UNIVERSITEIT GENT

FACULTEIT ECONOMIE
EN BEDRIJFSKUNDE

TWEEKERKENSTRAAT 2
B-9000 GENT
Tel. : 32 - (0)9 – 264.34.61
Fax. : 32 - (0)9 – 264.35.92

WORKING PAPER

A Panel Analysis of the Fisher Effect with an Unobserved $I(1)$ World Real Interest Rate

Gerdie Everaert

SHERPPA, Ghent University

April 2012

2012 / 782

D/2012/7012/15

A Panel Analysis of the Fisher Effect with an Unobserved $I(1)$ World Real Interest Rate

Gerdie Everaert*

SHERPPA, Ghent University

April 12, 2012

Abstract

The Fisher effect states that inflation expectations should be reflected in nominal interest rates in a one-for-one manner to compensate for changes in the purchasing power of money. Despite its wide acceptance in theory, much of the empirical work fails to find favorable evidence. This paper examines the Fisher effect in a panel of 21 OECD countries over the period 1983-2010. A first generation panel test finds cointegration between nominal interest rates and inflation. However, a non-stationary common factor in the error terms of this alleged cointegrating relation is detected using the Panel Analysis of Non-stationarity in Idiosyncratic and Common Components (PANIC). This implies that the regression results are spurious. A possible interpretation for the non-stationary common factor is that it reflects permanent common shifts in the real interest rate induced by e.g. shifts in time preferences, risk aversion and the steady-state growth rate of technological change. We next control for an unobserved non-stationary common factor in estimating the Fisher equation using both the Common Correlated Effects (CCE) and the Continuously Updated (Cup) estimation approach. The impact of inflation on the nominal interest rate is found to be insignificantly different from 1.

JEL Classification: C23, E31, E43

Keywords: Fisher effect, panel cointegration, cross-sectional dependence, unobserved common factors

1 Introduction

The Fisher effect states that inflation expectations should be reflected in nominal interest rates in a one-for-one manner to compensate for changes in the purchasing power of money (Fisher, 1930). This implies that the ex ante real interest rate, defined as the difference between the nominal interest rate and expected inflation, is not affected by changes in inflation expectations. While probably not being valid in

*I thank Koen Inghelbrecht, Lorenzo Pozzi and the participants of the Amsterdam Econometric Seminar (Tinbergen Institute, November 2011), the 5th CSDA International Conference on Computational and Financial Econometrics (University of London, December 2011) and of the 20th Annual Symposium of the Society for Nonlinear Dynamics and Econometrics (Istanbul, April 2012) for helpful suggestions and constructive comments. I further acknowledge financial support from the Interuniversity Attraction Poles Program - Belgian Science Policy, contract no. P5/21.

the short run, the Fisher effect is expected to hold as a long-run equilibrium concept. Insofar as permanent changes in expected inflation originate from permanent shocks in the rate of money growth, this is in accordance with the so-called long-run superneutrality of money. Despite its wide acceptance in theory, most of the empirical work fails to find convincing evidence in favor of the Fisher effect. As nominal interest rates and inflation are typically found to be non-stationary, the long-run Fisher effect implies that these two variables should cointegrate with unit slope coefficient such that the real interest rate is stationary and therefore not affected by permanent shocks to inflation. A survey of this literature shows that unit root tests find real interest rates to be non-stationary (see e.g. Rose, 1988; Rapach and Weber, 2004; Lai, 2008) while cointegration analysis either finds no cointegration between nominal interest rates and inflation (see e.g. MacDonald and Murphy, 1989; Koustas and Serletis, 1999) or when cointegration is found the estimated slope is significantly less (see e.g. Evans and Lewis, 1995) or significantly greater than one (see e.g. Crowder and Hoffman, 1996).

