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OPTIMAL GDP-INDEXED BONDS

Yasin Kürşat Önder

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GHENT UNIVERSITY

Department of Economics

Faculty of Economics and Business Administration
Campus Tweekerken, St.-Pietersplein 5, 9000 Ghent - BELGIUM

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Yasin Kürşat Önder

Ghent University

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Abstract

I investigate the introduction of GDP-indexed bonds as an additional source of government borrowing in a quantitative default model. The idea of linking debt payments to developments in GDP resurfaced with the 1980s debt crisis and peaked with the COVID-19 outbreak. I show that the gains from this idea depend on the underlying indexation method and are highest if payments are symmetrically tied to developments in GDP. Optimized indexed debt can eradicate default risk, halve consumption volatility, and increase asset prices while raising the government's debt balances. These changes occur because an optimally chosen indexation method does a better job at completing the markets.

Keywords: GDP-indexed bonds, sovereign default, risk sharing, state-contingent assets
JEL Codes: G11, G23, F34

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1 Introduction

Facilitating the debt repayments of emerging market economies is now at the epicenter of macroeconomic debates following a huge increase in government indebtedness. This is a recurring theme with the debt crisis. GDP-indexed bonds initially became popular in the aftermath of the 1980s debt crisis. Following the debt crisis in Europe and more recently with the COVID-19 outbreak, a renewed focus emerged among policy makers and platforms, including the G20, which has issued statement requesting a further analysis and has arranged study groups to facilitate the usage of these financial securities (see (G20 (2016)).

Contradictory assessments emerge on the potential gains of arranging GDP-indexed bonds. Proposals of GDP-indexed bonds are motivated by mitigated debt services during economic downturns to provide relief for governments in order to avoid default. The main claim is that these bonds may serve as an automatic stabilizer through lower payments during adverse shocks and may provide room for a countercyclical policy.[1] An alternate view conjectures that the gains would be very limited or may even lead to losses. This view also stems from the previously unsuccessful GDP-indexed bond issuances that emerged as part of the debt restructuring process for Bulgaria, Bosnia, and Costa Rica in the 1990s and for Argentina in 2005. Table A1 lists a number of countries that have issued GDP-linked and commodity-linked bonds, together with brief explanations. A discussion on Argentina's and Portugal's experimentation with GDP warrants is also provided in the Appendix.[2]

This paper quantitatively investigates the effects of arranging GDP-indexed debt. The framework provided here extends the canonical sovereign default model in the Eaton and Gersovitz (1981) tradition with limited commitment to debt repayment by introducing long-term indexed debt contracts as an additional asset class.

The contributions of this study are threefold. First, I evaluate the gains of introducing GDP-indexed bonds as a new security in the government's portfolio by investigating the optimal GDP indexation method, and outline the price, borrowing, and welfare gain dynamics in a quantitative default model with a non-contingent asset. Bonds are modeled as perpetuities, and I consider three main widely discussed indexation methods one at a time with a functional form that I use to extend the baseline indexation methods to

[1] This view is more prevalent and is found in Shiller (1993), Shiller (2012), and Borensztein and Mauro (2004) to name just a few studies.

[2] Previous GDP-indexed bonds for the listed countries, except Portugal and Singapore, are considered, at best, to have very limited success partly because of poor contract design. I also show that some indexation methods indeed leave the government worse off or have only negligible welfare effects.

engineer the highest welfare-generating contract. I mainly investigate floored, unfloor and suspension indexation methods both for the coupon or for the principal payments in the main text.[3]

I find that the GDP indexation method is key for a successful arrangement of these new financial products. Some contract suggestions that are widely cited and implemented leave agents with negligible welfare gains. And, in fact, some Arrow-Debreu-like securities, the results of which are presented in the Appendix, may generate welfare losses if they are not optimally designed. A predecessor to this idea dates back to Hart's dissertation (see Hart (1974) and Hart (1975)). Hart has shown that, under certain conditions, making markets more (but not fully) complete, such as making bonds more state contingent in this study, may lead to a Pareto inferior allocation. Nevertheless, this paper features a contract that benefits the government and holders of previously issued non-contingent debt. In an optimal contract in which investors are subject to two-sided risks (claims are higher (lower) if GDP grows more (less) than the trend) and the returns are symmetrically linked to the coupon or principal, I show that the government's welfare gains increase by at least 2 percent in terms of the compensating variation in permanent consumption. I also present an analysis that an economy with a floor on payments or suspension on payments during downturns also generates welfare gains. Yet, the highest welfare-generating contract can be obtained in an economy with no floor or no suspension on payments. Also, these contracts provide a better hedge for a countercyclical policy than they do with non-contingent debt. Consumption volatility halves in an economy with unfloored payments. Intuitively, the government can use these assets to transfer resources from high-income states to low-income states. Note also that while unfloored payments provide a better hedge during downturns, floored payments also provide limited hedging because they promise a higher payment during upturns than non-contingent debt, which induces higher prices. Yet, the government does not issue too much debt to avoid paying high contingent claims, and floored payments lead to a reduction in the sovereign's outstanding debt obligations as well as default risk. In an economy with payment suspensions, however, consumption volatility heightens because the government does not make payments during suspensions, and thus consumption jumps. These results stem from the non-linearity of the price schedules.

After documenting that an economy with unfloored payments is the highest welfare-generating economy, I then examine the dynamics of this contract only for brevity. The price of unfloored instruments is higher during economic downturns than with non-

[3]Country experiences provided in Appendix link these indexation methods to what happened in the data. I also consider a fourth method in the Appendix, which is akin to "Arrow-Debreu" securities.

contingent debt despite a decline in future expected payments because default risk is mitigated following the decreased debt burden. Yet, default risk with such instruments remains flat during economic booms, even though the government now promises to make higher payments. So, different forces are working in opposite directions. With the quantitative exercise, this paper shows that the asymmetric gains from economic downturns are higher than the possible losses from economic booms, during which the price schedule is flatter. I also show that the government is active in debt markets; that is, the government relies more on non-contingent debt during economic booms. During economic downturns, however, the government's main source of borrowing becomes indexed debt.

The second contribution of this study is on the government's optimal debt management. The role of indexation for debt management dates back to Barro (1995), who shows that government debt should be indexed to consumption and government expenditure for smoother tax rates in a model of optimal debt management.[4] In this paper, I show that if the sovereign is introduced with two assets–a non-contingent asset and a state-contingent asset (GDP-indexed bond)–then the sovereign undertakes a buyback operation depending on the indexation method. For instance, the sovereign finds issuing indexed bonds optimal during low-income periods while purchasing back some of its outstanding non-contingent debt with the proceeds. Indeed, almost 50 percent of welfare gains are coming from the government's active debt management. This result is important from a fiscal and optimal debt management perspective.

Lastly, I quantitatively investigate how a government can avoid a costly default by actively managing its portfolio that is enhanced with indexed debt. This approach might have been crucial for emerging market economies that are currently struggling with the COVID-19 shock or for subsequent adverse shocks. It should be noted however that my analysis abstracts from scenarios that may affect the success of GDP-indexed bonds, including governments' inability to underwrite these contracts at a reasonable premium.

I should note some concerns about the feasibility of arranging GDP-indexed bonds, such as data revisions or a methodological change in GDP calculation. Borensztein and Mauro (2004) and Griffith-Jones and Sharma (2006) address these concerns by proposing the establishment of independent institutions to ensure that data revisions can be adjusted for subsequent payments, as in the case of inflation-indexed bonds issued by the United Kingdom (Council of Economic Advisers). Or, as in the case of Portugal's experimentation with these bonds, no adjustments (positive or negative) on payments can be made

[4] Kim and Ostry (2018) discuss the role of GDP-indexed bonds in generating fiscal space to facilitate debt management policies.

following data revisions. If changes are made to GDP calculations, the old series to which the indexed coupons are tied may be maintained. Workshops and policy initiatives led by the G20 are being organized with the hope of facilitating the issuance of such instruments by addressing potential operational drawbacks (see Bank of England (2015), European Central Bank (2017), and European Commission ECFIN - OECD (2018)).

Literature Review. This paper belongs to the literature on new financial instruments– in particular, instruments linking payments to variables such as output, exports, trade balance, or price of a country's principal export commodity as a way of facilitating the government's debt repayment (see Bailey (1983), Krugman (1988), and Froot, Scharfstein and Stein (1989)). A new wave of studies that emerged in the 1990s particularly supported the idea of indexing payments to GDP (see Shiller (1993) and Barro (1995)). Shiller (1993) recommended an instrument that has a long-term maturity, possibly perpetual, and which would have its coupon and principal linked to GDP. However, all of these studies are abstracted from a formal analysis in which the default decision of a sovereign is endogenously determined. Following the growing interest in GDP-indexed bonds, a number of policy papers and workshops discussing the benefits and costs of issuing these new financial instruments resurfaced under a one-period debt framework (see IMF (2017), Athanasoulis and Shiller (2001), and Durdu (2009). Again, they are all abstracted from an endogenous default decision. This paper fills this gap by using a class of quantitative endogenous sovereign default models (e.g., Aguiar and Gopinath (2006) and Arellano (2008)) with foundations that rely on Eaton and Gersovitz (1981).[5] By extending the framework provided here, one can also explore alternative forms of financial securities such as commodity-linked bonds or revenue bonds where payments are tied to tax revenues.

This paper also contributes to the literature on optimal contract design, which dates back to Allen and Gale (1988). Even though the history of tying debt repayments to GDP originates in the 1980s, the authors have little to say on the optimal design of these securities. I show that a number of indexation methods either have negligible effects or leave the government worse off. However, an unfloored contract provides welfare gains to the issuer.

This paper is also related to studies documenting the superiority of using long-term debt for a quantitative analysis. Chatterjee and Eyigungor (2012), Arellano and Ramanarayanan (2012), and Hatchondo and Martinez (2009) document the superiority of using

[5] Hatchondo and Martinez (2012) present the effects of such an arrangement using strategic defaults with one-period Arrow-Debreu securities. They allow the sovereign to choose a payment contingent on the endowment realization next period. Thus, the government is not committed to any indexation method ex ante. As the authors put it: "[T]he government only promises payments for which it would not choose to default."

long-term debt over short-term debt in order to match key stylized facts of sovereign debt moments such as the debt-to-GDP ratio and sovereign spreads. A particularly important feature of long-term debt is its functionality for exploring alternative forms of indexation methods. To this end, the framework provided in this analysis is more amenable to a quantitative analysis and is also appropriate for an evaluation of the effects of different indexation methods.

The remainder of the paper proceeds as follows. Section 2 presents the model, and Section 3 explains the benchmark calibration. Section 4 features the simulation results. Finally, Section 5 concludes.

2 Model

This section features a dynamic small open economy model with GDP-indexed bonds and non-contingent debt, which are both defaultable assets with long-term maturity. The economy is endowed with a single tradable consumption good. The shocks to the economy are uninsured, and time is discrete.

2.1 Indexation schemes

This section explains a wide variety of indexation schemes that are considered in the paper. The government offers periodic payments that are tied to the developments in GDP growth. I consider two main methods–coupon-linked and principal-linked–GDP indexation methods and for each of these methods, I study floored, unfloored, and suspension schemes. These methods all include a fixed payment and a contingent payment. I also consider these indexation methods without a fixed payment but with only a contingent payment, akin to "Arrow-Debreu" securities. For brevity, these results appear in the Appendix as some of these Arrow-Debreu-like securities generate welfare losses if they are not optimally designed and are shown to be inferior to the ones that include a fixed payment.

2.1.1 Coupon-indexed payments

Coupon payments vary proportionally with realized income. For instance, if realized income in a period is 3 percent higher than mean trend income, coupon payments would be 3 percent higher than the asset with non-contingent debt. Under this subsection, I investigate three alternative economies: an economy with a floor on payments, an economy with no floor (unfloored), and an economy in which payments are suspended when income realization is lower than its trend income.

2.1.1.1 Floored payments

The rationale behind the floored indexation method is to assure investors by promising a coupon amount that pays a lower bound that is similar to what they would have received with a non-contingent asset. Thus, periodic coupon payments are not negatively affected by adverse developments in GDP. These payments are similar to what Argentina, Greece, Ukraine and Portugal issued . They are also widely discussed within policy circles (see Griffith-Jones and Sharma (2006), Borensztein and Mauro (2004), IMF (2017), and the references therein).

2.1.1.2 Unfloored payments

One criticism of floored payments is they do not carry out the underlying motivation of a risk-sharing spirit during economic downturns. The novelty of these financial instruments, as discussed in Shiller (1993) and Shiller (2012), is to have a market that shares macroeconomic risks. In addition, unfloored payments are motivated to facilitate a quicker recovery from an economic slowdown without a default by lowering the government's debt burden.

In this variant of GDP-indexed bonds, investors fully participate in the development of a country's GDP. With this way of indexing, investors' receivables decline during downturns. Note that this way of linking coupons with GDP realizations ensures a non-negative payment. For the baseline analysis, investors take a symmetrical risk-sharing approach for both the upside and the downside.

I show in the results section (Section 4) that with this indexation scheme, gains are highest for both the government and investors of previously issued non-contingent debt. Proponents of floored payments argue that current investors would be worse off with unfloored payments as the introduction of such assets may lower the price of existing non-contingent debt. Relegating the details of the mechanism to Section 4, our analysis shows that these bonds reduce default risk and thus increase the price of the asset during downturns, which leaves the holders of previously issued non-contingent debt better off.

2.1.1.3 Suspension on payments

One of the central discussions following the European debt crisis and more recently with the COVID-19 pandemic is the reprofiling of sovereign debt, which refers to the suspension of government debt repayments or imposing a debt service standstill (see Bolton et al.

(2020)).[6] International organizations such as the IMF and the World Bank have launched the Debt Service Suspension Initiative (DSSI) for a temporary suspension of debt service payments. With these motivations, I also introduce an indexation scheme in which the government does not make any payments whenever its income realization is smaller than its trend, yet, its payments accrue a risk-free rate r during debt suspensions.

2.1.2 Principal-indexed payments

The government makes additional payments that are tied to the principal on top of coupons. This type of GDP-indexed bond was issued by Singapore in 2001 and is also commonly studied in papers without endogenous default risk (see Borensztein and Mauro (2004), Bank of England (2015) and Schroder et al. (2004)).[7] I study the same indexation schemes of coupon-linked bonds under here as well.

2.2 Environment

Preferences and endowment. This paper considers a small open economy in which the benevolent government maximizes the utility of the infinitely lived representative household and has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where E denotes the expectation operator, $0 < \beta < 1$ is the intertemporal discount factor, and c_t denotes aggregate consumption at time t. The utility function $u(.)$ is an increasing, continuous, and strictly concave function that belongs to the class of CRRA utility functions and can be written as

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{2}$$

[6] Hatchondo et al. (2022) investigate the effect of introducing automatic debt suspensions during a global liquidity crisis. They find that even though contingent convertible bonds (cocos) reduce the frequency of defaults triggered by liquidity shocks, they increase the overall default frequency.

[7] Singapore issued the New Singapore Shares, which pay fixed 3 percent dividend payments per annum plus an extra dividend if the real growth rate is higher than the predetermined baseline GDP (see Government of Singapore (2001)).

with a risk aversion parameter γ. Each period, households receive a stochastic income $y \in \mathcal{Y} \subset \mathbb{R}_{++}$ that follows a Markov process:

$$\log(y_t) = (1-\rho)\,\mu + \rho \log(y_{t-1}) + \varepsilon_t, \tag{3}$$

with $|\rho| < 1$, and $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$.

Asset space. The government has two bonds in its debt balances–non-contingent bonds and GDP-indexed bonds–both of which have long-term maturities. The government issues a perpetuity that pays a geometrically decreasing coupon κ at a rate $\delta \in (0,1]$.[8] A non-contingent bond issued in period t promises to pay $(\kappa, (1-\delta)\kappa, (1-\delta)^2\kappa, ..., 0)$ units of the tradable good in subsequent periods. This is a typical formulation for long-term bonds to avoid having to keep track of the entire distribution of different bond maturities (see Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012), and Arellano and Ramanarayanan (2012)). Hence, long-term debt dynamics can be represented as follows:

$$b_{t+1} = (1-\delta)b_t + l_t,$$

where b_t and b_{t+1} are the total number of outstanding non-contingent debt coupon claims at the beginning of time t and $t+1$, and l_t is the number of non-contingent long-term bonds issued in period t. Notice that $\delta = 1$ is a special framework of long-term bonds and corresponds to the economy with one-period debt and one can write $\kappa = \frac{\delta+r}{1+r}$ where r is the risk-free interest rate.

Each period, the government chooses how many non-contingent bonds b and GDP-indexed bonds b_g to issue from a finite set $\mathcal{B}$ and $\mathcal{G}$, respectively. A common assumption in the sovereign default literature is that the government is a net issuer, so cannot accumulate assets.[9] Throughout the text, x and x' denote any variable x in period t and $t+1$, respectively.

