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CAPITAL-AUGMENTING TECHNICAL CHANGE IN THE CONTEXT OF UNTAPPED AUTOMATION OPPORTUNITIES

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Capital-augmenting technical change in the context of untapped automation opportunities

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Abstract

In this study, I explore the effects of capital-augmenting technical change (CATC) in a simple task-based context where untapped automation opportunities exist. I contribute to the literature by showing analytically that regardless of the value of the elasticity of substitution between capital and labor, CATC lowers the labor share of income in this setting. In contrast to standard production functions, CATC is thus capital-biased even in the face of strong complementarity between capital and labor. The intuitive explanation for this result is that a rise in the effectiveness of capital has two first-round effects on the labor share, namely (1) the standard effect whose sign is fully determined by the elasticity of substitution, and (2) a contraction in the set of tasks in which labor is more cost-effective than capital. Furthermore, I show that CATC increases the wage rate unambiguously. I argue that CATC in the face of untapped automation opportunities can be regarded as a convenient modelling approach to automation and I show that the implications of this approach match recent empirical findings regarding the labor market impact of automation.

Keywords: automation; capital-augmenting technical change; task-based production function; factor shares
JEL classification codes: E25, O33, O40

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1 Introduction

Many authors have used capital-augmenting technical change (CATC) as a core modelling instrument to automation in contexts where the share parameters of the production factors are fixed (e.g., Basso & Jimeno, 2021; Nordhaus, 2015; Sachs & Kotlikoff, 2012; Stähler, 2021). However, it has been noted that the implications of this approach are not consistent with the empirical findings regarding the impact of automation on the labor share (Acemoglu & Restrepo, 2018a). Empirical studies on the labor market impact of robotics find that the induced rise in labor productivity outstrips the induced rise in wages such that the labor share of income falls as a result of robot adoption (Acemoglu et al., 2020; Acemoglu & Restrepo, 2020; Dauth et al., 2017; Graetz & Michaels, 2018). Besides the evidence based on robotics data, there are convincing indications that automation technologies as a whole have strongly contributed to the lowering of the labor share (Autor & Salomons, 2018; Bergholt et al., 2022; Guimarães & Gil, 2022). In standard settings, however, CATC will only decrease the labor share of income if the elasticity of substitution between capital and labor is larger than one. This assumption is untenable given that the bulk of empirical estimates point to an elasticity of substitution lower than one (e.g., Chirinko, 2008; Gechert et al., 2021; Knoblach et al., 2020; Oberfield & Raval, 2021). In a simple set-up with fixed share parameters for capital and labor, CATC is thus not an adequate modelling approach to automation since it cannot simultaneously acknowledge that automation lowers the labor share and that capital and labor are gross complements in the aggregate production function.

This inconsistency in the CATC approach to automation can be remedied somewhat by distinguishing between traditional capital and automation capital and only regarding the latter as a gross substitute for labor. This method has been used by Basso and Jimeno (2021), Cords and Prettner (2022) and Jacobs and Heylen (2021) for instance. In these frameworks however, the effect of the automation process on the distribution of income across production factors is still governed by the elasticity of substitution between labor and (one type of) capital. The task-based approach put forward by Acemoglu and Restrepo (2018a; 2018b) represents a more fundamental alternative to the CATC approach, since automation in these models takes the form of a contraction in the set of tasks which are executed by labor. From the aggregate production function point of view, this represents a fall in the share parameter of the labor input. This modelling approach guarantees that automation lowers the labor share of income, in accordance with the empirical literature.

Crucial for this study is that two distinct situations can arise in this task-based approach to automation. In the first situation, firms are technologically constrained in their choice whether to produce a task using capital or labor. This is the "$I < \tilde{I}$" case in the notation of Acemoglu and Restrepo (2018b). In this scenario, all "automation opportunities" have been fully used by firms such that capital is used instead of labor in any instance where it is technologically feasible. In the second situation, firms are not constrained by technology in their choice whether to produce a task using capital or labor. This is the "$\tilde{I} < I$" case in the notation of Acemoglu and Restrepo (2018b). In this context, there are tasks where capital can perfectly substitute for labor, but firms prefer to make use of labor since it is more cost-effective. This second situation of "untapped automation opportunities" is my focus here.

In this work, I study the effects of CATC in a simple task-based model in the context where untapped automation opportunities exist. Under these conditions, the share parameters of capital and labor in the aggregate production function are no longer fixed, but they are a function of the relative factor cost. In this context, I show analytically that — under quite general conditions — CATC lowers the labor share of income, regardless of the value of the elasticity of substitution between capital and labor. Intuitively, this is because CATC will have two distinct labor share effects in this framework. First, the standard effect whose sign is entirely determined by the elasticity of substitution between capital and labor. Second, CATC will also lead firms to re-evaluate which tasks are more cost-effectively produced with capital than with labor and this leads to a contraction in the set of non-automated tasks. This second, so-called displacement effect always lowers the labor share and I show that this effect dominates regardless of the value of the elasticity of substitution. From this perspective, I argue that CATC in the presence of untapped automation opportunities is a convenient modelling approach to automation with credible empirical implications for the labor share. Furthermore, I show that the effect of CATC on the wage rate is also twofold in this setting. First, it raises the productivity of capital at the production of already automated tasks and this increases the demand for the production of non-automated tasks and thus for labor. Second, it leads to a contraction in the set of non-automated tasks and this displacement effect lowers labor demand. I show that the productivity effect dominates irrespective of the parameterization such that CATC always increases the wage rate. This is fully in accordance with the recent empirical findings of Gregory et al. (2022) who find that the negative displacement effect of routine-replacing technical change is more than fully compensated by countervailing mechanisms stemming from productivity gains. To the best of my knowledge, this is the first time that the effects of CATC on the labor share and the wage rate are studied in this setting.[1] The paper to which mine is most closely related is that of Martinez (2021).[2]

I argue that the context of untapped automation opportunities is a highly relevant one. First, note that it is somewhat uneconomic to assume that production firms face a hard constraint and cannot respond to the rising effectiveness of existing automation technologies by further automating the production process. Of course, imposing that firms are technologically constrained in terms of the set of automated tasks only implies that there can be no endogenous response at the level of the production firm which makes use of the automation technology. There can still be a reaction to changing factor costs or productivities at the level of the development of new automation technologies. Acemoglu and Restrepo (2018b) proceed in this way and thus make the technological constraint to automation itself endogenous. However, there are several reasons to suppose that it is not only the development of new automation technologies that can respond to changing factor costs or productivities, but also the degree to which firms make use of existing automation technologies.

