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Learning and the Size of the Government Spending Multiplier

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Abstract

This paper examines the government spending multiplier when economic agents combine adaptive learning and knowledge about future fiscal policy to form their expectations. The analysis shows that the effects of a government spending shock substantially change when the rational expectations hypothesis is replaced by this learning mechanism. In contrast to the dynamics under rational expectations, a government spending shock in a small-scale new Keynesian DSGE model with learning crowds *in* private consumption and is associated with a positive co-movement between real wages and hours worked. In the baseline calibration, the output multiplier under learning is above one and about twice as large as under rational expectations.

Keywords: adaptive learning, DSGE, fiscal multipliers, government spending

1 Introduction

Since the outbreak of the global financial crisis in 2008, countries around the world have tried to fight the recession with a series of fiscal policy measures. Many governments have adopted a broad range of expansionary measures such as large tax cuts, boosts in direct spending and various investment programmes. Conversely, other countries have embarked on fiscal austerity measures, because of concerns about the sustainability of public finances. This revival of fiscal policy has renewed the debate on the effects of discretionary fiscal policy.

A central issue in this debate is the size of the government spending multiplier. Although the empirical estimates are dispersed over a broad range, the estimates are in many cases higher than those found in theoretical business cycle models. Based on a comprehensive review of the empirical literature Ramey (2011) concludes that the multiplier is probably between 0.8 and 1.5. Moreover, several studies find a large positive response of private consumption to a government spending shock – see for example Blanchard and Perotti (2002), Fatás and Mihov (2001), Galí et al. (2007), and Perotti (2008). This stands in sharp contrast with the crowding *out* of consumption and the much smaller multipliers in most theoretical models.

An important question is to what extent these small multipliers depend on the hypothesis of rational expectations. This paper addresses this question by comparing the rational expectations benchmark with

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a model where agents form expectations using an adaptive learning mechanism and knowledge about future fiscal policy.

The hypothesis of rational expectations presumes that agents fully oversee the structure of the model and do not face any computational limitations in deriving expectations. In contrast, the adaptive learning approach discussed in Evans and Honkapohja (2001), introduces a more plausible view of rationality. It is a convenient approach to model bounded rationality by assuming that agents form expectations based on estimated forecasting models and update the coefficients in these models over time as new data becomes available.

It seems very natural to assume that agents have no perfect knowledge concerning the general equilibrium effects of fiscal policy. On the other hand, they presumably understand the direct implications for their current and future after-tax incomes. Therefore we follow the approach of Evans et al. (2009) and assume that agents understand the future path of taxes and other fiscal instruments implied by the government financing structure, while using infinite-horizon learning to forecast other variables.

This paper extends the existing literature by assessing the role of this learning set-up for the effects of government spending shocks in a new Keynesian DSGE model. The key result is that the multiplier under learning is about twice as large as under rational expectations. Hence, this paper provides a theoretical argument for the large multipliers in the recent empirical literature. Moreover, in contrast to the dynamics under rational expectations, government spending crowds *in* private consumption when agents engage in learning behaviour. The intuition for this result is that, in the learning model, households' incomplete structural knowledge results in excessive optimism or pessimism regarding the general equilibrium effects of future government spending. In particular, agents are more optimistic about future wages and more pessimistic about future interest rates than under rational expectations which leads to a positive effect on current consumption.

The analysis confirms the importance of price rigidity and the complementarity between labour and consumption for explaining the positive consumption response, as emphasized by Bilbiie (2011) and Christiano et al. (2011), for example. However, under rational expectations these features alone are not sufficient for government spending to crowd *in* private consumption in our model. Only under adaptive learning does consumption rise after the fiscal shock, resulting in a multiplier greater than one, even if price rigidity is limited and the degree of consumption-labour complementarity is small. Another result is that learning is crucial for generating a positive co-movement between hours worked and real wages after a government spending shock.

This paper also considers different fiscal financing strategies in an extended version of the model with distortionary taxes. Not surprisingly, the government spending multipliers are substantially smaller, or even negative, when government spending is financed through capital or labour income taxes. However, the output and consumption multipliers under learning are always larger than under rational expectations, irrespective of the financing strategy.

The work presented here is related to several other papers that build on the learning set-up of Evans et al. (2009). All these contributions emphasize the substantial differences between the responses to fiscal policy changes under learning and under rational expectations. Mitra et al. (2013) consider permanent policy changes in a real business cycle (RBC) model where agents also have to estimate the new steady state values. The authors show that under learning oscillatory dynamics can emerge and that the effects under learning depend on the specific form of the policy change. Recently, Mitra et al. (2016) have analysed the effects of a surprise two-year increase in government spending. An interesting result is that their learning model can generate a hump-shaped response in consumption. Gasteiger and Zhang (2014) study the impact of fiscal policy in a deterministic version of the RBC model with distortionary taxation.

This paper generalizes the analysis of the cited works by examining the dynamics in a new Keynesian DSGE model with commonly used model features such as imperfect competition, price rigidity, and capital adjustment costs. The paper shows that these model features crucially affect the impact of adaptive learning on the dynamics of a government spending shock, in particular when it comes to the degree of price rigidity. The importance of these features is also emphasized in a recent contribution by Evans et al. (2016). The authors examine the possibility of stagnation in a new Keynesian model when the zero lower bound on the nominal interest rate is binding. They show that, under learning, pessimistic expectations can push the economy into recession. The results presented here, apply to "normal times" where the central bank maintains a standard Taylor rule.

The remainder of the paper is organized as follows. Section 2 sets forth the DSGE model that will be used throughout the paper. Section 3 defines the rational expectations quilibrium. The adaptive learning mechanism is set out in Section 4. In Section 5, the effects of a temporary increase in government spending in the learning model are compared with the effects under rational expectations. A distinction is made between a neoclassical specification with fully flexible prices and a new Keynesian specification of the model. The role of learning for the government spending multipliers is discussed in Section 6. As an extension, Section 7 adds a richer specification of fiscal policy to the baseline model and discusses the role of different financing strategies. The last section concludes.

2 The Model economy

This section briefly describes the new Keynesian DSGE model that we will use in this paper. The model is based on the standard sticky-price framework analysed, for instance, in Woodford (2003) and Christiano et al. (2011). More elaborate specifications can be found in Smets and Wouters (2003, 2007) and Christiano et al. (2005).

The economy is populated by a representative household, a perfectly competitive final goods producer, a continuum of monopolistically competitive intermediate goods producers, a central bank, and a fiscal authority.[1]

Household The representative household maximizes expected lifetime utility. Preferences are defined over consumption, C_t, and hours worked, N_t, and described by the following utility function:

$$E_0^* \sum_{t=0}^{\infty} \beta^t \frac{\left[C_t^{\phi}(1-N_t)^{1-\phi}\right]^{1-\sigma} - 1}{1-\sigma}, \tag{1}$$

with $\beta \in (0,1)$, $\sigma > 0$, $\sigma \neq 1$, and $\phi \in (0,1)$.[2] Here $E_t^*(\cdot)$ denotes the subjective expectations of the household at time t. We consider King, Plosser and Rebelo (1988) preferences, which is standard in business cycle analysis. If $\sigma > 1$ consumption and labour are complements, which is an important model

[1] Appendix A contains the derivations of the model equations.

[2] Money is not included explicitly in the analysis. A cashless limit economy is assumed. See Woodford (2003) for a detailed discussion of this approach.

feature for the analysis in the subsequent sections.

The household's flow budget constraint is given by

$$C_t + I_t + B_{t+1} \leq W_t N_t + r_t^k K_t + R_{t-1}\Pi_t^{-1} B_t + D_t - T_t, \tag{2}$$

where I_t, W_t, r_t^k, D_t, and T_t denote period t gross investment, real wage rate, real rental rate of capital, dividends from intermediate firms, and lump-sum taxes, respectively. In addition, the variable B_t represents the quantity of one-period bonds carried over from period $t-1$. The variable R_{t-1} denotes the gross nominal interest rate on bonds purchased in period $t-1$, and Π_t denotes the gross inflation rate. The stock of physical capital, K_t, is owned by the household and accumulates according to

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{\varsigma_I}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t, \tag{3}$$

were δ denotes the physical rate of depreciation, and $\varsigma_I > 0$ is the Lucas and Prescott (1971) capital adjustment cost parameter.

As shown in Appendix C.1, log-linearising the equilibrium conditions and substituting the consumption Euler equation into the household's inter-temporal budget constraint yields the following consumption function:[3]

$$\Gamma_1 \hat{C}_t = \beta^{-1}\bar{K}\hat{K}_t + \Gamma_2 \hat{W}_t + \bar{K}\bar{r}^k \hat{r}_t^k + \bar{D}\hat{D}_t - \bar{G}\hat{G}_t - \Gamma_3 \hat{R}_t + SW_t^e - SR_t^e + S\Pi_t^e + Sr_t^{k,e} + SD_t^e - SG_t^e, \tag{4}$$

where

$$SW_t^e \equiv \Gamma_4 \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{W}_{t+j}, \tag{5}$$

$$SR_t^e \equiv \Gamma_3 \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{R}_{t+j}, \tag{6}$$

$$S\Pi_t^e \equiv \Gamma_3 \beta^{-1} \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{\Pi}_{t+j}, \tag{7}$$

$$Sr_t^{k,e} \equiv \bar{K}\bar{r}^k \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{r}_{t+j}^k, \tag{8}$$

$$SD_t^e \equiv \bar{D} \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{D}_{t+j}, \tag{9}$$

$$SG_t^e \equiv \bar{G} \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{G}_{t+j}, \tag{10}$$

under the assumption that the transversality condition

$$\lim_{j \to +\infty} \mathscr{R}_{t,t+j-1} E_t^* B_{t+j} = 0, \tag{11}$$

[3]Throughout the paper, hatted variables denote log-deviations from the steady state. Barred variables refer to steady state values.

with $\mathscr{R}_{t,t+j} = \left(\prod_{s=0}^{j} E_t^* R_{t+s-1} E_t^* \Pi_{t+s}^{-1}\right)^{-1}$, holds.[4] The coefficients Γ_1, Γ_2, Γ_3, and Γ_4 are given in Appendix C.1.

Equation (4) implies that the household's choice of current consumption depends on subjective expectations of future factor prices, interest rates, inflation rates, dividends, and government expenditures. It is assumed that expectations are formed at time t.

Optimal investment requires that

$$\hat{Q}_t = \beta E_t^* \hat{Q}_{t+1} - \hat{R}_t + E_t^* \hat{\Pi}_{t+1} + \beta \bar{r}^k E_t^* \hat{r}_{t+1}^k, \tag{12}$$

where Q_t denotes Tobin's Q, the shadow value of existing capital.[5] Forward iteration gives the following infinite-horizon optimal investment rule

$$\hat{Q}_t = -\hat{R}_t + \sum_{j=1}^{\infty} \beta^j \left[\bar{r}^k E_t^* \hat{r}_{t+j}^k - E_t^* \hat{R}_{t+j} + \beta^{-1} E_t^* \hat{\Pi}_{t+j}\right]. \tag{13}$$

Firms A representative, perfectly competitive firm bundles a continuum of intermediate goods into a final good using the following CES technology:

$$Y_t = \left(\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{14}$$

where $\varepsilon > 1$, and $Y_t(i)$ is the input of intermediate good $i \in [0,1]$. The firm chooses the quantities of inputs so as to maximize its profit, taking as given the final goods price P_t and the intermediate goods prices $P_t(i)$, for all $i \in [0,1]$. Profit maximization implies the demand equation for intermediate good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t. \tag{15}$$

There is a continuum of monopolistically competitive intermediate goods producers populating the unit interval. Facing the real factor prices W_t and r_t^k, and the demand function (15), a typical intermediate goods firm $i \in [0,1]$ rents labour, $N_t(i)$, and capital, $K_t(i)$, in order to minimize costs. Its production function is given by

$$Y_t(i) = Z_t K_t(i)^{\alpha} N_t(i)^{1-\alpha}, \tag{16}$$

where Z_t represents technology that follows an exogenous process given by

$$Z_t = Z_{t-1}^{\rho_Z} \exp\left(\varepsilon_t^Z\right), \quad \varepsilon_t^Z \sim \mathscr{N}\left(0, \sigma_Z^2\right), \tag{17}$$

with $\rho_Z \in (0,1)$.

Following Calvo (1983), intermediate goods producers set nominal prices in a staggered fashion. Each period an intermediate goods producer can adjust its price with a constant probability $1-\theta$. A firm i that is permitted to adjust prices in period t, will choose a new optimal price, $P_t^*(i)$, to maximize the

[4] Evans et al. (2012) provide a detailed analysis of the role of this condition for the validity of the Ricardian equivalence proposition.

