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Beyond Rational Expectations: The Effects of Heuristic Switching in an Overlapping Generations Model

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Abstract

We explore the transitional dynamics in an Overlapping Generations framework with and without heuristic switching. Agents use simple heuristics to forecast the interest rate and the real wage. The fraction of agents using a specific heuristic depends on its relative forecasting performance. In the absence of heuristic switching, the results indicate that there is a lot of variation in the transitional dynamics over different parameter values and heuristics. When agents switch between heuristics, however, the variation in the transitional dynamics decreases significantly. We show that heuristic switching has a stabilising effect on transitional dynamics that would otherwise exhibit permanent oscillations.

Keywords: Heuristic Switching, Heterogeneous Agents, Fiscal Policy, Transitional Dynamics, Overlapping Generations

JEL classification codes: D83, D84, E60

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1 Introduction

This paper explores the stability and transitional dynamics of an Overlapping Generations model with heuristic switching. Accordingly, this paper contributes to a growing literature that goes beyond the rational expectations paradigm. After all, the transitional and equilibrium impact of public policy, for instance, on the macroeconomy depends crucially on the behavioural response of households to these policies. An important determinant of this behavioural response is the procedure households apply to form expectations regarding the future course of different variables. Moreover, these expectations themselves are typically a key determinant of the current realisation of these variables. Several authors have argued that assuming that economic agents have rational expectations (RE) is unrealistic. An alternative is provided by the learning literature (see e.g. Evans and Honkapohja, 2001). In this literature, boundedly rational agents form expectations using a perceived law of motion. In the lion's share of this literature, agents act as econometricians who update the coefficients of their perceived law of motion as new realisations of the variables of interest become available over time.

A number of papers studied the effects of least-squares adaptive learning within an Overlapping Generations framework (henceforth OLG) – see e.g. Bullard (1994), Schönhofer (1999), Adam (2003), Tuinstra (2003), and Tuinstra and Wagener (2007). In Schönhofer (1999), for example, it is shown that if one explicitly considers learning in a monetary OLG model, the dynamic system may exhibit chaotic behaviour. Tuinstra (2003), on the other hand, introduces the notion of beliefs equilibria. These are equilibria where the belief of the agents best fits the time series data, which itself is generated by the model where agents have this belief. Although the learning dynamics might converge, the author shows that the corresponding inflation dynamics might be erratic. Different from these studies, Chen et al. (2008) study the dynamic behaviour of an OLG model with capital accumulation under three different types of expectations: rational, myopic and adaptive expectations. They conclude that the dynamics can be complex when using the latter two types. Moreover, the dynamic properties of the model crucially depend on the value of the intertemporal elasticity of substitution in consumption and, in the case of adaptive expectations, on the weight agents attach to past observations when forming expectations.

All these papers have enriched our knowledge of the properties that characterise OLG models when moving beyond the scope of rational expectations. The majority of the papers using an OLG framework, however, focus on the (mostly local) stability properties of the equilibrium, not on the transitional dynamics following policy shocks. A second shortcoming is that often only one forecasting rule is studied. If the agents act as econometricians, they constantly update the coefficients of the same equation, but they cannot distinguish or switch between different rules. Furthermore, even if multiple rules are studied simultaneously, it is virtually always assumed that all agents use the same rule at a given point in time. In these papers, the focus often lies exclusively with one-period-ahead forecasts as well, ruling out the possibility of multi-period-ahead forecasts.

It is, however, plausible that a fraction of the economic agents does not have the cognitive capacities to act as an econometrician. Just as it can not be ruled out a priori that different forecasting rules are being used by the economic agents at one point in time. So then, what is the macroeconomic impact if one would assume that economic agents use simple rules to forecast wages and interest rates? Additionally, how do the transitional dynamics behave when individuals have multiple rules to choose from? And are the transitional dynamics sensitive to the rule being used? Finally, how do they compare to the transitional dynamics in the rational expectations case?

In this paper, we assume that agents use such simple rules, heuristics, to forecast the future course of the interest rate and the real wage. Agents use these heuristics because, in general, they do not possess the cognitive abilities as assumed by the rational expectations literature nor to act as econometricians. Instead, the agents have a certain number of different heuristics at their disposal. We follow the heuristic switching mechanism proposed by Brock and Hommes (1997) and assume that agents assess on a regular basis the predictive power of the heuristic they are currently applying. If it performs well, the probability that agents will use the same heuristic in the next period will be higher. If it does not perform well, there is a higher probability that they switch to another rule.

The framework used to answer these questions is in line with the evidence provided by several laboratory experiments – see e.g. Adam (2007), Hommes (2011), Heemeijer et al. (2012), and Hommes (2014). In Heemeijer et al. (2012), for example, the authors use a standard OLG framework to conduct an individual experiment in order to assess the ability of individuals to form expectations and the degree to which these individuals learn about the accuracy of their forecasts. In the experiment, participants are asked to submit fifty one-step-ahead forecasts for the inflation rate. Over time, the participants also observe the actual realisations of the inflation rate. These can be used by the participants to forecast the remaining future inflation rates. The authors argue that their experimental results cannot be explained using the rational expectations approach. Rather, they are consistent with the use of constant gain algorithms or average expectations. Their results also indicate that individuals switch between different heuristics according to the relative forecasting performance of these rules.

The aim of this paper is to contribute to the literature that goes beyond the rational expectations paradigm by exploiting the heuristic switching approach within an Overlapping Generations model. Triggered by a fiscal policy shock, the objective is to study the transitional dynamics of the model for a large number of settings including one or multiple heuristics and compare the behaviour of the dynamics with their rational expectations counterpart.

The simulations lead to three main findings. First, in a context without heuristic switching (i.e. a context where individuals only have one heuristic at their disposal to form expectations), the evolution of transitional dynamics can be substantially different from the rational expectations case, especially in the first periods of the transition. Rational expectations is thus not always a good approximation. Furthermore, there is a lot of variation in the dynamics over different parameter values and heuristics. This finding implies that if only one heuristic is used, the macroeconomic impact of fiscal policy is highly sensitive to the heuristic being used. What is more, as the discount rate and the degree of risk aversion decrease, oscillatory behaviour may arise.

Second, after activating the heuristic switching regime, the variation in the transitional dynamics decreases significantly. Consequently, the sensitivity of the transitional effects of fiscal policy is much lower now and its exact impact is thus less uncertain for policy makers.

Third and last, the heuristic switching has a stabilising effect on the transitional dynamics. For certain heuristic scenarios for which the model exhibits permanent oscillations in the absence of heuristic switching, the dynamics now converge to the steady state in most cases.

These findings are important. They show that in an economy where agents switch between forecasting heuristics, the transitional dynamics are very different from those under rational expectations and those under heterogeneous expectations without switching. Compared to the latter, the variation in the transitional dynamics is smaller as the dynamics are less affected by the (initial) distribution of heuristics. Furthermore, it turns out that rational expectations is a better approximation when the switching mechanism is activated.

The remainder of this paper is structured as follows. Section 2 outlines the different model blocks. Section 3 focuses on the heuristic switching. In Section 4, we provide some details on the timing in the model. The calibration and parameterisation of the model is described in Section 5. Section 6 consists of the description and a detailed look into the results of the different simulations. Section 7 concludes.

2 Model

We consider a closed-economy Overlapping Generations model that builds on the framework of Auerbach and Kotlikoff (1987). Time is discrete and runs from 0 to $+\infty$. Each period lasts for 4 years[1]. At each moment in time, the economy is populated by J overlapping generations. The model consists of three actors: heterogeneous agents, firms, and a fiscal government. Markets are incomplete meaning that individuals cannot explicitly insure themselves against productivity shocks[2].

2.1 Demographics

At the beginning of each period, a continuum of new agents with measure one enters the model. Individuals have an uncertain lifespan. They face an age-specific survival probability φ_j between the age of j and $j+1$. The demographics of the model are exogenous and given by:

$$N_{j+1,t+1} = \varphi_j N_{j,t}, \tag{1}$$

where $N_{j,t}$ represents the number of individuals of age j at time t. Every individual who survives J periods will die with certainty after the J-th period.

2.2 Individuals

Individuals enter the model at the age of 18. Ex-ante, before any decisions are made, individuals only differ with respect to the heuristic they apply to form expectations. Furthermore, the eco-

[1]Having shorter time periods compared to the OLG models with two generations will allow us to calibrate the model in line with the literature. Furthermore, it is our belief that the heuristic switching mechanism benefits from a shorter period length as the heuristics itself can be more advanced and the updating process occurs more often.

[2]For an example of a OLG model with idiosyncratic productivity shocks, see Conesa et al. (2009).

nomic agents face idiosyncratic income risk during their active period of life. At any given point in time, individuals are characterised by a state vector (j, a, η, h), where j is the age of the agent, a the accumulated non-human wealth at the beginning of period t, η the productivity shock and h the heuristic that the agent is currently applying. Let $\Phi_{j,t}(a, \eta, h)$ denote the share of agents aged j of type (a, η, h) at date t. For each t and j we have $\int \Phi_{t,j}(da \times d\eta \times dh) = 1$.