A number of theoretical explanations for the empirical failure of the Fisher effect have been put forward. First, inflation expectations are not observed and are therefore replaced by ex post observed inflation to calculate ex post real interest rates. Evans and Lewis (1995) argue that the alleged permanent component in these ex post real interest rates may be due to people incorporating anticipated shifts in the inflation process into their expectations implying a persistent deviation of observed inflation from expected inflation over the period these shifts do not materialise. Second, Darby (1975) argues that the presence of taxes on interest income implies that nominal interest rates have to rise by more than one-for-one in response to a change in inflation expectations in order to keep the after-tax real interest rate constant. These tax effects may thus explain why nominal interest rates and inflation cointegrate with a slope coefficient greater than one. Third, in the seminal papers of Mundell (1963) and Tobin (1965) higher inflation causes a substitution out of money balances into bonds and real assets, putting downward pressure on real interest rates. This may explain why nominal interest rates and inflation cointegrate with a slope coefficient less than one.

A plausible econometric explanation is that the existing empirical evidence on the Fisher effect is flawed as it is based on a country-by-country analysis often using at most 50 annual observations. Using such relatively small data sets results in low power of conventional unit root and cointegration tests especially when there is high persistence under the alternative hypothesis of stationarity. Westerlund (2008) therefore suggests to test the Fisher effect in a panel of quarterly data covering 20 OECD countries between 1980 and 2004. Taking into account error cross-sectional dependence when testing for cointegration, he shows that the null hypothesis of no panel cointegration between interest rates and inflation can be rejected while the hypothesis of a unit slope coefficient on inflation cannot be rejected.

An alternative, but yet unexplored, explanation is that the real factors behind the real interest rate are not stable over time. Standard neoclassical growth models with household intertemporal utility maximization imply that the real interest rate is a function of time preference, risk aversion and the steady-state growth rate of technological change. While time preference and risk aversion are generally believed to be fairly stable, or at least changing only slowly over extended periods of time, shifts in steady-state growth, such as the 'productivity slowdown' of the early 1970s and the 'New Economy' resurgence of growth in the late 1990s, have been widely documented in the literature (see e.g. Oliner and Sichel, 2000; Roberts, 2001). Additional determinants of real interest rates suggested in the literature are demographic changes, changes in the stance of fiscal policy and the evolution of public debt, changes in the taxation of profits, (de)regulation of financial markets, ... (see e.g. Blanchard and Summers, 1984; Chadha and Dimsdale, 1999; Ardagna, 2009). Permanent shifts in any of these factors induce a unit root in the real interest rate and by extension in the residuals of a regression of the nominal interest rate on inflation. However, this does not automatically invalidate the Fisher hypothesis of a one-for-one relation between nominal interest rates and inflation. Basically, the sources of the non-stationary behaviour of real interest rates are omitted non-stationary variables that should be added to a regression of nominal interest rates on inflation for this to be a cointegrating relation. Note that this argument not only explains the failure to find cointegration but also the large variety of estimated slope coefficients over empirical studies that do find cointegration as Everaert (2011) shows that omitting relevant non-stationary variables yields spurious estimation results with standard cointegration tests indicating these results to be a cointegration regression in far too many cases.

Ideally, the non-stationary determinants of real interest rates should be included in a regression of nominal interest rates on inflation. However, there is a large variety of possible determinants which are, moreover, not directly observable or at least hard to measure. A promising way out of this problem is to identify these determinants by exploiting the strong cross-section correlation between interest rates observed over countries. Increasing economic integration leads to a substantial degree of linkage between real interest rates of different countries and has led a number of authors to construct and analyze a world real interest rate (Barro and Sala-i Martin, 1990; Koedijk et al., 1994; Lee, 2002) or to relate national real interest rates to international rather than to domestic events (Blanchard and Summers, 1984).