First, there is the strong positive correlation between the labor cost and robot density across countries, both worldwide and for Europe specifically (Cséfalvay, 2020). Differences in the automation technology frontier are not a good explanation for these cross-country differences, since robotics technology is commercially available in Europe. This cross-country variation is more readily explained by differences in the adoption of existing automation technologies prompted by the differences in labor costs and thus the cost-effectiveness of these technologies. At least with respect to robotics, this suggests that the situation in which production firms have untapped automation opportunities — which they can pursue as the cost effectiveness of automation technology relative to labor improves — is quite commonplace. Second, there is some casuistic evidence that insufficient cost-effectiveness in particular is an important reason for firms not to automate. Based on 26 interviews with managers (and government and union representatives) in the South African apparel industry, Parschau and Hauge (2020) find that an insufficiently strong business case for the investment is a crucial reason for why automation technology is not used in more instances. Finally, the presence of "soft barriers" to the adoption of automation technology might ensure that untapped automation opportunities exist. Atkinson (2019) indicates that many western nations lag behind in robot adoption and puts forward culture, government policy and labor market institutions as possible explanations. Unlike technical constraints, these barriers can most likely be overcome once the cost improvement of automating a task becomes sufficiently salient.

The remainder of this paper is structured as follows. In the second section, I set out the model. In the third section, I analytically derive the effects of CATC on the labor share. In the fourth section, I derive the effects of CATC on the wage rate. The fifth section concludes.

[1] Acemoglu and Restrepo (2021) investigate the effects of CATC in a task-based model of automation, but they restrict their focus to the technologically constrained case where $I < \tilde{I}$. Since an increase in the productivity of capital cannot give rise to a contraction in the set of non-automated tasks in this context, only the standard effect of CATC on the labor share remains. As a result, they conclude that the labor share effect of CATC is fully governed by the elasticity of substitution between capital and labor.

[2] He finds that the parameters of the aggregate production function are endogenously determined by the distribution of automation technologies across firms. In this micro-founded setting, changes in the moments of the distribution can generate a rise in the share parameter of capital in the aggregate production function and as such lower the labor share. These results are distinct from mine, however, since his study considers CATC as a firm-level process and he finds that the effect of CATC on the labor share depends entirely on whether it takes place in the least automated firms or in the most automated firms.

2 Model

I assume that an infinite amount of representative firms are active on product markets. The production function of a firm is set out in equation 1 and it exhibits constant returns to scale. It denotes that aggregate output is the result of the joint execution of a continuous set of tasks. The continuous set is normalized to the 0-1 interval. The parameter σ denotes the elasticity of substitution between tasks.

$$Y = \left(\int_0^1 t_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

Firms would like to use capital rather than labor for the execution of any task for which the relative cost of labor versus capital ($\frac{w}{r}$) is larger than the relative productivity of labor versus capital in that task ($\frac{\gamma(i)}{\delta(i)}$). In this study, I assume that firms can choose at the margin whether a task is executed by capital or by labor.[3] This corresponds to the case where firms are not constrained by automation technology ($\tilde{I} < I$) in Acemoglu and Restrepo (2018b). As a result of this assumption, equation 2 holds.

$$\begin{aligned} &\forall\ 0 \leq i \leq 1\ \wedge\ \frac{\gamma(i)}{\delta(i)} > \frac{w}{r}: \quad t_i = \gamma(i)\, l_i \\ &\forall\ 0 \leq i \leq 1\ \wedge\ \frac{\gamma(i)}{\delta(i)} \leq \frac{w}{r}: \quad t_i = \delta(i)\, k_i \end{aligned} \tag{2}$$

As in Acemoglu and Restrepo (2018b), I impose more structure on the comparative advantage schedule by imposing that the productivity of labor $\gamma(i)$ rises exponentially in i (equation 3). I also follow them in their assumption that the productivity of capital is the same in all tasks (equation 4).

$$\gamma(i) = e^{Bi} \tag{3}$$

$$\delta(i) = A \tag{4}$$

I can define $\tilde{I}$ as the marginally automated task in equation 5, meaning that firms are indifferent between using capital and labor for this task.[4] Note that $\tilde{I}$ also denotes the share of tasks which are automated.

$$\frac{\gamma(\tilde{I})}{\delta(\tilde{I})} = \frac{w}{r} \iff \frac{e^{B\tilde{I}}}{A} = \frac{w}{r} \quad \Rightarrow \tilde{I} = \frac{ln(\frac{w}{r}A)}{B} \tag{5}$$

The general task-based production function from equation 1 can now be updated in the light of the automation structure (equation 6).

$$Y = \left(\int_0^1 t_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \text{with } t_i = e^{Bi} l_i \quad \forall\ \tilde{I} < i \leq 1, \quad t_i = Ak_i \quad \forall\ 0 \leq i \leq \tilde{I} \tag{6}$$

$$\iff Y = \left[\int_0^{\tilde{I}} (Ak_i)^{\frac{\sigma-1}{\sigma}} di + \int_{\tilde{I}}^1 (e^{Bi} l_i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \tag{7}$$

[3] It is important for the credibility of this framework to clarify that this assumption does not require that capital can substitute perfectly for labor in all tasks. The assumption made here is simply that $\tilde{I} < I$ (following the notation of Acemoglu and Restrepo (2018b)).

[4] Note that for the production function to be properly defined, the share of automated tasks $\tilde{I}$ should not be smaller than 0 or greater than 1. Based on equation 5, it can easily be shown that $0 < \tilde{I} < 1$ implies the following restriction on the relative factor costs $\frac{1}{A} \leq \frac{w}{r} \leq \frac{e^B}{A}$.

Since the productivity nor the cost of capital varies over tasks, firms allocate the same amount of capital k to the execution of each automated task. As a result, the total capital stock can be written as $K = \int_0^{\tilde{I}} k_i \, di = \int_0^{\tilde{I}} k \, di = \tilde{I}k$. When we use this information in equation 7, we can simplify it to equation 8.

$$(7) \Rightarrow \; Y = \left[\tilde{I}(Ak_i)^{\frac{\sigma-1}{\sigma}} + \int_{\tilde{I}}^{1} (e^{Bi} l_i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

$$\Longleftrightarrow Y = \left[\tilde{I}^{\frac{1}{\sigma}} (AK)^{\frac{\sigma-1}{\sigma}} + \int_{\tilde{I}}^{1} (e^{Bi} l_i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \tag{8}$$

I assume that workers have no preference regarding which task they execute such that the same cost of labor w applies to each non-automated task. In a perfectly competitive labor market, firms will demand labor l_i for the execution of a non-automated task j (with $\tilde{I} < j \leq 1$) up to the point where the marginal product of the execution of that task $\frac{\partial Y}{\partial l_j}$ is the same constant w for each non-automated task.