Individuals choose sequences of (n, c, a'), i.e. labour supply, consumption and accumulated non-human wealth, to maximise their expected lifetime utility. The latter is given by

$$U = E\left\{\sum_{j=1}^{J} \beta^{j-1} \frac{\left(c_j{}^{(1-\mu)}(1-n_j)^{\mu}\right)^{1-\theta}}{1-\theta}\right\}. \tag{2}$$

The share of consumption is given by $1 - \mu$. The degree of relative risk aversion is governed by θ. The time discount factor is denoted by β. Individuals reaching the age of J_R retire.

The dynamic budget constraint of an individual aged $j < J_R$ with state (a, η, h) at time t is given by

$$(1 + \tau_c)c_j + a'_j = w_t \eta \varepsilon_j n_j (1 - \tau) + (1 + r_t(1 - \tau_k))(a_j + Tr_t). \tag{3}$$

He or she earns an after-tax wage of $w_t \varepsilon_j \eta n_j (1-\tau)$, where w_t is the real wage per unit of effective labour at time t, ε_j is an age-specific productivity parameter, η is the labour productivity shock and τ is the average tax rate on labour income. The consumption tax rate is τ_c. The real interest rate is given by r_t. Individuals pay taxes on capital income where the capital tax rate is denoted by τ_k. Individuals enter the model without wealth and leave no intentional bequests. Due to accidental bequests, individuals receive a transfer Tr_t from the government. The accumulated non-human wealth at the end of the period is denoted by a'. We impose that individuals are not able to borrow: $a' \geq 0$. This individual maximises the following recursive problem:

$$V(j, a, \eta, h) = \max_{c_j, n_j, a'_j} U(c_j, n_j) + \beta \varphi_j \sum_{\eta'} \pi(\eta'|\eta) V(j + 1, a', \eta', h). \tag{4}$$

The stochastic process regarding the labour productivity shock is denoted by $\pi(\eta'|\eta)$. The heuristic used is given by h. From the age of J_R onwards, individuals receive a public pay-as-you-go pension. Their budget constraint for the ages $j \geq J_R$ is given by

$$(1 + \tau_c)c_j + a'_j = (1 + r_t(1 - \tau_k))(a_j + Tr_t) + pp_t. \tag{5}$$

The maximisation problem is now given by

$$V(j, a, h) = \max_{c, a'} U(c_j) + \beta \varphi_j V(j + 1, a', h). \tag{6}$$

A final note on the basic pay-as-you-go (PAYG) pension pp_t received by the retired households in the model. For simplicity, it is assumed that all individuals receive the same pension, i.e. a fraction b_p of the average after-tax wage in the economy.

2.3 Firms

The production function of the representative firm is given by

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}, \tag{7}$$

where A is the level of technology that assumed to be constant over time, K_t is the capital used by the firm, and L_t is given by

$$L_t = \sum_{j=1}^{J_R-1} N_{j,t} \int n_j(a,\eta,h)\eta\varepsilon_j \Phi_{t,j}(da \times d\eta \times dh). \tag{8}$$

2.4 Government

Government expenditures on goods and public pensions are financed by taxes on labour, capital and consumption. The fraction of output that is devoted to government consumption g_c is adjusted such that the government budget is balanced every period. Formally, the government budget constraint is given by

$$G_{c,t} + P_t = T_{n,t} + T_{c,t} + T_{k,t} \tag{9}$$

$$\text{with: } \begin{cases} G_{ct} = g_c Y_t \\ P_t = \sum_{j=j_R}^{J} N_{j,t} \int pp_t \Phi_{j,t}(da \times dh) \\ T_{kt} = \tau_k r_t K_t \\ T_{nt} = \tau \sum_{j=1}^{J_R-1} N_{j,t} \int w_t n_j(a,\eta,h)\eta\varepsilon_j \Phi_{j,t}(da \times d\eta \times dh) \\ T_{ct} = \tau_c \sum_{j=1}^{J} N_{j,t} \int c_j(a,\eta,h)\Phi_{j,t}(da \times d\eta \times dh) \end{cases}$$

3 Heuristic switching

In this paper, we take the view that agents form expectations by choosing between a number of simple rules, heuristics, according to their relative forecasting performance. We build upon the heuristic switching mechanism proposed by Brock and Hommes (1997) and extended in Anufriev and Hommes (2012)[3]. More specifically, economic agents have different heuristics at their disposal and they endogenously select the heuristic or forecasting rule that performed the best in previous periods. On a regular basis, individuals assess the predictive power of the heuristic they are currently using vis-à-vis the predictive power of the other rules. If the current rule performs well, the probability that an individual will keep on using the same rule is higher. If not, there is a higher probability that he or she will switch.

The different heuristics at the disposal of an individual to form expectations are the following[4]:

[3] For an in-depth analysis of the use of heuristics and heuristic switching, we refer the reader to De Grauwe (2012), Heemeijer et al. (2012) and Hommes (2006).

[4] In this section, we only provide the different heuristics for the evolution of the interest rate. Note that agents form

$$r^{1,e}_{t+s,t} = r^{1,e}_{t+s-1,t} + \psi_1 s^{-1}(r_t - r^{1,e}_{t,t-1}), \text{ (Adaptive)} \tag{10}$$

$$r^{2,e}_{t+s,t} = r^{2,e}_{t+s-1,t} + \psi_2 s^{-1}(r_t - r_{t-1}), \text{ (Trend)} \tag{11}$$

$$r^{3,e}_{t+s,t} = \frac{1}{\psi_3}\sum_{j=0}^{\psi_{3-1}} r_{t-j}, \text{ (Average)} \tag{12}$$

$$r^{4,e}_{t+s,t} = \sum_{j=0}^{\psi_4-1} \phi_j r_{t-j}, \text{ (Weighted Average)} \tag{13}$$

$$r^{5,e}_{t+s,t} = r^{5,e}_{t,t-1} + t^{-1}(r_t - r^{5,e}_{t,t-1}), \text{ (OLS)} \tag{14}$$

with $s \in \{1, ..., J-j\}$. In these equations, both actual realisations (r_t) and expected values ($r^{h,e}_{t+s,t}$) of the interest rate are given. The subscript t in r_t denotes the historical period t in which the realisation occurred. On the other hand, $r^{h,e}_{t+s,t}$ is the expectation at time t of the interest rate in period $t+s$ using heuristic h. For example, $r^{1,e}_{t,t-1}$ is the expectation at time $t-1$ using the first heuristic of the interest rate at time t.

The first heuristic boils down to an adaptive expectations approach to form forecasts. It states that the expectation of an individual regarding the evolution of the interest rate equals $r^{1,e}_{t,t-1}$, the expectation of the interest rate in the current period t made at time $t-1$, and a fraction $\psi_1 s^{-1}$ of the forecast error, i.e. the difference between the actual realisation of the interest rate r_t and $r^{1,e}_{t,t-1}$. The second one is a trend rule. Here, $r^{2,e}_{t+1,t}$ equals the actual realisation of the interest rate r_t plus a fraction $\psi_2 s^{-1}$ of the difference between the current and previous realisation of the interest rate. Agents expect higher interest rates in the future when the current interest rate r_t is higher than the previous interest rate r_{t-1} and vice versa. When ψ_1 and ψ_2 are low, individuals are less inclined to adjust their expectations. When ψ_1 and ψ_2 are high, individuals will be more inclined when adjusting their expectations. The third heuristic implies that the expected interest rate for the next period equals an unweighted average of the last ψ_3 realisations of the interest rate, while in the fourth heuristic $r^{4,e}_{t+1,t}$ is determined using a weighted average of the last ψ_4 realisations of the interest rate. Finally, the fifth heuristic is a recursive formulation of ordinary least squares (OLS) learning. Under this heuristic, agents run a least squares regression of interest rates on a constant. In other words, they form expectations about future interest rates by computing the sample mean $t^{-1}\sum_{s=0}^{t} r_s$.

Agents using heuristic h at time t use this heuristic to form $r^{h,e}_{t+s,t}$ ($s \in \{1, .., J-j\}$, J denoting the maximum age an individual can reach and j denoting the actual age), i.e. the expected values at time t of the interest rate and the real wage in the remaining periods of their life. In period $t+1$, they will update their expectations of these values as new information becomes available.

For the last three heuristics, Equations (12)–(14), we assume that the agents using these heur-

expectations about wages as well using the same heuristic, and the heuristics that they use to do so are equivalent to the ones stated in this section. Furthermore, these heuristics are based on the heuristics provided in Hommes (2014).

istics expect that the new value applies for the remainder of their life. Thus, for example, if the individual has at most three more periods to live after the current period, expectations for these periods held at time t are the following: $r^{h,e}_{t+1,t} = r^{h,e}_{t+2,t} = r^{h,e}_{t+3,t}$. In the next period, however, the updated expected values for the last two periods of life ($r^{h,e}_{t+2,t+1} = r^{h,e}_{t+3,t+1}$) might differ from the expected values for these periods at time t ($r^{h,e}_{t+2,t} = r^{h,e}_{t+3,t}$). In other words, agents assume that the expected value at time t applies for the remainder of their life. In time $t+1$, the expected value itself might change, but they still assume that this new expected value applies for the remainder of their life.