[8] If default is avoided, the price of the debt will be risk-free, and the average one-period bond price is equivalent to the present value of non-contingent long-term debt. Coupon payment κ is calculated such that the default-free price of the debt is equal to the average one-period debt price and is formally derived when explaining the investors' problem.

[9] Önder (2016) shows that allowing the government to have a saving technology generates the same results for the class of Eaton-Gersovitz models.

GDP-indexed payments function. The GDP-indexed payments function can be compactly written as follows:

$$
\begin{aligned}
\Gamma(y) = (1-\mathcal{I}_s)\Bigg[(1-\mathcal{I}_p)\Bigg(&(1-\mathcal{I}_f)\,Max\left\{0,\mathcal{I}_e\left(1+\theta\left(\frac{y}{y^*}-1\right)\right)+(1-\mathcal{I}_e)\theta\left(\frac{y}{y^*}\right)\right\} \\
&+\mathcal{I}_f Max\left\{y^*,\mathcal{I}_e\left(1+\theta\left(\frac{y}{y^*}-1\right)\right)+(1-\mathcal{I}_e)\theta\left(\frac{y}{y^*}\right)\right\}\Bigg) \\
&+\mathcal{I}_p\Bigg((1-\mathcal{I}_f)\left(\mathcal{I}_e\left(\theta\left(\frac{y}{y^*}-1\right)\right)+(1-\mathcal{I}_e)\,\theta\left(\frac{y}{y^*}\right)\right) \\
&+\mathcal{I}_f Max\left\{y^*,\mathcal{I}_e\theta\left(\frac{y}{y^*}-1\right)+(1-\mathcal{I}_e)\,\theta\left(\frac{y}{y^*}\right)\right\}\Bigg)\Bigg]+\mathcal{I}_s\mathcal{S}(y)\left(\eta\left(y\right)\right), \qquad (4)
\end{aligned}
$$

where $\mathcal{I}_s$ is an indicator function equal to 1 if realized payments are subject to a suspension and take the value of zero if a suspension scheme is not in place. Indicator function $\mathcal{I}_p$ turns on if payments are tied to the principal and turns off if payments are tied to the coupon. Indicator function $\mathcal{I}_f$ becomes 1 if payments are floored and becomes zero if payments are unfloored, and an indicator function $\mathcal{I}_e = 1$ implies that indexed bonds include a fixed payment as well as a contingent payment, while $\mathcal{I}_e = 0$ do not entail a fixed payment at all but only a contingent payment that becomes similar to an Arrow-Debreu type security. I analyze these indexation schemes that do not include a fixed payment at all, $\mathcal{I}_e = 0$, in the Appendix.[10] The function $\mathcal{S}(y)$ is also an indicator function that becomes unity when income y is smaller than y^*, which is the mean income μ in equation (3) and takes the value of zero otherwise. Thus, when both $\mathcal{I}_s$ and $\mathcal{S}(y)$ are equal to 1, then the government makes no payment under this scheme. The last term in the equation, $\eta(y)$, is equal to the term in brackets with which $(1-\mathcal{I}_s)$ is multiplied.

Indexation in the model is akin to an "income gap."[11] I also introduce a *multiplier* θ to be able to further investigate price dynamics and to find the optimal indexation scheme. The highest welfare generating multiplier is determined to be the optimal θ. This flexible representation of a GDP-indexed payment function will enable researchers to explore a wide variety of alternative indexation methods. A number of indexation methods can be studied by turning on indicator functions one by one. Alternative variants of this indexing function are explored in depth in subsequent sections.

Timing. The government starts a period with outstanding GDP-indexed bonds b_g, non-contingent debt b, and income y, which are all public information. The government

[10] In an earlier version of the model, a ceiling/cap on the payment was also considered but is dropped from the analysis here for brevity as it yields implications that are similar to those in the unfloored economy.

[11] Typically, these indexation methods depend on a threshold GDP level to tie payments (see Bank of England (2015) and Government of Singapore (2001), Borensztein and Mauro (2004), and Durdu (2009)).

then chooses to default, and if repaid, then investors offer a menu of prices depending on the government's amount of new debt issuance of each type and the current income.[12] The government then chooses its portfolio for its budget balances with which consumption reads

$$c = y - b_g\left(\left(1-\mathcal{I}_p\right)\kappa\Gamma(y) + \left(1-\mathcal{I}_s\mathcal{S}(y)\right)\mathcal{I}_p\left(I_e Max\left(0,\left(\kappa+\Gamma\left(y\right)\right)\right)\right)\right) + \\ \left(1-I_e\right)\left(\kappa+\Gamma\left(y\right)\right)) q_g(b', b_g', y) i_g' - \kappa b + q(b', b_g', y) i',$$

where $i_g' = b_g' - (1-\mathcal{I}_s\mathcal{S}(y))(1-\delta)b_g - \mathcal{I}_s\mathcal{S}(y)b_g e^r$ and $i' = b' - (1-\delta)b$. Notice that with a GDP indexation scheme that entails payment suspensions during downturns–that is, when $\mathcal{I}_s\mathcal{S}(y) = 1$–payments accrue an interest rate r and the government does not make any payments today (embedded in $\Gamma(y)$). With a GDP indexation scheme that ties payments to the principal– that is, when $\mathcal{I}_p = 1$–then the GDP-indexing function is added to coupon payments κ conditional on the payments are not like Arrow-Debreu securities–that is, $\mathcal{I}_e = 1$. Notice that with coupon-linked payments, $\mathcal{I}_p = 0$, we do not need to multiply coupon payments with $\mathcal{I}_e$ as it is already in the GDP-indexing function $\Gamma(y)$. The formulation ensures that negative payments are not allowed.

If the government defaults on its debt, then a penalty scheme is imposed; the government is not allowed to borrow from international markets for a stochastic number of periods with an income cost of defaulting during its exclusion. Its consumption during exclusion is given by

$$c = y - \phi(y),$$

where default cost $\phi(y)$ is equal to $Max\{0, d_0 y + d_1 y^2\}$. The government starts with zero debt when access to the credit markets is regained.

2.3 Recursive formulation

Let V_R and V_D denote the value of repayment and the value of default functions, respectively, and let V be the value function for the government that has the option to default. For any non-contingent price function q and GDP-indexed debt price function q_g, V satisfies the following functional equation:

$$V(b, b_g, y) = Max\left\{V_R(b, b_g, y), V_D(y)\right\}, \tag{5}$$

[12]This situation is identical to the government offering investors a menu of prices and investors choosing how much to lend in equilibrium. The majority of the studies in the quantitative default literature describe similar game setups.

where the government's value of repayment is given by

$$V_R(b, b_g, y) = \max_{c \geq 0, b' \in \mathcal{B}, b'_g \in \mathcal{G}} \left\{ u\left(c\right) + \beta \mathbb{E}_{y'|y} V(b', b'_g, y') \right\}, \tag{6}$$

subject to

$$\begin{aligned} c &= y - b_g \Bigg(\left(1 - \mathcal{I}_p\right) \kappa \Gamma(y) + \left(1 - \mathcal{I}_s \mathcal{S}(y)\right) \mathcal{I}_p \Big(I_e Max\left(0, \left(\kappa + \Gamma\left(y\right)\right)\right) \Big) \\ &+ \left(1 - I_e\right)\left(\kappa + \Gamma\left(y\right)\right) \Bigg) + q_g(b', b'_g, y) i'_g - \kappa b + q(b', b'_g, y) i', \\ i'_g &= b'_g - (1 - \mathcal{I}_s \mathcal{S}(y))(1 - \delta) b_g - \mathcal{I}_s \mathcal{S}(y) b_g e^r, \\ i' &= b' - (1 - \delta) b. \end{aligned}$$

The defaulting government regains access to the credit markets with a constant probability $\psi \in [0, 1]$ and has zero debt at the time of market reentry. The value of defaulting reads

$$V_D(y) = u\left(c\right) + \beta \mathbb{E}_{y'|y} \left[(1 - \psi) V_D(y') + \psi V(0, 0, y') \right], \tag{7}$$

subject to

$$c = y - \phi(y).$$

The solution to this problem yields a binary default decision rule $\hat{d}(b, b_g, y) \in \{0, 1\}$, 1 if the government defaults, and 0 otherwise, and borrowing rules for each asset type $\hat{b}(b, b_g, y)$ and $\hat{b_g}(b, b_g, y)$. In equilibrium, defined in Section 2.4, investors use these decision rules to price contracts. Now it remains to describe the investors' problem. The bond market is competitive, and investors are assumed to be risk-neutral. I relax this assumption in Appendix, and the qualitative findings remain the same.[13] Thus, investors take the price schedules $q(b', b'_g, y)$ and $q_g(b', b'_g, y)$ as given. The opportunity cost of funds is given by the exogenous risk-free interest rate $r > 0$. With the zero-profit assumption and no-arbitrage condition in place, price functions for non-contingent bonds q and GDP-indexed bonds q_g solve the following functional equations:

[13] The following stochastic discount factor (SDF) is used to introduce the risk aversion premium: $\mathcal{M}(y', y) = exp(-r - \left[\xi \epsilon' + 0.5 \xi^2 \sigma_\epsilon^2\right])$ and ξ determines the magnitude of risk premia. Notice that bond payoffs become more valuable to investors for the states in which the government faces low-income shocks ϵ within this formulation, which is a special case of the discrete time version of Vasicek (1977). This specification can also be particularly relevant for GDP-indexed bonds as risk-averse investors may dislike these assets during which the government faces low-income shocks. This risk premium is endogenous to the government's borrowing decision depending on the type of bonds available to the government. I undertake a detailed analysis for risk-averse lenders in Appendix A.6 and I find that the qualitative results to a large extent remain the same.

$$q_g(b', b'_g, y) = \frac{\mathbb{E}_{y'|y}\Bigg[(1-d')\,(1-\mathcal{I}_s\mathcal{S}(y'))\,\Big[(1-\mathcal{I}_p)\kappa\Gamma(y') + \mathcal{I}_p Max(0, (\kappa\mathcal{I}_e + \Gamma(y'))) + (1-\delta)q'_g\Big]}{1+r},$$
$$+ \frac{\mathcal{I}_s\mathcal{S}(y')e^r q'_g}{1+r}\Bigg], \tag{8}$$
$$q(b', b'_g, y) = \frac{\mathbb{E}_{y'|y}\left[\left(1-\hat{d}\left(b', b'_g, y'\right)\right)\left[\kappa + (1-\delta)q\left(b'', b''_g, y'\right)\right]\right]}{1+r}, \tag{9}$$

where $d' = \hat{d}\left(b', b'_g, y'\right)$ denotes the next-period equilibrium default decision, $b'' = \hat{b}(b', b'_g, y')$ denotes the next-period equilibrium non-contingent bond decision, $b''_g = \hat{b_g}(b', b'_g, y')$ denotes the next-period equilibrium GDP-indexed bond decision and q'_g satisfies $q_g\left(b'', b''_g, y'\right)$. In the absence of default risk, the price of the non-contingent long-term debt equals $q^* = \frac{\kappa}{r+\delta} = \frac{1}{1+r}$, so one can write $\kappa = \frac{r+\delta}{1+r}$.

In equations (8) and (9), the expected value of investing in a bond should be equal to the expected return of investing in a risk-free asset with a return of r. If an investor purchases non-contingent debt today, conditional on the government's repayment next period, the investor will receive κ units of goods plus the unmatured amount of its receivables, which are worth $(1-\delta)q\left(b'', b''_g, y'\right)$. In an economy with GDP-indexed bonds, if the underlying indexation scheme is coupon linked, then an investor receives $\kappa\Gamma(y')$. If the underlying indexation scheme is principal linked, then an investor receives $Max(0, (\kappa\mathcal{I}_e + \Gamma(y')))$ units of goods tomorrow. If the underlying scheme is one in which payments are suspended during economic downturns ($\mathcal{I}_s\mathcal{S}(y') = 1$), then an investor does not receive any payments the next period but receivables grow at a rate of r. If the underlying scheme is Arrow-Debreu-like securities ($\mathcal{I}_e = 0$), then an investor only receives contingent payments.

2.4 Definition of equilibrium

This paper focuses on a Markov perfect equilibrium (MPE). Hence, equilibrium conditions on the current state variables, not on the entire history.

Definition 1 (Markov perfect equilibrium) A *Markov perfect equilibrium* is characterized by

1. a collection of value functions V, V_R, and V_D;

2. rules for default $\hat{d}$, non-contingent debt borrowing $\hat{b}$ and $\hat{b_g}$; and

3. debt price functions q, q_g;

such that:

i. given price functions q_g, q; $\left\{V, V_R, V_D, \hat{d}, \hat{b}, \hat{b_g}\right\}$ solve the Bellman equations (5), (6), and (7).

ii. given policy rules $\left\{\hat{d}, \hat{b}, \hat{b_g}\right\}$, the price functions q_g, q satisfy conditions (8) and (9).

3 Calibration

A common practice in the literature is to calibrate the model economy to Argentina. Table 1 summarizes the model parameters used in the paper. The income process parameters are estimated using the detrended real GDP series of Argentina between the first quarter of 1980 and the last quarter of 2000. The auto-correlation coefficient for the output process and the standard deviation of innovations are standard estimates. The cyclical component of GDP from the estimated AR(1) process spins $\rho = 0.9$ and $\sigma_\epsilon = 0.027$. A period in the paper corresponds to a quarter. The risk-aversion rate $\gamma = 2$ and the risk-free rate $r = 1$ percent are common in the sovereign default literature. The probability of market reentry parameter ψ assumes an average exclusion of one year from the credit markets, which is also the value used by Arellano (2008). The average exclusion period is also within the range of the average exclusion period documented in Cruces and Trebesch (2013) and Gelos, Sahay and Sandleris (2011).

A typical approach in the literature is to have a quadratic loss function with which the cost of default increases more than proportionally with income. Following Chatterjee and Eyigungor (2012), I assume the following functional form for the cost of defaulting: $\phi(y) = Max\{0, d_0 y + d_1 y^2\}$. The cost parameters d_0 and d_1 are calibrated to match the average debt-to-GDP ratio and the mean spreads and are calibrated to be -0.66 and 0.997, respectively. Lastly, β is set to be 0.96, which is a standard value in the quantitative default literature to be able to match the default frequency. The debt statistics and the business cycle statistics are obtained from Hatchondo and Martinez (2009).

To match the duration of debt, δ is set to be 0.0375. With this value, average debt duration is around four years in the simulations, which is roughly the average debt duration for Argentina reported in Broner, Lorenzoni and Schmukler (2007).

The definition of duration in Macaulay (1938) is used to compute the duration of the long-term debt. Duration D is the weighted average maturity of future cash flows. A bond issued at time t that promises to make periodic payments κ for the subsequent periods at time $1, 2, \ldots, J$ years into the future with the final price of zero has duration D.[14]

Table 1: Parameter Values

	Parameter	Value
Risk aversion	γ	2
Risk-free rate	r	1%
Probability of reentry after default	ψ	0.282
Income autocorrelation coefficient	ρ	0.9
Standard deviation of innovations	σ_ϵ	2.7%
Mean log income	$\mu = y^*$	$(-1/2)\sigma_\epsilon^2$
Debt duration	δ	0.0375
Calibrated		
Discount factor	β	0.96
Income cost of defaulting	d_0	-0.66
Income cost of defaulting	d_1	0.997

3.1 Numerical Solution

This section briefly outlines the numerical algorithm undertaken in this paper to solve the model presented in the text while relegating the details to the Appendix. The main challenge lies in finding the optimal doubles of (b, b_g). Some portfolio doubles may imply very similar continuation values, which can be troublesome when attempting to obtain convergence.

I initially solve the model using the value function iteration approach with global search methods; value functions and price functions are iterated until the difference in two subsequent iterations remains the same. Then I switch to local search methods in

[14] $D = \frac{1+i}{i+\delta}$ where i is the periodic yield an investor would earn if the bond is held to maturity with no default and it satisfies

$$q = \sum_{j=1}^{\infty} \frac{\kappa(1-\delta)^{j-1}}{(1+i)^j}.$$

The sovereign spread r_s is computed as the difference between yield i and the risk-free rate r. The annualized spread reported in the tables is computed as

$$1 + r_s = \left(\frac{1+i}{1+r}\right)^4.$$

The debt levels obtained from the simulations are equivalent to the present value of future debt obligations and computed as $\frac{b'}{\delta+r}$.

the neighborhood of my candidate optima for b', b'_g that are obtained with a global search method. This also economizes grid points on assets and income. With this procedure, a convergence criterion of 10^{-5} is attained. With finer grids, in theory, one should be able to obtain convergence without local search methods. Note that when solving the model with a single asset, the global solution method suffices.

4 Quantitative results

I initially present the simulation results of the baseline model (economy without GDP-indexed bonds) and match the target statistics. Then I introduce GDP-indexed bonds with alternative forms of indexation schemes.