[5] Tobin's Q is defined as $Q_t \equiv q_t/\lambda_t$, where q_t is the Lagrangian multiplier with respect to the capital accumulation rule and λ_t the Lagrangian multiplier with respect to the household's budget constraint in the household's optimization problem. See Appendix A for the derivations.

expected present discounted value of future profits

$$E_t^* \sum_{j=0}^{\infty} (\beta\theta)^j \frac{U_{C,t+j}}{U_{C,t}} \left\{ \frac{P_t^*(i)}{P_{t+j}} Y_{t+j}(i) - MC_{t+j} Y_{t+j}(i) \right\}, \tag{18}$$

where $U_{C,t+j}$ is the j-period ahead marginal utility of consumption. At the end of each period, the intermediate firm distributes its profits as a real dividend, $D_t(i)$, to the representative household.

It is shown in Appendix C.1 that optimal price setting yields the following representation of the infinite-horizon Phillips curve:

$$\hat{\Pi}_t = \varphi\theta^{-1}\widehat{MC}_t + \beta\varphi \sum_{j=0}^{+\infty} (\beta\theta)^j \left[(1-\alpha) E_t^* \hat{W}_{t+j+1} + \alpha E_t^* \hat{r}^k_{t+j+1} - E_t^* \hat{Z}_{t+j+1} \right] + \beta(1-\theta) \sum_{j=0}^{+\infty} (\beta\theta)^j E_t^* \hat{\Pi}_{t+j+1}, \tag{19}$$

with $\varphi \equiv (1-\theta)(1-\beta\theta)$.[6]

Government Policies The fiscal authority finances expenditure through lump-sum taxes and bond sales. The government budget constraint is given by

$$T_t + B_{t+1} = G_t + R_{t-1}\Pi_t^{-1} B_t. \tag{20}$$

The central bank sets the nominal interest rate according to the following Taylor rule:

$$R_t = \Pi_t^{\rho_\Pi} u_t^R \tag{21}$$

with $u_t^R = \left(u_{t-1}^R\right)^{\rho_R} \exp\left(\varepsilon_t^R\right)$, $\varepsilon_t^R \sim \mathcal{N}(0, \sigma_R^2)$, and $\rho_R \in (0,1)$. It is assumed that the Taylor principle holds, i.e. $\rho_\Pi > 1$.

Market Clearing Market clearing in the goods market and the markets of production factors requires that the following conditions are met:

$$Y_t = C_t + I_t + G_t, \tag{22}$$

$$N_t = \int_0^1 N_t(i)\,di, \tag{23}$$

$$K_t = \int_0^1 K_t(i)\,di. \tag{24}$$

Linear Approximation The remainder of the paper considers the log-linear approximation of the model about its steady state. The equilibrium conditions of the linearised model are given in Appendix B.

[6] The results in this paper are independent of whether the forecasts of the technology shock $E_t^* \hat{Z}_{t+j}$ are determined by adaptive learning or by the shock process (17).

3 Rational Expectations Equilibrium

We begin with the standard case of rational expectations as a benchmark to compare against the adaptive learning model. In the rational expectations case, agents have full knowledge of the structure of the economy and the underlying equilibrium, and use this knowledge to forecast future variables. In the absence of a policy change, the rational expectations equilibrium of the linearised model can be written as a function of the capital stock $\hat{K}_t$ and the technology shock $\hat{Z}_t$

$$y_t = \bar{\Psi} \begin{bmatrix} \hat{K}_t \\ \hat{Z}_t \end{bmatrix}, \tag{25}$$

where y_t is the vector of log-linearised endogenous variables of the model.[7]

4 Adaptive learning

We now go beyond the rational expectations hypothesis and assume agents combine limited structural knowledge with adaptive learning to forecast future variables. In particular, agents understand the structure of government financing and use the government budget constraint (20) and the announced future change in government spending to forecast future taxes. In forecasting other variables they rely on forecasting models estimated using least-squares learning.

As argued by Evans et al. (2009) this set-up is a natural way to proceed. When it comes to the general equilibrium effects of fiscal policy, it is hard to believe that households and firms have perfect knowledge on how fiscal policy shocks affect future aggregate variables. On the other hand, agents presumably understand the direct implications of higher future taxes for their future disposable incomes.

This approach implies that $E_t^* \hat{G}_{t+j} = G_{t+j}$ in the consumption function (4), since households know the future path of government spending and understand the direct effect of this path on their future disposable incomes. Forecasts on wages $E_t^* \hat{W}_{t+j}$, interest rates $E_t^* \hat{R}_{t+j}$, inflation rates $E_t^* \hat{\Pi}_{t+j}$, rental rates of capital $E_t^* \hat{r}^k_{t+j}$, and dividends $E_t^* \hat{D}_{t+j}$, appearing in the conditions (4), (13), and (19), however, depend on the perceived laws of motion (PLMs) held by the agents, with coefficients updated over time using recursive least squares.[8] Following Mitra et al. (2013, 2016), it is assumed that the form of these laws correspond to (25) so that they are linear functions of the capital stock $\hat{K}_{t+1}$ and the technology shock $\hat{Z}_t$:

$$E_t^* y^f_{t+1} = \psi_t \begin{bmatrix} \hat{K}_{t+1} \\ E_t^* \hat{Z}_{t+1} \end{bmatrix} = \psi_t \begin{bmatrix} \hat{K}_{t+1} \\ \rho_Z \hat{Z}_t \end{bmatrix}, \tag{26}$$

where $E_t^* y^f_{t+1} = \left(E_t^* \hat{W}_{t+1}; E_t^* \hat{R}_{t+1}; E_t^* \hat{\Pi}_{t+1}; E_t^* \hat{r}^k_{t+1}; E_t^* \hat{D}_{t+1}; E_t^* \hat{K}_{t+2} \right)$.[9]

[7] The 13 endogenous log-linearized variables of the baseline new Keynesian model are private consumption ($\hat{C}_t$), dividends ($\hat{D}_t$), investment ($\hat{I}_t$), capital ($\hat{K}_t$), marginal cost ($\hat{MC}_t$), labour ($\hat{N}_t$), inflation ($\hat{\Pi}_t$), Tobin's Q ($\hat{Q}_t$), nominal interest rate ($\hat{R}_t$), rental rate of capital ($\hat{r}^k_t$), lump-sum taxes ($\hat{T}_t$), wage rate ($\hat{W}_t$), and output ($\hat{Y}_t$).

[8] Following the *infinite-horizon* learning approach, it is assumed that agents make forecasts infinitely many periods ahead. By contrast, the *Euler equation* learning approach assumes that agents make one-step ahead forecasts which are typically present in Euler equations. For a discussion of the two approaches see Honkapohja et al. (2013). An earlier version of this paper considered *Euler equation* learning in a model where agents did not incorporate the future path of government spending into their behavioural rules. The results of this approach, which are available upon request, are very similar to those presented here.

[9] As is standard in the learning literature, it is assumed that the agents know the parameters of the observed exogenous processes.

Equation (26) can be iterated forward to obtain the forecasts $E_t^* y_{t+j}^f$ for $j = 2, 3, \ldots$, which in turn are used to express the sums of future expected terms in (4), (13), and (19) as linear functions of $\hat{Z}_t$ and $\hat{K}_{t+1}$. These expressions, together with the other equilibrium conditions (cf. Appendix B), define the temporary equilibrium at time t given the coefficients ψ_t, the predetermined variables $\hat{B}_t$, $\hat{K}_t$, and $\hat{R}_{t-1}$, current government spending $\hat{G}_t$, and the exogenous shocks $\hat{u}_t^R$ and $\hat{Z}_t$. Note that agents have access to $\hat{Z}_t$ and $\hat{K}_{t+1}$ when making their forecasts at time t. This means that the equilibrium values of the endogenous variables and the agents' forecasts are simultaneously determined. See Appendix C.1 for further details.

Agents estimate the coefficients ψ_t using a constant-gain variant of Recursive Least Squares:

$$\begin{aligned} \psi_t &= \psi_{t-1} + \gamma S_t^{-1} X_{t-1} \left(y_{t-1}^f - \psi_{t-1}^T X_{t-1} \right)^T, \\ S_t &= S_{t-1} + \gamma \left(X_{t-1} X_{t-1}^T - S_{t-1} \right), \end{aligned} \tag{27}$$

where $X_t = \left(\hat{K}_{t+1}; \hat{Z}_t \right)$ is the data vector used to estimate the beliefs, S_t is the moment matrix for X_t, and $\gamma > 0$ is the gain parameter. The learning rule (27) specifies the evolution of the belief parameters over time. Taken together with the temporary equilibrium equations, it determines the dynamics under learning.

Because the gain parameter is assumed to be a positive constant, the learning algorithm weighs recent data more heavily. Orphanides and Williams (2005, 2007) refer to this approach as "perpetual learning" because agents forget past data over time and hence learn permanently. For this reason, this procedure is more robust to structural change such as changes in fiscal policy. Moreover, several studies, such as Eusepi and Preston (2011) and Evans and Honkapohja (2001, p. 49), show how a constant-gain learning rule can replicate the phenomenon of excess volatility. The constant-gain recursive least squares algorithm is therefore widely used in the adaptive learning literature (see Eusepi and Preston, 2011; Milani, 2007; Slobodyan and Wouters, 2012, for example).

Using the terminology of Evans and Honkapohja (2001, Chapter 13) the PLMs (26) are "restricted" or "underparameterised" because they do not include $\hat{G}_t$ and $\hat{u}_t^R$. Adding the monetary policy shock $\hat{u}_t^R$ does not alter the results of the paper. The exclusion of government spending $\hat{G}_t$ reflects the assumption that agents have imperfect knowledge on the general equilibrium effects of fiscal policy. If this variable were added to the PLMs, the impulse response functions under learning presented in this paper would coincide with those under rational expectations.

The restricted forecast rule (26) cannot converge to the rational expectations equilibrium, because it is not in the same space. However, it can converge to a distribution centred around the so-called restricted perceptions equilibrium (RPE). In this equilibrium the agents' forecasts are optimal relative to the restricted information set. That is, although agents use an underparameterised forecasting model, their forecast errors are uncorrelated with the (restricted) information set X_t used in the expectation formation. Guse (2008) provides a general technique to project the actual law of motion into the same class as the underparameterised forecasting model. The technique defines a projected T-map which maps the restricted forecast rule to the projected actual law of motion. The RPE can be found as a fixed point of this map. In the next section, the initial coefficients of the forecast rule, ψ_0, are pinned down to the RPE-implied coefficients. Note, however, that – in contrast to a decreasing-gain algorithm – the constant-gain algorithm employed here cannot converge to the coefficients ψ_0, but rather to a distribution centred around those values. An interesting implication is that the impact responses to a government spending shock will fluctuate around those presented here owing to fluctuations in the estimated coefficients ψ_t of the forecast rule – this topic is left to future research. In this paper, we restrict our attention to the impulse responses for the RPE coefficients ψ_0.[10]

5 The role of expectations for the effects of government spending shocks

This section examines the effects of a temporary increase in government spending under different assumptions with respect to agents' expectations. In particular, the macroeconomic effects of the shock under rational expectations are compared with those under adaptive learning. Because the role of price rigidity is of crucial importance, the new Keynesian model is examined in comparison with a neoclassical specification of the model where prices are fully flexible.

5.1 Calibration

The model is calibrated to quarterly periods. The parameters receive the values presented in Table 1. Most parameters are set to values that are typical in the business cycle literature. The elasticity of output with respect to capital, α, is fixed to 1/3. The subjective discount factor, β, is calibrated to match an annualized steady state real interest rate of 4.0%. The value of δ is 0.025 so that the depreciation rate of capital is 2.5% per quarter. The elasticity of substitution between intermediate goods, ε, is such that the mark-up of price over marginal cost is equal to 20% in steady state. The Calvo parameter, θ, is 0.75, implying an average frequency of price reoptimization of 4 quarters. The Taylor rule coefficient on inflation, ρ_Π, is 1.5, a standard value in the literature. The AR(1) coefficient of technology, ρ_Z, receives a value of 0.90. The coefficient of risk aversion, σ, is set to 2.0. This value is roughly in the middle of the range of the empirical estimates and consistent with the estimates obtained by Basu and Kimball (2002). Following Christiano et al. (2011), the capital adjustment cost parameter, ς_I, is equal to 17. The share of government expenditure in GDP, $\bar{G}/\bar{Y}$, is set at 0.20 to match the postwar U.S. government spending share. For the ratio $\bar{B}/\bar{Y}$ the average general government gross financial liabilities for the U.S. provided in the OECD (2014b) database over the period 2000–2013 are used. The preference parameter ϕ is calibrated such that the share of time devoted to work in the steady state is fixed to 1/3. As a benchmark, the gain parameter, γ, is set to 0.02, which is a value well within the range of estimates reported in the literature. However, the particular value of the gain parameter is not crucial for our impulse response analysis.[11] All error terms are assumed to have a standard deviation of 0.05. For simplicity, the AR(1) coefficient of the nominal interest rate, ρ_R, is assumed to be zero.