$$(8) \Rightarrow \; \forall \; \tilde{I} < j \leq 1 : w = \frac{\partial Y}{\partial l_j} \; \Longleftrightarrow w = \frac{\sigma}{\sigma - 1} \, Y^{\frac{1}{\sigma}} \, \frac{\sigma - 1}{\sigma} \, e^{Bj \frac{\sigma-1}{\sigma}} (l_j)^{\frac{-1}{\sigma}}$$

$$\Longleftrightarrow \forall \; \tilde{I} < j \leq 1 : \; l_j = w^{-\sigma} \; Y \; e^{Bj(\sigma-1)} \tag{9}$$

Note that $\frac{\partial Y}{\partial l_j}$ is a negative function of l_j such that non-automated tasks face diminishing returns. The aggregate labor market equilibrium is defined by equation 10: it expresses that the total labor demanded for the execution of non-automated tasks equals the total exogenous labor supply L. We replace the task-specific labor demand l_i by the expression set out in equation 9 such that equation 10 holds.

$$L = \int_{\tilde{I}}^{1} l_i \; di \; \Longleftrightarrow L = w^{-\sigma} \; Y \int_{\tilde{I}}^{1} e^{Bi(\sigma-1)} \; \; di \tag{10}$$

From the aggregate labor market equilibrium in equation 10, we find an expression for the wage rate w in equation 11. Based on w, firms choose how much labor l_i they demand for a non-automated task through equation 12.

$$(9) \Rightarrow w = \left(\frac{Y}{L} \right)^{\frac{1}{\sigma}} \left(\frac{e^{B(\sigma-1)} - e^{B\tilde{I}(\sigma-1)}}{B\,(\sigma - 1)} \right)^{\frac{1}{\sigma}} \tag{11}$$

$$(9) \text{ and } (11) \; \Rightarrow \; \forall \; \tilde{I} < i \leq 1 : l_i = \frac{e^{Bi(\sigma-1)} B\,(\sigma-1)\,L}{e^{B(\sigma-1)} - e^{B\tilde{I}(\sigma-1)}} \tag{12}$$

We insert expression 12 for l_i in the production function of equation 8 and simplify.

$$(8) \text{ and } (12) \; \Rightarrow Y = \left[\tilde{I}^{\frac{1}{\sigma}} (AK)^{\frac{\sigma-1}{\sigma}} + \int_{\tilde{I}}^{1} \left(\frac{e^{Bi} e^{Bi(\sigma-1)} B\,(\sigma-1)\,L}{e^{B(\sigma-1)} - e^{B\tilde{I}(\sigma-1)}} \right)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

$$\Longleftrightarrow Y = \left[\tilde{I}^{\;\frac{1}{\sigma}} (AK)^{\frac{\sigma-1}{\sigma}} + \left(\frac{B\,(\sigma-1)\,L}{e^{B(\sigma-1)} - e^{B\tilde{I}(\sigma-1)}} \right)^{\frac{\sigma-1}{\sigma}} \int_{\tilde{I}}^{1} e^{Bi(\sigma-1)} di \right]^{\frac{\sigma}{\sigma-1}}$$

$$\Longleftrightarrow Y = \left[\tilde{I}^{\;\frac{1}{\sigma}} (AK)^{\frac{\sigma-1}{\sigma}} + \left(\frac{B\,(\sigma-1)\,L}{e^{B(\sigma-1)} - e^{B\tilde{I}(\sigma-1)}} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{e^{B(\sigma-1)} - e^{B\tilde{I}(\sigma-1)}}{B(\sigma-1)} \right) \right]^{\frac{\sigma}{\sigma-1}}$$

$$\Longleftrightarrow Y = \left[\tilde{I}^{\frac{1}{\sigma}} \, (AK)^{\frac{\sigma-1}{\sigma}} + \left(\frac{e^{B(\sigma-1)} - e^{B\tilde{I}(\sigma-1)}}{B\,(\sigma-1)} \right)^{\frac{1}{\sigma}} L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \text{[5]} \tag{13}$$

[5]Note that if no technological restriction on the automatability of tasks were to exist and firms chose to automate all tasks ($\tilde{I} = 1$), the production technology specified in equation 13 would simplify to an AK-technology with $Y = AK$.

3 Capital-augmenting technical change and the labor share

I now study how CATC (an increase in A) affects the labor share in a static context in which the capital supply K and the labor supply L are fixed. In this context, the ratio of the factor remunerations $\frac{w}{r}$ determines the evolution of the labor share. The wage rate w was derived in the previous section and is given by equation 11. To find the interest rate, I assume that firms operate on perfectly competitive capital markets such that capital is paid its marginal product. For simplicity, I assume that the capital stock does not depreciate over time. Deriving equation 13 with regard to K thus yields the interest rate r equation 14.[6]

$$(13) \Rightarrow r = \frac{\partial Y}{\partial K} \iff r = Y^{\frac{1}{\sigma}} \tilde{I}^{\frac{1}{\sigma}} A^{\frac{\sigma-1}{\sigma}} K^{\frac{-1}{\sigma}} \iff r = \left(\frac{Y}{K}\right)^{\frac{1}{\sigma}} \left(A^{(\sigma-1)} \tilde{I}\right)^{\frac{1}{\sigma}} \tag{14}$$

We divide equation 11 by equation 14 to obtain an expression for $\frac{w}{r}$.

$$(11) \text{ and } (14) \Rightarrow \frac{w}{r} = \left(\frac{K}{L}\right)^{\frac{1}{\sigma}} \left(\frac{e^{B(\sigma-1)} - e^{B\tilde{I}(\sigma-1)}}{A^{(\sigma-1)} B\tilde{I}\,(\sigma-1)}\right)^{\frac{1}{\sigma}} \tag{15}$$

We can enrich equation 15 by acknowledging that the share of automated tasks $\tilde{I}$ is endogenously determined within this framework through equation 5. Crucially, $\tilde{I}$ is a function of A and $\frac{w}{r}$.

$$(15) \text{ and } (5) \Rightarrow \frac{w}{r} = \left(\frac{K}{L}\right)^{\frac{1}{\sigma}} \left(\frac{e^{B(\sigma-1)} - e^{B\tilde{I}(A,\frac{w}{r})(\sigma-1)}}{A^{(\sigma-1)} B\tilde{I}(A,\frac{w}{r})\,(\sigma-1)}\right)^{\frac{1}{\sigma}} \tag{16}$$

Since empirical estimates of the elasticity of substitution between capital and labor σ typically result in a value lower than one, it is more elegant to rewrite equation 16 to equation 17.

$$(16) \Rightarrow \frac{w}{r} = \left(\frac{K}{L}\right)^{\frac{1}{\sigma}} \left(\frac{A^{(1-\sigma)}\left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right)}{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)}\right)^{\frac{1}{\sigma}} \tag{17}$$