4.1 Results with non-contingent debt

Column (1) of Table 2 presents the data moments and column (2) presents the baseline moments that are obtained without GDP-indexed bonds using the parameters in Table 1. The simulation results of the baseline economy match both the long-run sovereign debt moments and the business cycle moments reasonably well. In the next subsections, I will discuss the role that GDP-indexed bonds play and compare the outcome of different indexation methods systematically with the one obtained here without recalibrating the baseline model.

4.2 Results with indexed payments

The effect of the coupon-linked floored method is reported in Table 2 under column (3). The government utilizes more of indexed debt, and the default frequency, which is measured as the number of defaults in 100 years, slightly increases. This can be attributed to a slight increase in the government's total debt. Even though the spread in floored debt seems to be higher than the spread in baseline debt, the price of floored indexed debt is in fact slightly higher than the price of non-contingent debt in the baseline economy. This is because necessary adjustments had to be made (details of which are provided in the Appendix) in order to be able to compute the spread of indexed bonds as expected payments now depend on income realization. Results for the coupon-linked unfloored method are presented under column (4). Similar to the floored method, the government relies more on indexed debt in its debt balances, and the government's total debt increases while the default frequency is falling. The effect of debt suspensions during downturns is illustrated under column (5). In an economy with debt suspensions, the suspensions make significant changes to the government's balance sheet. The government's total debt-to-GDP ratio becomes 62.5 percent, and the spreads of both debt instruments as well as the default rate soar. Yet, the government achieves a lower level of consumption volatility.

In general, economies with floored and unfloored payments under coupon-linked methods are highly similar to the baseline economy, mainly because payments are indexed to coupons. This result is related to the fact that payments are indexed to the coupons, and thus, the increase in expected payments because of the promised coupon payments is not substantial enough. For instance, in an economy where payments are tied to the principal, one obtains substantial portfolio dynamics. The outcome of this analysis is shown under columns (6)-(9). The government's reliance on indexed debt lessens, and the government can do a better job at smoothing its consumption. Also, both the default rate and the spread are lower than their coupon-linked counterparts. This suggests that the government's promises on how much more or less to pay change the optimal response of the government. In order to explore this channel and to be able to offer an optimal contract, a multiplier θ in the GDP-indexing function $\Gamma(y)$ is engineered. This will also lead to a better understanding of the dynamics of GDP-indexed bonds.

4.3 Optimal indexation scheme

In this section, I optimize the indexation schemes for each indexation method considered in Table 2 using the multiplier θ in the GDP-indexed function $\Gamma(y)$. The optimal value of θ is obtained in a welfare analysis (details of which are relegated to Section 4.4).

Table 3 presents the results with an optimal multiplier, and the following observations stand out. The moments for coupon-linked schemes (columns (3)-(5)) and principal-linked schemes (columns (6)-(8)) come closer than the one in Table 2. This is because the optimal multiplier θ for the coupon-linked unfloored indexation scheme is 9, whereas this number corresponds to 0.4 for the principal-linked unfloored indexation scheme. Thus, the economies promise similar expected payment schedules.

Table 3 further shows that the trade balance ($tb = y - c$) becomes procyclical with income; in other words, the borrowing policy becomes countercyclical across all schemes. Intuitively, the government avoids issuing indexed debt for high-income states and borrows during low-income states to smooth its consumption. That is, the government transfers resources from good income states by borrowing during low-income states. This channel will be further explored in Section 4.6.

Table 2: Long-run effects of introducing GDP-indexed bonds

	(1) Data	(2) Baseline	Coupon linked (3) Floor	Coupon linked (4) Unfloor	Coupon linked (5) Suspension	Principal linked (6) Floor	Principal linked (7) Unfloor	Principal linked (8) Suspension
Debt Statistics								
Non-contingent debt-to-GDP	39	39.38	3.12	1.11	8.22	19.12	35.08	27.28
GDP-indexed debt-to-GDP	*n.a*	*n.a*	36.65	39.98	54.34	19.09	21.69	17.72
Mean r_s	7.44	7.32	8.16	7.69	11.25	6.37	2.35	9.41
Mean r_s indexed	*n.a*	*n.a*	7.96	7.33	9.41	7.33	2.96	9.24
Default rate	4	3.51	3.68	3.41	5.02	2.89	0.26	4.19
Duration	4	3.97	3.92	3.87	3.58	4.09	3.77	4.75
Duration index	*n.a*	*n.a*	3.96	3.89	3.70	4.01	3.80	4.66
Business Cycle Statistics								
$\sigma(c)/\sigma(y)$	1.10	1.18	1.15	1.12	0.84	1.12	0.59	0.92
$\sigma(tb/y)$	1.35	0.91	0.83	0.73	1.64	2.08	4.18	2.83
$\rho(c,y)$	0.97	0.99	0.99	0.99	0.90	0.87	0.35	0.70
$\rho(r_s,tb/y)$	0.56	0.91	0.88	0.86	-0.50	0.04	0.28	-0.39
$\rho(r_s,y)$	-0.65	-0.85	-0.85	-0.85	-0.81	-0.79	0.20	-0.80
$\rho(tb/y,y)$	-0.88	-0.76	-0.66	-0.62	0.55	0.01	0.80	0.47
$\rho(b',y)$	*n.a*	0.37	-0.58	-0.31	-0.69	0.78	0.60	0.78
$\rho(b'_g,y)$	*n.a*	*n.a*	0.59	0.43	0.66	-0.78	-0.60	-0.80

The first column reports the data moments, and the second column presents the simulation results with non-contingent debt. Columns (3)-(5) show the results for GDP-indexed bonds with coupon-linked payments, while columns (6)-(8) show the results for GDP-indexed bonds with principal-linked payments. The standard deviation of variable h is denoted by $\sigma(h)$, and the coefficient correlation between variables h and m is denoted by $\rho(h,m)$. Consumption and income are reported by natural logs. I compute the yield (and the spreads) and the debt duration by using expected payments to incorporate uncertainty about the amount of payment as well as the time of the payment. In particular, the yield and duration as well as the spreads are computed by generating a debt covenant that is never defaulted on. To this end, the simulations incorporate the adjustment stemming from different GDP indexation methods. Details for these computations are provided in the Appendix.

Table 3: Long-run effects of indexed bonds with an optimal multiplier θ

			Coupon linked			Principal linked		
	(1) Data	(2) Baseline	(3) Floor	(4) Unfloor	(5) Suspension	(6) Floor	(7) Unfloor	(8) Suspension
Debt Statistics								
Non-contingent debt-to-GDP	39	39.38	15.50	7.92	22.61	11.03	7.11	33.94
GDP-indexed debt-to-GDP	*n.a*	*n.a*	22.16	46.69	23.55	26.99	47.54	8.12
Mean r_s	7.44	7.32	6.46	2.14	9.38	6.97	2.19	7.67
Mean r_s indexed	*n.a*	*n.a*	6.89	2.25	9.08	7.27	2.24	12.97
Default rate	4	3.51	2.91	0.14	4.31	3.16	0.14	3.33
Duration	4	3.97	4.08	4.80	3.78	4.01	4.79	3.95
Duration index	*n.a*	*n.a*	4.78	4.78	3.78	3.99	4.78	3.67
Business Cycle Statistics								
$\sigma(c)/\sigma(y)$	1.10	1.18	1.10	0.56	5.65	1.06	0.57	1.05
$\sigma(tb/y)$	1.35	0.91	2.12	3.53	35.87	1.90	3.31	2.02
$\sigma(r_s)$		3.00	2.54	0.32	5.19	2.82	0.34	3.64
$\sigma(r_s)$ indexed	*n.a*	*n.a*	2.60	0.56	7.50	2.97	2.82	4.15
$\rho(c,y)$	0.97	0.99	0.87	0.63	-0.58	0.89	0.70	0.86
$\rho(r_s,tb/y)$	0.56	0.91	0.05	-0.45	-0.54	0.09	-0.52	-0.23
$\rho(r_s,y)$	-0.65	-0.85	-0.80	-0.53	-0.80	-0.83	-0.62	-0.81
$\rho(tb/y,y)$	-0.88	-0.76	0.05	0.83	0.66	0.08	0.83	0.15
$\rho(b',y)$	*n.a*	0.37	0.75	0.60	0.68	0.67	0.53	0.82
$\rho(b'_g,y)$	*n.a*	*n.a*	-0.75	-0.60	-0.71	-0.66	-0.52	-0.83

The first column reports the data moments, and the second column presents the simulation results with non-contingent debt for each model's corresponding optimal multiplier θ. Columns (3)-(5) show the results for GDP-indexed bonds with coupon-linked payments, while columns (6)-(8) show the results for GDP-indexed bonds with principal-linked payments. The standard deviation of variable h is denoted by $\sigma(h)$, and the coefficient correlation between variables h and m is denoted by $\rho(h,m)$. Consumption and income are reported by natural logs. I compute the yield (and the spreads) and the debt duration by using expected payments to incorporate uncertainty about the amount of payment as well as the time of the payment. In particular, the yield and duration as well as the spreads are computed by generating a debt covenant that is never defaulted on. To this end, the simulations incorporate the adjustment stemming from different GDP indexation methods. Details for these computations are provided in the Appendix.

Table 3 under the unfloored columns of (4) and (7) reports the simulation results with the unfloored indexation method of coupon-linked and principal-linked methods, respectively. Proponents of this indexation method are right in their conjecture that more funds can be raised and the spreads are mitigated.

In this formulation, debt obligations would be mitigated if the current income realization is below the mean trend income. Thus, these contracts act like an automatic stabilizer by facilitating the best countercyclical borrowing policy among all considered schemes. The government is able to generate higher (lower) debt during economic downturns (upturns).

An interesting result is that unfloored payments, regardless of whether they are coupon or principal linked, eradicate defaults almost entirely. Yet, this is achieved without too much increase in the debt-to-GDP ratios. In an economy with unfloored payments, however, this is not the case. Intuitively, too much increase in indexed debt would increase the government's promised payments and reduce its consumption during good times. Likewise, too much debt increases the likelihood of defaulting during bad times. Thus, unfloored payments work like a stabilizer for the economy. The upper right panel of Figure 1 also confirms the conjecture of the proponents of this method: defaults are less likely. The figure shows that the necessary adverse shock for a government to default is larger in an economy with unfloored GDP-indexed bonds than in an economy with non-contingent debt at the time of the introduction of these assets.

With floored payments under columns (3) and (6), these assets also lead to a lower number of defaults in equilibrium. This can be attributed to the government's lower total debt balances. With floored payments, the government's reliance on indexed debt is lower than in the unfloored economy. As will be discussed in Section 4.6, the asset price of the indexed debt increases as the expected stream of payments increases, and the risk of defaulting remains generally flat during high-income states. Even though the government can raise higher revenues through an indexed debt auction, future payments also increase with higher income realization. To this end, the government abstains from issuing too much indexed debt to avoid making extra payments during high-income states. This explains why the government has a higher share of non-contingent debt in its portfolio than in an economy with unfloored payments. This also shows that the floored indexation method helps mitigate the debt dilution problem in the presence of long-term debt through avoiding higher debt issuance during upturns.

Except in an economy with a principal-linked suspension method, the share of GDP-indexed debt is greater than the share of non-contingent debt in the government's budget balances. The cost of borrowing declines for all indexed debt except for the debt suspension scheme. Thus, economies with indexed debt typically observe higher asset prices with lower asset price volatility (picked up by $\sigma(r_s)$ and $\sigma(r_s)$ indexed).

Indexed debt also has important implications for consumption volatility. While both floored and unfloored methods lead to a decline, the decline halves with unfloored methods. A coupon-linked suspension scheme, on the other hand, leads to a significant jump in volatility. The intuition is that consumption shoots up substantially with a multiplier when income payments are suspended. I included Figure A1 in the Appendix to help readers visualize this anomaly.

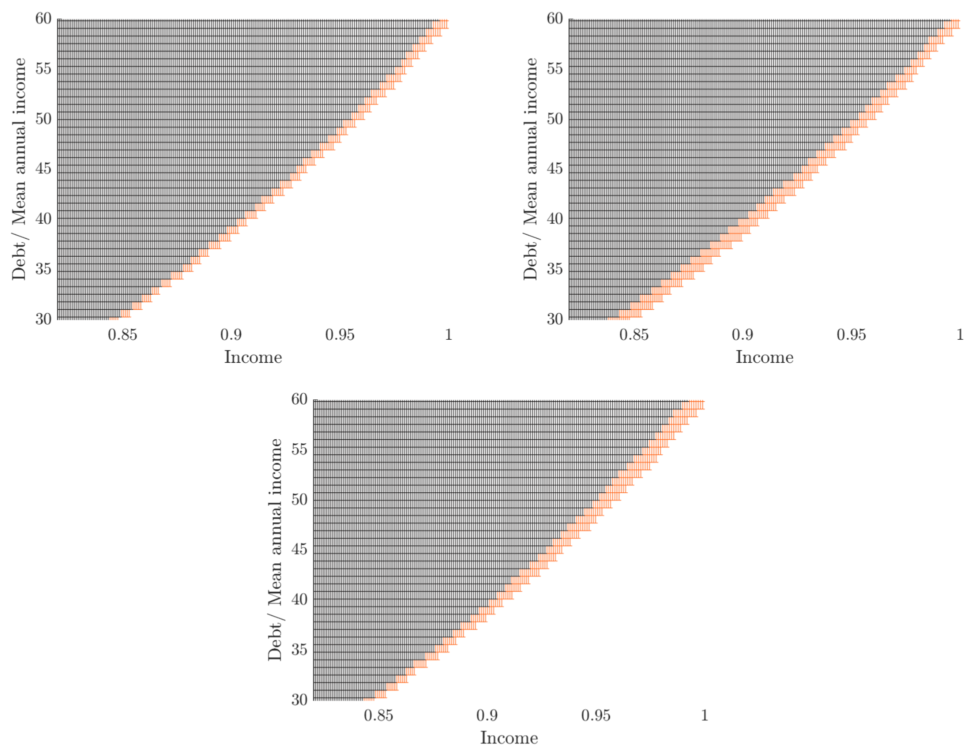

Figure 1: Default thresholds are presented with optimal θ. The figure shows the combinations of non-contingent debt and income for which the government finds defaulting optimal with no initial indexed debt. The horizontal axis represents assets normalized by mean trend income. Defaulting (repayment) is optimal in all black (white) areas. The upper left and right panels display the government's equilibrium default decisions in the GDP-indexed economy with a floored coupon-linked method and with an unfloored coupon-linked method compared to the baseline economy, respectively. The bottom panel displays the government's equilibrium default decisions in an economy with a suspension in the coupon-linked method compared to the baseline economy. The colored area denotes the states where defaulting is optimal to the baseline economy only.

4.4 Welfare gains

This section investigates the welfare gains from the introduction of GDP-indexed bonds using non-contingent debt economy as the baseline framework. To this end, welfare gains are computed in terms of compensating variations in consumption that would leave a household indifferent between staying in an economy with non-contingent debt and moving into an economy where GDP-indexed bonds is an alternative borrowing option. We measure consumption-equivalent welfare gains denoted by η as,

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u\left(c_\tau^B[1+\eta] | b_t, y_t\right) = E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u\left(c_\tau^G | b_t, b_t^g = 0, y_t\right), \tag{10}$$

in which the consumption streams $\{c_\tau^B\}_{\tau=t}^{\infty}$ and $\{c_\tau^G\}_{\tau=t}^{\infty}$ are attained in the baseline economy and in the economy with GDP-indexed bond, respectively. Welfare gain measure η is evaluated conditional on initial non-contingent debt b, initial zero GDP-indexed bonds b^g and endowment and is derived from equilibrium value functions with

$$\eta(b_t, b_t^g, y_t) = \left(\frac{V^G(b_t, b_t^g = 0, y_t)}{V^B(b_t, y_t)}\right)^{\frac{1}{1-\gamma}} - 1. \tag{11}$$

Given the CRRA utility function nature of the households, compensating variations can be computed at particular income and non-contingent debt doubles/triplets at no initial indexed debt. $V^B(b_t, y_t)$ and $V^G(b_t, b_t^g = 0, y_t)$ are value functions evaluated for doubles/triplets of non-contingent debt b_t, GDP-indexed bonds b_t^g and output y_t in the baseline economy and in the economy with GDP-indexed bonds, respectively. Thus, a positive valuation would mean that agents prefer living in an economy with GDP-indexed bonds.

The left panel of Figure 2 plots the welfare implications of three baseline coupon-linked methods with $\theta = 1$ for a better visualization as a function of the calibration target for long-term debt stock in the baseline economy and alternative income levels.