Table 2 shows the model values of some important macroeconomic aggregates. The calibration produces shares of private consumption and investment in GDP close to those observed in most industrialized countries. The steady-state labour's share of total income is 0.56, a value roughly consistent with the observed U.S. labour income share.[12]

[10] The parameterisation considered guarantees a unique stationary rational expectations equilibrium. As noted by Guse (2008) there is no general method to determine uniqueness of the restricted perceptions equilibrium. This is a topic for future research.

[11] Orphanides and Williams (2005, 2007) found that a gain parameter in the range 0.01–0.04 provides the best fit between the agents' forecasts in the model and the expectations data from the Survey of Forecasters. Using a similar strategy, Branch and Evans (2006) obtain a value of 0.0345. The estimate of Milani (2007) equals 0.0183 and hence lies within the same range. However, the estimated gain of 0.0029 in Eusepi and Preston (2011) is much smaller. The estimation results from Slobodyan and Wouters (2012) provide values for γ going from 0.001 to 0.06 depending on the particular learning scheme. Within the range of values mentioned here, the effect of a different value for the gain parameter is negligible.

[12] The U.S. labour income share in the industrial sector over the period 2000-2010 was on average 57% (OECD, 2014b).

Parameter	Description	Value
α	Output elasticity with respect to capital	1/3
β	Households subjective discount factor	$1.04^{-0.25}$
γ	Gain parameter	0.02
δ	Rate of physical capital depreciation	0.025
ε	Elasticity of substitution between intermediate goods	6.0
θ	Degree of nominal price rigidity	0.75
ρ_Π	Taylor rule inflation rate coefficient	1.5
ρ_R	Interest rate AR(1) coefficient	0.00
ρ_Z	Technology shock AR(1) coefficient	0.90
σ	Coefficient of risk aversion	2.0
σ_R	Standard deviation of the interest rate disturbance ε^R	0.05
σ_Z	Standard deviation of the technology disturbance ε^Z	0.05
ς_I	Capital adjustment cost parameter	17
ϕ	Preference parameter	0.35
$\bar{B}/\bar{Y}$	Steady state government debt to output ratio	0.74
$\bar{G}/\bar{Y}$	Steady state government expenditure to output ratio	0.20

Table 1: Model parameters.

Variable	Value	Variable	Value
$\frac{C}{Y}$	0.601	$\frac{r^k K}{Y}$	0.278
$\frac{I}{Y}$	0.199	Annualised r	0.040
$\frac{WN}{Y}$	0.556	Annualised r^k	0.147

Table 2: Steady-state values of main variables in the baseline model. Annualised r and r^k are defined as $\bar{R}^{1/4}-1$ and $\left(1+\bar{r}^k\right)^{1/4}-1$, respectively.

5.2 Impulse responses after a government spending shock

We now turn to the impulse responses of economic variables following a temporary increase in government spending of 1% of GDP at the beginning of period $t = 1$ that is financed through an increase in lump-sum taxes. After the shock, government spending gradually converges back to its steady-state value according to the following autoregressive process:

$$\hat{G}_t = \rho_G \hat{G}_{t-1}, \quad t > 1, \tag{28}$$

where ρ_G is assumed to be equal to 0.9. Then the expression (10) for the present value of future government expenditures becomes

$$SG_t^e = \bar{G} \frac{\beta \rho_G}{1 - \beta \rho_G} \hat{G}_t. \tag{29}$$

In this section, it is assumed that real public debt remains constant and lump-sum taxes adjust to maintain budget balance in each period. Financing with distortionary taxation and government bonds is discussed in the next section.

Neoclassical specification Figure 1 shows the responses to the government spending shock when prices are fully flexible ($\theta \to 0.00$). The solid and dotted lines depict the impulse responses under rational expectations and adaptive learning, respectively.[13]

Under rational expectations the effects of fiscal policy in a neoclassical model are well-understood – see for instance Aiyagari et al. (1992) and Baxter and King (1993). The wealth and inter-temporal substitution effects triggered by a temporary increase in government spending lead to a reduction in consumption and leisure on impact. Intuitively, agents adjust their consumption and labour supply because higher future taxes reduce their overall wealth. At the same time, the government absorption of resources reduces private investment. The increase in labour supply and the drop in investment imply a decline in the capital-labour ratio, resulting in a fall in the wage W_t and a rise in the rental rate of capital r_t^k and the interest rate R_t. The high (but declining) real interest rate leads to a rising path of consumption. The increase in the real interest rate is also a source of the declining path of labour supply through an inter-temporal substitution effect: it pays to work harder in periods when the interest rate is high and to use part of the earnings to build up the capital stock. Along the transition path, private consumption and employment gradually return to the original steady state; the capital-labour ratio converges to its steady-state value as investment recovers and both W_t and r_t^k return to their steady-state values.

Under learning, consumption falls less than under rational expectations. The reason is that agents anticipate the increase in future taxes, but fail to correctly foresee the paths of lower future wages and higher future interest rates. Agents are too optimistic about future wages and underestimate the rise in interest rates that will result from the policy change. More generally, the responses of expected future interest rates, factor prices, and dividends to the fiscal shock are only limited because these expectations are determined by adaptive learning and only gradually adjust to the observed fall in the capital stock. Those expectations determine the household's consumption choice, as they appear in the expectational terms SW_t^e, SR_t^e , $Sr_t^{k,e}$, and SD_t^e in the consumption function (4). The mechanism is as follows. In period $t = 1$ expected wages (SW_t^e) are higher and expected interest rates (SR_t^e) are lower under learning than

[13] The impulse responses of all variables in the model and the expectations on all forward-looking variables are included in Appendix D.

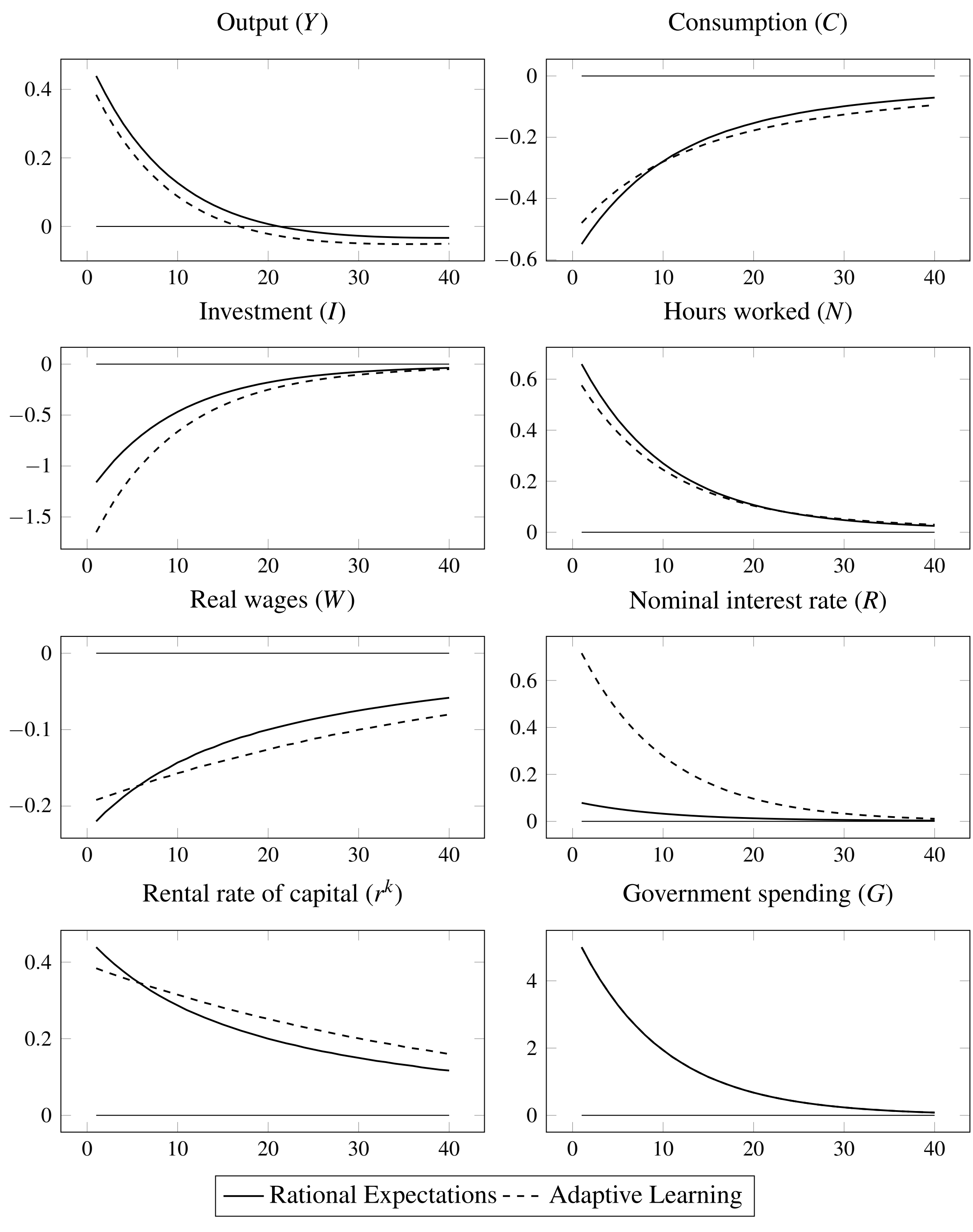

Figure 1: Impulse responses to an increase in government spending of 1% of GDP in the neoclassical specification of the model. The impulse response functions are measured in percentage deviations from steady-state. The horizontal axis measures quarters.

under rational expectations. That is why the fall in consumption is smaller, even though expected future rental rates ($Sr_t^{k,e}$) and dividends (SD_t^e) are lower than under rational expectations. Because consumption falls less, labour supply will also rise less. The smaller drop in consumption induces a smaller rise in the marginal utility of consumption, which is a key element in the agents' intra-temporal optimality between labour and leisure. The utility gain of a marginal increase in labour supply will be smaller. Moreover, since under learning the drop in disposable income is associated with a smaller decrease in consumption, the representative household must dissave more and, as a consequence, investment declines more sharply. This leads to a larger increase in the interest rate. In the aggregate, the net impact of a government spending shock on output is slightly smaller under adaptive learning than under rational expectations. In the periods following the shock, agents' forecasts adjust in response to the observed fall in the capital stock. The lower capital stock leads to lower forecasts of future wages and dividends, on the one hand, and higher forecasts of future interest rates and rental rates of capital on the other. In later periods, as the capital stock reverts back to normal, all expectations converge to the steady state.

New Keynesian specification Figure 2 depicts the impulse responses after a government spending shock in the economy where prices are rigid. When agents have rational expectations, the effects of a fiscal expansion are similar to those under fully flexible prices. Quantitatively, however, the effect on hours worked is stronger because the rise in labour supply is accompanied by an outward shift in labour demand. As set forth by Linnemann and Schabert (2003), Perotti (2008) and others, nominal rigidities generate a fall in the mark-up when the government boosts aggregate demand. This induces a rise in labour demand, which amplifies the increase in employment and reduces the fall in the real wage rate.

When agents form expectations using an adaptive learning mechanism, the effects of a government spending shock change substantially, especially with respect to the response of private consumption and real wages. In contrast to the neoclassical specification, government spending crowds *in* private consumption. This finding is particularly interesting since it is in accordance with the empirical evidence found in Blanchard and Perotti (2002), Fatás and Mihov (2001), Fragetta and Gasteiger (2014), Galí et al. (2007), and Perotti (2008), for example.

The main mechanisms that generate a rise in consumption can be described as follows. Under learning, lower expected future real interest rates weaken the incentive to postpone consumption. Moreover, over-optimism concerning future wages dampens the conventional negative wealth effect of the fiscal expansion on current consumption. These weaker negative wealth and substitution effects on consumption in period $t = 1$ also lead to a smaller increase in labour supply on impact which, together with the higher demand for labour, permits the real wage to rise. Consequently, agents experience a positive substitution effect from leisure into labour and consumption and overcome the negative wealth effect of the fiscal shock. In the periods following the shock the expectations $E_t^*\hat{W}_{t+j}$, $E_t^*\hat{R}_{t+j}$, $E_t^*\hat{\Pi}_{t+j}$, $E_t^*\hat{r}_{t+j}^k$, $E_t^*\hat{D}_{t+j}$ gradually respond to the data. Eventually, as explained in the above discussion of the neoclassical case, all expectations return to their steady state values.