It is easily verified that as long as capital has the comparative advantage in some, but not all tasks ($0 < \tilde{I} < 1 \iff 0 < \frac{ln(\frac{w}{r}A)}{B} < 1$), the ratio of factor remunerations $\frac{w}{r}$ is strictly positive and finite. We now implicitly derive equation 17 with regard to A and try to determine the sign of $\frac{\partial(\frac{w}{r})}{\partial A}$.

$$(17) \Rightarrow \frac{\partial\left(\frac{w}{r}\right)}{\partial A} = \left(\frac{K}{L}\right)^{\frac{1}{\sigma}} \frac{1}{\sigma} \left(\frac{A^{(1-\sigma)}\left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right)}{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)}\right)^{\frac{1-\sigma}{\sigma}} \frac{1}{\left(B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)\right)^2}$$

$$\left\{ \left[B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)\right] \left[(1-\sigma)\,A^{-\sigma}\left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right) - A^{(1-\sigma)}(1-\sigma)e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)}\left(\frac{w}{r}A\right)^{-1}\left(\frac{\partial\left(\frac{w}{r}\right)}{\partial A}A + \frac{w}{r}\right)\right]\right.$$
$$\left. -A^{(1-\sigma)}\left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right)(1-\sigma)\left(\frac{w}{r}A\right)^{-1}\left(\frac{\partial\left(\frac{w}{r}\right)}{\partial A}A + \frac{w}{r}\right)\right\}$$

[6] Note that we can also find the wage rate w by deriving output Y in equation 13 to the aggregate input of labor L. If we do this, we find equation 11 again.

$$\iff \frac{\partial\left(\frac{w}{r}\right)}{\partial A} = \left(\frac{K}{L}\right)^{\frac{1}{\sigma}} \frac{1}{\sigma} \left(\frac{A^{(1-\sigma)}\left(e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}\right)}{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)} \right)^{\frac{1-\sigma}{\sigma}} \frac{1}{B\tilde{I}(A,\frac{w}{r})}$$
$$\left\{ \left[A^{-\sigma}\left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right) - \frac{A^{-\sigma}e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)}}{\frac{w}{r}} \left(\frac{\partial\left(\frac{w}{r}\right)}{\partial A}A + \frac{w}{r}\right)\right] - \frac{A^{-\sigma}\left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right)}{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)\,\frac{w}{r}} \left(\frac{\partial\left(\frac{w}{r}\right)}{\partial A}A + \frac{w}{r}\right)\right\}$$

$$\iff \frac{\partial\left(\frac{w}{r}\right)}{\partial A} = \left(\frac{K}{L}\right)^{\frac{1}{\sigma}} \frac{1}{\sigma} \left(\frac{A^{(1-\sigma)}\left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right)}{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)} \right)^{\frac{1-\sigma}{\sigma}} \frac{A^{-\sigma}}{B\tilde{I}(A,\frac{w}{r})}$$
$$\left\{ \left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right) - \frac{e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)}}{\frac{w}{r}} \left(\frac{\partial\left(\frac{w}{r}\right)}{\partial A}A + \frac{w}{r}\right) - \frac{e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}}{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)\,\frac{w}{r}} \left(\frac{\partial\left(\frac{w}{r}\right)}{\partial A}A + \frac{w}{r}\right)\right\}$$

$$\iff \frac{\partial\left(\frac{w}{r}\right)}{\partial A} = F\left\{ \left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right) - \frac{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)\,e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} + \left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right)}{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)\,\frac{w}{r}} \left(\frac{\partial\left(\frac{w}{r}\right)}{\partial A}A + \frac{w}{r}\right)\right\} \tag{18}$$

With $F = \left(\frac{K}{L}\right)^{\frac{1}{\sigma}} \frac{1}{\sigma} \left(\frac{A^{(1-\sigma)}\left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right)}{B\tilde{I}(A,\frac{w}{r})(1-\sigma)} \right)^{\frac{1-\sigma}{\sigma}} \frac{A^{-\sigma}}{B\tilde{I}(A,\frac{w}{r})}$

It is uncomplicated to see that $F > 0$ as long as $0 < \tilde{I} < 1 \iff 0 < \frac{ln(\frac{w}{r}A)}{B} < 1$.

For now, let us only look at the first-round effect of CATC on the labor share. This means that we disregard the fact that any change in the relative factor costs $\frac{w}{r}$ will also have repercussions for the choice that firms make between capital and labor ($\tilde{I}(A)$ instead of $\tilde{I}(A,\frac{w}{r})$). This implies setting $\frac{\partial(\frac{w}{r})}{\partial A}A$ equal to zero on the right-hand side of equation 18.

$$(18) \Rightarrow \left(\frac{\partial\left(\frac{w}{r}\right)}{\partial A}\right)^{*} = F\left[\left(e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}\right) - \frac{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)\,e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} + \left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right)}{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)}\right] \tag{19}$$

From equation 19, we can see that CATC affects the labor share in two distinct ways. The first part $(e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)})$ originates from the direct dependence of $\frac{w}{r}$ on A in equation 17. It is thus the typical labor share effect of CATC that would prevail in a scenario where the set of automated tasks $\tilde{I}$ were constant. It stems from the fact that — because of the increased productivity of capital — the effective execution of already automated tasks increases. This effect coincides with what Acemoglu and Restrepo (2019) have labelled the "deepening of automation" or "automation at the intensive margin". As is well-known, the sign of this effect is entirely dependent on the value of the elasticity of substitution σ. If capital and labor are substitutes ($\sigma > 1$) the effect on the labor share is negative. If capital and labor are complements ($0 < \sigma < 1$) the effect is positive. Since the share of automated tasks $\tilde{I}$ depends on A in this setting with untapped automation opportunities, there is a second labor share effect present. In equation 19, the second part $\left(-\frac{B\tilde{I}(A,\frac{w}{r})(1-\sigma)e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} + \left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right)}{B\tilde{I}(A,\frac{w}{r})(1-\sigma)}\right)$ originates from the indirect dependence of $\frac{w}{r}$ on A through $\tilde{I}$. This is the so-called "displacement effect" of automation.

This effect is strictly negative as long as $0 < \tilde{I} < 1$, since CATC leads to a contraction in the set of non-automated tasks (see Appendix A) and the production of non-automated tasks faces diminishing returns.

Rearranging equation 19, we obtain equation 20.

$$(19) \Rightarrow \left(\frac{\partial\left(\frac{w}{r}\right)}{\partial A}\right)^* = F\left[-e^{-B(1-\sigma)} - \frac{e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}}{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)}\right] \tag{20}$$

Based on equation 20, it is clear that net first-round effect $\left(\frac{\partial\left(\frac{w}{r}\right)}{\partial A}\right)^*$ of CATC on the relative factor remuneration $\frac{w}{r}$ is unambiguously negative as long as $0 < \tilde{I} < 1$. This is the case for all values of the elasticity of substitution between capital and labor σ. In a static context with fixed factor inputs K and L, this implies that the first-round effect of CATC on the labor share is negative.