This paper uses recent advances in panel data econometrics to identify and account for unobserved common factors in a panel of quarterly data for nominal interest rates and inflation covering 21 OECD countries between 1983 and 2010. The analysis consists of two steps. In the first step, we investigate the integration properties of the data using the Panel Analysis of Non-stationarity in Idiosyncratic and Common Components (PANIC) of Bai and Ng (2004). The most important conclusion from this analysis is that real interest rates and the error terms of a fixed effects (FE) regression of nominal interest rates on inflation can both be decomposed in a single non-stationary common factor and a stationary idiosyncratic component. The latter finding is consistent with Westerlund (2008) who also shows that real interest rates and Fisher equation regression errors are stationary after filtering out common factors. This leads him to conclude that nominal interest rates and inflation cointegrate. However, Westerlund does not analyze the integration properties of the common factors but simply assumes these to be stationary. Our finding of a non-stationary common factor invalidates his conclusion and implies that regression results ignoring this common factor are spurious (see Urbain and Westerlund, 2011). In the second step we therefore estimate the relation between nominal interest rates and inflation taking into account a common non-stationary component in real interest rates. In particular, we use the common correlated effects pooled (CCEP) estimation approach proposed by Pesaran (2006) and Kapetanios et al. (2011) and the continuously-updated (Cup) estimation approach proposed by Bai et al. (2009). The advantage of both approaches is that they can consistently estimate the relationship between nominal interest rates and inflation under very general integration properties of the data without the need to identify and measure the determinants of real interest rates as long as these determinants are common to all countries. Thus, rather the treating the cross-section correlation as a nuisance, which requires adjustment of standard unit root and cointegration tests, we exploit the comovement of interest rates to identify unobserved common determinants of real interest rates. This allows us to test the Fisher effect in the presence of a non-stationary world real interest rate. Endogeneity of observed inflation induced by a rational expectations forecasting error is taken into account using CupBC, a bias-corrected version of the Cup estimator, and CCEP_GMM, a GMM version of the CCEP estimator. We also propose how to test for cointegration from the error terms of the CCEP and CUP estimators. A small-scale Monte Carlo simulation shows that these two estimators and cointegration tests perform reasonably well for the modest sample size $T = 112$, $N = 21$ that is available for our empirical analysis. From the estimation results, the hypothesis of a one-for-one relation between the nominal interest rate and inflation cannot be rejected using either the CupBC or the CCEP_GMM estimator.

The paper is organized as follows. Section 2 outlines the standard Fisher equation. Section 3 analyses the time series properties of the data. Section 4 augments the standard Fisher equation with a non-stationary factor and discusses how this factor-augmented equation can be estimated. Section 5 analyses the small sample properties of the proposed estimators using a Monte Carlo simulation and Section 6 presents the estimation results. Section 7 concludes.

2 The standard Fisher equation

Fisher (1930) hypothesized that inflation expectations should be reflected in the nominal interest rate in a one-for-one manner to compensate for changes in the purchasing power of money. This implies that the real interest rate should be invariant to changes in expected inflation. Formally, the Fisher hypothesis can be stated as $\beta = 1$ in

$$i_{it} = r_{it}^e + \beta \pi_{it}^e, \qquad i = (1, \ldots, N), \quad t = (1, \ldots, T), \tag{1}$$

where i_{it} is the nominal interest rate observed in country i at time t, r_{it}^e is the ex ante real interest rate and π_{it}^e the expected rate of inflation.

The validity of the Fisher effect cannot be directly analyzed using (1) as r_{it}^e and π_{it}^e are unobserved ex ante variables. The Fisher equation can be written in terms of ex post observed variables after making two assumptions. First, the ex ante real interest rate is driven by real factors which are typically assumed to be more or less stable over time such that r_{it}^e can be written as

$$r_{it}^e = \alpha_i + \nu_{it}, \tag{2}$$

where α_i is a country-specific constant and ν_{it} is a stationary error term which captures temporary fluctuations in r_{it}^e. Second, assuming rational expectations

$$\pi_{it} = \pi_{it}^e + \zeta_{it}, \tag{3}$$

where ζ_{it} is a mean zero stationary forecast error orthogonal to any information known at time t. Inserting (2) and (3) in (1) yields

$$i_{it} = \alpha_i + \beta\pi_{it} + \epsilon_{it}, \tag{4}$$

where ϵ_{it} is a composite error term, comprised of the forecast error $-\beta\zeta_{it}$ and the term ν_{it}.