Comparing the neoclassical and the new Keynesian specifications of the learning model, it is apparent that price rigidity is crucial for generating crowding *in* of private consumption. In particular, as argued by Bilbiie (2011) and Christiano et al. (2011), staggered price setting by monopolistically competitive firms and complementarity between consumption and labour provides a channel by which private consumption can react positively to a government spending shock. If prices are sticky, a government spending shock induces an outward shift in the demand for labour, strengthening the rise in employment. Because

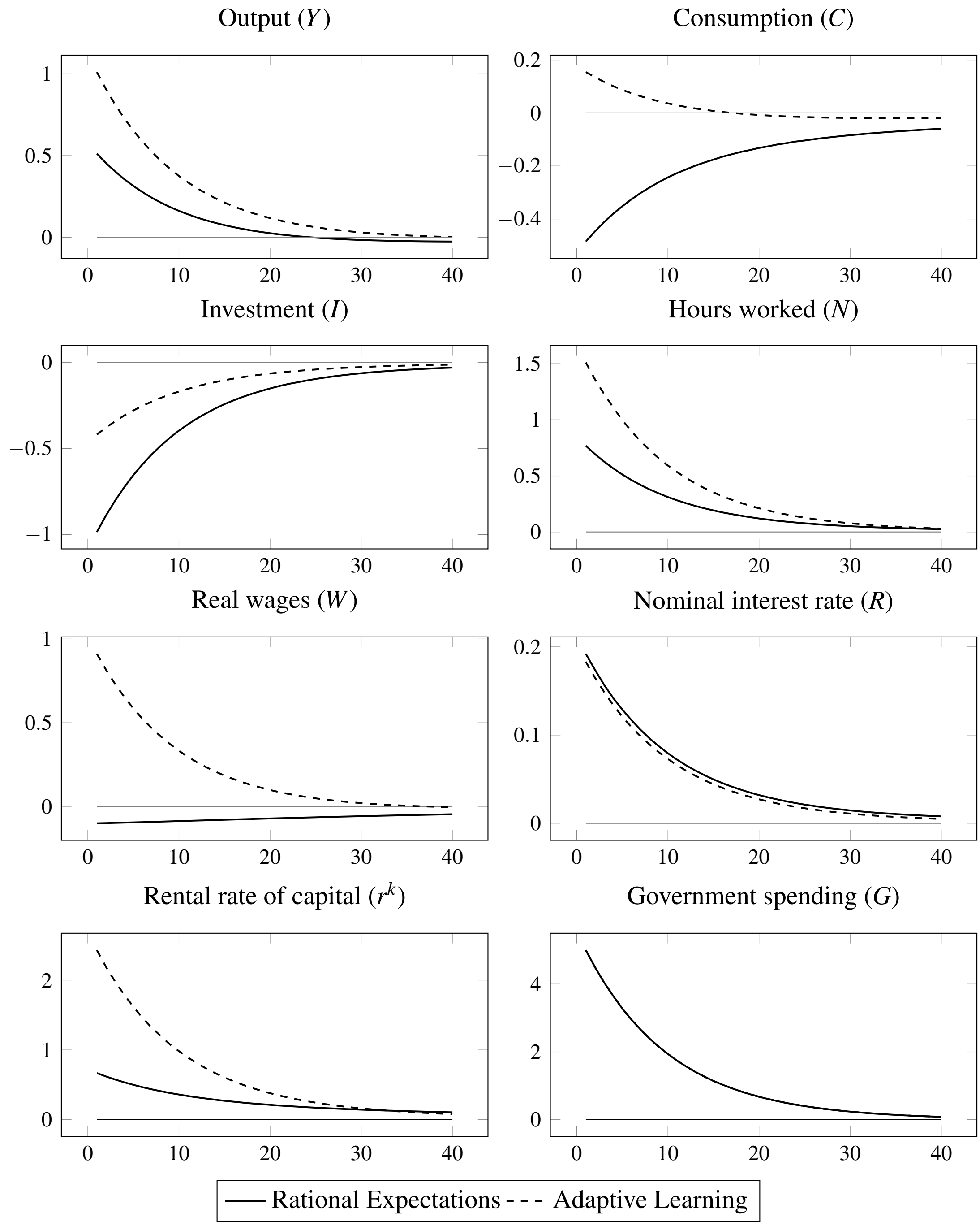

Figure 2: Impulse responses to an increase in government spending of 1% of GDP in the new Keynesian model. The impulse response functions are measured in percentage deviations from steady-state. The horizontal axis measures quarters.

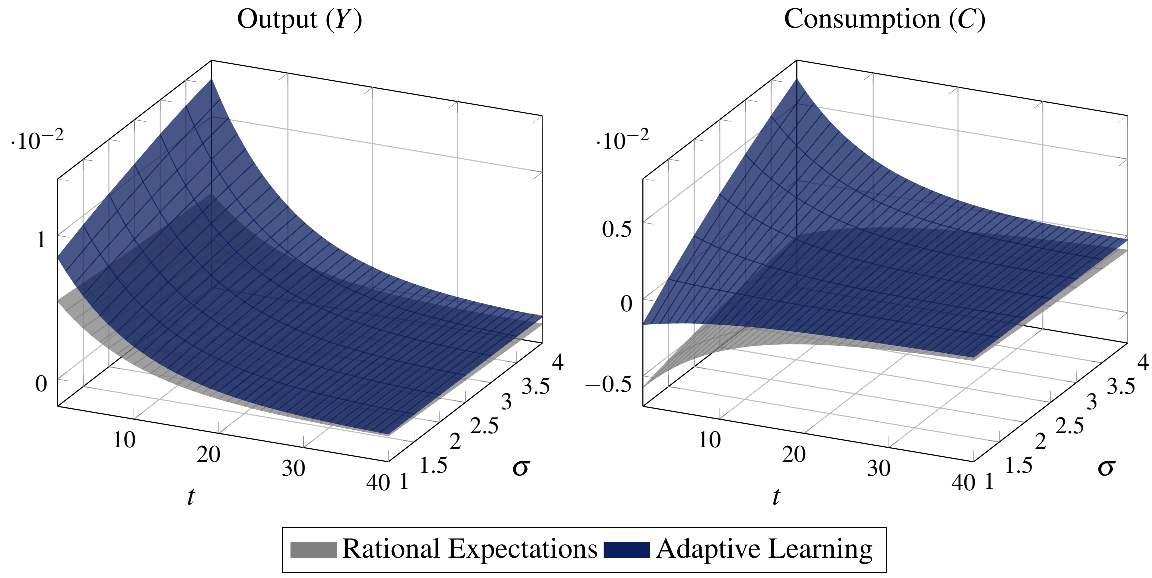

Figure 3: Impulse responses to an increase in government spending of 1% of GDP for different degrees of non-separability (σ) in the new Keynesian model. The impulse response functions are measured in percentage deviations from steady-state. The x-axis measures quarters.

consumption and labour are complements, i.e. $\sigma > 1$, this increase in employment raises the marginal utility of consumption. Under learning, this channel is strong enough for private consumption to crowd *in* after the government spending shock, whereas under rational expectations it is not.

Given our preference specification, higher values of σ imply stronger complementarity between consumption and labour. At the same time, a higher value of σ, i.e. a lower inter-temporal elasticity of substitution, makes households less willing to postpone consumption in response to the expected real interest rate. Thus, the response of consumption is stronger, the larger the value of σ.[14] Figure 3 illustrates this result. The figure shows the impulse responses of private consumption and output for different values of σ. The grey (light) shaded area in the right-hand plot shows that when economic agents have rational expectations, the impact of government spending on private consumption is negative for all considered values of σ. This is in sharp contrast with the impulse responses of the adaptive learning model depicted by the blue (dark) shaded area. It is clear that under the learning mechanism, the crowding *in* effect on consumption occurs for every $\sigma > 1$. However, in the limit case of $\sigma = 1$, when preferences are separable over leisure and consumption, this effect does not occur.

Another notable observation is the positive response of real wages under learning. Only when agents use the adaptive learning mechanism, the increase in aggregate hours after a positive government spending shock coexists with an increase in real wages. That is because the learning behaviour reduces the labour supply effect of the government spending shock, while price rigidity leads to a rise in labour demand. Considering this, the adaptive learning mechanism brings the theoretical impulse responses again

[14]This result is particularly interesting given the discussion in the literature on preference-based explanations for government spending crowding *in* private consumption and the positive co-movement of consumption and hours worked. Linnemann (2006) argues that a certain type of non separable utility, where labour and consumption are complements, can generate these results in a standard real business cycle model. However, Bilbiie (2009, 2011) points out that the preferences considered by Linnemann rely on a downward-sloping labour supply schedule. By contrast, a standard King et al. (1988) utility specification is considered here and we find that in the adaptive learning model government spending crowds *in* private consumption even if the degree of complementarity, σ, is small.

into line with those observed empirically. Evidence on the positive co-movement between real wages and hours worked after a government spending shock can be found in Rotemberg and Woodford (1992), Galí et al. (2007), and Fatás and Mihov (2001), for example. However, the empirical evidence is not entirely unambiguous (see, for instance, Ramey and Shapiro (1998), and Perotti (2008)). Another difference with the neoclassical specification, is the dampening effect of learning on the fall in investment.

6 The government spending multiplier

We now turn to the analysis of the government spending multiplier in the new Keynesian model. The question of the size of the government spending multiplier has been addressed by many authors in the literature. In a comprehensive review of the literature, Ramey (2011) concludes that the range of estimates of the output multiplier is probably between 0.8 and 1.5. Furthermore, as noted above, a number of empirical studies find that the consumption multiplier is positive. Replicating these empirical findings represents an important challenge for most theoretical rational expectations models. In response to this, several authors have proposed different mechanisms such as alternative preference specifications (Linnemann, 2006), the existence of rule-of-thumb consumers (Galí et al., 2007), different kinds of rigidities, and the stance of monetary policy (Coenen et al., 2012; Leeper et al., 2015).

Against that background, the discussion in the previous section shows that expectation formation too is a key factor for the impact of fiscal policy. Adaptive learning can amplify this impact substantially, even in the absence of accommodative monetary policy. As explained in the previous section, because under learning agents underestimate the general equilibrium effects of tax-financed government spending expansion in their expectation formation, private consumption can respond positively and in this way amplify the response of aggregate economic activity.

Table 3 illustrates this result. It reports the present-value government spending multipliers for output, consumption, and investment in the rational expectations model and the adaptive learning model. Following Mountford and Uhlig (2009), the present-value multiplier for variable X over a k-period horizon is calculated as

$$\frac{PV(\Delta X)}{PV(\Delta G)} = \frac{\sum_{t=0}^{k} \bar{R}^{-t} X_t}{\sum_{t=0}^{k} \bar{R}^{-t} G_t} \frac{1}{\bar{G}/\bar{X}}, \tag{30}$$

where X_t is the response of variable X in period t, G_t is government spending in period t, $\bar{R}$ is the steady state gross nominal interest rate, and $\bar{G}/\bar{X}$ is the steady state government expenditure to X ratio.

Table 3 shows that the present-value multipliers for output are significantly bigger under learning than under rational expectations, also at longer horizons. Thus, the learning model is capable of generating multipliers that are well within the range of empirical estimates reviewed by Ramey (2011). Moreover, the short- and longer-term consumption multiplier is always positive under learning, whereas it is always negative under rational expectations. An important result is that it is possible to achieve this outcome even if the degree of complementarity between labour and consumption in the utility function is weak, whereas in a model with rational expectations it is often necessary to assume high values for this parameter (see Linnemann, 2006; Bilbiie, 2009, 2011, for example).

In addition, adaptive learning provides a theoretical mechanism for generating government spending multipliers bigger than one, even if price stickiness is relatively small. This is particularly relevant since the discussion in Nakamura and Steinsson (2008) points out that the extent of price rigidity is often overestimated. Figure 4 reports the multipliers for output, consumption, and investment for different degrees

	Rational expectations				Adaptive learning			
	Impact	1 year	4 years	6 years	Impact	1 year	4 years	6 years
$\frac{PV(\Delta Y)}{PV(\Delta G)}$	0.51	0.50	0.46	0.43	1.01	1.00	0.99	0.97
$\frac{PV(\Delta C)}{PV(\Delta G)}$	−0.29	−0.30	−0.34	−0.37	0.09	0.09	0.07	0.06
$\frac{PV(\Delta I)}{PV(\Delta G)}$	−0.20	−0.20	−0.20	−0.20	−0.08	−0.08	−0.09	−0.09

Table 3: Present-value multipliers in the new Keynesian model under rational expectations and under adaptive learning.

of price rigidity θ. Moreover, the figure allows to compare the multipliers under rational expectations with those under adaptive learning. As noted earlier, throughout this analysis the central bank maintains a standard Taylor rule. The figure shows that for the benchmark case with $\theta = 0.75$, the output multiplier under learning is bigger than one and about twice as large as the multiplier under rational expectations.

