. Nevertheless, our prediction is noteworthy because it is different from what one would obtain in our Pasinetti framework in the absence of endogenous automation. If the share of automated tasks $\tilde{I}$ was not decided upon by firms, but technologically fixed, then z and $\tilde{I}$ would be independent such that the wealth Gini z would be a linear function of the income Gini $z\tilde{I}$. Intuitively speaking,

[7] A change in any parameter, except for s_w, induces a change of z and $\tilde{I}$ in the same direction in the Pasinetti steady state.

[8] Note that these model predictions are only valid for the long-run steady-state, which is important since the wealth distribution is a stock variable which can only adjust slowly to new equilibrium conditions.

the general theoretical perspective above required us to consider wealth inequality as a result of income inequality, and to be agnostic about the source of income inequality. In the Pasinetti model, however, the opposite is true: all income inequality in the model is generated by the distinction between capitalists and workers such that income inequality is the result of wealth inequality. Since capitalists do not work and thus cannot claim any of the labour income, their income share is capped at the total capital share of income. In the "fixed technology", Cobb-Douglas version of the Pasinetti economy, the capital share of income is a constant $\tilde{I}$. In such a model, the income Gini is capped at $\tilde{I}$, even as the wealth Gini reaches 1. The relationship between wealth and income inequality is thus linear in models where all income inequality comes from the capitalist-worker distinction, as long as the capital share of income is fixed.

In the model *with* endogenous automation, however, the capital share of income is no longer fixed[9], and thus the income Gini is no longer capped. This is because changes that shift the wealth distribution in favour of capitalists also shift the income distribution in favour of capital. Put even simpler, our model suggests that, when the wealthy get wealthier, this is also accompanied by an increase in the relative return to wealth. As a result, capitalist-worker income inequality should increase more than the capitalist-worker wealth inequality. In the long run, the intuitive result that the wealth Gini should be a concave function of the income Gini is also obtained in the Pasinetti steady state through the integration of automation into the analysis.

We conduct a simple empirical exercise to test whether there is evidence for such a concave association in the data. We use yearly data of the World Inequality Database for wealth and income inequality, and obtain an unbalanced panel of 212 countries starting in 1980. Wealth inequality data is on net personal wealth, and income inequality data is on pre-tax national income. All data is for adults, and under the assumption that income and wealth is distributed equally among household members. In Table 1, the results of simple, vanilla OLS regressions are summarized: the dependent variable is the wealth inequality indicator (top 1% share or Gini). In columns (1) and (2), the inequality indicator used is the share of the top 1% in total wealth or income. In columns (3) and (4), the Gini is used as the inequality indicator. Evidently, I do not claim to estimate a causal relationship here. The purpose here is merely

[9] Of course, a task-based model of automation like mine is not the only way to allow the capital share of income to vary over time. An elasticity of substitution between capital and labour higher than one, as in Zamparelli (2017), would also succeed at this. In section 6, however, we argue based on the literature that automation is an empirically more plausible mechanism to endogenize the capital share of income than general capital-labour substitutability (e.g., Bergholt et al., 2022).

to test whether the macroeconomic relation between inequality in wealth and income is in accordance with the model prediction.

Table 1: OLS Regressions for Wealth Inequality (Data Source: World Inequality Database)

Dependent Variable: Wealth Inequality	(1) Top 1% share	(2) Top 1% share	(3) Gini	(4) Gini
Lagged Wealth Inequality	0.974*** (0.003)	0.967*** (0.003)	0.995*** (0.002)	0.976*** (0.003)
Income Inequality	0.046*** (0.005)	0.085*** (0.009)	0.006*** (0.002)	0.057*** (0.006)
Square of Income Inequality		-0.130*** (0.023)		-0.042*** (0.005)
Observations	5480	5480	5460	5460
Centered R-Squared	98%	98%	98%	98%

Newey-West standard errors between parentheses (lag length of two).
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

Regardless of the inequality indicator, there is sizeable evidence of a positive long-run relationship between income and wealth inequality. More important for our purposes is that there is also evidence in favour of wealth inequality being a concave function of income inequality: in either specification, the square term is significantly negative. We have shown that this concavity is not necessarily in contradiction with an explanation of the evolution in income inequality which focusses on the income inequality generated by wealth inequality. Such concavity may not be reconcilable with a "wealth inequality"-based explanation of income inequality under rigid technological assumptions where the capital share of income is fixed, but it is the natural result if one explains income and wealth inequality in a joint framework where firms choose to what extent the production process is automated.