Based on equation 18, it is also possible to find an expression for the final net effect $\frac{\partial\left(\frac{w}{r}\right)}{\partial A}$ of CATC on the labor share.

$$\begin{aligned}(18) \Rightarrow \frac{\partial\left(\frac{w}{r}\right)}{\partial A}\left\{1 + F\left[\frac{Ae^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)}}{\frac{w}{r}} + \frac{A(e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)})}{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)\,\frac{w}{r}}\right]\right\} = \\ F\left\{-e^{-B(1-\sigma)} - \frac{\left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right)}{B\tilde{I}(A,\frac{w}{r})\,(1-\sigma)}\right\}\end{aligned}$$

$$\Longleftrightarrow \frac{\partial\left(\frac{w}{r}\right)}{\partial A} = \frac{\left(\frac{\partial\left(\frac{w}{r}\right)}{\partial A}\right)^*}{1 + \left(\frac{K}{L}\right)^{\frac{1}{\sigma}}\frac{1}{\sigma}\left(\frac{\left(e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}\right)}{(1-\sigma)}\right)^{\frac{1-\sigma}{\sigma}}\frac{A^{\frac{1-\sigma}{\sigma}}}{\left(B\tilde{I}(A,\frac{w}{r})\right)^{\frac{1}{\sigma}}\frac{w}{r}}\left[e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} + \frac{e^{-B\tilde{I}(A,\frac{w}{r})(1-\sigma)} - e^{-B(1-\sigma)}}{B\tilde{I}(A,\frac{w}{r})(1-\sigma)}\right]} \tag{21}$$

The denominator of the expression on the right-hand side of equation 21 is positive as long as $0 < \tilde{I} < 1$. From this, we can conclude that the final net effect $\frac{\partial\left(\frac{w}{r}\right)}{\partial A}$ of CATC on the labor share has the same negative sign as the first-round effect $\left(\frac{\partial\left(\frac{w}{r}\right)}{\partial A}\right)^*$. Since the denominator is also strictly greater than one as long as $0 < \tilde{I} < 1$, we can conclude that the final net effect of CATC on the labor share is not as negative as the first-round effect. The intuitive reason for this is that when CATC lowers $\frac{w}{r}$ in the first-round effect, this also implies that the relative cost of producing a task with labor falls. As a result, firms react to the increased cost effectiveness of labor by re-expanding the set of non-automated tasks somewhat in the second round. In Appendix A, I show that the CATC will always increase the set of automated tasks $\tilde{I}$, however, such that the final net effect of CATC on the set of non-automated tasks always goes in the direction of a contraction. In the end, this displacement effect (the contraction in the set of non-automated tasks) still dominates and the final net effect of CATC on the labor share is always negative. This is consistent with the empirical consensus that (progress in) automation technologies is linked with a lowering of the labor share (Acemoglu et al., 2020; Acemoglu & Restrepo, 2020; Autor & Salomons, 2018; Bergholt et al., 2022; Dauth et al., 2017; Graetz & Michaels, 2018; Guimarães & Gil, 2022).

4 Capital-augmenting technical change and the wage rate

In a static framework with fixed factor inputs K and L, we can also determine how CATC affects the wage rate w by deriving equation 11 with regard to A.

$$(11) \Rightarrow \frac{\partial w}{\partial A} = \frac{1}{\sigma}\left(\frac{Y}{L}\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}}{B\,(1-\sigma)}\right)^{\frac{1-\sigma}{\sigma}} \left[\frac{-Y}{L}\frac{1}{B\left(\frac{w}{r}A\right)^{2-\sigma}}\frac{\partial\left(\frac{w}{r}A\right)}{\partial A} + \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}}{B\,(1-\sigma)}\frac{1}{L}\frac{\partial Y}{\partial A}\right]$$

$$\iff \frac{\partial w}{\partial A} = G\left[\frac{-Y}{L}\frac{1}{B\left(\frac{w}{r}A\right)^{2-\sigma}}\frac{\partial\left(\frac{w}{r}A\right)}{\partial A} + \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}}{B\,(1-\sigma)}\frac{1}{L}\frac{\partial Y}{\partial A}\right] \tag{22}$$

With $G = \frac{1}{\sigma}\left(\frac{Y}{L}\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}}{B(1-\sigma)}\right)^{\frac{1-\sigma}{\sigma}}$ such that G strictly positive as long as $0 < \tilde{I} < 1$

Based on equation 22, we can decompose the absolute wage effect of CATC into two distinct effects. The first term captures the displacement effect: CATC causes the set of non-automated tasks to contract (since $\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}$ is strictly positive: see Appendix A) and this unambiguously lowers the demand for labor. The second term captures the productivity effect of CATC: CATC increases the effective execution of automated tasks and this unambiguously increases the demand for the execution of non-automated tasks (and thus the demand for labor), since automated tasks and non-automated tasks are q-complements in the aggregate production function with constant returns to scale. To evaluate the sign of the absolute wage effect $\frac{\partial w}{\partial A}$, we have to determine which of these two effects dominate. Based on equation (A2) in Appendix A, we can find an expression for the displacement effect term (equation 23). In Appendix B, we derive an expression for the productivity effect term (equation 24).

$$(A2) \Rightarrow \frac{-Y}{L}\frac{1}{B\left(\frac{w}{r}A\right)^{2-\sigma}}\frac{\partial\left(\frac{w}{r}A\right)}{\partial A} = \frac{-\frac{w}{r}\frac{Y}{L}\frac{1}{B\left(\frac{w}{r}A\right)^{2-\sigma}}}{\sigma + \frac{(1-\sigma)}{\left(e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}\right)\left(\frac{w}{r}A\right)^{1-\sigma}} + \frac{1}{B\tilde{I}(\frac{w}{r},A)}} \tag{23}$$

$$(B3) \Rightarrow \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}}{B\,(1-\sigma)}\frac{1}{L}\frac{\partial Y}{\partial A} = r\,\frac{K}{L}A^{-1}\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}}{B(1-\sigma)} \tag{24}$$

Following equation 22, the sign of the wage effect of CATC is determined by the relative strength of the displacement effect and the productivity effect. We insert the obtained expressions 23 and 24 for these effects and evaluate the sign of $\frac{\partial w}{\partial A}$.