Equation (4) forms the basis for testing the Fisher hypothesis. Given that i_{it} and π_{it} are typically found to be $I(1)$ series, this is nowadays done using unit root testing and cointegration analysis (see e.g. MacDonald and Murphy, 1989). In fact, this alleged non-stationarity significantly simplifies the job of testing the Fisher hypothesis. First, when i_{it} and π_{it} are cointegrated, super consistency of the LS estimator implies that (4) can be estimated ignoring the correlation between π_{it} and ϵ_{it} and any dynamics in ϵ_{it}. Note that cointegration between i_{it} and π_{it} requires ϵ_{it} to be stationary, but does not depend on the specific value of β with $\beta = 1$ then being denoted as the full Fisher effect and $\beta \neq 1$ as the partial Fisher effect. Popular theoretical explanations for $\beta \neq 1$ are (i) taxes on interest income which imply that the nominal interest rate has to raise by more than one-for-one ($\beta > 1$) in response to a change in inflation expectations to keep the after-tax real interest rate constant and (ii) portfolio shifts out of money balances into interest bearing assets in response to an increase in inflation expectations which puts downward pressure on real interest rates ($\beta < 1$).

Second, defining the ex post observed real interest rate r_{it} as

$$r_{it} \equiv i_{it} - \pi_{it} = \alpha_i - (1-\beta)\,\pi_{it} + \epsilon_{it}, \tag{5}$$

the Fisher hypothesis boils down to a simple unit root test on r_{it}. This is a test for the full Fisher effect as stationarity of r_{it} requires $\beta = 1$ when π_{it} is found to be non-stationary.

3 Time series properties of the data

This section analyses the time series properties of the data. We start with country-by-country and first generation panel unit root and cointegration tests. As strong evidence of cross-sectional dependence is found, we next use second generation panel tests and decompose all series in a common factor and an idiosyncratic component and analyze the time series properties of these components separately using PANIC.

3.1 Data

We use quarterly data taken from the International Financial Statistics database of the International Monetary Fund. The sample includes 21 OECD countries (see Table 2 for the full list of countries) covering the period from 1983Q1 to 2010Q4. The nominal interest rate is either the three months treasury bill rate, if available, or the three months money market rate. Expected inflation is proxied by the ex post observed inflation rate calculated as the year-on-year percent change in the consumer price index (CPI). We use year-on-year percent changes as this attenuates the strong noise in annualized quarter-on-quarter percent changes (also see Bekaert and Wang, 2010). Year-on-year changes are also the most prominent way inflation is reported and is also the subject of most professional inflation forecasts (central bank forecasts, survey forecasts, ...). Studies examining the forecasting power of alternative methods (see e.g. Stock and Watson, 1999; Ang et al., 2007) typically also focus on a one-year inflation horizon. One disadvantage of using annual inflation at a quarterly frequency is that we will have to take into account that the forecast errors ζ_{it} follow a MA(3) process due to overlapping observations.

3.2 Country-by-country unit root and cointegration tests

As outlined in Section 2, a first way to test the Fisher effect is to analyze the time series properties of i_{it}, π_{it} and r_{it}. Table 1 presents country-by-country ADF-GLS unit root tests for a model with a constant and no trend. This ADF-GLS test is the modified augmented Dickey and Fuller (1979) (ADF) test based on generalized least squares (GLS) demeaning of the data as suggested by Elliott et al. (1996). Compared to the standard ADF test, removal of the constant term by means of GLS demeaning yields substantial power improvements, especially in small samples. More details on the exact implementation of the test are provided as a note to Table 1. First looking at i_{it} and π_{it}, the unit root hypothesis cannot be rejected at the 5% level of significance for any of the individual countries. Given this finding, the full Fisher effect requires r_{it} to be stationary. At the 5% level of signifcance, the null hypothesis of a unit root in r_{it} can only be rejected for Norway and Portugal. Thus, there is no clear support of the full Fisher.