In total agents have five different heuristics at their disposal. Consistent with the literature on Heuristic Switching Models, the performance $\Theta_{t,h}$ of heuristic $h \in (1,2,3,4,5)$ is based on the squared prediction error of that specific rule in a specific period t:

$$\Theta_{t,h} = -\left(r_t - r^{h,e}_{t,t-1}\right)^2 + \upsilon\Theta_{t-1,h} \tag{15}$$

All that is left to be specified is the fraction of agents using a specific heuristic h. Using a discrete choice model, this is given by

$$\gamma_{h,t} = \xi\gamma_{h,t-1} + (1-\xi)\frac{\exp\left(k\Theta_{t-1,h}\right)}{\sum_{h=1}^{5}\exp\left(k\Theta_{t-1,h}\right)} \tag{16}$$

Here, $\gamma_{h,t}$ measures the fraction of individuals using heuristic h at period t. This means that $\sum_{h=1}^{5}\gamma_{h,t} = 1$. Furthermore, $\upsilon \in$[0,1] is a parameter measuring the memory of the economic agents. The lower υ, the less economic agents take past periods into account when comparing heuristics. Furthermore, $k \geq 0$ is the intensity of choice. The larger k, the faster agents switch between heuristics. The last parameter is $\xi \in [0,1]$, measuring inertia. If this parameter is high, economic agents switch less to other heuristics even if they clearly perform better. In other words, the habit of using a certain heuristic is stronger.

4 Timing

Each period, a number of decisions have to be made by the individuals populating the economy. These sequential steps are:

1. Given their expectations, $r^{h,e}_{t+k,t}$ and $w^{h,e}_{t+k,t}$, individuals decide on the amount of labour they want to supply to the labour market. They do so by maximizing their utility (2) with respect to the budget constraint (3).

2. Factor prices r_t and w_t are determined by the demand equations of the firm for capital and labour

$$r_t = \alpha A\left(\frac{L_t}{K_t}\right)^{(1-\alpha)} - \delta, \tag{17}$$

$$w_t = (1-\alpha)A\left(\frac{K_t}{L_t}\right)^{\alpha}. \tag{18}$$

These values of w_t and r_t do not change the value of n. Labour has already been supplied to the labour market.

3. The pension pp_t and the transfer Tr_{t+1} are determined by:

$$pp_t = b_{p,t}\left(\frac{\sum_{j=1}^{J_{R-1}} L_{j,t} \int w_t n(j,a,\eta,h)\eta\varepsilon_j(1-\tau)\Phi_{j,t}(da \times d\eta \times dh)}{\sum_{j=1}^{J_{R-1}} L_{j,t}}\right), \tag{19}$$

$$Tr_{t+1} = \left(\frac{\sum_{j=1}^{J}(1-\varphi_j)N_{j,t} \int a'(j,a,\eta,h)\Phi_{j,t}(da \times d\eta \times dh)}{\sum_{j=1}^{J} N_{j,t+1}}\right), \tag{20}$$

where $b_{p,t}$ is the net replacement rate for the individuals.

4. Having observed the actual realisation of the interest rate r_t and the wage w_t, individuals update their expectations $r^{h,e}_{t+s,t}$ and $w^{h,e}_{t+s,t}$, $s \in \{1, ..., J-j\}$, using Equations (10–14).

5. Individuals receive their labour income (based on n as determined in step 2) and capital income. Afterwards, they decide on c and a'.

6. They evaluate the heuristic they are applying using Equation (15). Afterwards, the heuristic switching takes place.

7. The capital market, the labour market and the goods market clear every period t:

$$K_{t+1} = \sum_{j=1}^{J} N_{j,t} \int a'(j,a,\eta,h)\Phi_{j,t}(da \times d\eta \times dh), \tag{21}$$

$$L_t = \sum_{j=1}^{J_{R-1}} N_{j,t} \int n(j,a,\eta,h)\varepsilon_j\eta\Phi_{j,t}(da \times d\eta \times dh), \tag{22}$$

$$Y_t = \sum_{j=1}^{J} N_{j,t} \int c(j,a,\eta,h)\Phi_{j,t}(da \times d\eta \times dh) + G_{c,t} + (K_{t+1} - (1-\delta)K_t). \tag{23}$$

8. Government policies $\{G_c, T_{n,t}, T_{k,t}, T_{c,t}\}$ are determined using Equation (9) and the fraction of Y_t used for government spending (g_c) is endogenously determined such that the government budget is balanced each period:

$$g_c = \frac{T_{n,t} + T_{c,t} + T_{k,t} - PP_t}{Y_t} \tag{24}$$

9. Y_t is determined by Equation (7).

10. $\Phi_{j+1,t+1} = Z_{j,t}(\Phi_{j,t})$ where $Z_{j,t}$ is the law of motion induced by the exogenous mortality rates, the exogenous Markov process for labour productivity, the endogenous asset accumulation and the heuristic switching regime.

5 Data and calibration

In this section, the parameterisation and calibration of the model is outlined in detail. The lion's share of the calibration is in line with the literature on quantitative OLG models with idiosyncratic risk. The model is calibrated to Belgium, a typical core country for the euro area, for the period 2000–2007, i.e. the years preceding the crisis that began in 2008.

5.1 Demographics

Agents enter the economy at the age of 18 (model age = 1), retire at the age of 66 (model age = 13) and live at most until the age of 94 years. Each period in the model lasts for four years. The conditional survival probabilities $\{\varphi_j\}$ are taken from the Human Mortality Database and are for 2000.

5.2 Technology and employment

The parameters regarding technology are $\{\alpha, A, \delta\}$. The capital share in production α equals 0.36. The level of technology A is constant and normalised such that the equilibrium real wage rate in the benchmark model w is equal to 1. The depreciation rate δ is calibrated using a target for the equilibrium annual real interest rate of $r = 4.5\%$.

5.3 Labour productivity shocks and parameters

In the model, an individual of age j and idiosyncratic shock η who works n hours will earn a pre-tax wage of

$$w n_j \eta \varepsilon_j. \tag{25}$$

We use the specification reported by Cournède and Gonand (2006) to calibrate the age-specific productivity profile ε_j. The resulting profile is hump-shaped.

The productivity shock η can take three values: $\eta \in \{\eta_1, \eta_2, \eta_3\}$. The Markov transition matrix is a (3x3)-matrix:

$$\Omega = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{bmatrix}, \tag{26}$$

where ρ_{ij} is the probability $Pr(j|i)$ to end up in state j in the next period given state i in the current period. Taking all this information together, the states of the Markov chain $\{\eta_1, \eta_2, \eta_3\}$ and the Markov transition matrix Ω still have to be determined. For the labour productivity states $\{\eta_1, \eta_2, \eta_3\}$ in the labour earnings process, we use a discretised Markov chain for a continuous

Table 1: Calibration summary

Weight on leisure in utility function	μ	0.6164	Average fraction of time spent working = 1/3
Discount factor in utility function	β	0.98	
Coefficient of risk aversion	θ	4	Conesa and Krueger (2006); Krueger and Ludwig (2013)
Level of technology	A	4.478	$w = 1$
Capital share in production	α	0.36	
Depreciation rate	δ	0.36	$r = 4.5\%$
Age-specific component of wages	ε_j		Cournède and Gonand (2006)
Tax rate on labour	τ	52.2%	Heylen and Van de Kerckhove (2013)
Tax rate on consumption	τ_c	13.4%	Heylen and Van de Kerckhove (2013)
Tax rate on capital	τ_k	26.8%	McDaniel (2007)
Net replacement rate pension	b_p	0.631	OECD, Pensions at a Glance (2005)

AR(1)-process with persistence ζ_s and variance σ_η^2. The persistence is chosen to be 0.969 and the variance 0.01 (Krueger and Ludwig, 2013).

The Markov transition matrix for the idiosyncratic productivity risk is then given by

$$\Omega = \begin{bmatrix} 0.8851 & 0.1113 & 0.0035 \\ 0.0557 & 0.8887 & 0.0557 \\ 0.0035 & 0.1113 & 0.8851 \end{bmatrix},$$

while the values for η_1, η_2 and η_3 are respectively 0.6029, 1 and 1.6587.

5.4 Preferences

The instantaneous utility function of the individuals is given by Equation (2)[5]. The parameters to be calibrated are $\{\beta, \mu, \theta\}$. As in Conesa and Krueger (2006) and Krueger and Ludwig (2013), θ is chosen to be 4. The relative weight on leisure μ, on the other hand, is determined such that Belgian employed individuals work on average $1/3$ of their time. Using the values of μ and θ, a coefficient of relative risk aversion of approximately 2 is obtained. Finally, β is set to 0.98.

5.5 Fiscal policy variables

The government in the model finances spending on goods and PAYG-pensions with taxes on consumption, capital and labour. For the tax rates τ_c and τ, we use the same data as Heylen and Van de Kerckhove (2013). For details on the construction of these fiscal policy variables, we refer to Heylen and Van de Kerckhove (2013, their Appendix 1). The value for the capital tax rate τ_k is determined using the tax series constructed by Cara McDaniel. we use the average between 2000–2007[6]. Regarding the basic PAYG-pension received by the retired households in the model, we use data on the average net replacement rate after retirement obtained from the OECD (Pensions at a Glance, 2005).