The consumption multiplier is increasing with the degree of price rigidity. As noted earlier, it is optimal for an intermediate firm that cannot change its price, to hire more labour when the demand for its intermediate good increases. This amplifies the rise in employment after a government spending increase, and encourages the household to consume more when preferences are non-separable. Figure 4 shows that government spending crowds *in* private consumption when prices are sufficiently rigid. For example, if $\theta = 0.75$ the consumption multiplier equals 0.09. Moreover, notice that the crowding out of investment becomes smaller as prices become more sticky. Nevertheless, the investment multiplier always remains negative.

7 Alternative specification of fiscal policy

In the baseline impulse response analysis, the increase in government spending was financed through an increase in lump-sum taxes. This makes the results comparable with the policy experiments typically considered in the literature. As an extension, this section considers a richer specification of fiscal policy in which the fiscal authority finances expenditure, interest payments, and lump-sum transfers through the emission of one-period debt and through taxation on private consumption and capital and labour income.

The government budget constraint is now given by

$$B_{t+1} + \tau_t^c C_t + \tau_t^w W_t N_t + \tau_t^k r_t^k K_t = G_t + R_{t-1} \Pi_t^{-1} B_t + TR_t, \tag{31}$$

where τ_t^c, τ_t^w, and τ_t^k are the tax rates on private consumption, labour income, and capital income, respectively, and TR_t are lump-sum transfers.[15]

The steady state tax rates are set equal to U.S. averages over the period 2000–2013: $\bar{\tau}^c = 0.01$, $\bar{\tau}^w = 0.39$, and $\bar{\tau}^k = 0.39$. The consumption tax rate is calculated as in Appendix B of Leeper et al. (2010). The labour income tax rate and the corporate income tax rate are retrieved from the OECD (2014c) and OECD (2014a) databases.[16]

[15]In contrast to the baseline model, a lump-sum *transfer* TR_t is considered instead of a lump-sum *tax* T_t, but this is just a matter of definition since $T_t = -TR_t$. It is more natural to proceed in this way since, with this alternative fiscal policy specification, the parameterisation of the model implies a *negative* lump-sum tax.

[16]The labour income tax is the combined central and sub-central government income tax rate plus employee social security contribution, as a percentage of average gross wage earnings. The capital income tax rate is the basic combined central and

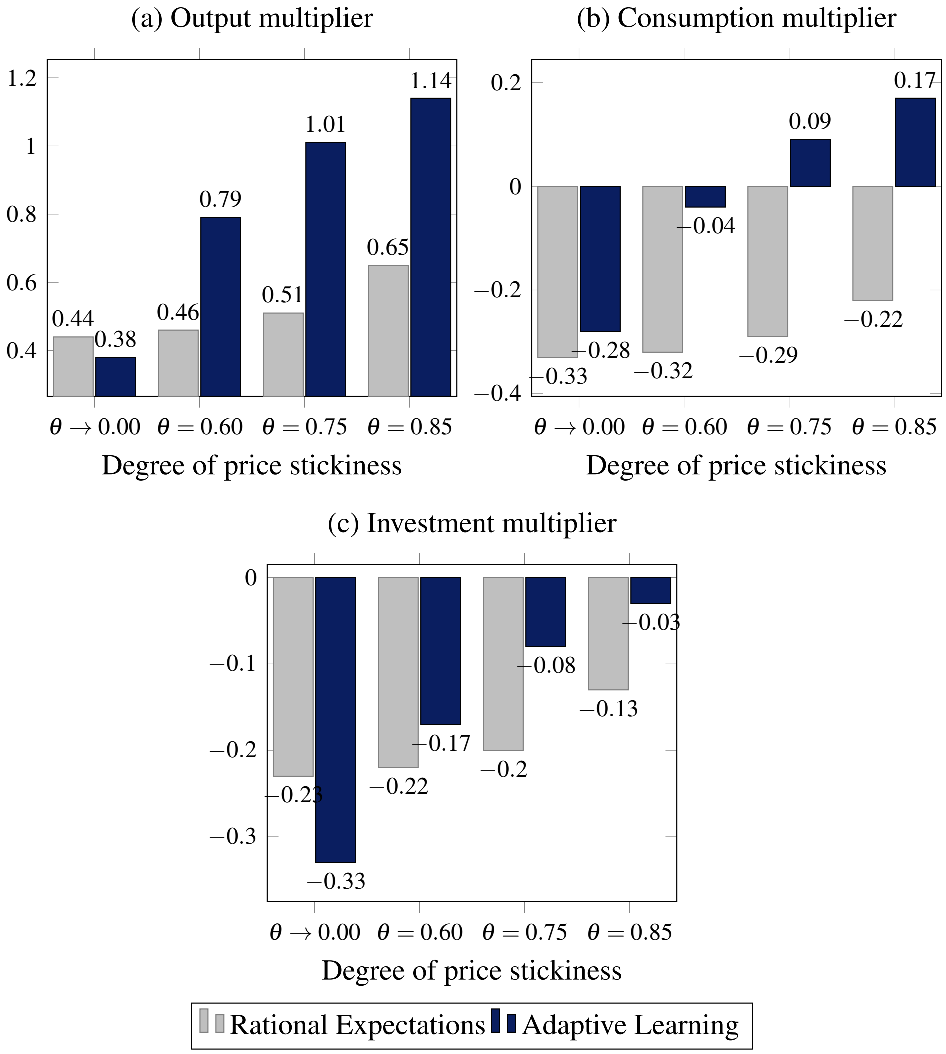

Figure 4: Impact multipliers for different degrees of price rigidity in the rational expectations model and the adaptive learning model.

The rich specification of fiscal policy allows us to compare government spending multipliers for different fiscal financing strategies. Table 4 includes the results for three strategies in the new Keynesian model. "Strategy 1" corresponds to the baseline analysis of a government spending increase financed through lump-sum taxation. In "Strategy 2" the spending increase is associated with a rise in the capital income tax. In particular, the government raises the tax rate on capital such that in each period the spending increase is matched by an equal increase in steady-state tax revenues from capital. In the same manner, "Strategy 3" corresponds to an increase in the labour income tax. For the sake of consistency with the approach adopted in the previous sections, real public debt remains constant throughout all simulations and lump-sum transfers adjust to ensure that the period-by-period government budget constraint (31) is satisfied. Note, however, that the results are identical if we instead allow for debt financing. As shown in Appendix C.2, the particular paths of debt and lump-sum transfers are irrelevant for the equilibrium allocation.[17] This is a generalisation of the Ricardian equivalence result of Evans et al. (2012). The derivation of the equations governing the dynamics under learning is detailed in Appendix C.2.

Table 4 shows that the effects of a government spending shock depend quite dramatically on the fiscal financing strategy. First, consider the multipliers when the rise in government spending is associated with an increase in the capital tax rate (Strategy 2). With capital taxes temporarily higher, agents with rational expectations want to reduce investment and consume more. Therefore, investment declines more strongly than under lump-sum financing and the short-term consumption multipliers under rational expectations are less negative. The adverse effects on the capital stock suppress the multipliers in the medium and long run. In the learning model, the output multiplier at impact is less than half of the multiplier under lump-sum financing. The consumption multiplier under learning is now slightly negative at impact and reaches a value of -0.37 after six years.

The most striking difference between rational expectations and learning occurs in the presence of labour income tax financing (Strategy 3). Under rational expectations, the output multiplier is negative at every horizon. Labour supply falls considerably at impact as the temporary increase in the labour income tax rate generates a strong incentive to postpone work to periods with lower tax rates. This intertemporal substitution effect dominates the wealth effect on labour supply and the rise in labour demand associated with the rise in government spending. Now the consumption-labour complementarity works in the opposite direction as before: the sharp drop in employment lowers the marginal utility of consumption significantly, resulting in deeply negative consumption multipliers. The same mechanism is at play in the learning model, but the drop in consumption is much weaker. As in the model without distortionary taxes, agents underestimate the general equilibrium effects of the policy change, in particular those on future real interest rates. This results in a smaller decline in consumption in period $t = 1$ and weakens the negative contemporaneous effect of low consumption on labour supply caused by consumption-labour complementarity. In fact, under learning employment *rises* after the policy shock. As a consequence, learning completely reverses the sign of the output multipliers. Under rational expectations, the impact multiplier is negative and equals -0.64, whereas under learning it is positive and equal to 0.48. The impulse responses for the different financing strategies are depicted in Figure 5 of the Appendix.

sub-central (statutory) corporate income tax rate given by the adjusted central government rate plus the sub-central rate. Both tax rates are marginal rates. See OECD (2014c) and OECD (2014a) for explanatory notes.

[17] Alternatively, one could assume that debt adjusts to satisfy the government budget constraint (31) in each period and lump-sum transfers stabilise debt according to $\hat{TR}_t = -\rho_{TR}\hat{B}_t$, with $\rho_{TR} > 0$. The results under this approach are identical to those reported in the paper, except for the dynamics of lump-sum transfers and debt, of course. A sufficiently large ρ_{TR} ensures a stable path of government debt.

	Rational expectations				Adaptive learning			
	Impact	1 year	4 years	6 years	Impact	1 year	4 years	6 years
Strategy 1: lump-sum financing (baseline model)								
$\frac{PV(\Delta Y)}{PV(\Delta G)}$	0.51	0.50	0.46	0.43	1.01	1.00	0.99	0.97
$\frac{PV(\Delta C)}{PV(\Delta G)}$	−0.29	−0.30	−0.34	−0.37	0.09	0.09	0.07	0.06
$\frac{PV(\Delta I)}{PV(\Delta G)}$	−0.20	−0.20	−0.20	−0.20	−0.08	−0.08	−0.09	−0.09
Strategy 2: capital tax financing								
$\frac{PV(\Delta Y)}{PV(\Delta G)}$	0.32	0.29	0.12	0.02	0.43	0.38	0.20	0.08
$\frac{PV(\Delta C)}{PV(\Delta G)}$	−0.23	−0.26	−0.40	−0.48	−0.08	−0.12	−0.27	−0.37
$\frac{PV(\Delta I)}{PV(\Delta G)}$	−0.68	−0.69	−0.70	−0.71	−0.80	−0.80	−0.81	−0.82
Strategy 3: labour tax financing								
$\frac{PV(\Delta Y)}{PV(\Delta G)}$	−0.64	−0.68	−0.85	−0.95	0.48	0.47	0.43	0.40
$\frac{PV(\Delta C)}{PV(\Delta G)}$	−1.07	−1.10	−1.24	−1.33	−0.37	−0.38	−0.41	−0.43
$\frac{PV(\Delta I)}{PV(\Delta G)}$	−0.71	−0.71	−0.73	−0.74	−0.17	−0.18	−0.18	−0.18

Table 4: Present-value multipliers for different specifications of fiscal policy in the new Keynesian model with rational expectations and with adaptive learning. See main text for a description of the different financing strategies.

8 Conclusion

This paper assesses the role of expectations for the macroeconomic dynamics of a government spending shock and, in particular, for the size of the government spending multiplier. There is no doubt that it is implausible to assume that agents have complete knowledge of the structure of the economy. Therefore, this paper considers a model where agents understand the direct wealth effects from the change in government spending and taxes, but fail to fully foresee the general equilibrium effects on factor prices and other aggregate variables. To forecast these variables they rely on small forecasting models estimated using least-squares learning. The impulse responses under this type of learning show that the effects of expansionary fiscal policy crucially depend on the agents' beliefs about the future.

Expectations significantly influence the size of the short- and longer-term multipliers of output, private consumption, and investment. The new Keynesian adaptive learning model generates an output multiplier at impact of 1.01, a value that is about twice as large as the multiplier under rational expectations. Expectations of future real interest rates are crucial for understanding this result. Under rational expectations, the inter-temporal substitution effect of higher future interest rates causes consumption to fall. Under learning, however, agents underestimate the increase in future interest rates and consumption rises at impact. Additionally, the learning mechanism induces a positive co-movement between real wages and hours worked after a government spending shock in the new Keynesian model. Also the investment multiplier for this model is larger than for the rational expectations model, but remains negative.

This paper confirms the findings of Bilbiie (2011), Christiano et al. (2011), and others, that emphasize the importance of sticky prices and consumption-labour complementarity for government spending to crowd *in* private consumption. However, in the parameterisation considered, these model features alone are not sufficient to generate a rise in consumption. Only in the learning model, government spending crowds *in* private consumption. Hence, this paper provides a new explanation for a positive consumption response to a temporary government spending increase.