$$(22) \Rightarrow sgn\left(\frac{\partial w}{\partial A}\right) = sgn\left[\frac{-\frac{w}{r}\frac{Y}{L}\frac{1}{B\left(\frac{w}{r}A\right)^{2-\sigma}}}{\sigma + \frac{(1-\sigma)}{\left(e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}\right)\left(\frac{w}{r}A\right)^{1-\sigma}} + \frac{1}{B\tilde{I}(\frac{w}{r},A)}} + r\,\frac{K}{L}A^{-1}\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}}{B(1-\sigma)}\right]$$

$$\iff sgn\left(\frac{\partial w}{\partial A}\right) = sgn\Bigg[-\ \frac{w}{r}\frac{1}{B(\frac{w}{r}A)^{2-\sigma}}\frac{Y}{L} +\ \sigma\ r\ \frac{K}{L}\ A^{-1}\ \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}}{B(1-\sigma)}$$
$$+\ r\ \frac{K}{L}\frac{1}{B}\frac{1}{A^{2-\sigma}}\left(\frac{w}{r}\right)^{\sigma-1} +\ r\ \frac{K}{L}A^{-1}\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)} - e^{-B(1-\sigma)}}{B(1-\sigma)}\frac{1}{B\tilde{I}(\frac{w}{r},A)}\Bigg]$$

$$\iff sgn\left(\frac{\partial w}{\partial A}\right) = sgn\Bigg[-\ \frac{Y}{L}\ \left(\frac{w}{r}\right)^{\sigma-1}\frac{1}{BA^{2-\sigma}} +\ \sigma\ r\ \left(\frac{w}{r}\right)^{\sigma}\frac{1}{BA^{2-\sigma}}B\tilde{I}(\frac{w}{r},A) +\ r\ \frac{1}{B}\frac{K}{L}\frac{1}{A^{2-\sigma}}\left(\frac{w}{r}\right)^{\sigma-1} +\ r\ \left(\frac{w}{r}\right)^{\sigma}\frac{1}{B}A^{\sigma-2}\Bigg]$$

$$\iff sgn\left(\frac{\partial w}{\partial A}\right) = sgn\left[\sigma\ r\left(\frac{w}{r}\right)^{\sigma}\frac{1}{BA^{2-\sigma}}B\tilde{I}(\frac{w}{r},A) + \frac{1}{BA^{2-\sigma}}\left[-\frac{Y}{L}\left(\frac{w}{r}\right)^{\sigma-1} + \frac{K}{L}\ w^{\sigma-1}\ r^{2-\sigma} + w^{\sigma}\ r^{1-\sigma}\right]\right]$$

$$\iff sgn\left(\frac{\partial w}{\partial A}\right) = sgn\left[\sigma\ w^{\sigma}r^{1-\sigma}\frac{1}{BA^{2-\sigma}}B\tilde{I}(\frac{w}{r},A) + \frac{1}{BA^{2-\sigma}}\left(\frac{w}{r}\right)^{\sigma-1}\left[-\frac{Y}{L} + \frac{K}{L}\ r\ +\ w\ \right]\right]$$

$$\iff sgn\left(\frac{\partial w}{\partial A}\right) = sgn\left[\sigma\ w^{\sigma}r^{1-\sigma}\frac{1}{BA^{2-\sigma}}B\tilde{I}(\frac{w}{r},A)\ +\ \frac{1}{BA^{2-\sigma}}\left(\frac{w}{r}\right)^{\sigma-1}\frac{1}{L}[\ -Y\ +\ rK\ +\ wL\ ]\right] \tag{25}$$

The second term in equation 25 simplifies to zero since $Y = rK + wL$ holds given the constant returns to scale production function in equation 13 and the assumption of perfectly competitive factor markets. One can easily prove this equality based on equations 11, 13 and 14.

$$(27) \Rightarrow sgn\left(\frac{\partial w}{\partial A}\right) = sgn\left[\sigma\ w^{\sigma}r^{1-\sigma}\frac{\tilde{I}}{A^{2-\sigma}}\right] \tag{26}$$

In equation 26, one can observe that the absolute wage effect of CATC is positive regardless of the value of the elasticity of substitution between capital and labor. In other words, the productivity effect of CATC always dominates the displacement effect such that an increase in the effectiveness of capital K always increases the wage rate w. This result echoes the empirical findings of Gregory et al. (2022) who study the labor demand effects of routine-replacing technological change and find that the negative displacement effect is more than fully compensated by countervailing mechanisms stemming from productivity gains.

5 Concusion

In this work, I demonstrated the attractive properties of a simple, yet largely unstudied process of automation that reconciles elements from different strands in the literature. In standard settings, the effect of capital-augmenting technical change on the distribution of income between the production factors depends entirely on whether capital and labor are gross complements or substitutes in the aggregate production function. By adopting a task-based approach, I analytically showed that this is not the case when untapped automation opportunities exist. More precisely, when firms are not constrained by technology — such that they can choose at the margin whether they use capital or labor for the production of a task, based on cost effectiveness —, the effect of capital-augmenting technical change on the labor share is negative regardless of the value of the elasticity of substitution between capital and labor. This is because the standard effect (whose sign is governed by the elasticity of substitution) is overridden by the the contraction in the set of non-automated tasks which capital-augmenting technical change elicits. Furthermore, this simple framework predicts that the effect of capital-augmenting technical change on the wage rate is twofold. First, it raises the productivity of capital at the production of already automated tasks and this increases the demand for the execution of non-automated tasks and thus for labor. Second, it leads to a contraction in the set of non-automated tasks and this displacement effect lowers labor demand. The productivity effect dominates irrespective of the parameterization such that capital-augmenting technical change always increase the wage rate. I argue that the implications of capital-augmenting technical change in this setting are consistent with the recent empirical findings regarding the labor market impact of automation technologies.

References

Acemoglu, D., Lelarge, C., & Restrepo, P. (2020). Competing with robots: Firm-level evidence from france. *AEA Papers and Proceedings*, *110*, 48-53.

Acemoglu, D., & Restrepo, P. (2018a). Modeling automation. *AEA Papers and Proceedings*, *108*, 48-53.

Acemoglu, D., & Restrepo, P. (2018b). The race between man and machine: Implications of technology for growth, factor shares, and employment. *American Economic Review*, *108*(6), 1488-1542.

Acemoglu, D., Restrepo, P. (2019). Artificial Intelligence, Automation, and Work, in A. Agrawal, J. Gans & A. Goldfard (Eds),*The Economics of Artificial Intelligence*, University of Chicago Press, Ch. 8, pp. 197-236.

Acemoglu, D., & Restrepo, P. (2020). Robots and jobs: Evidence from US labor markets. Journal of Political Economy, 128(6), 2188-2244.

Acemoglu, D., & Restrepo, P. (2021). Tasks, automation, and the rise in US wage inequality. *NBER Working Paper*, National Bureau of Economic Research, No. w28920.