The following important implications stand out from this analysis. First, households have a higher preference for an economy where investors fully participate in developments in GDP relative to the one without full participation. The discrete fall in the welfare chart is the outcome of inevitable default when the income is below the default threshold. To provide some intuition behind these welfare gains, I focus on the highest welfare-generating contract, the unfloored scheme. For that, the chart in the right panel of Figure 2 is provided. Particularly, the chart presents the impact of the arrangement of the unfloored indexation method on the bond pricing schedule of non-contingent debt along with the price of indexed debt, plotted against the ratio of net debt issuance $(b' - (1-\delta)b + b'_g)$ to

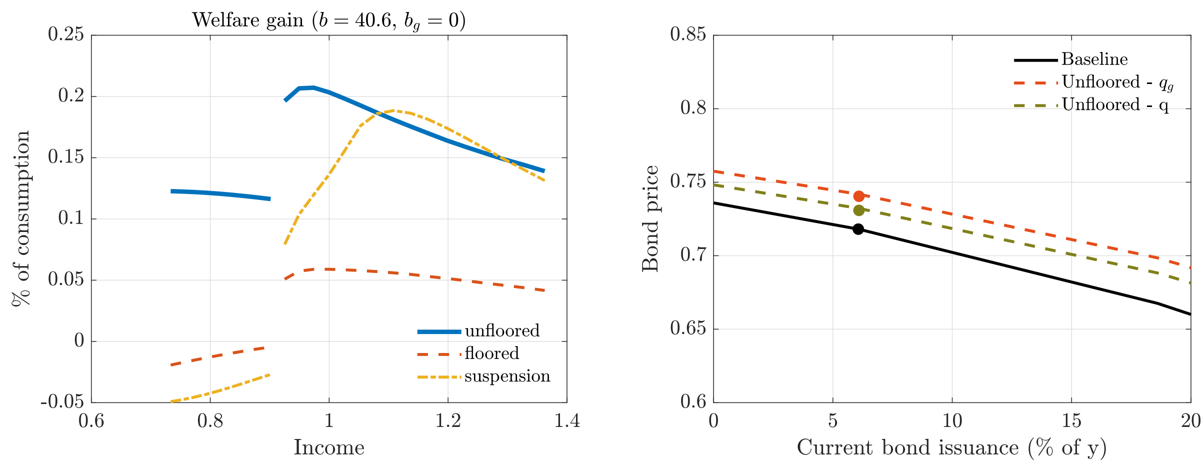

Figure 2: The left panel plots welfare gains measured in consumption-equivalent terms from the introduction of indexed bonds with $\theta = 1$. The initial debt portfolio at the time of inception entails no indexed bonds ($b_g = 0$) and a stock of non-contingent debt that is equal to the mean debt level observed in the baseline simulations. The right panel shows the effect of the introduction of GDP-indexed bonds with unfloored payments on the GDP-indexed bond pricing and non-contingent debt pricing schedules as a function of the ratio of net debt issuance $(b' - (1-\delta)b + b'_g)$ to mean income (normalized to one) at the time of arrangement. Thick dots reflect equilibrium realizations.

mean income (normalized to one) at the time of asset introduction. Notice that the asset prices for indexed debt and non-contingent debt in the unfloored economy shift upward, reflecting the effect of expected increased payments on the bond price and the decline in default likelihood during economic downturns, as depicted in the upper right panel of Figure 1. This finding also confirms the disciplining role of indexed debt in the model economies.

Next, I plot the welfare gains of indexed economies with alternative multipliers θ. In this way, I can find the optimal contract by choosing the highest welfare-generating multiplier. The outcome of this analysis is presented in Figure 3, which shows that the highest welfare gains can be attained under an unfloored scheme. In fact, both coupon-linked and principal-linked unfloored methods generate very similar high welfare gains.

The intuition lies in the government's declined debt service during an adverse income shock, which reduces default risk, and the sovereign borrows to lower consumption volatility. Two opposite forces are affecting the price of the asset and welfare during good income states that are suitably high. The sovereign's promised payments are amplified, and the heightened debt burden does not lead to enough of an increase in default risk during high-income states; thus, the price of the asset remains high, as depicted in Figure 5. Even though promised future payments increase the asset price, they reduce consumption.

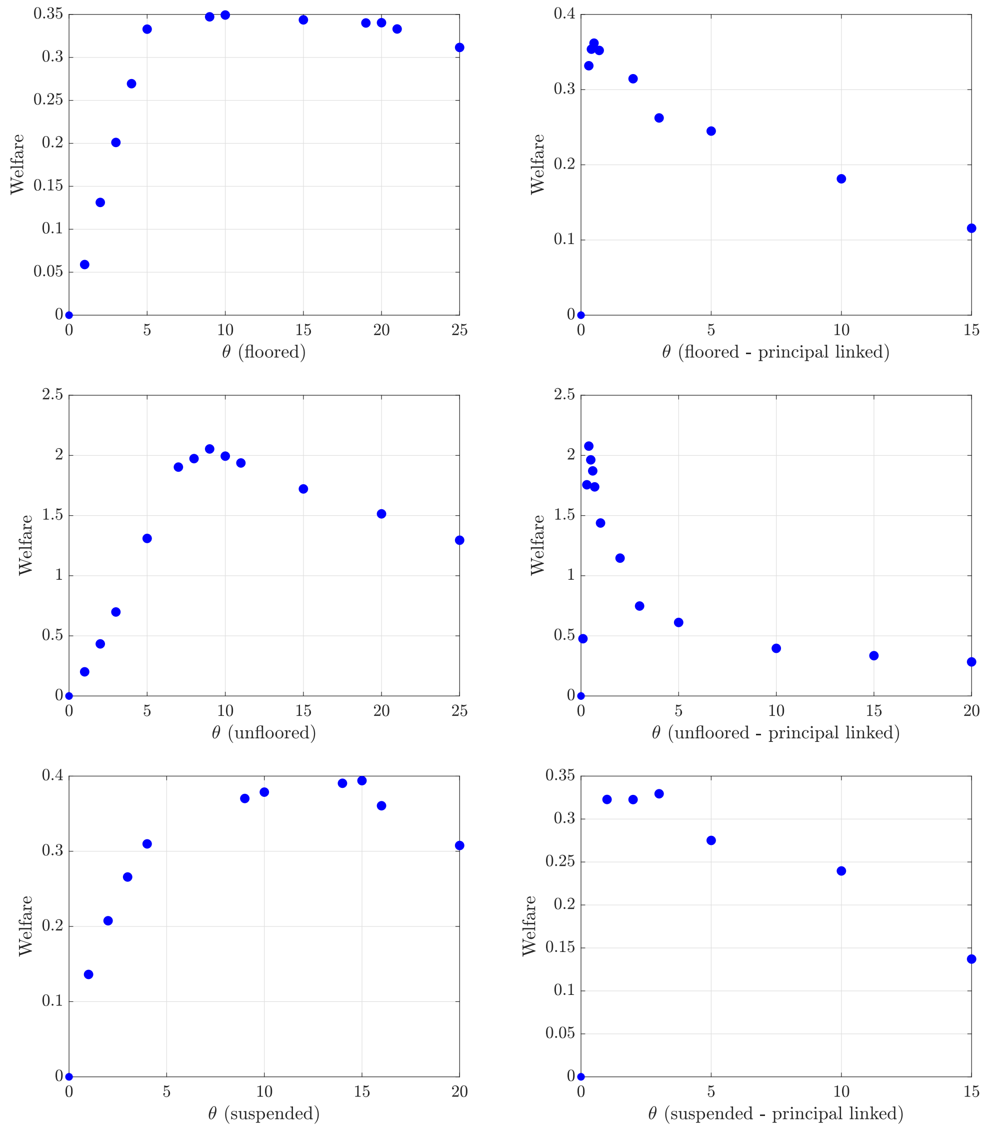

Figure 3: This figure shows the welfare gains from alternative indexation methods for different values of θ in equation (4). To compute the welfare gains the initial stock of indexed bonds is set to zero, that is $b_g = 0$. Non-contingent debt holdings are set to the ergodic level of the non-contingent debt stock in the baseline economy; that is, $b = 40$ percent of mean income which is normalized to one. The charts on the left display coupon-linked schemes, while the charts on the right show principal-linked schemes.

Thus, the benefit of high revenue generation via increased prices through government auction, measured as $q_g * (b'_g - (1-\delta)b_g)$, is outweighed by the cost of payment promises made in the previous period, measured as $b_g \kappa \Gamma(y')$. This mechanism leads the government to reshuffle its portfolio toward non-contingent debt during high-income states.

Figure 3 suggests that the government also benefits from a floored indexation method even though the gains are limited compared to an economy with an unfloored indexation scheme. Important implications stand out from this analysis regarding the dynamics of long-term debt. Notice the important forces determining the price of debt q_g in equation (8). During economic upturns, both the state-contingent payment $\Gamma(y')$ and the default probability $d(b', b'_g, y')$ rise because of the heightened debt burden during economic booms. In a quantitative analysis, the former effect dominates the latter one; thus, promised payments during upturns lift the price of the debt. Since floored payments promise a higher payment during upturns, this indexing method facilitates higher debt prices compared to the benchmark economy, which reduces both the default risk and the total debt issuance that is consistent with the floored column of Table 3. In the next section, I further explore these dynamics.

4.5 Welfare decomposition

To better understand the composition of welfare changes, Table 4 reports the sources of welfare gains following the approach of Aguiar, Amador and Fourakis (2020). The details of this decomposition are relegated to Appendix A.2. In essence, Aguiar, Amador and Fourakis (2020) present that welfare gains can be decomposed into three: (i) gains from changes in the consumption paths, (ii) gains from changes in default behavior, and (ii) gains from changes in consumption volatility.

Table 4 reports the average welfare change of 2 percent at the time of the switch from baseline economy to to the economy with GDP-indexed bonds.[15] Further, Table 4 reports the sources of welfare gains. The table shows that the most of the welfare gains are accounted for by the lower default frequency attained in the economy with GDP-indexed bonds which reduces deadweight loss of income cost of defaulting, which generates a welfare gain of 1.05 percent. The second most contributor to the welfare gains is a high front-loaded consumption profile which leads to a rise in the welfare around 0.72 percent. The contribution of lower consumption volatility attained by the indexed bonds comes last with a welfare gain of 0.25 percent.

Figure 4 plots the heterogeneity in the welfare changes in the ergodic distribution of the baseline economy. Despite a sizable heterogeneity in the welfare changes that ranges between 1.5 percent and 2.5 percent, they are all in the positive territory across all states.

[15] The averages are computed using the formula $\int \eta(b_t, b_t^g, y_t) d\Gamma^{AI}(b_t, b_t^g, y_t)$, where Γ^B is the steady-state distribution of states for the sovereign in the AI economy. Welfare gains from consumption paths are computed using simulations of 100,000 samples of 150 periods each. Simulations start from the ergodic distribution of non-contingent debt, income and default that is attained by the baseline economy 150 periods after initialization with zero non-contingent and hidden debt and mean income. Then, in one simulated economy we keep using the decision rules of the baseline economy while in the other, we switch to using the decision rules of the one obtained in the GDP-indexed economy.

Table 4: Decomposition of welfare gains at the time of switch

Welfare gain from cons. paths (%)	2.03
From tilting consumption (%)	0.72
From lowering income cost of defaulting (%)	1.05
From lowering consumption volatility (%)	0.25

Notes: The table shows the decomposition of average welfare changes at the time of switch from the baseline economy to the economy with GDP-indexed bonds. I follow the methodology developed in Aguiar, Amador and Fourakis (2020), details of which are relegated to Appendix A.2.

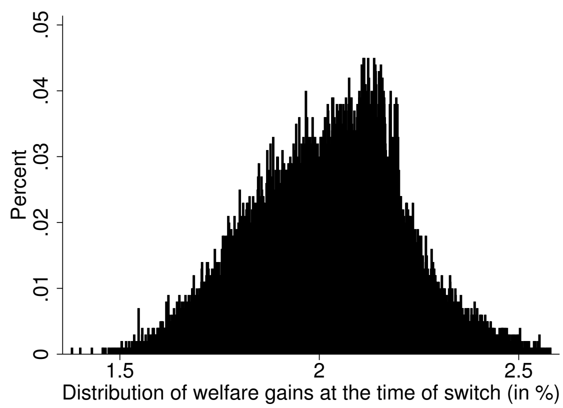

Figure 4: The figure shows the distribution of welfare changes at the time of the switch across steady-states.

4.6 Price dynamics

This section explores the price dynamics of the coupon-linked unfloored method with an optimal multiplier, as this contract turns out to be the highest welfare-generating one. This exploration will sharpen our understanding of the price equations in the text and the model dynamics. Figure 5 shows equilibrium pricing schedules of indexed debt (Unfloored q_g), non-contingent debt (Unfloored q) in an indexed economy along with the non-contingent debt in the baseline economy as a function of non-contingent debt with zero indexed debt for each of the mean income, low-income and high-income states where low- and high-income correspond to two standard deviations below and above the mean income, respectively. The bottom right chart plots these prices evaluated at the ergodic level of non-contingent debt and indexed debt. That is, the non-contingent debt level is set to be 39.4 percent in the baseline economy and 8.1 percent in the indexed economy, while ratio of unfloored indexed debt to mean income (normalized to one) is set to be 46.5 percent.

The figure reveals important dynamics. The price of indexed debt, denoted as Unfloored - g_g in the charts, responds strongly to income levels. Notice that the price of indexed debt remains the highest across all debt levels for high-income states in the upper left chart but remains the lowest for low-income levels for low debt levels in the upper right chart. Two important terms stand out for the pricing function of indexed debt in equation (8): coupon payments and the probability of default. To make the argument simpler, I will avoid the discussion of future prices. The promised payments term, $(\kappa\Gamma(y'))$, is a determinant factor of the bond prices. For low-income levels, even though the default likelihood lessens as the government's debt liability is lower, which is depicted in Figure 1, asset prices decline because the promised payments fall, which in turn reduces the asset prices of indexed debt. For high-income levels, asset prices rise as the promised payments rise, while the probability of defaulting remains flat.

This now introduces an interesting portfolio allocation problem for the government. I plot the next-period equilibrium borrowing rules for the indexed debt and non-contingent debt of the coupon-linked unfloored method in Figure 6 as a function of income evaluated at the ergodic level of non-contingent debt and indexed debt, the same as in the bottom right chart of Figure 5.[16] The figure reveals that even though the government mainly relies on indexed debt for financing, it uses an active portfolio rebalancing according to its income realization. For instance, for high-income states, the government resorts to using more non-contingent debt despite its lower price (displayed in the bottom right chart of Figure 5) because the debt burden intensifies with promised payments during high-income states. For lower-income states, despite the lower price of indexed debt, the government utilizes more of this debt and less non-contingent debt because the government's debt burden eases during low income states under indexed debt.

4.7 Indexed debt can avert debt crises

In this section, I quantitatively show that an optimized coupon-linked unfloored indexation payment can play a pivotal role in maintaining debt sustainability and can avert a costly default. The analysis entails feeding into the baseline and indexed model economies the same hypothetical path of income shocks, and it investigates how the endogenous variables of the model compare in the two specifications.

[16] The chart for the floored method is highly similar to the one with the unfloored method presented in Figure 6. I avoid adding it for clarity.

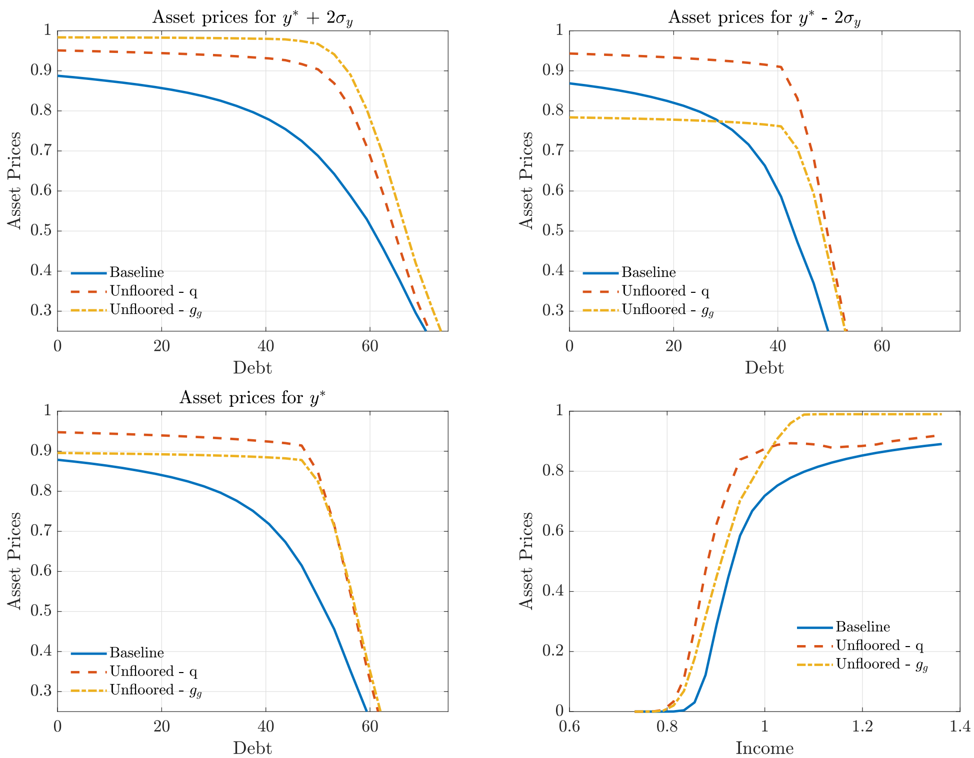

Figure 5: The upper left (right) panel depicts the bond prices of an economy with a coupon-linked unfloored indexation method and the baseline economy with two standard deviations above (below) mean income as a function of non-contingent debt and with no initial indexed debt. The bottom left panel depicts the bond prices for the mean income level. The bottom right panel plots the bond prices as a function of the ergodic debt levels reported in Table 3 at alternative income levels.