Finally, the learning perspective provides new insights on the desirability of different fiscal financing strategies. For policy makers who seek to stimulate aggregate demand, the results demonstrate that lump-sum financing is preferred over capital or labour tax financing, both under learning and under rational expectations. However, adaptive learning significantly alters the effects of a government spending shock when financed by distortionary taxes. Unlike in the rational expectations model, the government spending multipliers for output are still positive with labour tax financing.

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Appendices

A Derivations of model equations

A.1 Household's optimization problem

The Lagrangian associated with the household's optimization problem is given by

$$\begin{aligned}\mathscr{L} = E_0^* \sum_{t=0}^{\infty} \beta^t \Bigg\{ & U\left(C_t, 1-N_t\right) \\ & + \lambda_t \left[W_t N_t + r_t^k K_t + R_{t-1}\Pi_t^{-1} B_t + D_t - T_t - C_t - I_t - B_{t+1} \right] \\ & + q_t \left[(1-\delta) K_t + I_t - \mathscr{S}\left(K_t, I_t, I_{t-1}\right) - K_{t+1} \right] \Bigg\}.\end{aligned}$$

The associated optimality conditions are

$$\frac{\partial \mathscr{L}_t}{\partial C_t} = 0 \quad \Leftrightarrow \quad U_{C,t} = \lambda_t, \tag{32}$$

$$\frac{\partial \mathscr{L}_t}{\partial (1-N_t)} = 0 \quad \Leftrightarrow \quad U_{1-N,t} = \lambda_t W_t, \tag{33}$$

$$\frac{\partial \mathscr{L}_t}{\partial B_{t+1}} = 0 \quad \Leftrightarrow \quad \beta E_t^* \left(\lambda_{t+1} R_t \Pi_{t+1}^{-1} \right) = \lambda_t \Leftrightarrow R_t = E_t^* \left(\frac{\lambda_t \Pi_{t+1}}{\beta \lambda_{t+1}} \right), \tag{34}$$

$$\frac{\partial \mathscr{L}_t}{\partial I_t} = 0 \quad \Leftrightarrow \quad \lambda_t = q_t (1 - \mathscr{S}_{I_t,t}) - \beta E_t^* \left(q_{t+1} \mathscr{S}_{I_{t-1},t+1} \right), \tag{35}$$

$$\frac{\partial \mathscr{L}_t}{\partial K_{t+1}} = 0 \quad \Leftrightarrow \quad \beta E_t^* \left(\lambda_{t+1} r_{t+1}^k \right) + \beta E_t^* \left[q_{t+1} \left(1 - \delta - \mathscr{S}_{K_t,t+1} \right) \right] = q_t, \tag{36}$$

$$\frac{\partial \mathscr{L}_t}{\partial \lambda_t} = 0 \quad \Leftrightarrow \quad W_t N_t + r_t^k K_t + R_{t-1} \Pi_t^{-1} B_t + D_t - T_t - C_t - I_t - B_{t+1} = 0,$$

$$\frac{\partial \mathscr{L}_t}{\partial q_t} = 0 \quad \Leftrightarrow \quad (1-\delta) K_t + I_t - \mathscr{S}\left(K_t, I_t, I_{t-1}\right) - K_{t+1} = 0.$$

Combining conditions (32) and (33) yields the labour supply equation

$$W_t = \frac{U_{1-N,t}}{U_{C,t}}. \tag{37}$$

Conditions (32) and (34) allow us to derive the following Euler equation for consumption

$$U_{C,t} = \beta R_t E_t^* \left(\Pi_{t+1}^{-1} U_{C,t+1}\right). \tag{38}$$

Optimality conditions (35) and (36) can be further simplified using condition (34). We get that

$$1 = Q_t \left[1 - \mathscr{S}_{I_t,t}\right] - R_t^{-1} E_t^* \left[\Pi_{t+1} Q_{t+1} \mathscr{S}_{I_{t-1},t+1}\right], \tag{39}$$

$$Q_t = R_t^{-1} E_t^* \left\{\Pi_{t+1} \left[r_{t+1}^k + Q_{t+1} \left(1 - \delta - \mathscr{S}_{K_t,t+1}\right)\right]\right\}, \tag{40}$$

where Tobin's $Q_t \equiv q_t/\lambda_t$.

Functional Form Assumptions The following specifications of preferences and the capital adjustment cost function are considered:

$$U(C_t, 1-N_t) = \frac{\left[C_t^{\phi} (1-N_t)^{1-\phi}\right]^{1-\sigma} - 1}{1-\sigma},$$

$$\mathscr{S}(\cdot) = \frac{\varsigma_I}{2} \left(\frac{I_t}{K_t} - \delta\right)^2 K_t,$$

where $\varsigma_I > 0$ is the Lucas and Prescott (1971) capital adjustment cost parameter. For these functional forms, the optimality conditions (37), (38), (39), and (40) become

$$W_t = \frac{1-\phi}{\phi} \frac{C_t}{1-N_t},$$

$$C_t^{\phi(1-\sigma)-1} (1-N_t)^{(1-\phi)(1-\sigma)} = \beta R_t E_t^* \left[\Pi_{t+1}^{-1} C_{t+1}^{\phi(1-\sigma)-1} (1-N_{t+1})^{(1-\phi)(1-\sigma)}\right],$$

$$1 = Q_t \left[1 - \varsigma_I \left(\frac{I_t}{K_t} - \delta\right)\right],$$

$$Q_t = R_t^{-1} E_t^* \left\{\Pi_{t+1} \left[r_{t+1}^k + Q_{t+1} \left(1 - \delta - \varsigma_I \left(\frac{I_{t+1}}{K_{t+1}} - \delta\right) \left(\frac{1}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta\right) - \frac{I_{t+1}}{K_{t+1}}\right)\right)\right]\right\}.$$

Written in terms of deviations from steady state, the Euler equation (38) becomes

$$\begin{aligned} [\phi(1-\sigma)-1]\hat{C}_t - \frac{(1-\sigma)(1-\phi)\bar{N}}{1-\bar{N}} \hat{N}_t = \hat{R}_t - E_t^* \hat{\Pi}_{t+1} + [\phi(1-\sigma)-1] E_t^* \hat{C}_{t+1} \\ - \frac{(1-\sigma)(1-\phi)\bar{N}}{1-\bar{N}} E_t^* \hat{N}_{t+1}. \end{aligned} \tag{41}$$

A.2 Firms' optimization problem

A.2.1 Final goods sector

The profit maximization problem of the final goods firm is represented as

$$\max_{\{Y_t(i)\}} P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj, \quad \forall i \in [0,1], \tag{42}$$

where both the final goods price P_t and the prices for the intermediate goods $P_t(j)$, $j \in [0,1]$, are taken as given. Profit maximization yields the following demand schedule for intermediate good i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t. \tag{15}$$

The final goods producers are perfectly competitive. Thus, we have the following zero-profit condition

$$P_t \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t(i) Y_t(i) di = 0. \tag{43}$$

This leads to the following expression for the final goods price

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}. \tag{44}$$

In the symmetric equilibrium all intermediate goods producers set the same price. Therefore, the aggregate price P_t and the intermediate goods prices $P_t(i)$ for all i will be the same.

A.2.2 Intermediate goods sector

The Lagrangian for the expenditure minimization problem for the intermediate goods producer i is given by

$$\mathscr{L} = W_t N_t(i) + r_t^k K_t(i) + \mu_t(i)\left[Y_t(i) - Z_t K_t(i)^{\alpha} N_t(i)^{1-\alpha}\right], \tag{45}$$

and the corresponding first-order conditions

$$\begin{aligned} W_t &= \mu_t(i)(1-\alpha) Z_t K_t(i)^{\alpha} N_t(i)^{-\alpha}, \\ r_t^k &= \mu_t(i) \alpha Z_t K_t(i)^{\alpha-1} N_t(i)^{1-\alpha}. \end{aligned}$$

Here the Lagrange multiplier is also the real marginal cost. Therefore we will define the real marginal cost of firm i as $MC_t(i) \equiv \mu_t(i)$. In the symmetric equilibrium real marginal cost is common to all firms and given by

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}\left(R_t^k\right)^{\alpha} W_t^{1-\alpha} Z_t^{-1} \tag{46}$$

Intermediate goods producers choose the price $P_t^*(i)$ that maximizes discounted real profits

$$E_t^* \sum_{j=0}^{\infty} (\beta\theta)^j \frac{U_{C,t+j}}{U_{C,t}} \left\{\frac{P_t^*(i)}{P_{t+j}} Y_{t+j}(i) - MC_{t+j} Y_{t+j}(i)\right\} \tag{47}$$

subject to

$$Y_{t+j}(i) = \left(\frac{P_t^*(i)}{P_{t+j}}\right)^{-\varepsilon} Y_{t+j}. \tag{48}$$

The corresponding first-order condition is

$$E_t^* \sum_{j=0}^{\infty} (\beta\theta)^j \frac{U_{C,t+j}}{U_{C,t}} P_{t+j}^{\varepsilon} Y_{t+j} \left\{(1-\varepsilon)\left(P_t^*(i)\right)^{-\varepsilon} P_{t+j}^{-1} + \varepsilon MC_{t+j}\left(P_t^*(i)\right)^{-\varepsilon-1}\right\} = 0 \tag{49}$$

Given Calvo pricing, the price index (44) can be written as

$$P_t^{1-\varepsilon} = (1-\theta)\left(P_t^*(i)\right)^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon}. \tag{50}$$

B Log-linearisation

B.1 New Keynesian specification

For the new Keynesian model we have the following log-linearised equilibrium conditions:

$$\hat{Y}_t = \left(1 - \frac{\bar{G}}{\bar{Y}} - \frac{\bar{I}}{\bar{Y}}\right)\hat{C}_t + \frac{\bar{G}}{\bar{Y}}\hat{G}_t + \frac{\bar{I}}{\bar{Y}}\hat{I}_t, \tag{51}$$

$$\hat{Y}_t = \hat{Z}_t + \alpha\hat{K}_t + (1-\alpha)\hat{N}_t, \tag{52}$$

$$\hat{\Pi}_t = \varphi\theta^{-1}\widehat{MC}_t + \beta\varphi\sum_{j=0}^{+\infty}(\beta\theta)^j\left[(1-\alpha)E_t^*\hat{W}_{t+j+1} + \alpha E_t^*\hat{r}_{t+j+1}^k - E_t^*\hat{Z}_{t+j+1}\right] + \beta(1-\theta)\sum_{j=0}^{+\infty}(\beta\theta)^j E_t^*\hat{\Pi}_{t+j+1}, \tag{53}$$

$$\hat{W}_t = \alpha\hat{K}_t + \hat{Z}_t + \widehat{MC}_t - \alpha\hat{N}_t, \tag{54}$$

$$\hat{r}_t^k = (1-\alpha)\hat{N}_t + \hat{Z}_t + \widehat{MC}_t + \hat{K}_t(\alpha-1), \tag{55}$$

$$\hat{K}_{t+1} = \hat{K}_t(1-\delta) + \hat{I}_t\delta, \tag{56}$$

$$\hat{R}_t = \rho_\Pi\hat{\Pi}_t + \hat{u}_t^R, \tag{57}$$

$$\hat{u}_t^R = \rho_R\hat{u}_{t-1}^R + \varepsilon_t^R, \tag{58}$$

$$\hat{Z}_t = \rho_Z\hat{Z}_{t-1} + \varepsilon_t^Z, \tag{59}$$

$$\bar{T}\hat{T}_t = \bar{G}\hat{G}_t + \beta^{-1}\bar{B}\hat{R}_{t-1} - \beta^{-1}\bar{B}\hat{\Pi}_t, \tag{60}$$

$$\hat{C}_t + \hat{N}_t\frac{\bar{N}}{1-\bar{N}} = \hat{W}_t, \tag{61}$$

$$\Gamma_1\hat{C}_t = \beta^{-1}\bar{K}\hat{K}_t + \Gamma_2\hat{W}_t + \bar{K}\bar{r}^k\hat{r}_t^k + \bar{D}\hat{D}_t - \bar{G}\hat{G}_t - \Gamma_3\hat{R}_t + SW_t^e - SR_t^e + S\Pi_t^e + Sr_t^{k,e} + SD_t^e - SG_t^e, \tag{62}$$

$$\hat{Q}_t = -\hat{R}_t + \sum_{j=1}^{\infty}\beta^j\left[\bar{r}^k E_t^*\hat{r}_{t+j}^k - E_t^*\hat{R}_{t+j} + \beta^{-1}E_t^*\hat{\Pi}_{t+j}\right], \tag{63}$$

$$\hat{Q}_t = \delta\varsigma_I\left(\hat{I}_t - \hat{K}_t\right), \tag{64}$$

where a circumflex denotes log-deviations from the steady state.