[5] This functional form is often used in the quantitative OLG literature with idiosyncratic risk: see e.g. Conesa and Krueger (2006) and Krueger and Ludwig (2013).

[6] The updated tax series can be downloaded from www.caramcdaniel.com/researchpapers. The methodology is discussed in McDaniel (2007).

6 Simulations

The goal of this paper is go beyond the rational expectations paradigm and use heuristics to study the transitional dynamics following fiscal policy shocks. In this section, we explore the effects of an unanticipated permanent labour tax decrease financed by government spending.

To illustrate the general properties of the model, we present the results under two alternative choices for the discount factor β and the coefficient of risk aversion θ: the benchmark environment where $\beta = 0.98$ and $\theta = 4$, and an alternative environment where $\beta = 0.86$ and $\theta = 2$. In the latter environment the heuristics switching model is able to produce constant or dampened oscillations in the endogenous variables – a finding that is in line with the results of other researchers who have examined heuristic switching in asset pricing and OLG models (see Agliari et al., 2016, Anufriev and Hommes, 2012, Bullard, 1994, Gaunersdorfer et al., 2008, Schönhofer, 1999, and Tuinstra and Wagener, 2007, for example). We also examine the sensitivity of the transitional dynamics with respect to several of the heuristics' parameters. All the results discussed in the following subsections are for the scenario in which τ is reduced from 52.2% to 42.2%.

6.1 Results for $\beta = 0.98$ and $\theta = 4$

In this section, we discuss the results for the environment with very common values for the discount factor and the coefficient of risk aversion, i.e. $\beta = 0.98$ and $\theta = 4$. To gain intuition it is useful to consider three cases: homogeneous expectations, heterogeneous expectations with constant fractions, and heuristic switching. In the first case, we assume that economic agents have only one of the available forecasting heuristics at their disposal. It it useful to first investigate the dynamics of these different homogeneous expectations models and compare them with the rational expectations outcome before turning to heterogeneity in expectations. Next, we consider the case of heterogeneous expectations with constant fractions. In this case several forecasting heuristics are used, but the fractions of agents using different forecasting heuristics stay constant. Thus, at this point there is no heuristic switching. Finally, in the third case, we allow the fractions of the heuristics to change over time based upon relative past performance, i.e. we assume that agents evaluate the performance of the heuristic they are currently using. According to its relative performance to other heuristics, individuals might switch to a different heuristic.

Homogeneous expectations. We begin by investigating the dynamics under homogeneous forecasting rules. The transitional dynamics of output, the real wage rate, and the forecast errors for wages are shown in Figure 1[7]. The heuristics' parameters ψ_1 and ψ_2 for the Adaptive and Trend heuristic are set to 0.65, which corresponds to the gain in the adaptive heuristic used in Anufriev and Hommes (2012). We set $\psi_3 = \psi_4 = 5$, which means that individuals using the Average or Weighted Average heuristic take an average over the past 20 years. The weights ϕ_j are determined using the formula $\phi_j = \frac{(2\psi_4 - 1) - 2j}{\psi_4^2}$. When $\phi_4 = 5$ this yields the following weights: 9/25 for the current period, 7/25 for the previous period, and 5/25, 3/25 and 1/25 for the other periods.

[7] The permanent decrease in the labour tax occurs in period 1. The value for output in the old steady state is 16.6. The initial steady state level for the pre-tax wage is 1.

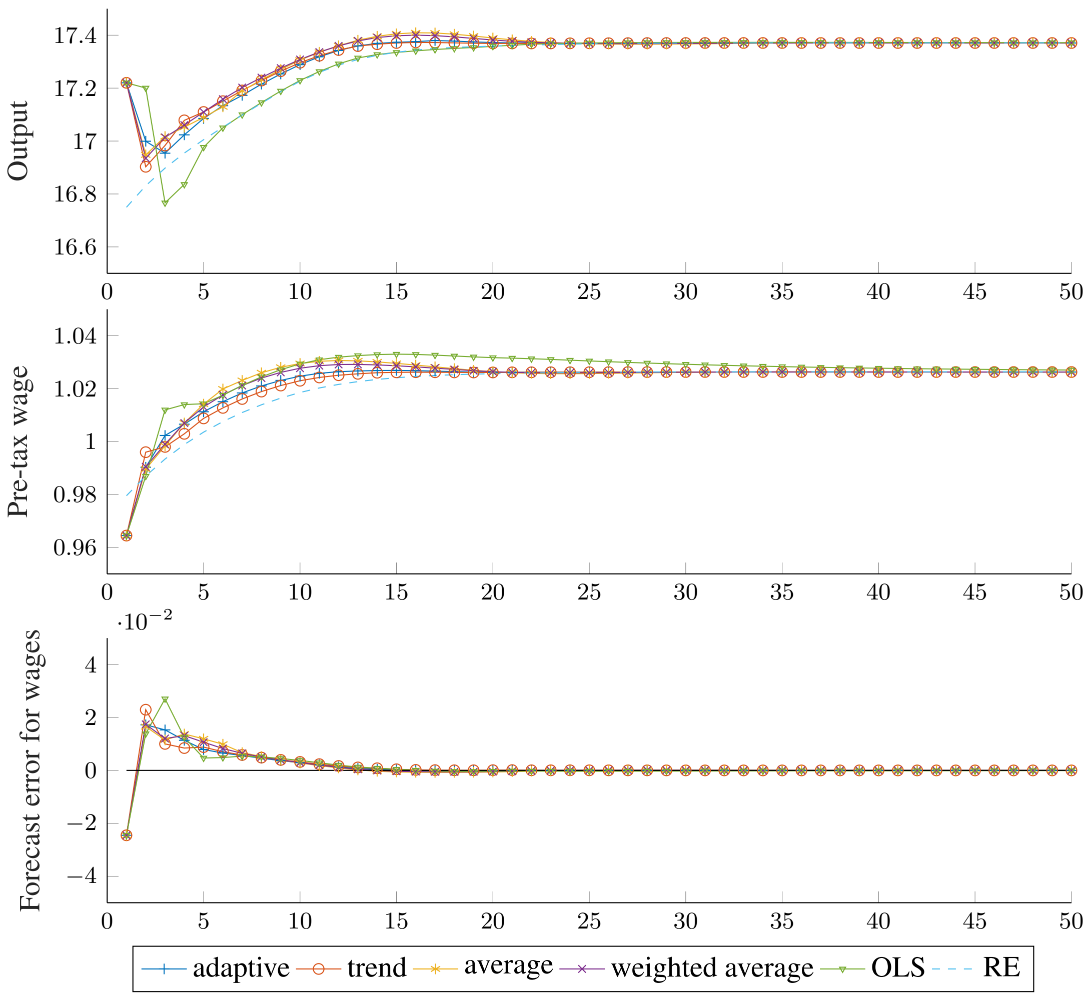

Figure 1: Transitional dynamics after a labour tax decrease under homogeneous expectations for the benchmark parameterisation $\beta = 0.98$ and $\theta = 4$.

As the top panel of Figure 1 shows, the difference between output in the rational expectations (RE) case (dashed line) and the heuristic scenarios in the first period is most pronounced. In the rational expectations case, the value for w_t is lower than the value in the old steady state due to the higher labour supply. For the different heuristics, however, the expectation for the wage equals the value for the real pre-tax wage rate in the old steady state. As the individuals base their labour supply on their expectation for w, they supply more labour relative to the rational expectations case leading to a higher level of output. Afterwards, output decreases as employment decreases, but the increase in K_t counteracts this decrease to some extent. The transitional dynamics indicate that substantial output gains can be achieved compared to the rational expectations case. In all heuristic cases, output overshoots its rational expectations counterpart at impact. In the first years after the shock, the expectations based on OLS learning are adjusted strongly. This leads to significant deviations from the RE outcome in the beginning of the transition period, but a much faster convergence to the new steady steady compared to the other heuristics. For the other heuristics, in particular when the Average and Weighted Average heuristics are used, output remains above its new steady state level for a long time.

Unsurprisingly, the dynamics of the actual real wage rate are closely related to the heuristic used to form expectations. At first, the real wage rate is lower than its RE counterpart, but it quickly catches up. The bottom panel shows the forecast errors for wages. The forecast error is negative when the expected value is bigger than the actual value and positive otherwise. This is the case in the first period as all forecasting heuristics over-estimate future wages. The forecast errors change in sign in the second period and then gradually converge to zero.

Most importantly, these panels indicate that even for very common parameter values for β and θ the resulting dynamics, both for output and wages but also for all the other variables, are substantially different from the dynamics in the rational expectations case. Second, they show that the resulting dynamics depend on the heuristic that is used. Thus, there is a certain degree of variation in the dynamics.

Heterogeneous expectations without heuristic switching. Before turning to the heuristic switching model, it is useful to analyse heterogeneity in expectations in the absence of heuristic switching. This corresponds to setting the inertia parameter ξ in Equation (16) equal to one. The transitional dynamics for this case are given in Figure 2. The solid lines depict the dynamics for eight different distributions $\{\gamma_{h,0}\}$, $1 \leq h \leq 5$, over the heuristics, i.e. fractions of individuals using the different heuristics. These are given in Table 2. When one or more heuristics are used more extensively than others, this could lead to different dynamics compared to more equal distributions over the heuristics. For example, in $\Gamma_{1,0}$ all agents use the Adaptive heuristic to form expectations. As Figure 2 shows, different distributions over the heuristics have a significant effect on the evolution of output and wages along the transition path.