Autor, D., & Salomons, A. (2018). Is automation labor-displacing? Productivity growth, employment, and the labor share. *NBER Working Paper*, National Bureau of Economic Research, No. w24871.

Atkinson, R. D. (2019). Robotics and the Future of Production and Work. *Information Technology and Innovation Foundation Working Paper.*

Basso, H. S., & Jimeno, J. F. (2021). From secular stagnation to robocalypse? Implications of demographic and technological changes. *Journal of Monetary Economics*, *117*, 833-847.

Bergholt, D., Furlanetto, F., & Maffei-Faccioli, N. (2022). The Decline of the Labor Share: New Empirical Evidence. *American Economic Journal: Macroeconomics.* Forthcoming.

Chirinko, R. S. (2008). Sigma: The long and short of it. *Journal of Macroeconomics*, *30*(2), 671-686.

Cords, D., & Prettner, K. (2022). Technological unemployment revisited: automation in a search and matching framework. *Oxford Economic Papers*, *74*(1), 115-135.

Cséfalvay, Z. (2020). Robotization in Central and Eastern Europe: catching up or dependence?. *European Planning Studies*, *28*(8), 1534-1553.

Dauth, W., Findeisen, S., Südekum, J., & Woessner, N. (2017). German robots-the impact of industrial robots on workers. *CEPR Discussion Paper*, No. DP12306.

Gechert, S., Havranek, T., Irsova, Z., Kolcunova, D. (2021). Measuring capital-labor substitution: The importance of method choices and publication bias. *Review of Economic Dynamics.* Forthcoming

Gregory, T., Salomons, A., & Zierahn, U. (2022). Racing with or Against the Machine? Evidence on the Role of Trade in Europe. *Journal of the European Economic Association*, 20(2), 869-906.

Guimarães, L., & Gil, P. M. (2022). Explaining the labor share: Automation vs labor market Institutions. *Labor Economics*, 102146.

Jacobs, A., Heylen, F. (2021). Demographic change, secular stagnation and inequality: automation as a blessing? (No. 21/1030). *Ghent University, Faculty of Economics and Business Administration Working Papers.*

Karabarbounis, L., & Neiman, B. (2014). The global decline of the labor share. *The Quarterly Journal of Economics*, *129*(1), 61-103.

Knoblach, M., Roessler, M., Zwerschke, P. (2020). The Elasticity of Substitution Between Capital and Labour in the US Economy: A Meta-Regression Analysis. *Oxford Bulletin of Economics and Statistics*, *82*(1), 62-82.

Martinez, J. (2021). Putty-Clay Automation. *CEPR Discussion Paper*, No. DP16022.

Nordhaus, W. D. (2015). Are we approaching an economic singularity? information technology and the future of economic growth. *NBER Working Paper*, National Bureau of Economic Research, No.w21547.

Oberfield, E., & Raval, D. (2021). Micro data and macro technology. *Econometrica*, 89(2), 703-732.

Parschau, C., & Hauge, J. (2020). Is automation stealing manufacturing jobs? Evidence from South Africa's apparel industry. *Geoforum*, *115*, 120-131.

Sachs, J. D., & Kotlikoff, L. J. (2012). Smart machines and long-term misery. *NBER Working Paper*, National Bureau of Economic Research, No. w18629.

Stähler, N. (2021). The impact of aging and automation on the macroeconomy and inequality. *Journal of Macroeconomics*, *67*, 103278.

Appendix A: The effect of CATC on the share of automated tasks

It is uncomplicated to show that CATC will always increase the share of tasks which are automated $\tilde{I}$. We start from the expression for $\tilde{I}$ in equation 5 and we derive it with respect to A.

$$(5) \Rightarrow sgn\left(\frac{\partial \tilde{I}}{\partial A}\right) = sgn\left(\frac{\partial\left(\frac{ln\left(\frac{w}{r}A\right)}{B}\right)}{\partial A}\right) = sgn\left(\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}\right) \tag{A1}$$

Based on equation (A1), it suffices to determine the sign of $\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}$ to check how CATC affects the share of automated tasks $\tilde{I}$. We replace $\frac{w}{r}$ by its expression (set out in equation 17) and we derive $\frac{w}{r}A$ with regard to A.

$$(17) \Rightarrow \frac{\partial\left(\frac{w}{r}A\right)}{\partial A} = \left(\frac{K}{L}\right)^{\frac{1}{\sigma}} \frac{\partial\left(\left(\frac{A\left(e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}\right)}{B\tilde{I}(\frac{w}{r},A)(1-\sigma)}\right)^{\frac{1}{\sigma}}\right)}{\partial A}$$

$$\iff \frac{\partial\left(\frac{w}{r}A\right)}{\partial A} = \left(\frac{K}{L}\right)^{\frac{1}{\sigma}} \frac{1}{\sigma}\left(\frac{A\left(e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}\right)}{B\tilde{I}(\frac{w}{r},A)\,(1-\sigma)}\right)^{\frac{1-\sigma}{\sigma}} \frac{1}{\left(B\tilde{I}(\frac{w}{r},A)\,(1-\sigma)\right)^2}\left\{\left[B\tilde{I}(\frac{w}{r},A)\,(1-\sigma)\right]\right.$$
$$\left[\left(e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}\right) - A(1-\sigma)e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}\left(\frac{w}{r}A\right)^{-1}\left(\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}\right)\right]$$
$$\left. -A\left(e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}\right)(1-\sigma)\left(\frac{w}{r}A\right)^{-1}\left(\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}\right)\right\}$$

$$\iff \frac{\partial\left(\frac{w}{r}A\right)}{\partial A} = \left(\frac{K}{L}\right)^{\frac{1}{\sigma}} \frac{1}{\sigma}\left(\frac{A\left(e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}\right)}{B\tilde{I}(\frac{w}{r},A)\,(1-\sigma)}\right)^{\frac{1-\sigma}{\sigma}} \frac{1}{B\tilde{I}(\frac{w}{r},A)}$$
$$\left\{\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{(1-\sigma)} - \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}}{\frac{w}{r}}\left(\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}\right) - \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{\frac{w}{r}B\tilde{I}(\frac{w}{r},A)\,(1-\sigma)}\left(\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}\right)\right\}$$

$$\iff \frac{\partial\left(\frac{w}{r}A\right)}{\partial A} = \frac{w}{r}\frac{1}{\sigma}\left(\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B\tilde{I}(\frac{w}{r},A)(1-\sigma)}\right)^{-1} \frac{1}{B\tilde{I}(\frac{w}{r},A)}$$
$$\left\{\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{(1-\sigma)} - \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}}{\frac{w}{r}}\left(\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}\right) - \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{\frac{w}{r}B\tilde{I}(\frac{w}{r},A)\,(1-\sigma)}\left(\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}\right)\right\}$$