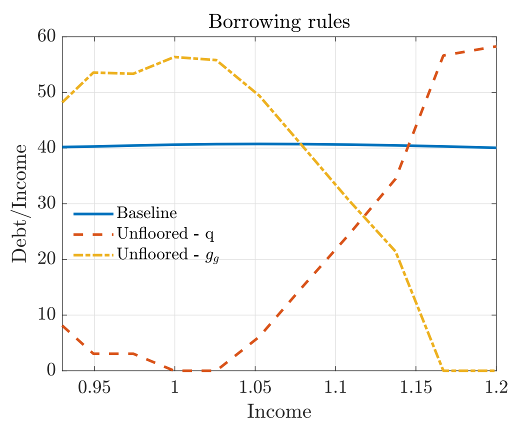

Figure 6: Borrowing decision rules of unfloored coupon-linked scheme under the optimal multiplier θ. The functionals are plotted with the ergodic level of non-contingent and indexed debt presented in Table 3.

Figures 7 and 8 plot the consumption, debt, and asset price dynamics 500 quarters after the arrangement of indexed bonds so that the long-run averages for all studied economies can be attained. I resort to a coupon-linked method because I obtain an almost perfect overlap in consumption dynamics with the principal-linked unfloored economy.

Figure 7 plots the time-series evolution of consumption as well as the income shocks for the economy with indexed debt and the baseline economy. The left panel of Figure 7 visualizes the consumption differential between the economy with coupon-linked unfloored debt and the baseline economy. On the right axis the underlying sequence of income shocks is presented, and the transparent gray-shaded area denotes the periods in which the baseline economy defaults but the economy with indexed debt does not.

The left panel of Figure 7 highlights that the consumption in an economy with indexed debt is generally higher than consumption in the baseline economy during adverse income shocks, whereas it is lower during positive income shocks. The economy with indexed debt avoids a costly default, whereas the baseline economy defaults. The right panel highlights how much smoother the consumption dynamics are in the economy with indexed debt.

Notice that starting from the 50^{th} quarter to the 65^{th} quarter, the consumption differential between two economies in the left panel is relatively flat at around zero despite positive shocks. This is also the period in which the consumption volatility of the government heightens, as shown in the right panel. The intuition for these anomalies is provided in Figure 8.

The left panel of Figure 8 visualizes the time-series paths for non-contingent debt as a percentage of annual mean income (normalized to one) for the baseline economy (dash-dotted yellow line), the non-contingent debt (solid blue line) of the indexed debt economy as well as the unfloored debt of the indexed economy (dashed red line). Notice that the government mainly relies on indexed debt but is very active in the debt markets as it reshuffles its portfolio depending on income realization. As the government draws positive income shocks starting from the 40^{th} period, the government starts issuing more non-contingent debt, which is consistent with the borrowing rules presented in Figure 6. In fact, the government goes into a buyback operation. It purchases back indexed debt using the proceeds from the issuance of non-contingent debt. Through this portfolio rebalancing, the government manages to avoid paying higher payments during a stream of positive income shocks. Yet, this operation results in a level of consumption volatility that is similar to the one observed in the baseline economy.

The right panel of Figure 8 displays the time-series paths for the asset prices. The plots are consistent with the charts presented in Figure 5. The asset prices of the indexed economy always remain above the asset prices of the baseline economy. It is also evident that the price volatility of non-contingent debt in the indexed economy is considerably lower than in the baseline economy, which is consistent with the $\sigma(r_s)$ moment presented in Table 3.

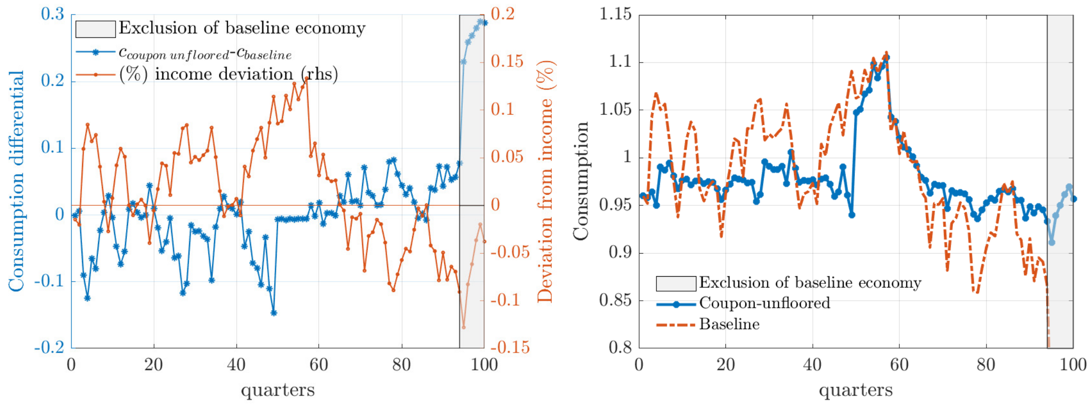

Figure 7: Evolution of consumption dynamics in the baseline economy and in an economy with a coupon-linked unfloored asset. The left panel presents the consumption differential between the indexed economy and the baseline economy. Endowment shocks are scaled on the right axis as percentage deviations from mean annual income. The first period in the figure corresponds to the 501^{st} quarter after the introduction of indexed debt.

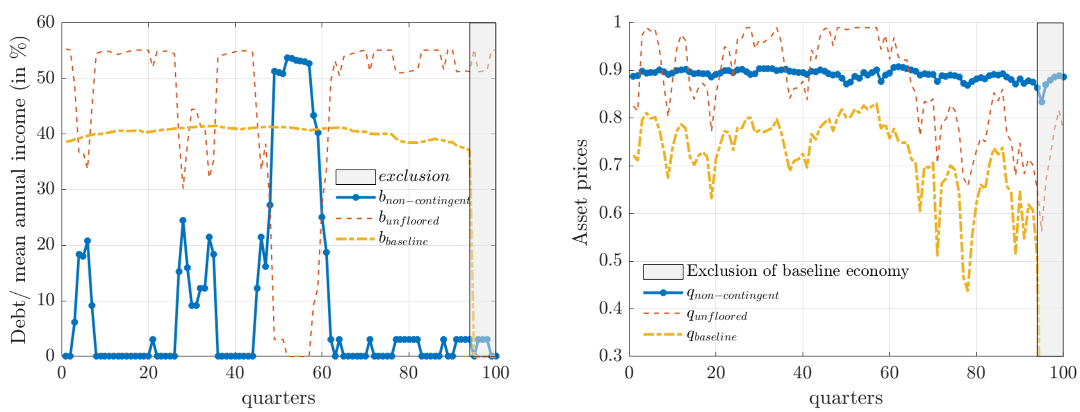

Figure 8: Evolution of debt and spread dynamics in the baseline economy and in an economy with a coupon-linked unfloored asset. The first period in the figure corresponds to the 501^{st} quarter after the introduction of indexed debt. Mean annual income is normalized to one.

4.8 Transition

In this section, I analyze an unanticipated announcement such that from now on, the government would have access to indexed debt. In this way, I can explore how long it takes for the economy with indexed debt to attain its long-run averages. In period 0, both the baseline economy and the economy with indexed debt are initiated with zero indexed debt and the ergodic level of non-contingent debt in the baseline economy. I assume two different initial income shocks: high- and low-income shocks, which correspond to two standard deviations above and below the mean income. The rest of the shocks follow the estimated process in equation (3). The simulated trajectories reflect the average of 30,000 sample paths without a default from periods 0 and onward.

The upper panel of Figure 9 illustrates the transition dynamics of debt and spread for the baseline economy (dashed lines) and the economy with indexed debt (solid lines) with an initial low-income shock. The charts show that the government goes through an immediate indexed debt issuance and uses the proceeds of this issuance to purchase back non-contingent debt. As discussed before, despite facing lower indexed debt prices than non-contingent debt during low-income states, the government relies on indexed debt during these low-income states as the debt burden declines with low-income shocks. As the severity of the shock fades away over time, the government converges to its long-run averages within 30 quarters. The right panel shows that immediately after the introduction of indexed debt, the spreads of non-contingent debt and indexed debt plunge, mainly because of the effects of promised payments as well as the decline in the likelihood of default that is discussed in Section 4.6. The charts are also consistent with the long-run moments presented in Table 3

The lower panels of Figure 9 visualize the transition dynamics of relative consumption (left panel) and its relative volatility (right panel). The left panel reveals that the consumption of a government with indexed debt initially jumps and stays above the baseline economy for about eight quarters. The intuition for an initial jump in consumption relies on the government's new debt issuance in indexed bonds and a significant drop in the cost of debt issuance. In the long run, however, the action of promised payments during upturns takes its toll, consistent with Figure 7, and the government ends up with slightly lower consumption. Yet, the bottom right panel of Figure 9 displays that consumption volatility in an economy with indexed debt halves permanently. Thus, the economy with indexed debt can do a better job at smoothing consumption, which is consistent with the $\sigma(c)/(\sigma(y)$ moment presented under the coupon-linked unfloored column of Table 3.

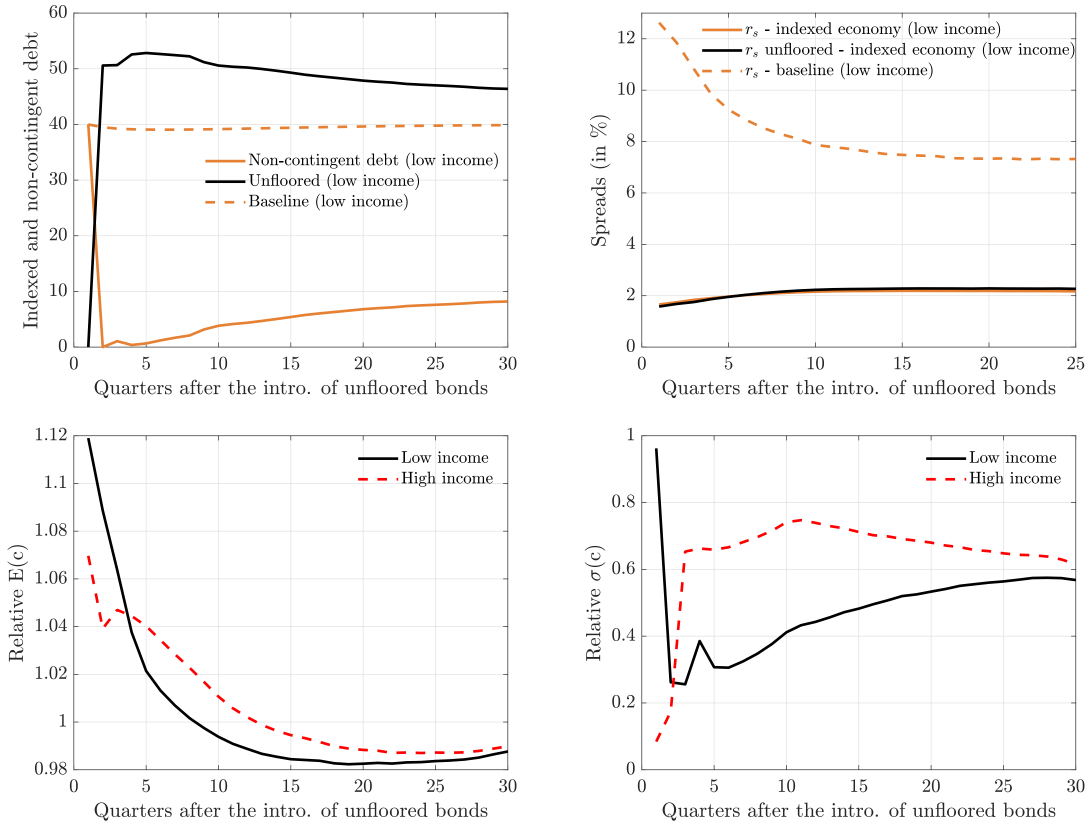

Figure 9: The charts depict transitions from the baseline economy to an indexation economy with coupon-linked unfloored payments. The indexation economy starts with no indexed debt and a stock of non-contingent debt that coincides with the ergodic level of debt in the baseline economy. High- and low-income shocks correspond to two standard deviations above and below mean income, and income shocks follow the estimated process in equation 3 for the rest of the periods.

4.9 Capital gains

This section explores the impact of optimized indexed debt arrangements for the holders of previously issued non-contingent debt b at the time of the unanticipated announcement.

I present the market value of debt to better understand the indexation dynamics. Figure 10 depicts the market value of non-contingent debt for mean income with no initial indexed debt for coupon-linked schemes as a function of non-contingent debt. The figure presents that the market value of the debt shifts up following the arrangement of indexed debt for all indexation schemes. This occurs because these assets reduce default risk, which raises the price of non-contingent debt. Thus, the market value of the debt surges, and holders of previously issued non-contingent debt enjoy capital gains. The figure also shows that the market value of the debt is the highest in an economy where investors fully participate in GDP developments (unfloored economy). Recall that investors agree to receiving lower coupons during economic downturns, higher coupons during economic upturns.

This graph shows that the government's gains from the introduction of indexed debt can be further improved through a "voluntary debt exchange" by transferring investors' capital gains to the government through debt exchange prior to the inception of indexed debt (see Hatchondo, Martinez and Önder (2017)).

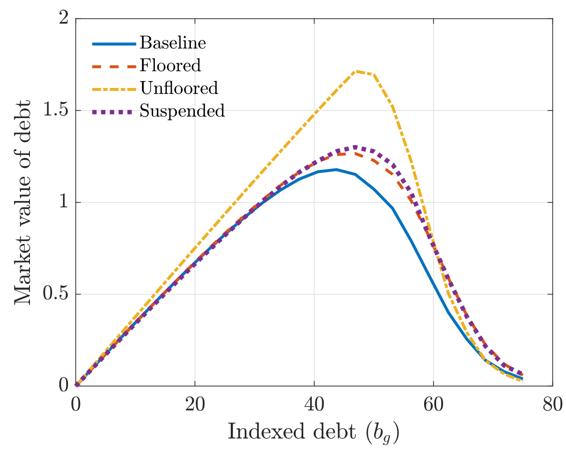

Figure 10: Market value of debt for mean income. The market value at time t is computed by $q(b', b'_g, y)b'$ where b is the total number of outstanding non-contingent debt coupon claims, y denotes the mean trend income and $q(b', b'_g, y)$ is the corresponding non-contingent debt price.

5 Conclusion

This paper analyzes the potential gains from the introduction of GDP-indexed bonds by undertaking different GDP indexation method proposals. I show that GDP-indexed bonds should be carefully designed to avoid the mistakes of previous experiences. Governments might be severely hit with a poor contract design and may end up with negligible gains or even losses. I present a contract that benefits both the government and investors of previously issued debt. I show that both coupon- and principal-indexed methods in economies with floored, unfloored- or suspended payments feature welfare gains when these assets promise both fixed and contingent payments. The gains are highest if the contract offers symmetric risks where payments decline (increase) during downturns (upturns). However, unoptimized contracts that promise only contingent payments, akin to Arrow-Debreu securities, may generate welfare losses. I show that investors of previously issued debt would also benefit from this contract design as the price of the asset increases following a decline in the likelihood of default, which raises the market value of the debt. These results remain the same with risk-averse lenders as well.

Quantitatively, this paper also solves a portfolio allocation problem of the sovereign where two assets have long-term maturity and are subject to default risks. This solution is particularly important in shedding light on recent discussions on the sovereign's optimal debt management. I show that buyback behavior plays a significant role to attain higher welfare figures. However, it should be noted that this study is abstracted from modeling a country's inability or competency to issue these bonds at a reasonable premium.

So far, none of the countries that have issued GDP-linked bonds have undertaken an indexation method where the parties of the contract take on symmetrical risk sharing during both economic downturns and upturns. This study could inform current policy discussions with a formal analysis and pave the way for an optimal contract design with which both the current investors and the sovereign would be better off. Otherwise, the issuance of previously unsuccessful GDP-indexed bonds can be repeated.