Heterogeneous expectations with heuristic switching. The transitional dynamics under heuristic switching are given in Figure 3. Each line corresponds to a different initial distribution over the heuristics (see Table 2). The figure plots the dynamics for the following parameter values gov-

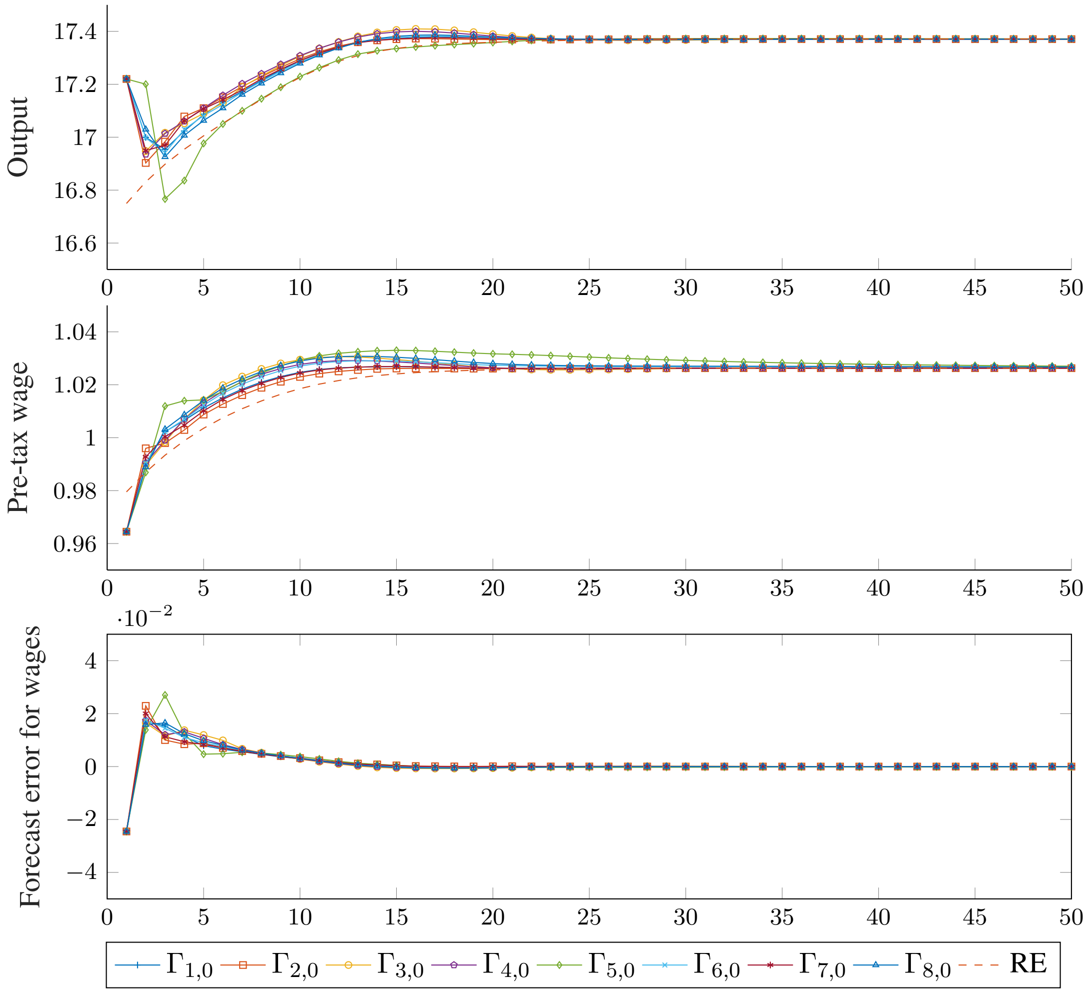

Figure 2: Transitional dynamics after a labour tax decrease under heterogeneous expectations without heuristic switching for the benchmark parameterisation $\beta = 0.98$ and $\theta = 4$. The solid lines depict the dynamics under heterogeneous expectations for the distributions over the heuristics given in Table 2. The dashed lines depict the dynamics under rational expectations.

Table 2: Different initial distributions over the heuristics.

	$\Gamma_{1,0}$	$\Gamma_{2,0}$	$\Gamma_{3,0}$	$\Gamma_{4,0}$	$\Gamma_{5,0}$	$\Gamma_{6,0}$	$\Gamma_{7,0}$	$\Gamma_{8,0}$
$\gamma_{1,0}$ (Adaptive)	1	0	0	0	0	1/5	1/2	0
$\gamma_{2,0}$ (Trend)	0	1	0	0	0	1/5	1/2	0
$\gamma_{3,0}$ (Average)	0	0	1	0	0	1/5	0	1/3
$\gamma_{4,0}$ (Weighted Average)	0	0	0	1	0	1/5	0	1/3
$\gamma_{5,0}$ (OLS)	0	0	0	0	1	1/5	0	1/3

erning the switching mechanism: $k = 0.4$, $\xi = 0.9$, and $\upsilon = 0.7$. These values correspond to the benchmark values of Agliari et al. (2016).

Comparing Figure 3 with Figure 2, we observe that under heuristic switching the transitional dynamics are less affected by the initial distribution of heuristics compared to the case without switching. The differences between the different heuristic scenarios is smaller and both output and the wage rate converge more quickly to the new steady state. Agents switch between different heuristics based upon their relative past forecasting performance and this leads to an equal distribution over the heuristics, even if the initial distribution is very unequal. Figure 4 shows that, for all initial distributions considered, the impacts of the different heuristics gradually converge to an equal distribution. Thus, in a framework without rational expectations and with heuristic switching, the effects of a labour tax decrease become much more predictable and monotonic even though a lot of individuals have different ways of forming expectations and even considering different initial distributions. The heuristic switching framework enables policy makers to better anticipate the effects of tax changes.

The welfare effects along the transition path for the different heuristics are depicted in Figure 5. The effects are calculated using the Consumption Equivalent Variation (CEV) measure

$$CEV_t = \left[\frac{V_{h,t}\,(j=0,a,\eta,h)}{V_{RE,t}\,(j=0,a,\eta)}\right]^{\frac{1}{(1-\mu)(1-\theta)}} - 1,$$

where $V_{h,t}\,(j=0,a,\eta,h)$ is the expected lifetime utility of a newborn at time t using heuristic $h \in (1,2,3,4,5)$, and $V_{RE,t}\,(j=0,a,\eta)$ is the expected lifetime utility of a newborn under rational expectations. This indicator measures the percentage change in consumption that would make the expected lifetime utility of a newborn under rational expectations equivalent to the expected lifetime of a newborn using heuristic h in the heuristic switching model. Figure 5 shows that in the first decades after the labour tax reduction, welfare in the heuristic switching model is lower than under rational expectations, irrespective of the heuristic used. These negative welfare consequences arise because labour supply is too high as wage expectations at impact are equal to the high pre-tax wage rate from the old steady state. In subsequent periods, wage expectations adjust and welfare improves. For agents using the OLS heuristic, however, welfare remains low compared to the rational expectations outcome and only catches up slowly.

The main message here is that after the heuristic switching regime has been activated, the variation in the transitional dynamics decreases significantly, meaning that the transitional effects of a decrease in labour taxes become more predictable and monotonic over all initial distributions considered. Thus, allowing for alternative expectations and a lot of heterogeneity in terms of initial fractions does not lead to a wide range of possible transitional paths, but decreases in fact the range in which the transitional dynamics are located.