An alternative way to test the Fisher effect is to infer whether there is a one-for-one cointegration relation between i_{it} and π_{it}. Table 1 reports OLS coefficient estimates for β in equation (4) along with an ADF cointegration test (without constant) on the OLS residuals. The results are clearly not in support of the Fisher effect. Although, the estimated slope coefficients are relatively close to one in a lot of countries, the null hypothesis of a unit coefficient is clearly rejected in most countries. More importantly, the ADF cointegration test results show that the null hypothesis of no cointegration cannot be rejected in 19 out of 21 countries. This implies that the OLS regression results should be considered spurious.

3.3 First generation panel unit root and cointegration tests

As the individual county data span a relatively short period of 28 years, the failure to find evidence in favor of the Fisher effect may be due to a lack of power to reject the null hypothesis of a unit root in either r_{it} or $\widehat{\epsilon}_{it}$. Power can be increased substantially by exploiting the panel dimension of the data. In particular, we use the Maddala and Wu (1999) (MW) panel unit root test which is a combination of the p-values from the country-specific unit root tests. The advantages of the MW test are that (i) one can use different lag lengths in the individual ADF regressions (as implied by lag optimization) and (ii) it can be calculated from p-values of any country-specific type of unit root test.

Table 1: Country-by-country unit root and cointegration tests

Sample period: 1983:Q1-2010:Q4, 21 countries

	ADF-GLS unit root tests									Cointegration analysis						
	i_{it}			π_{it}			r_{it}			Fisher regressions				ADF on $\widehat{\epsilon}_{it}^{OLS}$		
	k	test	p-val	k	test	p-val	k	test	p-val	$\widehat{\beta}$	se	t-stat	p-val	k	test	p-val
Australia	9	−0.31	0.56	4	−0.32	0.56	4	−1.90	0.05	1.26	0.09	2.98	0.00	0	−2.94	0.11
Austria	1	−1.09	0.25	4	−1.82	0.06	3	−1.65	0.09	1.34	0.13	2.54	0.01	0	−1.93	0.54
Belgium	1	0.23	0.75	8	−0.14	0.63	0	−0.84	0.35	1.18	0.14	1.30	0.20	0	−1.19	0.85
Canada	7	−0.44	0.51	4	−0.07	0.65	4	−1.35	0.16	1.56	0.14	4.07	0.00	4	−1.79	0.61
Denmark	2	0.21	0.74	8	0.38	0.79	0	−0.85	0.34	1.54	0.19	2.86	0.01	0	−1.97	0.52
Finland	0	−0.16	0.62	4	−0.50	0.49	2	−0.99	0.29	1.80	0.11	7.58	0.00	4	−1.76	0.63
France	1	0.27	0.76	9	−0.13	0.63	0	−0.80	0.37	1.32	0.11	2.96	0.00	0	−1.25	0.83
Germany	1	−1.34	0.17	0	−1.40	0.15	0	−1.36	0.16	1.33	0.11	2.95	0.00	0	−2.11	0.45
Greece	0	0.03	0.68	5	−0.31	0.56	0	−1.23	0.20	0.96	0.05	−0.74	0.46	0	−2.27	0.36
Ireland	2	−0.26	0.58	5	0.12	0.71	6	−1.39	0.15	0.80	0.14	−1.42	0.16	4	−1.74	0.64
Italy	1	0.27	0.76	5	0.09	0.70	0	−1.30	0.17	1.48	0.07	6.73	0.00	0	−2.83	0.14
Japan	3	−0.58	0.46	4	−1.25	0.19	4	−0.31	0.56	1.68	0.13	5.35	0.00	4	−2.28	0.36
Netherlands	1	−1.56	0.11	4	−1.90	0.05	4	−1.13	0.23	0.57	0.19	−2.31	0.02	1	−1.23	0.84
New Zealand	12	−1.03	0.27	12	−0.44	0.51	2	−1.65	0.09	1.10	0.05	1.88	0.06	1	−2.51	0.25
Norway	0	−1.03	0.28	4	0.06	0.69	0	−2.50	0.01	1.37	0.11	3.39	0.00	0	−3.63	0.02
Portugal	2	−0.61	0.44	1	−0.50	0.49	3	−2.19	0.03	0.91	0.04	−2.48	0.01	4	−1.98	0.52
Spain	3	−0.21	0.60	4	0.34	0.78	4	−1.34	0.16	1.29	0.08	3.60	0.00	4	−1.86	0.58
Sweden	8	0.01	0.68	8	−0.47	0.50	9	−1.01	0.28	1.14	0.07	1.89	0.06	4	−2.03	0.49
Switzerland	6	−1.82	0.06	6	−0.72	0.40	6	−0.71	0.40	1.23	0.10	2.41	0.02	0	−4.44	0.00
UK	1	−0.27	0.58	8	−1.68	0.08	1	−0.25	0.58	1.29	0.10	2.90	0.00	4	0.22	0.99
US	1	−0.88	0.33	8	−1.66	0.09	5	−0.61	0.44	1.19	0.15	1.24	0.22	4	−1.73	0.64