6 In defence of Piketty's account of a rising capital share and rising wealth inequality

Based on his two so-called 'fundamental laws of capitalism', Piketty (2014) argues that the recently observed increases in the capital share of income (and the consequently rising levels of wealth inequality) in advanced economies can be explained by increases in the capital-output ratio. Simplified, his story is that the fall in the growth rate of labour productivity and in the population growth rate (combined with a rise in saving rates) lead to an increase in the capital-output ratio. More contested, however, is the premise that this increase in the capital-output ratio generates a rise in the capital share of income. This requires an elasticity of substitution between capital and labour σ that is larger than 1, while most empirical estimates point to a value for σ below 1 (e.g., Chirinko, 2008; Gechert et al., 2022; Knoblach et al., 2020; Oberfield & Raval, 2021). Moreover, Rognlie (2014) argues that for the rise in the capital-output ratio to increase the *net* share of capital income, the lower bound for σ is even larger than one.

As I will show, a fall in the population or labour productivity growth rate n or a rise in the capitalist saving rate s_c can generate a rise in the capital-output ratio and subsequently increase the capital share of income without requiring $\sigma > 1$. It is not general capital-labour substitution, but the displacement of labour by capital from the execution of tasks which will drive the rise in the capital share of income. This is conceptualized as automation by Acemoglu & Restrepo (2018). Automation in general and robotization specifically have been shown to lower the labour share of income (Acemoglu et al., 2020; Acemoglu & Restrepo, 2020; Autor & Salomons, 2018; Dauth et al., 2017; Eden & Gaggl, 2018; Graetz & Michaels, 2018; Guimarães & Gil, 2022). Bergholt et al. (2022) find that automation is the main responsible of the fall in the US labour share, while finding evidence for capital-labour complementarity.

Dividing equation (11) by equation (12), we obtain equation (32) for the capital-output ratio.[10]

$\frac{k}{y} = \frac{\tilde{I}}{A} e^{-\frac{B}{2}\left(1-\tilde{I}\right)^2} = \frac{\tilde{I}}{r}$ (32)

From the description of the Pasinetti equilibrium in section 3, it becomes clear that $\tilde{I}^* = 1 - \sqrt{\frac{2}{B}\ln\left(\frac{n}{s_c A}\right)}$ depends negatively on n and positively on s_c, while $r^* = \frac{n}{s_c}$ depends positively on n and negatively on s_c. As a result, the capital output ratio $\left(\frac{k}{y}\right)^* = \frac{s_c}{n}\Big(1 -$

[10] What I obtain is essentially Piketty's first fundamental law of capitalism: the share of capital in national income is the rate of return to capital multiplied by the capital-output ratio: $\tilde{I} = r\frac{k}{y}$.

$\sqrt{\frac{2}{B}\ln\left(\frac{n}{s_c A}\right)}\Bigg)$ depends negatively on n and positively on s_c.[11] Lower population growth rates and lower labour productivity growth rates will thus, through n, imply both capital deepening and a fall in the labour income share. Likewise, an increase in the tendency of capitalists to save out of their income will, through s_c, generate both capital deepening and a fall in the labour income share. At the same time, the model indicates that the fall in the labour share of income will lead to a rise in the share of wealth held by capitalists $z^* = \frac{s_c - s_w/\tilde{I}^*}{s_c - s_w}$. If the macroeconomic state of advanced economies is best described by the Pasinetti steady state, the model implies that the Pikettian explanations for lowering of the labour share and rising wealth inequality hold up in the absence of aggregate capital-labour substitutability. If one integrates the notion of automation in a task-based setting into the analysis, there is a strong case to be made for the common explanation of capital deepening, a falling labour income share and rising wealth inequality, even if σ is not larger than 1.

Finally, my analysis in section 4 also provides cautious support for the effectiveness of Piketty's (2014) policy recommendations in the fight against wealth inequality. Of course, I only consider capital income taxation and not capital taxation itself, so one must be careful in over-interpreting my results on this front. Nevertheless, my theoretical findings imply that fiscal measures could be effective in pushing back both wealth inequality and the fall in the labour share of income.