Figure 6 illustrates that the transitional dynamics under heuristic switching do not change much for alternative values for heuristics' parameters. The left panel depicts the transitional dynamics of output for different values of the gain parameter ψ_1 in the Adaptive heuristic – see Equation (10). A higher (lower) gain corresponds to a stronger (weaker) adjustment of the ex-

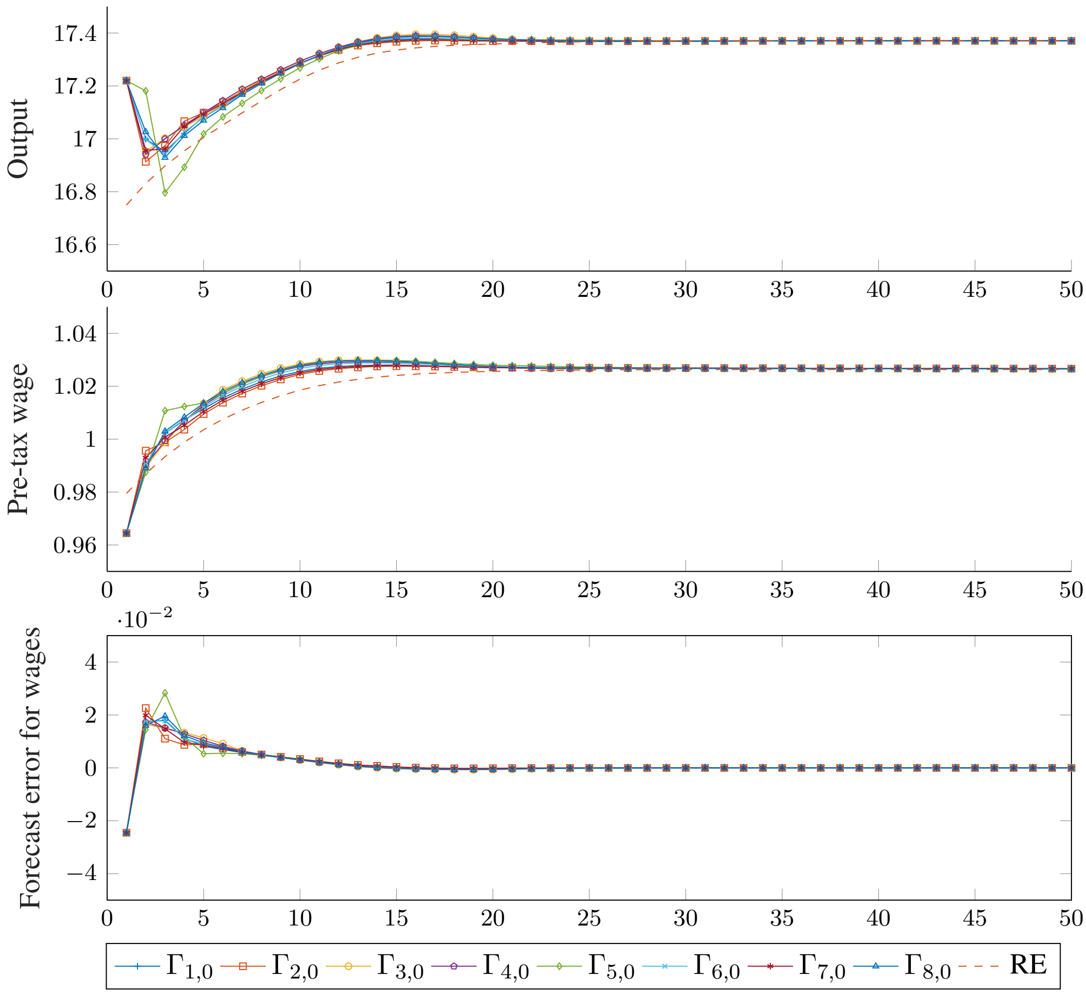

Figure 3: Transitional dynamics after a labour tax decrease under heuristic switching for the benchmark parameterisation $\beta = 0.98$ and $\theta = 4$. The solid lines depict the dynamics under heuristic switching for the initial distributions over the heuristics given in Table 2. The dashed lines depict the dynamics under rational expectations.

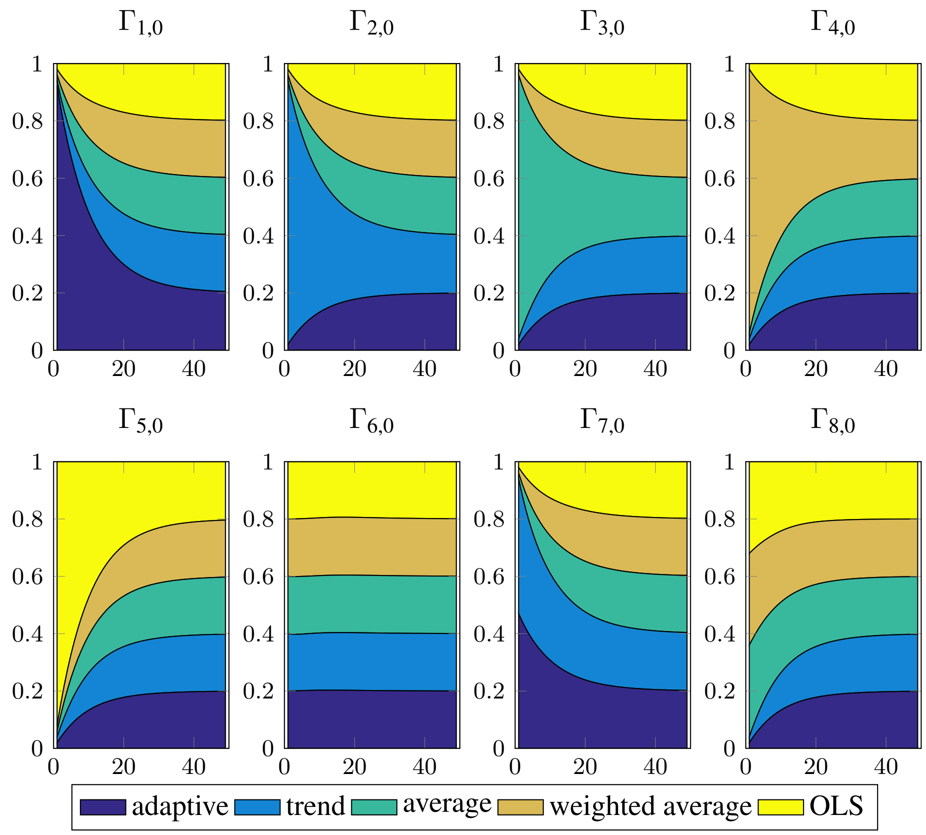

Figure 4: Evolution of the distribution over heuristics when $\beta = 0.98$ and $\theta = 4$ for the initial distributions given in Table 2.

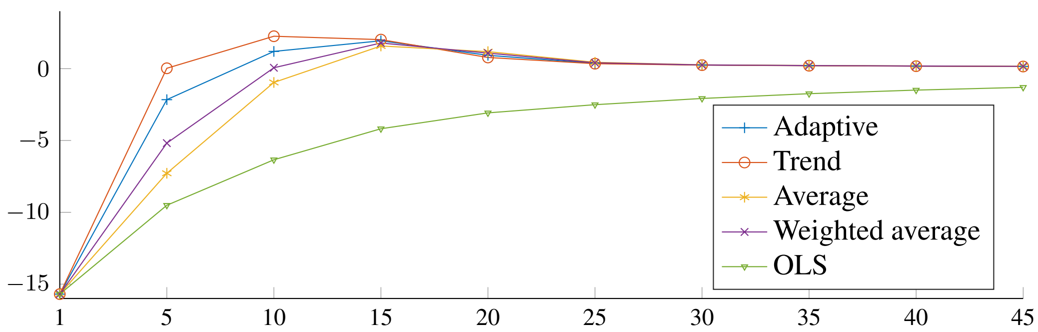

Figure 5: Welfare effects for different heuristics measured in consumption-equivalent variation relative to rational expectations equilibrium. The lines depict the effects for newborn agents along the transition path under heuristic switching. Parameter values are $\beta = 0.98$ and $\theta = 4$.

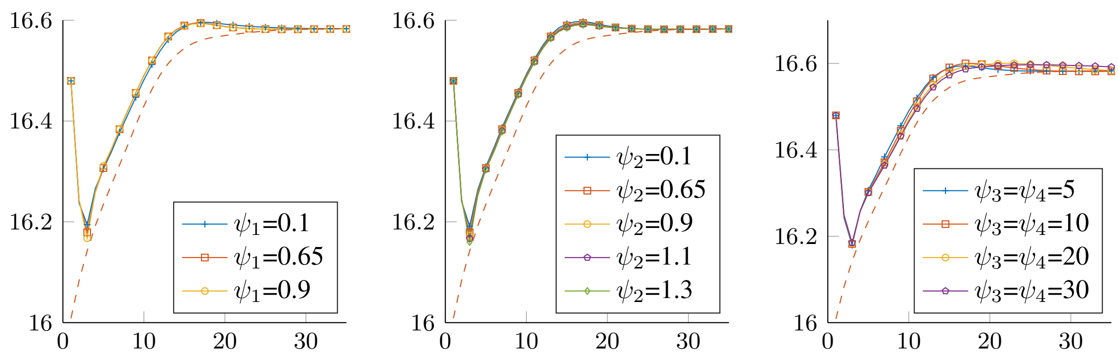

Figure 6: Sensitivity of the results to different values of the heuristics' parameters under the benchmark parameterisation $\beta = 0.98$ and $\theta = 4$. The solid lines depict the transitional dynamics of output under heuristic switching for different values of the gain parameters ψ_1 and ψ_2, and the parameters ψ_3 and ψ_4 governing the number of periods considered for calculating the averages in (12) and (13). The initial distribution over the heuristics is $\Gamma_{0,6}$. The dashed lines depict the dynamics under rational expectations.

pectations in the direction of the last observation. The middle panel shows the output dynamics for different values of ψ_2. A large (small) value for ψ_2 corresponds to a relatively strong (weak) extrapolation of the trend in the Trend heuristic – see Equation (11). The largest value for ψ_2 (i.e. 1.3) corresponds to the value of the gain in the "strong trend-following rule" considered by Anufriev and Hommes (2012). Finally, the right panel considers different horizons for the Average and Weighted Average heuristics, i.e. different values for ψ_3 and ψ_4. We have also experimented with other values for the memory parameter (υ), the intensity of choice parameter (k), and the inertia parameter (ξ) and find that the results do not change much[8].