Notes: ADF-GLS refers to the Elliott et al. (1996) augmented Dickey-Fuller (ADF) generalised least squares test. The lag length k is selected using the modified Akaike information criterion (MAIC) suggested by Ng and Perron (2001) with the maximum lag length k_{max} set according to the Schwert (1989) rule: $k_{max} = \text{int}\{12\,(T/100)^{0.25}\} = 12$. The Fisher regressions report country-specific OLS coefficient estimates for β in equation (4). The t-stat is calculated under the null hypothesis that $\beta = 1$. The ADF cointegration tests are ADF tests (with no deterministic terms) on the estimated residuals of the country-specific OLS regressions ($\widehat{\epsilon}_{it}^{OLS}$). The p-values for the country-specific unit root and cointegration tests are obtained by simulating their finite-sample distributions (based on 20,000 Monte Carlo iterations) taking into account, as in Cook and Manning (2004), that the augmentation of the test equation is optimized using the MAIC criterion. The 1% and 5% critical values taken from these distributions are -2.59 and -2.02 for the ADF-GLS test and -3.88 and -3.30 for the ADF test on the OLS residuals.

The top panel of Table 2 reports results for a MW panel unit root test on i_{it}, π_{it} and r_{it} calculated from the ADF-GLS p-values reported in Table 1. For i_{it} and π_{it}, the null hypothesis of a unit root is not rejected while for r_{it} it is rejected well below the 1% level of significance. Thus, the full Fisher effect seems to be strongly supported by the panel unit root results.

Turning to cointegration between i_{it} and π_{it}, the bottom panel of Table 2 reports fixed effects (FE) estimates for β in equation (4) and a MW cointegration test calculated from the p-values of an ADF test on the estimated residuals $\widehat{\epsilon}_{it}^{FE}$. The panel FE results are also supportive for the Fisher effect. First, the null hypothesis of no cointegration is clearly rejected using the MW panel test. This suggests that there is a cointegrating relation between i_{it} and π_{it}. Second, the point estimate of the slope coefficient is 1.09 and is significantly different from one. These results support the partial but not the full Fisher effect. However, despite being consistent and asymptotically normally distributed (Phillips and Moon, 1999), the FE estimator and especially its t-statistic are biased in small samples (see e.g. Kao and Chiang, 2000). The main reason for the latter is that autocorrelation in the error terms invalidates the standard asymptotic variance of the FE estimator. Therefore, we bootstrap the standard error of the FE estimator by resampling whole cross-sectional units with replacement as suggested by Kapetanios (2008). The advantage of this resampling scheme is that it preserves (i) the autocorrelation structure in the data and the errors, (ii) the endogeneity of π_{it} and (iii) the cross-sectional dependence[1]. Using the bootstrapped standard error, the FE estimator is now not significantly different from 1.[2]