B.2 Neoclassical specification

The log-linearised equilibrium conditions characterizing the dynamics of the neoclassical specification of the model are the following:

$$\hat{Y}_t = \left(1 - \frac{\bar{G}}{\bar{Y}} - \frac{\bar{I}}{\bar{Y}}\right)\hat{C}_t + \frac{\bar{G}}{\bar{Y}}\hat{G}_t + \frac{\bar{I}}{\bar{Y}}\hat{I}_t, \tag{65}$$

$$\hat{Y}_t = \hat{Z}_t + \alpha\hat{K}_t + (1-\alpha)\hat{N}_t, \tag{66}$$

$$\hat{W}_t = \alpha\hat{K}_t + \hat{Z}_t - \alpha\hat{N}_t, \tag{67}$$

$$\hat{r}^k_t = (1-\alpha)\hat{N}_t + \hat{Z}_t + \hat{K}_t(\alpha - 1), \tag{68}$$

$$\hat{K}_{t+1} = \hat{K}_t(1-\delta) + \hat{I}_t\delta, \tag{69}$$

$$\hat{Z}_t = \rho_Z\hat{Z}_{t-1} + \varepsilon^Z_t, \tag{70}$$

$$\bar{T}\hat{T}_t = \bar{G}\hat{G}_t + \beta^{-1}\bar{B}\hat{R}_{t-1}, \tag{71}$$

$$\hat{C}_t + \hat{N}_t\frac{\bar{N}}{1-\bar{N}} = \hat{W}_t, \tag{72}$$

$$\Gamma_1\hat{C}_t = \beta^{-1}\bar{K}\hat{K}_t + \Gamma_2\hat{W}_t + \bar{K}\bar{r}^k\hat{r}^k_t + \bar{D}\hat{D}_t - \bar{G}\hat{G}_t - \Gamma_3\hat{R}_t + SW^e_t - SR^e_t + Sr^{k,e}_t + SD^e_t - SG^e_t, \tag{73}$$

$$\hat{Q}_t = -\hat{R}_t + \sum_{j=1}^{\infty}\beta^j\left[\bar{r}^k E^*_t\hat{r}^k_{t+j} - E^*_t\hat{R}_{t+j}\right], \tag{74}$$

$$\hat{Q}_t = \delta\varsigma_I\left(\hat{I}_t - \hat{K}_t\right). \tag{75}$$

C Learning dynamics

C.1 Baseline model

C.1.1 Household

In this appendix we derive the linearised consumption function under learning. We apply the approach of Evans et al. (2009) and assume agents combine structural knowledge on the government budget constraint with expectations based on small forecasting models.

Forward iteration of the Euler equation (41) yields

$$-\sigma E^*_t\hat{C}_{t+j} = [\phi(1-\sigma) - 1]\hat{C}_t + (1-\phi)(1-\sigma)\left(E^*_t\hat{W}_{t+j} - \frac{\bar{N}}{1-\bar{N}}\hat{N}_t\right) - \hat{R}_t - \sum_{k=1}^{j-1}E^*_t\hat{R}_{t+k} + \sum_{k=1}^{j}E^*_t\hat{\Pi}_{t+k}, \tag{76}$$

where $\hat{N}_{t+j}$ is substituted out using (61).

Since real government debt is constant under the policy experiment considered in Section 5.2 and by (3) we can write the household's flow budget constraint (2) as

$$\beta^{-1}\bar{K}\hat{K}_t = \phi^{-1}\bar{C}\hat{C}_t + \bar{K}\hat{K}_{t+1} - \bar{W}\hat{W}_t - \bar{r}^k\bar{K}\hat{r}^k_t - \bar{B}\beta^{-1}\left(\hat{R}_{t-1} - \hat{\Pi}_t\right) - \bar{D}D_t + \bar{T}\hat{T}_t, \tag{77}$$

where we have used (61) to substitute out labour $\hat{N}_t$. Combining the government budget constraint (60) with the flow budget constraint (77) we can get the expected value inter-temporal budget constraint of the household

$$\eta \hat{C}_t + \eta \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{C}_{t+j} = \beta^{-1} \bar{K} \hat{K}_t + \bar{W} \hat{W}_t + \bar{K} \bar{r}^k \hat{r}_t^k + \bar{D} \hat{D}_t - \bar{G} \hat{G}_t + \sum_{j=1}^{+\infty} \beta^j \left[\bar{W} E_t^* \hat{W}_{t+j} + \bar{r}^k E_t^* \hat{r}_{t+j}^k + \bar{D} E_t^* \hat{D}_{t+j} - \bar{G} E_t^* \hat{G}_{t+j} \right] \quad (78)$$

by assuming that the transversality condition (11) holds. Here $\eta = \bar{C}\phi^{-1}$.

Substituting the Euler equation (76) into this inter-temporal budget constraint yields the consumption function

$$\Gamma_1 \hat{C}_t = \beta^{-1} \bar{K} \hat{K}_t + \Gamma_2 \hat{W}_t + \bar{K} \bar{r}^k \hat{r}_t^k + \bar{D} \hat{D}_t - \bar{G} \hat{G}_t - \Gamma_3 \hat{R}_t + SW_t^e - SR_t^e + S\Pi_t^e + Sr_t^{k,e} + SD_t^e - SG_t^e, \quad (79)$$

where SW_t^e, SR_t^e, $S\Pi_t^e$, $Sr_t^{k,e}$, SD_t^e, and SG_t^e are defined by (5)–(10) and

$$\Gamma_1 \equiv \frac{\eta}{1-\beta}, \quad (80)$$

$$\Gamma_2 \equiv \bar{W} - \frac{\beta \eta (1-\phi)(1-\sigma)}{\sigma (1-\beta)}, \quad (81)$$

$$\Gamma_3 \equiv \frac{\beta \eta}{\sigma (1-\beta)}, \quad (82)$$

$$\Gamma_4 \equiv \bar{W} + \frac{(1-\phi)(1-\sigma)\eta}{\sigma}. \quad (83)$$

Since the future path of government spending is assumed to be known and given by (28), the term SG_t^e can be obtained as

$$SG_t^e = \bar{G} \sum_{j=1}^{+\infty} \beta^j \rho_G^j \hat{G}_t = \bar{G} \frac{\beta \rho_G}{1 - \beta \rho_G} \hat{G}_t.$$

As described in the main text, the forecasts $E_t^* \hat{W}_{t+j}$, $E_t^* \hat{R}_{t+j}$, $E_t^* \hat{\Pi}_{t+j}$, $E_t^* \hat{r}_{t+j}^k$, and $E_t^* \hat{D}_{t+j}$ depend on the perceived laws of motion (26). In particular, for every variable y^f forecasted under learning agents use the forecast function $E_t^* \hat{y}_{t+1}^f = \psi_{y,t} X_t$ with $X_t = \left[\hat{K}_{t+1}; \hat{Z}_t\right]$. Here $\psi_{y,t}$ is the vector of beliefs in the PLM for y^f. In particular, the perceived laws of motion for capital and the technology shock are given by

$$\begin{bmatrix} E_t^* \hat{K}_{t+2} \\ E_t^* Z_{t+1} \end{bmatrix} = H_t X_t, \quad (84)$$

with

$$H_t = \begin{bmatrix} \psi_{kk,t} & \psi_{kz,t} \\ 0 & \rho_Z \end{bmatrix}, \quad (85)$$

where it is assumed that the shock process (59) is known to the agents. From this it follows that $E_t^* \hat{y}_{t+j+1}^f = \psi_{y,t} H_t^j X_t$, for $j \geq 0$, which allows us to obtain the following expressions for the sums in

(79):

$$
\begin{aligned}
SW_t^e &= \Gamma_4 \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{W}_{t+j} = \Gamma_4 \psi_{w,t} \beta \left(I - \beta H_t\right)^{-1} X_t, && (86)\\
SR_t^e &= \Gamma_3 \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{R}_{t+j} = \Gamma_3 \psi_{r,t} \beta \left(I - \beta H_t\right)^{-1} X_t, && (87)\\
S\Pi_t^e &= \Gamma_3 \beta^{-1} \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{\Pi}_{t+j} = \Gamma_3 \psi_{\pi,t} \left(I - \beta H_t\right)^{-1} X_t, && (88)\\
Sr_t^{k,e} &= \bar{K}\bar{r}^k \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{r}_{t+j}^k = \bar{K}\bar{r}^k \psi_{r^k,t} \beta \left(I - \beta H_t\right)^{-1} X_t, && (89)\\
SD_t^e &= \bar{D} \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{D}_{t+j} = \bar{D} \psi_{d,t} \beta \left(I - \beta H_t\right)^{-1} X_t. && (90)
\end{aligned}
$$

In the neoclassical specification of the model, the consumption rule is the same as (79) but without the term $S\Pi_t^e$.

The sums of future expected terms in (13) can be handled in the same way as the sums in the consumption function. By virtue of (84) we obtain the following expression

$$
\hat{Q}_t = -\hat{R}_t - \psi_{r,t} \beta \left(I - \beta H_t\right)^{-1} X_t + \psi_{\pi,t} \left(I - \beta H_t\right)^{-1} X_t + \beta \bar{r}^k \psi_{r^k,t} \left(I - \beta H_t\right)^{-1} X_t. \qquad (91)
$$

C.1.2 Firms

Log-linearisation of the first-order condition (49) yields

$$
\hat{p}_t^*(i) = (1 - \beta\theta) \widehat{MC}_t + (1 - \beta\theta) \beta\theta \sum_{j=0}^{\infty} (\beta\theta)^j E_t^* \widehat{MC}_{t+j+1} + \beta\theta \sum_{j=0}^{+\infty} (\beta\theta)^j E_t^* \hat{\Pi}_{t+j+1}, \qquad (92)
$$

where $p_t^*(i) = P_t^*(i)/P_t$. In the symmetric equilibrium all intermediate goods producers have identical marginal costs

$$
\widehat{MC}_t = (1 - \alpha) \hat{W}_t + \alpha \hat{r}_t^k - \hat{Z}_t. \qquad (93)
$$

Combining this expression with (92) and the log-linear approximation of the price index (50) we obtain condition (19). The sums of future expected terms in (19) can be handled in the same way as the sums in the consumption function. From (84) we have

$$
\begin{aligned}
\hat{\Pi}_t = \varphi\theta^{-1} \widehat{MC}_t + \beta\varphi \left[(1 - \alpha) \psi_{w,t} \left(I - \beta\theta H_t\right)^{-1} X_t + \alpha \psi_{r^k,t} \left(I - \beta\theta H_t\right)^{-1} X_t - \frac{\beta\theta\rho_Z}{1 - \beta\theta\rho_Z} Z_t \right] \\
+ \beta (1 - \theta) \psi_{\pi,t} \left(I - \beta\theta H_t\right)^{-1} X_t. \qquad (94)
\end{aligned}
$$

C.2 Alternative specification of fiscal policy

In all strategies considered in Section 7 the consumption tax rate remains constant. Hence, the linearised version of the government budget constraint (31) is

$$\bar{B}\hat{B}_{t+1} + \bar{\tau}^c\bar{C}\hat{C}_t + \bar{\tau}^w\bar{W}\bar{N}\left(\hat{\tau}_t^w + \hat{W}_t + \hat{N}_t\right) + \bar{\tau}^k\bar{r}^k\bar{K}\left(\hat{\tau}_t^k + \hat{r}_t^k + \hat{K}_t\right) = \bar{G}\hat{G}_t + \bar{B}\beta^{-1}\left(\hat{R}_{t-1} - \hat{\Pi}_t + \hat{B}_t\right) + \bar{TR}\hat{TR}_t. \quad (95)$$

Strategy 2: capital tax financing In this case, the household's flow budget constraint reads

$$\beta^{-1}\bar{K}\hat{K}_t = \left[(1+\bar{\tau}^c)\bar{C} + (1-\bar{\tau}^w)\bar{W}(1-\bar{N})\right]\hat{C}_t + \bar{K}\hat{K}_{t+1} - (1-\bar{\tau}^w)\bar{W}\hat{W}_t - (1-\bar{\tau}^k)\bar{r}^k\bar{K}\hat{r}_t^k - \bar{B}\beta^{-1}\left(\hat{R}_{t-1} - \hat{\Pi}_t + \hat{B}_t\right) + \bar{\tau}^k\bar{r}^k\bar{K}\hat{\tau}_t^k - \bar{D}\hat{D}_t - \bar{TR}TR_t + \bar{B}\hat{B}_{t+1}. \quad (96)$$

The reaction of the capital income tax rate is given by $\bar{\tau}^k\bar{r}^k\bar{K}\hat{\tau}_t^k = \hat{G}_t\bar{G}$. Combining this expression with (95) and (96), and following the same steps as in Appendix C.1 we can derive the consumption function under learning. In particular, the inter-temporal budget constraint of the household and the consumption function can be written, respectively, as (78) and (79), with β^{-1} replaced by $\tilde{\beta}^{-1} \equiv \bar{r}^k + 1 - \delta$ and $\eta = \left[1 + (1-\phi)\phi^{-1}(1+\bar{\tau}^c)(1-\bar{\tau}^w)^{-1}\right]\bar{C}$.