6.2 Results for $\beta = 0.86$ and $\theta = 2$

Several studies have shown that complex dynamics, such as dampened or permanent oscillations around a locally stable steady state, can arise in Overlapping Generations models with heterogeneous expectations (see Bullard, 1994, Schönhofer, 1999, and Tuinstra and Wagener, 2007, for example). In this section, we illustrate this in the context of our model when $\beta = 0.86$ and $\theta = 2$.

In the top panel of Figure 7, we include the transitional dynamics for output in the heterogeneous expectations model without heuristic switching when $\beta = 0.86$ and $\theta = 2$ for different (fixed) distributions over the heuristics as given in Table 2. For all distributions, the transitional dynamics oscillate in the first periods of the transition path. The oscillations are very large when a large fraction of agents use the Trend heuristic (cf. $\Gamma_{0,2}$ and $\Gamma_{0,7}$). When only the Average or the Weighted Average heuristic is used, oscillations are dampened but persist for a considerably long time (cf. $\Gamma_{0,3}$ and $\Gamma_{0,4}$). On the other hand, when the OLS learning heuristic is used, convergence occurs much faster (cf. $\Gamma_{0,5}$, $\Gamma_{0,6}$ and $\Gamma_{0,8}$). The variation in the dynamics across distributions is thus very big.

[8]We have considered the following values for the different parameters: $\upsilon = \{0, 0.25, 0.5, 0.7, 1\}$, $k = \{0, 0.4, 0.8, 1.2\}$, $\xi = \{0, 0.25, 0.5, 0.75, 0.9, 1\}$.

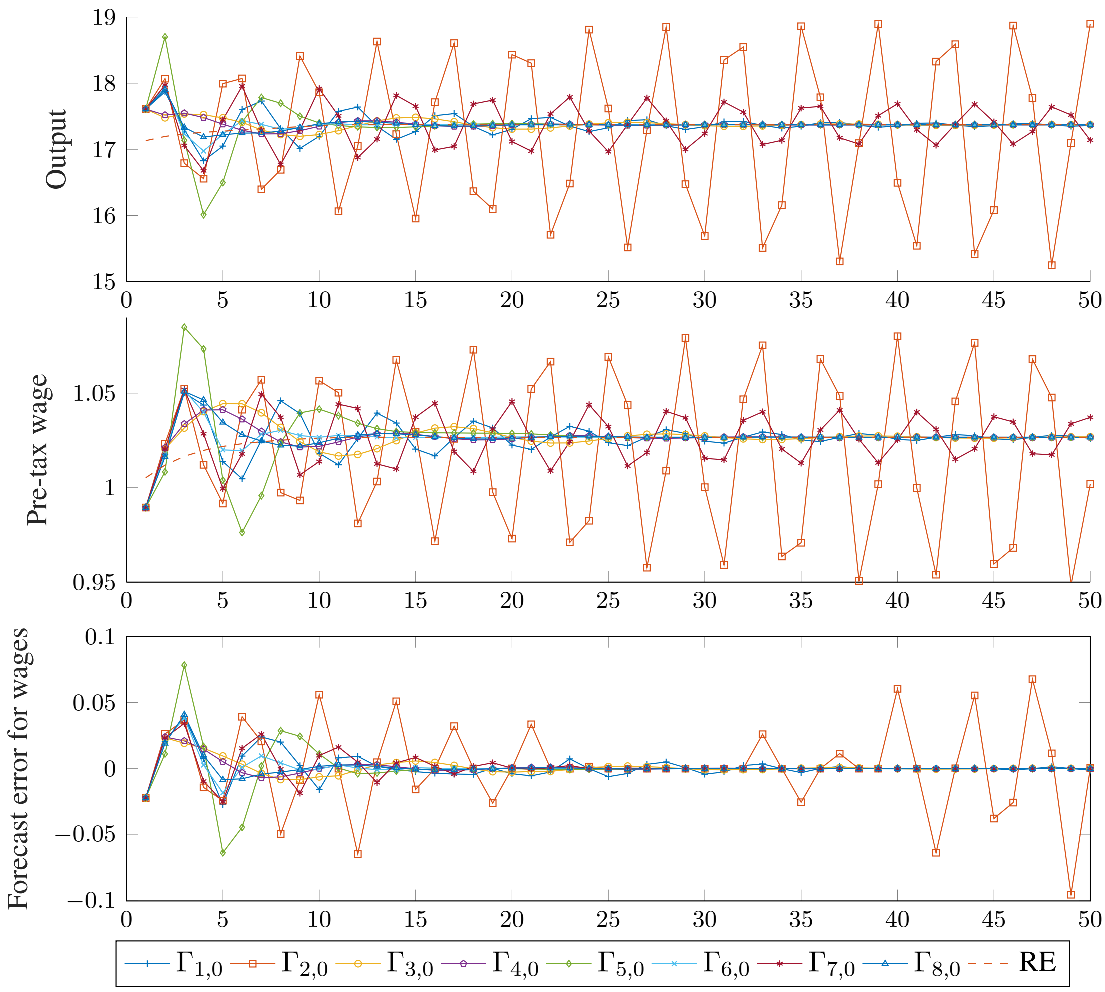

Figure 7: Transitional dynamics after labour tax decrease under heterogeneous expectations without heuristic switching for the alternative parameterisation $\beta = 0.86$ and $\theta = 2$. The solid lines depict the dynamics for the different distributions over the heuristics given in Table 2. The dashed lines depict the dynamics under rational expectations.

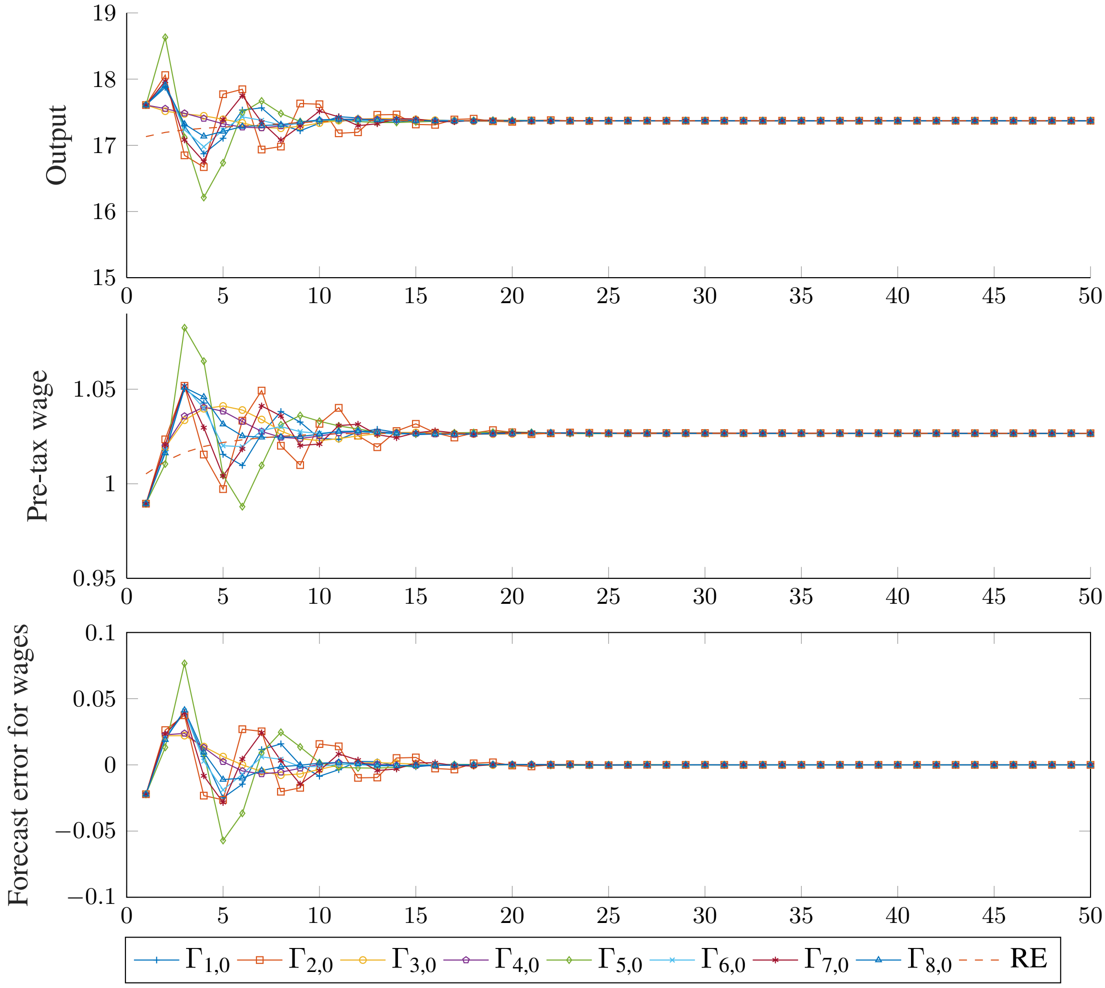

Figure 8: Transitional dynamics after labour tax decrease under heuristic switching for the alternative parameterisation $\beta = 0.86$ and $\theta = 2$. The solid lines depict the dynamics for the different initial distributions over the heuristics given in Table 2. The dashed lines depict the dynamics under rational expectations.