[11] But the relationship between the capital-output ratio and the $\frac{s_c}{n}$-ratio is not linear as in Piketty's second fundamental law of capitalism. In fact, $\frac{\partial\left(\frac{k}{y}\right)^*}{\partial\left(\frac{s_c}{n}\right)} = \tilde{I} + \frac{1}{B(1-\tilde{I})}$ and thus it depends on the share of automated tasks $\tilde{I}$. If $B < 1$, the effect of $\frac{s_c}{n}$ on $\left(\frac{k}{y}\right)^*$ is more than proportional for all values of $\tilde{I}$. If $B \geq 1$, the effect of $\frac{s_c}{n}$ on $\left(\frac{k}{y}\right)^*$ is proportional only for $\tilde{I} = 1 - \frac{\sqrt{B}}{B}$, less than proportional for $\tilde{I} < 1 - \frac{\sqrt{B}}{B}$ and more than proportional for $\tilde{I} > 1 - \frac{\sqrt{B}}{B}$. As $\tilde{I}$ approaches one and the economy approaches the AK-technology, the effect of $\frac{s_c}{n}$ on $\left(\frac{k}{y}\right)^*$ becomes infinitely strong: $\lim_{\tilde{I} \xrightarrow[<]{} 1} \frac{\partial\left(\frac{k}{y}\right)^*}{\partial\left(\frac{s_c}{n}\right)} = +\infty$. If we are in the dual steady state, however, it is easy to see that $\left(\frac{k}{y}\right)^* = \frac{s_w}{n}$ as in the standard, proportional formulation of the second fundamental law by Piketty (2014). Note that Madsen et al. (2018) find empirical evidence for a less-than-proportional effect of the savings-growth rate ratio on $\left(\frac{k}{y}\right)^*$. This could be interpreted as evidence in favour of the relevance of the Pasinetti steady state with $B > 1$ to describe recent evolutions in advanced economies.

7 Conclusion

In this study, I studied the wealth distribution between capitalists and workers in an economy where the distribution of income between capital and labour is determined by the rational automation choice of firms. More specifically, firms face a quite general task-based production function and can choose which tasks they wish to produce with capital, and which with labour.

I find that rising capital productivity will incite firms to expand the set of tasks which are best produced by capital. This implies a fall in the labour share of income. In terms of the wealth distribution between capitalists and workers, we can distinguish two phases of rising capital productivity. At first, when the share of automated tasks is still sufficiently low (lower than the ratio of the saving rates of the classes $\frac{s_w}{s_c}$), rising capital productivity does not generate the possibility of a durable capitalist class. When the share of automated tasks (or equivalently, the capital share of income) is pushed above the threshold of $\frac{s_w}{s_c}$, a durable capitalist class emerges. This is the shift from the dual steady state to the Pasinetti steady state. From then onwards, rising capital productivity will also increase the wealth share of the capitalist class. Eventually, if capital productivity becomes large enough, the economy is fully automated and capitalists own the totality of the capital stock.

I also consider the implications of capital income taxation of which the revenue is transferred to workers. I find that positive capital income taxation implies that the economy has to be more automated before capitalists can emerge: the new threshold for the share of automated tasks is $\frac{s_w}{(1-\tau)s_c}$. Moreover, even under full automation capitalists will own less than the total capital stock. For very large values for the capital income tax rate τ, capitalists will not even exist under full automation.

In the Pasinetti steady state, inequality in income is based on the degree of wealth inequality between capitalists and workers. I illustrated how the integration of endogenous automation (and thus an endogenous capital share of income) into the Pasinetti framework can lead to a coherent, wealth-based explanation of income inequality, in contrast to what happens in technological settings with a fixed capital share.

Finally, I also examined what my findings mean for the credibility of Piketty's (2014) joint explanation of the fall in the labour share of income and rising wealth inequality through the increase in the capital-output ratio (itself driven by changes in growth and savings rates). I find that, in my framework where the distribution of income between capital and labour is determined by the rational automation choice of firms, this mechanism no longer requires an elasticity of substitution larger than one. As a result, one major criticism against his account of the story seems to collapse.

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