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A Appendix

A.1 Previous experiences

Table A1 summarizes a number of previous experiences that have indexed their payments either to GDP or to a commodity with a brief explanation. While a number of them are designed as part of a debt-restructuring scheme, Portugal and Singapore have experimented with these bonds outside of the debt-restructuring process. Even though the shares issued by Singapore were moderate and were accessible only to lower income groups to redistribute wealth, it was more sizable in Portugal's case (around 6.7 percent of total government debt (see Pina (2022)). Yet, both Singapore's and Portugal's certificates were not tradable. I elaborate below on the experiences of Argentina and Portugal. All cases have their uniqueness. Argentina's case highlights the importance of the contract design as it is deemed to be too complicated and is thought to be one of the reasons behind its failure. Portugal's experience with state-contingent assets, on the other hand, can be considered successful but Portugal's warrants were not tradable, unlike Argentina's, and further steps might be necessary in the design.

Linking the model to the country experiences, they all typically include a floor on payments and a growth threshold to trigger payments. The threshold in the model corresponds to the country's long-run trend growth. Some countries, such as Ukraine, Argentina, and later Portugal also introduced some form of a cap on payments. The earlier version of the model also introduced caps (without a floor), but was later dropped for brevity as it generates similar moments to the with unfloored indexation method.

Another growing example is catastrophe bonds which is similar to the "suspension on payments" indexation method analyzed in this paper. Suspension on these bonds can be triggered following a disaster which is moral hazard free than those of GDP-indexed bonds where investors may be wary of policymakers' ill intentions. Since these natural disasters are potentially costlier for the state, these bonds offer a natural state contingency as well. These bonds were issued by Mexico in 2006 and 2009. Grenada as well as Peru, Colombia and Chile issued similar catastrophic bonds with disaster/hurricane clauses which can carry out 6-12 months deferral on the payment.

Yet, no countries have issued GDP-indexed bonds with an unfloored indexation method which reflects the spirit of these assets the most and which is the highest welfare generating financial instrument.

Argentina's experience. One of the reasons behind the failure of Argentina's experience lies in the complexity of the contract design. For instance, its coupon payments, when it was first introduced in June 2005 as a resolution to the debt renegotiation process to its 2001 default, relied on a number of contingencies. These warrants were issued in 4 different denominations (Argentine pesos, US dollars, Euros and Yen) under 4 different jurisdictions (Argentine, British, Japanese and New York jurisdictions). To summarize, coupon payments can be triggered only iff the following three conditions are satisfied: (*i*) real GDP must be greater than the baseline real GDP, (*ii*) real GDP growth rate must exceed the real GDP growth rate of the baseline year, and (*iii*) cumulative payments cannot exceed 0.48 per unit of security in its corresponding currency.[17] The payment function can

[17] The Republic of Argentina (2004) provides the entire prospectus for the issued GDP-indexed bonds. Costa, Chamon and Ricci (2008) provides a brief overview of Argentina's experimentation with GDP-indexed bond issuance.

then be summarized as

$$\kappa_{j,t+1} = \frac{0.05\left(Y_t - y_t^b\left(1+\pi_t\right)\right)}{ExchangeRate_{j,t}} Unit_j \mathcal{I}_1(y > y_t^b)\mathcal{I}_2(\Delta y_t > \Delta y_t^b)\mathcal{I}_3\left(\sum_{\tau=1}^{t} C_{\tau,j} < 0.48\right)$$

where $\kappa_{j,t+1}$ is the coupon payment made in time $t+1$ that is indexed to the currency j, Y_t denotes the nominal GDP in period t measured in current prices, y_t denotes real GDP in 1993 prices in period t, y_t^b denotes the baseline real GDP in period t obtained from the Debt Exchange Prospectus, π_t is the GDP deflator using the 1993 series as the baseline, $ExchangeRate_{j,t}$ denotes the exchange rate per unit of the issued underlying currency j with respect to Argentine peso in period t, $Unit_j$ is the unit of currency coefficient explained by The Republic of Argentina (2004) as "the unit of currency coefficient represents the proportion that one GDP-linked security with a notional amount of one unit of currency bears to the aggregate Eligible Amount of all Eligible Securities outstanding as of the date of this prospectus supplement (approximately U.S.$81.8 billion), calculated using exchange rates in effect on December 31, 2003.". That means that Indicator functions $\mathcal{I}_1$, $\mathcal{I}_2$ and $\mathcal{I}_3$ turn on when the following three conditions are met. $\mathcal{I}_1$ turns on if real GDP in 1993 prices in period t is higher than the baseline real GDP in a baseline year. This baseline year is explicitly defined over the horizon of the warrant. It starts with a real growth rate of 4.3 percent and gradually converges to the real growth rate of 3 percent. $\mathcal{I}_2$ turns on if the growth rate of current real GDP exceeds the growth rate in the relevant reference year. Lastly, $\mathcal{I}_3$ is a payment cap which turns off if cumulative payments exceed 0.48, measured per unit of currency. That is, if all these conditions are satisfied, total payment is a fraction of the excess nominal GDP to be distributed among the units of the debt exchange agreement. The fraction would have been 5 percent if every party had participated in the debt exchange arrangement. The participation rate was 76 percent, thus, the fraction is set to be 3.8 percent $(0.76 \times 5\%)$.

This complicated design structure made the pricing of the asset particularly difficult. When the detached Argentinean warrant started trading six months after its issuance for the first time, the bonds were traded with a huge 50 percent discount. Even though spreads receded by 400 basis points, the bonds were not trading according to the fair value assessed by consensus expectations.

Portugal's experience. Portugal's experimentation with these assets is evaluated to be successful (see Pina (2022)). Portuguese GDP-indexed bonds were subscription based. That is, the Treasury sets the terms and conditions of the contract and investors then decide whether and how much to hold. For instance, the Treasury offered no additional payments if GDP statistics are revised. Portugal issued two GDP-indexed bonds, initially in 2013 with a maturity of 5 years and later on in 2017 with a maturity of 7 years. These certificates promised a floor and extra payments depending on the realization of the real GDP growth and were not tradable. Except in 2020 and 2021, these bonds made extra payments to its investors as real GDP growth realization exceeded expectations. To further improve the markets for GDP-indexed bonds, policymakers can make these bonds available in primary dealer markets while allowing for payments to decline during downturns so that these bonds can facilitate sustainable borrowing and make resilient budget balances to economic shocks.

A.2 Welfare Decomposition

This section provides the details of the welfare decomposition we conducted in Section 4.5 using the methodology developed by Aguiar, Amador and Fourakis (2020). Let $\{c_t^B, c_t^G\}$ denote consumption functions derived in the baseline economy and in the the economy with GDP-indexed bonds, respectively. We can then compute the welfare of the government in the baseline economy as

$$W^B(s_0) = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^B\right),$$

where $s_0 \equiv \{b_0, b_0^c, y_0\}$, with y_0 denoting the existing income, b_0 is the existing non-contingent debt and b_0^g is the existing GDP-indexed bonds which is zero in the baseline economy. Average welfare is then calculated by

$$\hat{W}^B = \int W^B(s_0)\, d\Gamma^B(s_0)$$

where Γ^B denotes the steady-state distribution of the income and non-contingent debt levels for the government.

Similarly, in the economy with GDP-indexed bonds

$$\hat{W}^G = \int W^G(s_0)\, d\Gamma^G(s_0)$$

where

$$W^G(s_0) = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^G\right)$$

and Γ^G is the steady-state distribution in the economy with GDP-indexed bonds.

We next define we define counter-factual consumption functions $c_t^{B,ND}$ and $c_t^{G,ND}$ to isolate the induced deadweight loss of income cost of defaulting on welfare

$$c_t^{B,ND}(s_t) = c_t^B(s_t) + d_t^B(s_t)\,\phi(y_t)$$
$$c_t^{G,ND}(s_t) = c_t^G(s_t) + d_t^G(s_t)\,\phi(y_t)$$

where d_t^B and d_t^G are the default decisions in the baseline economy and in the economy with GDP-indexed bonds, respectively. Recall that the term $\phi(y_t)$ refers to the output losses during the time of exclusion. These counterfactual consumption functions compute the corresponding consumption profiles for the sovereign if the sovereign follows the same debt and default rules but not incur output losses during periods of exclusion.

Lastly, to compute the welfare change that is attributed to the changes in consumption volatility, expected consumption that is free from output losses are calculated:

$$\bar{c}_t^{B,ND}(s_0) = E_t\left[c_t^{B,ND}(s_t)\,|s_0\right]$$
$$\bar{c}_t^{G,ND}(s_0) = E_t\left[c_t^{G,ND}(s_t)\,|s_0\right]$$

Given these consumption functions, we define

$$W^{i,ND} = \int E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^{i,ND}\right) d\Gamma^i(s_0)$$
$$\bar{W}^{i,ND} = \int \sum_{t=0}^{\infty} \beta^t u\left(\bar{c}_t^{i,ND}\right) d\Gamma^i(s_0)$$

where $i \in \{B, G\}$. Note that $\bar{W}^{B,ND}$ is the average expected welfare computed using the *expected* consumption functions that are free from income losses following a default while $W^{B,ND}$ denotes the average expected welfare computed using the consumption functions that are free from income losses following a default.

Then, the consumption-based welfare gain of being in the GDP-indexed bonds economy instead of remaining the baseline economy, λ, can be decomposed into

$$\underbrace{\left(\frac{W^G}{W^B}\right)^{\frac{1}{1-\gamma}}}_{1+\lambda} = \underbrace{\left(\frac{W^G/W^{G,ND}}{W^B/W^{B,ND}}\right)^{\frac{1}{1-\gamma}}}_{1+\lambda_D} \times \underbrace{\left(\frac{W^{G,ND}/\bar{W}^{G,ND}}{W^{B,ND}/\bar{W}^{B,ND}}\right)^{\frac{1}{1-\gamma}}}_{1+\lambda_V} \times \underbrace{\left(\frac{\bar{W}^{G,ND}}{\bar{W}^{B,ND}}\right)^{\frac{1}{1-\gamma}}}_{1+\lambda_T}.$$

The first factor λ_D captures the role of income costs of defaulting and computes the necessary consumption increase to compensate consumers for deadweight income loss in the baseline economy relative to deadweight income loss in the economy with GDP-indexed bonds. The second factor computes the relative gains obtained from removing consumption volatility in respective economies while the last factor λ_T captures the welfare change due to consumption tilting in consumption levels across two economies.

A.3 Indexed bond synthetic risk-free bond yields

A synthetic and risk-free indexed bond would satisfy the condition

$$q_g^*(b', b_g', y) = \frac{\mathbb{E}_{y'|y}\left[(1-\mathcal{I}_s\mathcal{S}(y'))\left[(1-\mathcal{I}_p)\kappa\Gamma(y') + \mathcal{I}_p Max(0, (\kappa\mathcal{I}_e + \Gamma(y'))) + (1-\delta)q_g^{*'}\right]\right]}{1+r}, + \frac{\mathcal{I}_s\mathcal{S}(y')e^r q_g^{*'}}{1+r}, \tag{12}$$

as it would repay in all states of the world. The pricing functional on the right-hand side of (12) satisfies $q_g^{*'} = q_g^*\left(b_g'', b'', y'\right)$ with $b'' = \hat{b}\left(b_g', b', y'\right)$, $b'' = \hat{b}\left(b_g', b', y'\right)$. Equation (12) can be solved by using equilibrium default, indexed bonds borrowing and non-contingent debt borrowing rules that are obtained to solve the optimization problem in the text. Given the synthetic risk-free pricing schedule, one can compute the corresponding yield that would make holding non-contingent promises indifferent. With that yield, spread and duration can be computed as in Footnote 14.

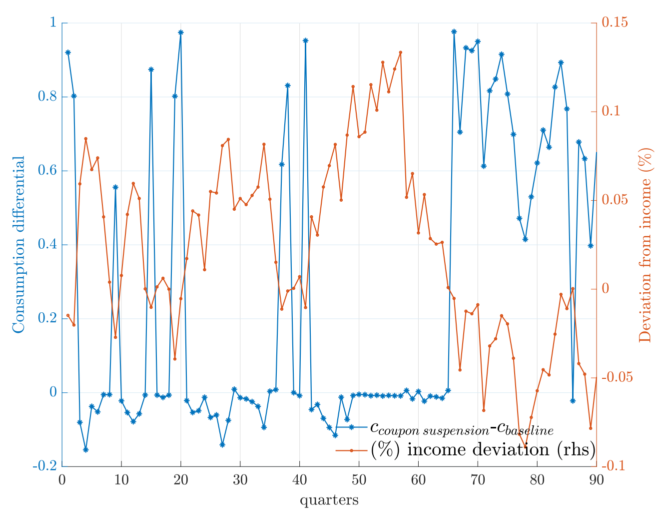

Figure A1: Evolution of consumption dynamics in the baseline economy and in an economy with a coupon-linked suspension asset. The left panel presents the consumption differential between the indexed economy and the baseline economy. Endowment shocks are scaled on the right axis as percentage deviations from mean annual income. The first period in the figure corresponds to the 501^{st} quarter after the introduction of indexed debt.

A.4 Numerical algorithm

The computational algorithm used in this paper requires iterating on the value and price functions until a convergence criteria of 10^{-5} is obtained. To evaluate the functions, equally spaced grid points; to approximate the functions outside the grid points linear interpolation is used. For non-contingent bonds b, GDP-indexed bonds b_g and income y 25 grid points, to evaluate the expectations 50 Gauss-Legendre quadrature points are utilized.[18] The numerical algorithm in the paper works as follows:

i. Initial guesses of V_R, V_D, q and q_g are set as the last period of finite-horizon economy as follows: $V_R(b, b_g, y) = u(y - \kappa b - \kappa b_g)$, $v_D(y) = u(y - \phi^d(y))$, and $q = 0$, $q_g = 0$.

ii. Optimization problem defined in equation (6) is solved for each grid point on assets and income and then I search for a globally optimum point for next period's borrowing decisions for non-contingent debt and indexed debt. This requires generating 50 grid points both for b' and b'_g to find the maximizing candidates. Initially, for a fixed b'_g, I find the corresponding optimal grid for b'. I feed that particular grid point into a one-dimensional minimization routine as an initial guess to pinpoint the optimal b' for a fixed b'_g with a double precision. Finally, I feed my fixed b'_g and the corresponding optimal b' into bi-dimensional optimization Powell routine to solve my optimal portfolio allocation doubles of (b', b'_g) for each grid point of (b, b_g, y) triples.

iii. Iterate the procedure defined above for equations (6) to (9) until the ergodic distribution between two iterations remains the same.

iv. Invoke local search methods within the neighborhood of the obtained candidate optima (b', b'_g) for each grid point of (b, b_g, y) in the previous step.

v. Iterate the local search methods for equations (6) to (9) until convergence criteria of 10^{-5} is obtained.

With the equilibrium value and price functions as well as decision rules for non-contingent debt borrowing, indexed debt borrowing and defaulting, I simulate the model. In particular:

- Set the number of samples $N = 250$, number of periods $T = 1501$ and $T_0 = 500$.

[18] See Hatchondo, Martinez and Sapriza (2010) on the superiority of interpolation methods over discrete state space techniques for the evaluation of value and price functions for one-period assets. Önder (2022*b*) shows the superiority of interpolation techniques with long-term debt and their superiority in the model's convergence properties.

- Use a random number generator to draw sequences of ε_t and ψ for $t = 1, 2, \ldots, T$ to compute the income of the subsequent periods and to evaluate the continuation value of default. It is recommended to keep the draws so that the same values can be used for each sample $n \in N$.

- Set the initial endowment y to be mean y and debt holdings b, b_g to be zero.

- Cut the first T_0 periods of each sample before computing the moments of the simulation so that randomly chosen initial values will not have any influence on moments.

The moments reported in all tables are computed from the 250 simulated sample paths such that each sample includes 80 periods (20 years) without a default observation. The sample period begins at least 20 periods after having access to the credit markets following a default episode. Default frequency is calculated using the entire simulation. Business cycle moments are reported after HP detrended with a smoothing parameter of 1600. I also make sure that both global search methods and local search methods generate very similar moments. Local search methods matter for the computation of welfare gains as one needs to make sure to achieve a finer convergence criterion for value functions. Yield and duration as well as spreads are computed by generating a debt covenant that it is never defaulted on, details of which are provided in Section A.3. To this end, the simulations incorporate the adjustment stemming from different GDP-indexation methods.