$$\iff \frac{\partial\left(\frac{w}{r}A\right)}{\partial A} = \frac{w}{r}\frac{1}{\sigma}\left(\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{(1-\sigma)}\right)^{-1}$$
$$\left\{\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{(1-\sigma)} - \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}}{\frac{w}{r}}\left(\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}\right) - \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{\frac{w}{r}B\tilde{I}(\frac{w}{r},A)\,(1-\sigma)}\left(\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}\right)\right\}$$

$$\iff \frac{\partial\left(\frac{w}{r}A\right)}{\partial A} = \frac{1}{\sigma}\left\{\frac{w}{r} - \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}}{\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{(1-\sigma)}}\left(\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}\right) - \frac{1}{B\tilde{I}(\frac{w}{r},A)}\left(\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}\right)\right\}$$

$$\iff \frac{\partial\left(\frac{w}{r}A\right)}{\partial A} = \frac{\frac{w}{r}}{\sigma + \frac{(1-\sigma)}{\left(e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}\right)(\frac{w}{r}A)^{1-\sigma}} + \frac{1}{B\tilde{I}(\frac{w}{r},A)}} \tag{A2}$$

As long as $0 < \tilde{I} < 1 \iff 0 < \frac{ln(\frac{w}{r}A)}{B} < 1$, one can observe that $\frac{\partial\left(\frac{w}{r}A\right)}{\partial A}$ is strictly positive. Based on equation (A2), this also implies that the share of automated tasks $\tilde{I}$ is positively affected by CATC.

Appendix B: The productivity effect of CATC on the wage rate

In section 4, we found that the effect of CATC on the wage rate depends on the relative strength of the displacement effect and the productivity effect (equation 22). The latter is represented by the term $\left(\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B(1-\sigma)}\frac{1}{L}\frac{\partial Y}{\partial A}\right)$. In this Appendix B, we derive an expression for this productivity effect. We start off by deriving Y — set out in equation 13 — with respect to A and updating our expression for the productivity effect.

$$(13) \Rightarrow \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B\,(1-\sigma)}\frac{1}{L}\frac{\partial Y}{\partial A} = \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B\,(1-\sigma)}\frac{1}{L}\left(\frac{\sigma}{\sigma-1}\right)Y^{\frac{1}{\sigma}}$$
$$\Bigg[(AK)^{\frac{\sigma-1}{\sigma}}B^{\frac{-1}{\sigma}}\left(\frac{1}{\sigma}\right)\left(B\tilde{I}(\frac{w}{r},A)\right)^{\frac{1-\sigma}{\sigma}}\left(\frac{1}{\frac{w}{r}A}\right)\frac{\partial(\frac{w}{r}A)}{\partial A}+\left(\tilde{I}(\frac{w}{r},A)\right)^{\frac{1}{\sigma}}K^{\frac{\sigma-1}{\sigma}}\left(\frac{\sigma-1}{\sigma}\right)A^{\frac{-1}{\sigma}}$$
$$+L^{\frac{\sigma-1}{\sigma}}\left(\frac{1}{\sigma}\right)\left(\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B(1-\sigma)}\right)^{\frac{1-\sigma}{\sigma}}\left(\frac{-1}{B(\frac{w}{r}A)^{2-\sigma}}\right)\frac{\partial(\frac{w}{r}A)}{\partial A}\Bigg] \tag{B1}$$

Based on equation (B1), we can further decompose the productivity effect of CATC in three distinct effects. The first term indicates the effect on output Y generated by the expansion of the set of automated tasks $\tilde{I}$. The second term indicates the effect on output Y generated by the increased productivity of capital at the production of already automated tasks. The third term indicates the effect on output Y generated by the contraction in the set of non-automated tasks $(1-\tilde{I})$. We simplify equation (B1) by replacing expressions by w, r and $\frac{w}{r}$ according to equations 11, 14 and 17.

$$(B1) \Rightarrow \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B\,(1-\sigma)}\frac{1}{L}\frac{\partial Y}{\partial A} =$$
$$\left(\frac{Y}{K}\right)^{\frac{1}{\sigma}}A^{\frac{\sigma-1}{\sigma}}\left(\tilde{I}(\frac{w}{r},A)\right)^{\frac{1}{\sigma}}\frac{K}{L}\frac{1}{B\tilde{I}(\frac{w}{r},A)\frac{w}{r}A}\frac{1}{\sigma-1}\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B(1-\sigma)}\frac{\partial(\frac{w}{r}A)}{\partial A}$$
$$+\left(\frac{Y}{K}\right)^{\frac{1}{\sigma}}A^{\frac{\sigma-1}{\sigma}}\left(B\tilde{I}(\frac{w}{r},A)\right)^{\frac{1}{\sigma}}\frac{K}{L}A^{-1}\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B(1-\sigma)}$$
$$+\left(\frac{Y}{L}\right)^{\frac{1}{\sigma}}\left(\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B(1-\sigma)}\right)^{\frac{1}{\sigma}}\frac{1}{\sigma-1}\left(\frac{-1}{B(\frac{w}{r}A)^{2-\sigma}}\right)\frac{\partial(\frac{w}{r}A)}{\partial A}$$

$$\iff \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B\,(1-\sigma)}\frac{1}{L}\frac{\partial Y}{\partial A} = \frac{r}{(\frac{w}{r})^{1-\sigma}A^{2-\sigma}B(\sigma-1)}\frac{\partial(\frac{w}{r}A)}{\partial A}$$
$$+\,r\,\frac{K}{L}A^{-1}\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B(1-\sigma)}+\frac{w}{\sigma-1}\left(\frac{-1}{B(\frac{w}{r}A)^{2-\sigma}}\right)\frac{\partial(\frac{w}{r}A)}{\partial A} \tag{B2}$$

In equation (B2), the first and third term cancel out implying that the positive productivity effect of the expansion in the set of automated tasks cancels out the negative productivity effect of the contraction in the set of non-automated tasks. Intuitively, this is because capital and labor are equally cost-effective at the production of the marginally automated task $\tilde{I}$, as indicated by equation 5. As a result, the productivity effect of CATC is entirely driven by the increased effectiveness of capital at the production of already automated tasks. Equation (B2) represents the second term in the right-hand side of equation 22.

$$(B2) \Rightarrow \frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B\,(1-\sigma)}\frac{1}{L}\frac{\partial Y}{\partial A} = r\,\frac{K}{L}A^{-1}\frac{e^{-B\tilde{I}(\frac{w}{r},A)(1-\sigma)}-e^{-B(1-\sigma)}}{B(1-\sigma)} \tag{B3}$$

Equation (B3) establishes a useful expression for the second term in equation 22 (the productivity effect).