With heuristic switching, the conclusion is different. Figure 8 reveals that the heuristic switching approach has a stabilising effect on the dynamics. For all initial distributions over the heuristics considered, the heuristic switching stabilises the dynamics and the transitional dynamics converge to the steady state. Figure 9 shows that this finding is robust to different choices of the heuristics' parameters. Larger gains ψ_1 in the Adaptive heuristic and stronger extrapolation values ψ_2 in the Trend heuristic amplify the oscillations, but the series still exhibit convergence to the new steady state. Larger values for ψ_3 and ψ_4 in the Average and Weighted Average heuristics also slow down convergence. As in Section 6.1 we have also found that the results are very similar for alternative values for the memory parameter (υ), the intensity of choice parameter (k), and the inertia parameter (ξ)[9].

Gaunersdorfer et al. (2008) and Agliari et al. (2016) have shown that in asset pricing models

[9]We have considered the parameter values given in footnote 8.

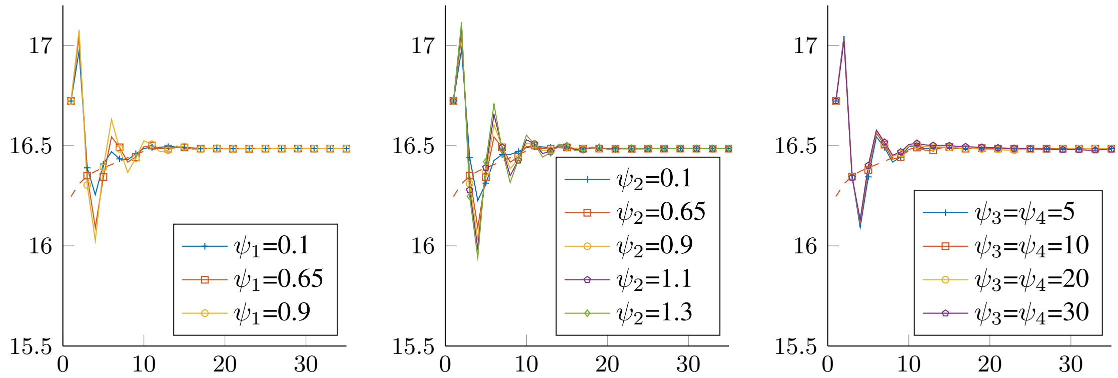

Figure 9: Sensitivity of the results to different values of the heuristics' parameters under the alternative parameterisation $\beta = 0.86$ and $\theta = 2$. The solid lines depict the transitional dynamics of output under heuristic switching for different values of the gain parameters ψ_1 and ψ_2, and the parameters ψ_3 and ψ_4 governing the number of periods considered for calculating the averages in (12) and (13). The initial distribution over the heuristics is $\Gamma_{0,6}$. The dashed lines depict the dynamics under rational expectations.

with heuristic switching permanent oscillations around a locally stable steady state may emerge due to strong trend extrapolators. Here we investigate if these dynamics can also occur in the context of our model. For this exercise we follow Agliari et al. (2016) and specify the Adaptive and Trend heuristics as follows:

$$r^{1,e}_{t+s,t} = r^{1,e}_{t+s-1,t} + \psi_5(r_t - r^{1,e}_{t,t-1}), \text{ (Adaptive 2)} \tag{27}$$

$$r^{2,e}_{t+s,t} = r^{2,e}_{t+s-1,t} + \psi_6(r_t - r_{t-1}), \text{ (Trend 2)} \tag{28}$$

where $\psi_5 = 0.65$ and $\psi_6 = 1.3$. Compared to our original specification given in Equations (10) and (11), the updating terms between brackets are not discounted in multi-period ahead forecasts[10]. Consistent with the analysis of Agliari et al. (2016) we set the persistence parameter $\xi = 0$. Figure 10 shows that under this parameterisation with a strong trend-following heuristic (i.e. $\psi_6 = 1.3$) the dynamics under heuristic switching lead to an invariant closed circle around the steady state[11]. The attractor in the left panel of Figure 10 shows the long run behaviour of output under heuristic switching. The middle and right panel show the time series of output and the fractions of the Adaptive heuristic (27) and Trend heuristic (28) for 50 periods. Thus, we can conclude that for certain parameterisations complex dynamics such as damped oscillations and persistent endogenous cycles can emerge in our model.

[10]In the working paper version of this paper we added the heuristics (27) and (28) to the pool of heuristics and reached the same general conclusions as reported here.

[11]For the sake of consistency with the approach adapted in the preceding sections, the agents are still allowed to switch between five forecasting heuristics, i.e. (12)–(14), (27), and (28). We have also simulated the model with only the latter two heuristics and found that permanent oscillations also emerge in that set-up.

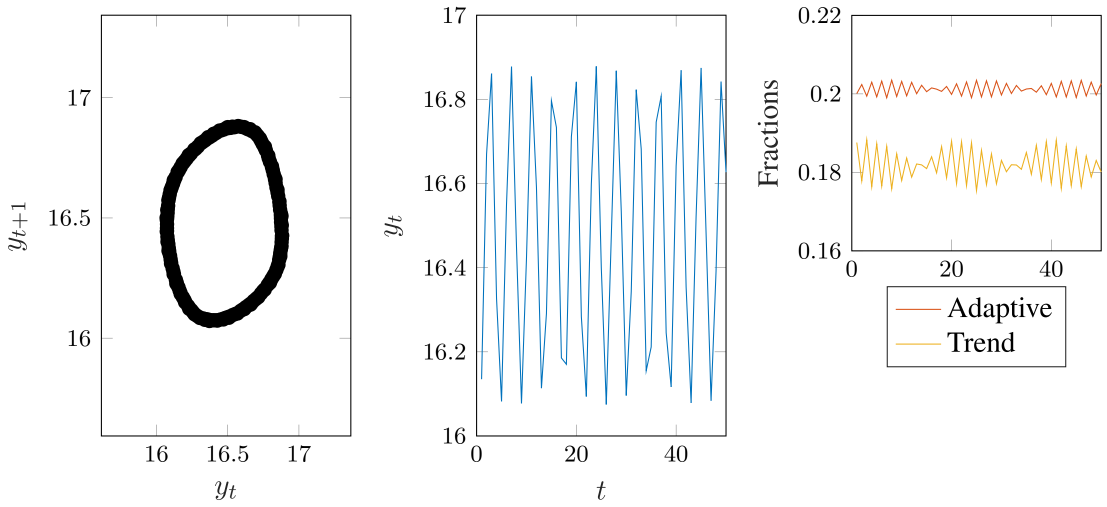

Figure 10: Attractor and time series for the heuristic switching model with strong trend extrapolators, i.e. $\psi_5 = 1.3$. Parameter values are $\beta = 0.86$, $\theta = 2$, and $\xi = 0$.

7 Conclusion

This paper explores the stability and transitional dynamics of an Overlapping Generations model with heuristic switching. Accordingly, this paper contributes to a growing literature that goes beyond the rational expectations (RE) paradigm. We assume that agents use simple rules, heuristics, to forecast the future course of the interest rate and the real wage. Agents use these heuristics because, in general, they do not possess the cognitive abilities as assumed by the RE literature nor to act as econometricians. Instead, the agents have a certain number of different heuristics at their disposal. On a regular basis, they assess the predictive power of the heuristic they are currently applying. If it performs well, the probability that agents will use the same heuristic in the next period will be higher. If it does not perform well, there is a higher probability that they switch to another rule.

The aim of this paper is to contribute to the literature that goes beyond the rational expectations paradigm by exploiting the heuristic switching approach within an Overlapping Generations model. Triggered by a fiscal policy shock, the objective is to study the transitional dynamics of the model for a large number of settings including one or multiple heuristics and compare the behaviour of the dynamics with their rational expectations counterpart.

The simulations lead to three main findings. First, in a context without heuristic switching (i.e. a context where individuals only have one heuristic at their disposal to form expectations), the evolution of transitional dynamics can be substantially different from the rational expectations case, especially in the first periods of the transition. Furthermore, there is a lot of variation in the dynamics over different parameter values and heuristics. This finding implies that if only one heuristic is used, the macroeconomic impact of fiscal policy is highly sensitive to the heuristic being used. What is more, as the discount rate and the degree of risk aversion decrease, the corresponding transitional dynamics may oscillate permanently around the new steady state under rational expectations. Second, after activating the heuristic switching regime, the variation in the transitional dynamics decreases significantly. Consequently, the sensitivity of the transitional effects of fiscal policy is much lower now and its exact impact is thus less uncertain for policy makers. Third and last, the heuristic switching has a stabilising effect on the transitional dynamics. For certain distributions over the heuristics for which the dynamics exhibit permanent oscillations in the absence of heuristic switching, the dynamics now converge to the steady state in most cases.

These findings are important. They imply that allowing individuals to choose from a wide range of forecasting rules is actually a better option than constraining them to use only one forecasting rule. It allows them to select the better performing rules, a feature that not only enhances the stability of the model, but reduces the uncertainty in the transitional dynamics as well.

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