A.5 Role of debt management

This section investigates the role of active debt management in the model and particularly in welfare calculations. To this end, I undertake alternative experiments using only unfloored GDP-indexed bonds tied to coupon payments with the optimal multiplier $\theta = 9$ found in the main text. In simulations, although rarely, the government finances the buyback of non-contingent bonds by issuing GDP-indexed bonds or vice versa. This behavior may particularly benefit the government as it is less costly to issue GDP-indexed bonds during adverse income shocks since the government is not entitled to pay coupons with unfloored payments. Or, the government may also benefit from purchasing back GDP-indexed bonds by issuing non-contingent bonds to avoid paying extra coupon payment during upturns. To find out the impact of this portfolio reallocation I introduce a no-buyback constraint. In particular, I impose $b_g^{prime} - (1-\delta)b_g \geq 0$ should be satisfied when $b^{prime} - (1-\delta)b > 0$, and $b^{prime} - (1-\delta)b \geq 0$ should be satisfied when $b_g^{prime} - (1-\delta)b_g > 0$. Results of this analysis are provided in column 2 of Table A2. Column 1 corresponds to the optimal multiplier $\theta = 9$ found in the main text and is reprinted for convenience.

In the second experiment I fix the share of each type of debt at the average values obtained from optimal multiplier simulations and only let the government adjust the total amount of debt, keeping shares constant. That is, the share of GDP-indexed bonds is set to be 85 percent 100*(46.69/(46.69+7.92)) per period while allowing total debt to adjust. This would provide the upper limit on welfare gains resulting from per-period optimal portfolio allocation. Results of this analysis are printed in Column 3 of Table A2. Lastly, I provide the results of the welfare analysis comparing the outcome of these experiments in Figure A2. Following results stand out. It appears that even though no-buyback constraint does not influence long-run simulation moments significantly, it still has important effects on the welfare as is visible in Figure A2. The figure plots the welfare dynamics of three economies: solid line corresponds to the economy with unfloored GDP-indexed bonds with optimal multiplier $\theta = 9$, dashed lines correspond to the economy where we introduce no-buybacks to this unfloored economy, and lastly dash-dotted lines correspond to the economy where the share of each type of debt are fixed at the average values. The left chart plots welfare gains measured in consumption-equivalent terms from the introduction of indexed bonds. The initial debt portfolio at the time of inception entails no indexed bonds ($b_g = 0$) and a stock of non-contingent debt that is equal to the mean debt level observed in the baseline simulations. Welfare is computed with respect to the baseline economy, the economy in which GDP-indexed bonds cannot be issued, as in equation (11). The welfare graph reveals that the gains are more than halved in the economy with a no-buyback constraint which shows that buyback operations play a significant role in optimal debt management and to attain higher welfare gains. Next, when we completely shut the government's ability to reshuffle its portfolio shares in the optimal multiplier economy, we observe significant swings in welfare depending on the state of the economy even though average welfare remains in the positive territory and still significant. This also highlights the importance of a country's ability to undertake an active optimal debt management policy. The discrete fall in the welfare chart is the outcome of inevitable default when the income is below the default threshold. Turning to Table A2, the economy with fixed shares does relatively poorly compared to the economy with GDP-indexed bonds without fixing shares (the first column).

This analysis highlights the importance of active debt management to boost the gains from these assets. Yet, again, this analysis is abstracted from modeling the government's competency to be able to issue these bonds at a reasonable premium or other considerations such as the liquidity for these bonds. For instance, Moretti (2020) argues that the market for these bonds tends to be shallow and countries may have to pay hefty liquidity premiums.

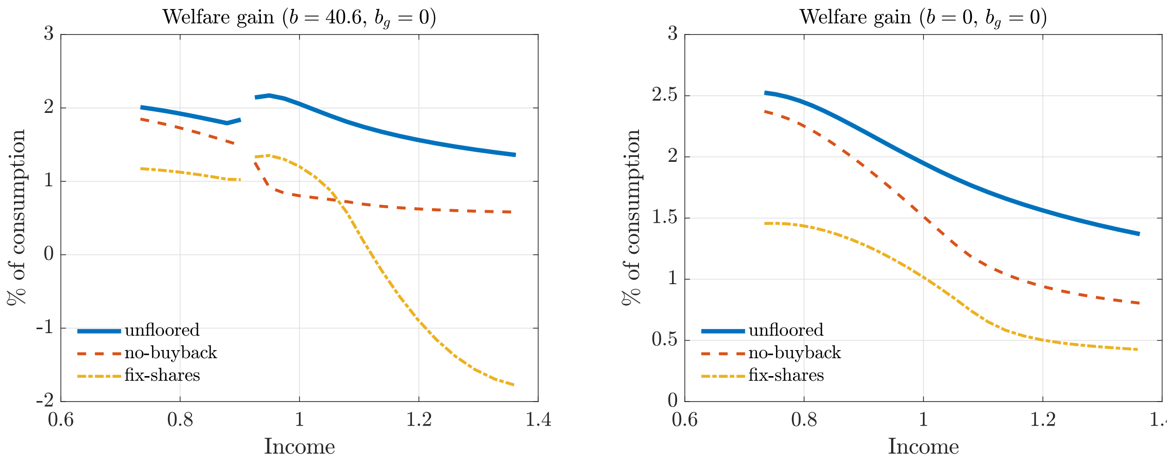

Figure A2: The left panel plots welfare gains measured in consumption-equivalent terms from the introduction of indexed bonds. The initial debt portfolio at the time of inception entails no indexed bonds ($b_g = 0$) and a stock of non-contingent debt that is equal to the mean debt level observed in the baseline simulations. The right panel shows the welfare gains when the initial debt of all kinds is set to zero. Solid line corresponds to the economy with unfloored GDP-indexed bonds where payments are tied to coupon with optimal multiplier. Dashed lines correspond to the economy where we introduce no-buybacks to this unfloored economy and dash-dotted lines correspond to the economy where the share of each type of debt is fixed at the average values.

A.6 Risk-averse lenders

Below I present the outcome of my analysis with risk-averse international investors. The price of sovereign bonds satisfies a no-arbitrage condition with the following stochastic discount factor (SDF) $\mathcal{G}(y', y) = exp(-r - \xi\epsilon' - 0.5\xi^2\sigma_\epsilon^2)$, where ξ denotes the risk aversion parameter for lenders.

This model of the discount factor is a special case of the discrete-time version of the Vasicek (1977) one-factor model of the term structure and has often been used in models of sovereign default (e.g., Arellano and Ramanarayanan, 2012; Bianchi, Hatchondo and Martinez, 2018; Önder, 2022*a*).

With this SDF, price functionals for non-contingent bonds q and GDP-indexed bonds q_g reads:

$$q_g(b', b'_g, y) = \frac{\mathbb{E}_{y'|y}\left[\mathcal{G}(\varepsilon')\,(1-d')\,(1-\mathcal{I}_s\mathcal{S}(y'))\left[(1-\mathcal{I}_p)\kappa\Gamma(y') + \mathcal{I}_p Max(0, (\kappa\mathcal{I}_e + \Gamma(y'))) + (1-\delta)q'_g\right]\right.}{1+r},$$

$$+ \left.\frac{\mathcal{I}_s\mathcal{S}(y')e^r q'_g}{1+r}\right], \quad (13)$$

$$q(b', b'_g, y) = \frac{\mathbb{E}_{y'|y}\left[\mathcal{G}(\varepsilon')\left(1-\hat{d}\left(b', b'_g, y'\right)\right)\left[\kappa + (1-\delta)q\left(b'', b''_g, y'\right)\right]\right]}{1+r}. \quad (14)$$

Table A3 reports our findings. For brevity, we focus our analysis on unfloored GDP-indexed bonds where the payments are tied to coupons. To make the baseline economy and the economy with GDP-indexed bonds comparable, I run both economies with risk-averse lenders for each randomly chosen risk-averse parameter ξ. The table shows that for small values of risk-aversion parameters, the table mainly confirms the existing results that are obtained with risk-neutral lenders in the text. In particular, columns 1 and 2 report the simulation results when risk aversion parameter $\xi = 1$ for the baseline economy and for an economy with GDP-indexed bonds under optimal unfloored coupon with $\theta = 9$. Following observations stand out. For a low risk-aversion parameter, results are similar to those obtained in the main text. Yet, when lenders are assumed to be more risk averse, as in columns 2-6, the gains from introducing GDP-indexed bonds diminish. In the case of $\xi = 10$, it appears that the government does not keep GDP-indexed bonds in its balances. The intuition is as follows. Risk-averse lenders do not like their receivables to fall particularly during downturns when they value them the most. So, they would require a higher premium for compensation. As lenders' degree of aversion rises, the required premium for lenders' compensation rises as well which makes these assets not-amenable for the government to issue. Thus, they end up not issuing them for very high risk-averse lenders.

Table A1: Previous indexed bond issuances

Country	Time issued	Indexation	Note
Argentina	2005	GDP	Part of debt restructuring, payments over a threshold GDP , see The Republic of Argentina (2004), Costa, Chamon and Ricci (2008)
Bosnia and Herz.	1990s	GDP	Part of Brady debt restructuring, payments over a threshold GDP
Bulgaria	1990s	GDP	Part of Brady debt restructuring, payments over a threshold GDP
Costa Rica	1990s	GDP	Part of Brady debt restructuring, payments over a threshold GDP
France	1973	Gold	Following a sharp depreciation of Franc against gold, the cost of burden has increased by 700 percent in 10 years
Greece	2012	GDP	Payment schedule depends on a number of conditions, see Hellenic Republic Ministry of Finance (2012) and Zettelmeyer, Trebesch and Gulati (2014)
Mexico	1970s	Oil	First to issue such bunds, Petrobonds
Mexico	1990s	Oil	Part of Brady debt restructuring
Nigeria	1990s	Oil	Part of Brady debt restructuring
Portugal	2013, 2017	GDP	Treasury Certificates Savings Growth, Payments over a threshold GDP, linked to coupons with floor, see Portuguese Treasury and Debt Management Agency (2017)
Ukraine	2015	GDP	Part of debt restructuring, annual payments if economic growth crosses certain thresholds, see Ministry of Finance of Ukraine (27 August, 2015), Reuters (2017)
Uruguay	1990s	Oil	Part of Brady debt restructuring
Singapore	2001	GDP	Investors are guaranteed to receive coupons each quarter and promised to receive extra returns if GDP exceeds its long-run average. Designed as part of a redistribution scheme, see Government of Singapore (2001)
Venezuela	1990s	Oil	Part of Brady debt restructuring

Table A2: Long-run statistics: debt management effects of introducing GDP-indexed bonds

	Unfloored	No-buyback	Fix shares
	Debt Statistics		
Non-contingent debt-to-GDP	7.92	2.93	7.74
GDP-indexed debt-to-GDP	46.69	51.32	43.91
Mean r_s	2.14	2.46	4.06
Mean r_s indexed	2.25	2.6	3.94
Default rate	0.14	0.19	0.28
Duration	4.80	4.73	4.43
	Business Cycle Statistics		
$\sigma(c)/\sigma(y)$	0.56	0.32	0.48
$\sigma(r_s)$	0.32	0.38	0.81
$\rho(c, y)$	0.63	0.87	0.91

First column reports the simulation results with unfloored GDP-indexed bonds with the optimal multiplier of $\theta = 9$. The second column introduces a no-buyback constraint to the economy presented in the first column. The third column presents the results when the share of each type of debt presented in the first column are fixed at its average values per period. The standard deviation of variable h is denoted by $\sigma(h)$, and the coefficient correlation between variables h and m is denoted by $\rho(h, m)$. Consumption and income are reported by natural logs. I compute the yield (and the spreads) and the debt duration by using expected payments to incorporate uncertainty about the amount of payment as well as the time of the payment. In particular, the yield and duration as well as the spreads are computed by generating a debt covenant that is never defaulted on. Details for these computations are provided in the Appendix.

Table A3: Long-run statistics and welfare gains: effects of introducing GDP-indexed bonds with risk-averse lenders

	(1) Baseline $\xi = 1$	(2) Unfloored $\xi = 1$	(3) Baseline $\xi = 5$	(4) Unfloored $\xi = 5$	(5) Baseline $\xi = 10$	(6) Unfloored $\xi = 10$
Debt Statistics						
Non-contingent debt-to-GDP	38.46	8.91	36.59	17.35	33.86	34.01
GDP-indexed debt-to-GDP	*n.a*	49.68	*n.a*	36.14	*n.a*	0.01
Mean r_s	7.05	1.28	7.25	5.39	7.46	7.51
Mean r_s indexed	*n.a*	1.64	*n.a*	9.09	*n.a*	18.02
Default rate	2.96	0.3	1.86	0.53	0.88	0.87
Duration	4	5	3.97	4.21	3.93	3.92
Business Cycle Statistics						
$\sigma(c)/\sigma(y)$	1.16	0.54	1.16	0.66	1.12	1.13
$\sigma(tb/y)$	0.91	3.78	0.92	2.07	0.81	0.84
$\sigma(r_s)$	2.81	0.51	2.47	1.11	2.06	2.1
$\rho(c, y)$	0.99	0.58	0.99	0.93	0.99	0.99
$\rho(r_s, tb/y)$	0.89	0.08	0.87	-0.42	0.83	0.83
$\rho(r_s, y)$	-0.86	0.07	-0.87	-0.71	-0.89	-0.89
$\rho(tb/y, y)$	-0.71	0.84	-0.69	0.85	-0.62	-0.63
$\rho(b', y)$	0.35	0.74	0.4	0.66	0.35	0.37
$\rho(b'_g, y)$	*n.a*	-0.73	*n.a*	-0.44	*n.a*	-0.04
Welfare gains						
Welfare gains (in %)	*n.a*	2.32	*n.a*	0.24	*n.a*	0.001

Columns (1), (3), and (5) report the simulation results of the baseline model with risk aversion parameter ξ of 1, 5 and 10, respectively. Columns (2), (4) and (6) present the simulation results with unfloored GDP-indexed bonds with optimal multiplier of $\theta = 9$ using risk aversion parameters of 1, 5 and 10, respectively. The standard deviation of variable h is denoted by $\sigma(h)$, and the coefficient correlation between variables h and m is denoted by $\rho(h, m)$. Consumption and income are reported by natural logs. I compute the yield (and the spreads) and the debt duration by using expected payments to incorporate uncertainty about the amount of payment as well as the time of the payment. Moments incorporate the spread and duration adjustment stemming from different GDP indexation methods. Details for these computations are provided in the Appendix.

Next, to put some structure into the formulation of the risk-aversion parameter and discipline the calibration of the risk-aversion parameter, I then use the framework of Bianchi, Hatchondo and Martinez (2018). They calibrate their model to the Mexican economy where they assume three high risk-premium episodes every twenty years and that each episode lasts on average for 1.25 years. During high-risk premium episodes, lenders become more risk averse and they document that the average spread rise during these episodes is 2 percent to which they calibrate their risk aversion premium to. Further, to capture the negative feedback between developments in local GDP and high-risk premium episodes, which is documented to be 4 percent lower during these episodes in Calvo, Izquierdo and Talvi (2006), a functional form governing the lenders' SDF is introduced as in Hatchondo et al., 2022. This formulation is also relevant because it is likely to capture investors' aversion to purchasing these assets particularly during the periods of low global liquidity and low local GDP as they are formulated to be highly correlated.

Following the identical parameter space of Bianchi, Hatchondo and Martinez (2018) and Hatchondo et al. (2022) and relegating the details of the calibration to these studies, I redo my analysis with GDP-indexed bonds. The new ingredients are the bondholders' risk-premium shock $p_t \in \{p_L, p_H\}$ which follows a Markov process. In particular, a high-risk-premium episode starts with probability $\pi_{LH}(y) \in [0,1]$ and ends with probability $\pi_{HL} \in [0,1]$ where π_{LH} is a decreasing function of y: $\pi_{LH}(y) = Min\left\{\pi_{LH0}e^{-\pi_{LH1}\log(y)-0.5\pi_{LH1}^2\sigma_\varepsilon^2}, 1\right\}$. One can rewrite a no-arbitrage condition with SDF as $M(y', y, p) = exp(-r - p\varepsilon' - 0.5p^2\sigma_\varepsilon^2)$.

Also, following these studies and their notation for ease of exposition, the recovery rate α, government expenditures g and a utility cost of defaulting $U^D(y)$ are introduced. Again, the calibration and the discussion of these parameters are relegated to Bianchi, Hatchondo and Martinez (2018) and Hatchondo et al. (2022).