Optimal investment now requires that

$$\hat{Q}_t = \beta E_t^*\hat{Q}_{t+1} - \hat{R}_t + E_t^*\hat{\Pi}_{t+1} + \beta\left(1-\bar{\tau}^k\right)\bar{r}^k E_t^*\hat{r}_{t+1}^k - \beta\bar{\tau}^k\bar{r}^k E_t^*\hat{\tau}_{t+1}^k. \quad (97)$$

By iterating forward and using $\bar{\tau}^k\bar{r}^k\bar{K}\hat{\tau}_t^k = \hat{G}_t\bar{G}$ we obtain the infinite-horizon optimal investment rule

$$\hat{Q}_t = -\hat{R}_t + \sum_{j=1}^{\infty}\beta^j\left[\left(1-\bar{\tau}^k\right)\bar{r}^k E_t^*\hat{r}_{t+j}^k - E_t^*\hat{R}_{t+j} + \beta^{-1}E_t^*\hat{\Pi}_{t+j} - \bar{K}^{-1}\bar{G}E_t^*\hat{G}_{t+j}\right]. \quad (98)$$

Strategy 3: labour tax financing In this case, the optimality condition for labour is given by

$$\hat{N}_t = \frac{1-\bar{N}}{\bar{N}}\left(\hat{W}_t - \hat{C}_t - \frac{\bar{\tau}^w}{1-\bar{\tau}^w}\hat{\tau}_t^w\right), \quad (99)$$

instead of (61). The household flow budget constraint is

$$\beta^{-1}\bar{K}K_t = \left[(1+\bar{\tau}^c)\bar{C} + (1-\bar{\tau}^w)\bar{W}(1-\bar{N})\right]\hat{C}_t + \bar{K}\hat{K}_{t+1} - (1-\bar{\tau}^w)\bar{W}\hat{W}_t + \bar{W}\bar{\tau}^w\hat{\tau}_t^w - (1-\bar{\tau}^k)\bar{r}^k\bar{K}\hat{r}_t^k - \bar{B}\beta^{-1}\left(\hat{R}_{t-1} - \hat{\Pi}_t + \hat{B}_t\right) - \bar{D}\hat{D}_t - \bar{TR}\hat{TR}_t + \bar{B}\hat{B}_{t+1}. \quad (100)$$

The dynamics of the labour income tax rate are determined by $\bar{\tau}^w\bar{W}\bar{N}\hat{\tau}_t^w = \hat{G}_t\bar{G}$. Combining this expression with (95) and iterating forward yields the inter-temporal budget constraint of the household. When (76) is substituted in this constraint, the following consumption function is obtained:

$$\Gamma_1 C_t = \tilde{\beta}^{-1}\bar{K}\hat{K}_t + \Gamma_2\hat{W}_t + \bar{K}\bar{r}^k r_t^k + \bar{D}\hat{D}_t - \Gamma_5\bar{G}\hat{G}_t - \Gamma_3\hat{R}_t + SW_t^e - SR_t^e + S\Pi_t^e + Sr_t^{k,e} + SD_t^e - \Gamma_6 SG_t^e. \quad (101)$$

where Γ_1, Γ_2, Γ_3, Γ_4, SW_t^e, SR_t^e, $S\Pi_t^e$, $Sr_t^{k,e}$, SD_t^e, and SG_t^e are defined above, again with β^{-1} replaced by $\tilde{\beta}^{-1} \equiv \bar{r}^k + 1 - \delta$ and $\eta = \left[1 + (1-\phi)\,\phi^{-1}\,(1+\bar{\tau}^c)\,(1-\bar{\tau}^w)^{-1}\right]\bar{C}$. The coefficients Γ_5 and Γ_6 are defined as

$$\begin{aligned}
\Gamma_5 &\equiv \frac{1-\bar{\tau}^w\bar{N}}{(1-\bar{\tau}^w)\,\bar{N}} - \frac{\tilde{\beta}\,(1-\sigma)\,\phi\,(1-\bar{N})\,\eta\bar{C}^{-1}}{\sigma\left(1-\tilde{\beta}\right)(1+\bar{\tau}^c)\,\bar{N}}, \\
\Gamma_6 &\equiv \frac{(1-\sigma)\,(1-\bar{N})\,\phi\eta\bar{C}^{-1}}{\sigma\,(1+\bar{\tau}^c)\,\bar{N}} + \frac{1-\bar{\tau}^w\bar{N}}{(1-\bar{\tau}^w)\,\bar{N}}.
\end{aligned}$$

The optimality conditions for investment are given by (97) and (98) where government spending and the capital tax rate drop out.

D Alternative specification of fiscal policy

Figure 5 and Figure 6 show the impulse responses to an increase in government spending of 1% of GDP and the expectations formed by the learning mechanism for different specifications of fiscal policy.

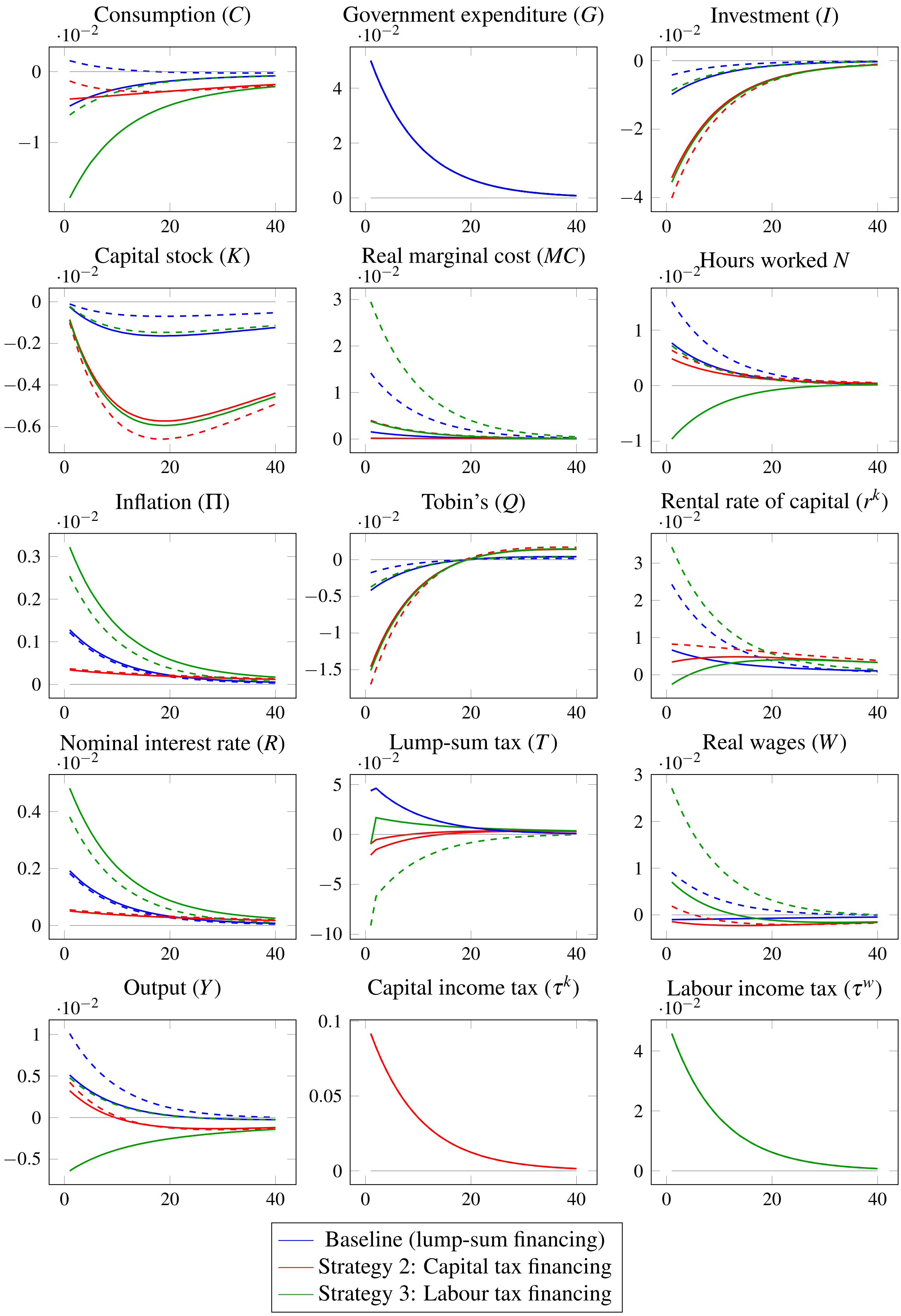

Figure 5: Impulse responses to an increase in government spending of 1% of GDP of the new Keynesian model for different fiscal policy specifications. The solid lines are the responses under rational expectations; the dashed lines are those under adaptive learning. The impulse response functions are measured in percentage deviations from steady-state. The horizontal axis measures quarters.

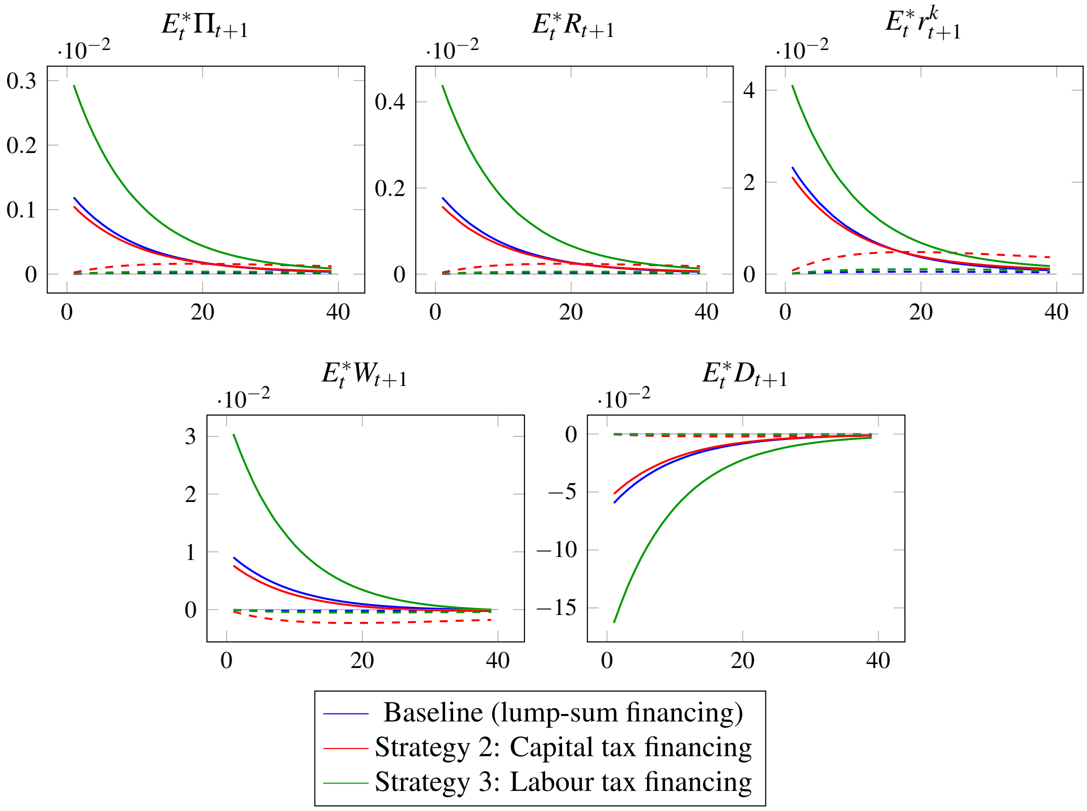

Figure 6: Expectations on forward-looking variables after a government spending shock of 1% of GDP of the new Keynesian model for different fiscal policy specifications. The solid lines are the responses under rational expectations; the dashed lines are those under adaptive learning. The impulse response functions are measured in percentage deviations from steady-state. The horizontal axis measures quarters.