We can rewrite the recursive formulation in the text as follows. Let $s \equiv (y, p)$ denote the vector of exogenous states.

$$V(b, b_g, s) = \max\left\{V_R(b, b_g, s), V_D(b, b_g, s)\right\}, \tag{15}$$

where the government's value of repaying is given by

$$V_R(b,b_g,s) = \max_{i\geq 0, i_c \geq 0, c\geq 0} \left\{ u\left(c\right) + \beta \mathbb{E}_{(s'|s)} V(b',b_g',s') \right\}, \tag{16}$$

subject to

$$\begin{aligned} c &= y - b_g \Big(\left(1-\mathcal{I}_p\right)\kappa\Gamma(y) + \left(1-\mathcal{I}_s\mathcal{S}(y)\right)\mathcal{I}_p \Big(I_e Max\left(0, \left(\kappa + \Gamma\left(y\right)\right)\right) \Big) \\ &+ \left(1-I_e\right)\left(\kappa + \Gamma\left(y\right)\right) \Big) + q_g(b',b_g',y)i_g' - \kappa b + q(b',b_g',y)i', \\ i_g' &= b_g' - (1-\mathcal{I}_s\mathcal{S}(y))(1-\delta)b_g - \mathcal{I}_s\mathcal{S}(y)b_g e^r, \\ i' &= b' - (1-\delta)b. \end{aligned}$$

The value of defaulting is given by:

$$V_D(b,b_c,s) = \quad u\left(y-g\right) - U^D(y) + \beta \mathbb{E}_{s'|s}\left[V(\alpha b, \alpha b_C, s')\right]. \tag{17}$$

The price of the non-contingent bond will be

$$q(b',b_g',s) \quad = \quad \mathbb{E}_{s'|s}\left[M(\varepsilon',p)\left[d'\alpha q\left(\alpha b',\alpha b_g',s'\right)\left(1-d'\right)\left[\kappa + (1-\delta)q\left(b'',b_g'',s'\right)\right]\right]\right]. \tag{18}$$

Similarly, the price of GDP-indexed bonds will be

$$\begin{aligned} q_g(b',b_g',s) &= \frac{1}{1+r}\mathbb{E}_{s'|s}\Bigg[\mathcal{G}(\varepsilon',p)\left[d'\alpha q_c\left(\alpha b',\alpha b_c',s'\right) + (1-d')\left[\left(1-\mathcal{I}_s\mathcal{S}(y')\right)\left[(1-\mathcal{I}_p)\kappa\Gamma(y')\right.\right.\right.\Bigg. \\ &+ \left.\left.\left.\mathcal{I}_p Max(0,(\kappa\mathcal{I}_e + \Gamma(y'))) + (1-\delta)q_g'\right] + \frac{\mathcal{I}_s\mathcal{S}(y')e^r q_g'}{1+r}\right]\right]\Bigg], \end{aligned} \tag{19}$$

where $d' = \hat{d}\left(b',b_g',s'\right)$ denotes the next-period equilibrium default decision, $b'' = \hat{b}\left(b',b_g',s'\right)$ denotes the next-period equilibrium non-contingent debt decision and $b_g'' = \hat{b}_g\left(b',b_g',s'\right)$ denotes the next-period equilibrium GDP-indexed bonds decision.

Long-run simulation results of this analysis are presented in Table A4. The first column reports the data moments and the second column presents the results of the baseline economy. These values are also printed in Bianchi, Hatchondo and Martinez (2018) and Hatchondo et al. (2022). Next, I introduce unfloored GDP-indexed bonds where payments are tied to coupons. Columns 3 and 4 report these moments for the economy with GDP-indexed bonds with the baseline and optimal multipliers of $\theta = 1$ and $\theta = 7$, respectively. Optimal multiplier is obtained using the equation (11) with a welfare analysis, the outcome of which is produced in Figure A3. The figure also yields similar dynamics to that of the middle left chart of Figure 3. Returning to the results in Table A4, 45 percent

(36.94/(36.94+44.3)) of the government debt portfolio is comprised of GDP-indexed bonds under the optimal multiplier $\theta = 7$ and the interest rate spread on the asset is significantly lower than the baseline economy. Yet, this significant decline in borrowing costs leads to a rise in the borrowing amounts and exacerbates the debt dilution problem which results in higher default rates. Importantly, the effect of risk aversion on the pricing of GDP-indexed bonds is visible when we compare the moments of "Δr_s with high-risk-premium shock" and "Δr_s cocos with high-risk-premium shock". It appears that lenders are becoming more risk averse particularly to the GDP-indexed bonds as these are episodes of low global liquidity and low domestic income. That is, lenders are not likely to receive coupons when they need them the most so they ask for a higher premium. The last column presents the simulation results when a no-buyback constraint is introduced to the economy in column 4. That is, the government cannot issue new non-contingent debt to buyback GDP-indexed bonds or cannot issue GDP-indexed bonds to buyback non-contingent debt. The moments in our economy with the no-buyback constraint resembles the one without the constraint. The welfare gains are slightly lower in the economy with the constraint highlighting the role of optimal debt management.

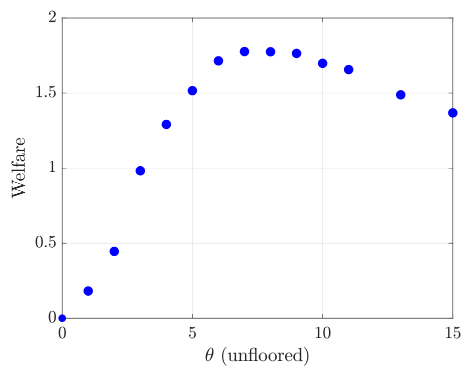

Figure A3: This figure shows the welfare gains from unfloored coupon indexation method for different values of θ in equation (4). To compute the welfare gains the initial stock of indexed bonds is set to zero, that is $b_g = 0$. Non-contingent debt holdings are set to the ergodic level of the non-contingent debt stock in the baseline economy (column 1) of Table A4; that is, $b = 42$ percent of mean income which is normalized to one.

Table A4: Long-run statistics and welfare gains: effects of introducing GDP-indexed bonds with risk-averse lenders

	(1) Data	(2) Baseline	(3) Unfloored $\theta = 1$	(4) Unfloored $\theta^* = 7$	(5) No-buyback $\theta^* = 7$
Debt Statistics					
Non-contingent debt-to-GDP	43	42.6	5.05	44.3	39.46
GDP-indexed debt-to-GDP	*n.a*	*n.a*	42.45	36.94	37.27
Mean r_s	2.4	2.32	2.42	1.23	1.08
Mean r_s indexed	*n.a*	1.86	1.67	0.71	0.78
Default rate	*n.a*	6.19	6.36	8.36	8.65
Duration	3	3.04	3.03	3.1	3.11
Duration indexed bonds	*n.a*	3.65	3.67	3.71	4.83
Probability high-risk-premium starts (%)	15.0	15.04	15.04	15.04	15.04
Lower income during high-risk-premium (%)	4.0	4.07	4.04	4.23	3.98
$\Delta\ r_s$ with high-risk-premium shock	2.0	2.09	2.05	0.13	0.09
$\Delta\ r_s$ cocos with high-risk-premium shock	*n.a.*	*n.a.*	1.88	1.66	1.78
Business Cycle Statistics					
$\sigma(c)/\sigma(y)$	1.0	1.04	0.98	0.94	1.50
$\sigma(r_s)$	0.9	1.44	1.44	0.48	0.46
$\rho(c,y)$	0.80	0.95	0.96	0.24	0.17
Welfare gains					
Welfare gains (in percent)	*n.a*	*n.a*	0.18	1.78	1.20

First column reports the simulation results of the baseline model and the second and the third columns present the simulation results with unfloored GDP-indexed bonds with the baseline and optimal multipliers of $\theta = 1$ and $\theta = 7$, respectively. The last column reports the results of the economy with a no-buyback constraint. The standard deviation of variable h is denoted by $\sigma(h)$, and the coefficient correlation between variables h and m is denoted by $\rho(h,m)$. Consumption and income are reported by natural logs. I compute the yield (and the spreads) and the debt duration by using expected payments to incorporate uncertainty about the amount of payment as well as the time of the payment. Moments incorporate the spread and duration adjustment stemming from different GDP indexation methods. Details for these computations are provided in the Appendix.

A.7 Arrow-Debreu (AD) like securities

In this section, I explore the effects of model dynamics when indexed debt is arranged to pay only contingent payments but no fixed payments, that is, $\mathcal{I}_e$ is equal to zero in equation (4). Note that this is not a typical AD security as the stream of payments is a perpetuity. In a typical AD setting, the asset's payoff is realized only iff a particular state of nature occurs some time in the future. In my setup, the asset is a perpetuity and payments are state-contingent which can be made at every state except default. In one-period debt context, this could have been considered as the government choosing a portfolio of K defaultable AD securities. For all $j \in \{1,2, \dots, K\}$, the AD security j promises to deliver one unit of goods only iff the income realization $y = y_j$. Then the price will be equal to the expected receivables of the investor.

Prior to further investigating the effects of such securities, I first investigate the effects of these bonds as in Alvarez and Jermann (2000) where they "focus on constraints that are tight enough to prevent default but allow as much risk sharing as possible" but with long-term debt as their analysis relies on to one-period debt contracts. Following a similar strategy I have restricted the government's next-period's choice set such that the government would never default while allowing as much risk sharing as possible with a single asset. Also, to make my analysis comparable, I have also calculated a baseline scenario where I also restrict the government's next-period's choice set such that the government would never default with non-contingent debt securities. Thus, the difference can be attributed to the risk-sharing structure of the underlying asset. The intuition is that this structure shuts down the main inefficiency in sovereign default models, which is default, and also shuts down the effects of debt management policy induced by two assets in the government's portfolio. This model could then provide an upper bound for welfare attained by risk-sharing property of GDP-indexed bonds with Arrow-Debreu (AD) like securities.

In the left chart of Figure A4, I first find the optimal multiplier θ by plotting welfare gains from switching to the economy with GDP-indexed bonds evaluated at zero initial debt. The highest welfare gain can be attained when $\theta = 0.5$.

Long-term simulations of this analysis are presented in Table A5 with the optimal $\theta = 0.5$. Because of the construction, default rates as well as spreads in both economies are now equal to zero. The government, however, can raise more debt in the economy with GDP-indexed bonds than the baseline economy. In fact, the total debt level in the economy with GDP-indexed bonds is more than doubled. The reason for that is the optimal multiplier is set to be 0.5. Thus, the government pays half of a unit coupon.

As each economy is restricted to have a single asset in their portfolio, plotting transitional welfare gains is not possible. Instead, I plot welfare gains evaluated at zero initial debt. The right chart of Figure A4 shows that welfare gains are always in the positive territory across different income levels and agents prefer to live in an economy with AD-like securities when their initial debt holdings are zero.

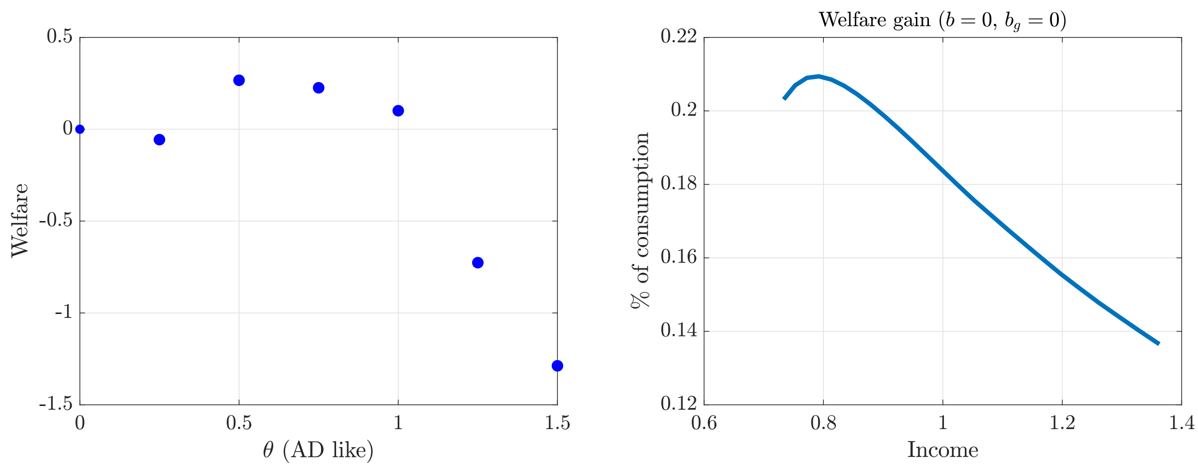

Figure A4: The left panel plots average welfare gains measured in consumption-equivalent terms from the introduction of indexed bonds for different values of θ. The initial debt portfolio at the time of inception entails no assets for each corresponding economy. The right panel shows the welfare gains for optimal θ, which is found to be 0.5, across different income levels.

Next, I drop the constraints that are tight enough to prevent default but allow as much risk sharing as possible. That is, I move back to Eaton and Gersovitz (1981) world and rerun my model with $\mathcal{I}_e = 0$ and allow the government to hold both assets simultaneously in its portfolio. This would also allow for computing welfare when these assets are introduced for the first time. For that, I present welfare gains in Figure A5 computed as in equation (11) under the welfare gain section. The figure shows that these assets do not generate as much welfare gains as in the main text and may even generate welfare losses if the multiplier θ is not optimized. This shows the importance of contract design to facilitate gains and to avoid previous unsuccessful experiences.

Table A5: Long-run statistics: Arrow-Debreu (AD) like securities

	Baseline	Arrow-Debreu $\theta^* = 0.5$
Debt Statistics		
Non-contingent debt-to-GDP	20.82	*n.a*
GDP-indexed debt-to-GDP	*n.a*	44.95
Mean r_s	0.0	*n.a*
Mean r_s indexed	*n.a*	0.0
Default rate	0.0	0.0
Duration	5.3	5.32
Business Cycle Statistics		
$\sigma(c)/\sigma(y)$	1.43	1.43
$\sigma(r_s)$	0.0	*n.a*
$\sigma(r_s)$ indexed	*n.a*	0
$\rho(c, y)$	0.91	0.89

First column reports the simulation results with the baseline economy where the government's next-period's choice set is restricted such that the government would never default with non-contingent debt securities and the second columns presents the results with AD like securities only.

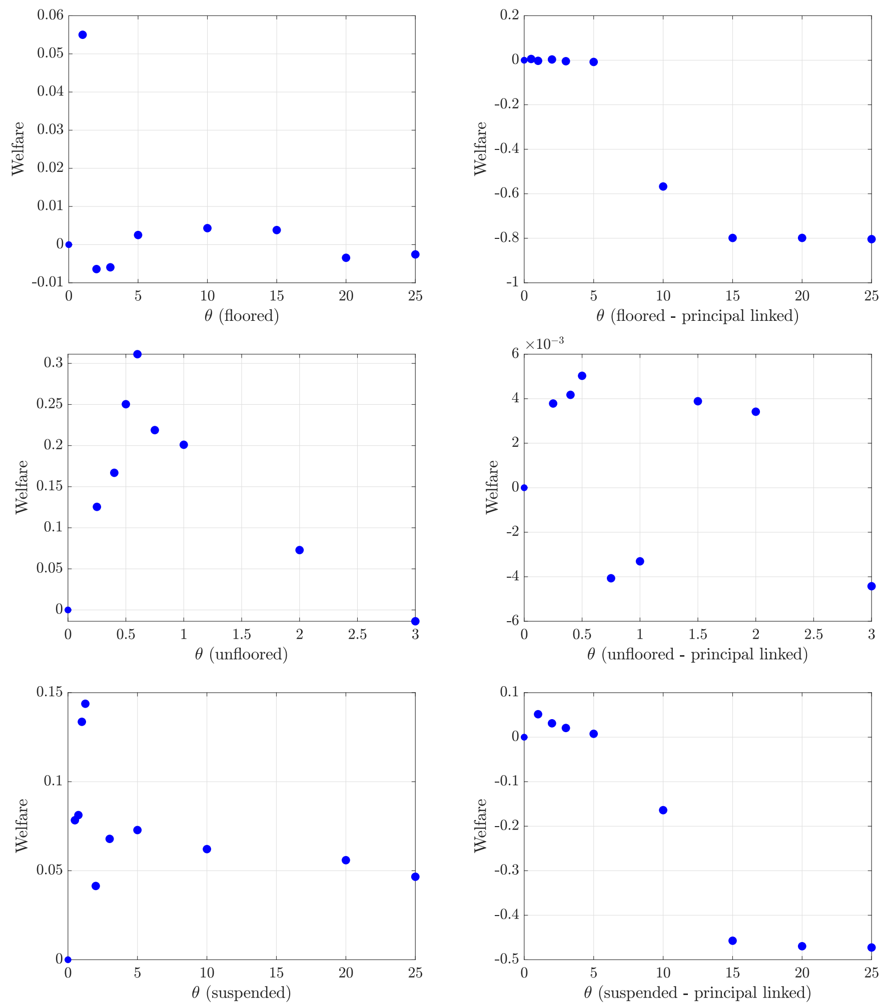

Figure A5: This figure shows the welfare gains from Arrow-Debreu-like securities for different values of θ in equation (4). To compute the welfare gains the initial stock of indexed bonds is set to zero, that is $b_g = 0$. Non-contingent debt holdings are set to the ergodic level of the non-contingent debt stock in the baseline economy; that is, $b = 40$ percent of mean income, which is normalized to one. The charts on the left display coupon-linked schemes, while the charts on the right show principal linked-schemes.