Table 2: Panel unit root and cointegration tests

Sample period: 1983:Q1-2010:Q4, 21 countries

Panel unit root tests	i_{it}	π_{it}	r_{it}
MW test	37.42	43.92	72.81
p-val	0.67	0.39	0.00

Panel cointegration analysis	Fisher regression							MW on $\widehat{\epsilon}_{it}^{FE}$	
		analytical			bootstrap				
	$\widehat{\beta}$	se	t-stat	p-val	se	t-stat	p-val	test	p-val
FE	1.09	0.02	5.22	0.00	0.08	1.14	0.26	70.81	0.00

Notes: The panel unit root test is the Maddala and Wu (1999) (MW) test defined as $-2\sum_{i=1}^{N}\ln(p_i)$ where p_i is the p-value corresponding to the unit root test of the ith country reported in Table 1. The p-value of the MW test is obtained from the χ^2 distribution with $2N$ degrees of freedom. The Fisher regression reports the coefficient estimate for β in equation (4) using FE. The t-stat is calculated under the null hypothesis that $\beta = 1$. The bootstrapped standard error is obtained as the standard deviation of the FE estimator over 5000 bootstrap iterations with bootstrap samples obtained by resampling cross-sections as in Kapetanios (2008). The MW cointegration test is calculated from the p-values of ADF unit root tests (with no deterministic terms) on the estimated residuals of the panel FE regression ($\widehat{\epsilon}_{it}^{FE}$). The p-values for the country-specific cointegration tests are obtained by simulating the finite-sample distribution of the ADF cointegration test (based on 20,000 Monte Carlo iterations) taking into account (i) that $\widehat{\epsilon}_{it}^{FE}$ are residuals from a panel FE regression and (ii) as in Cook and Manning (2004) that the augmentation of the test equation is optimized using the MAIC criterion. The 1% and 5% critical values taken from this simulated distribution are -3.29 and -2.69.

[1]Note that the cross-sectional resampling scheme is not valid in the case of local cross-sectional dependence but is appropriate in the presence of the below assumed factor structure which introduces global cross-sectional dependence which is symmetric across all panel units.

[2]Bootstrapping the pivotal t-statistic for $\beta = 1$ yields highly similar results.

3.4 Cross-sectional dependence and common factors

The panel results in Section 3.3 support the full Fisher effect. However, there are at least two reasons for why these results may not be trustworthy. First, the MW panel unit root and cointegration tests are only valid for combining p-values from cross-sectionally independent tests. O'Connell (1998) documents that the alleged power gain of panel unit root tests developed under cross-sectional independence may in practice very well be the consequence of nontrivial size distortions induced by cross-sectional dependence, raising the real size of tests with a nominal size of 5% to as much as 50%. A similar conclusion can be found in Banerjee et al. (2004, 2005). Second, insofar as the cross-sectional dependence is induced by non-stationary omitted common factors that are relatively small compared to the stationary component in the data, unit root tests are biased towards rejection of the null hypothesis of a unit root (Bai and Ng, 2004). In this section we therefore test for cross-sectional dependence and the presence of (non-stationary) common factors in the data and in the residuals of the FE Fisher regression.

Testing for cross-sectional dependence

Table 3 presents information on the extent of the cross-sectional dependence in the original data, the residuals of the ADF-GLS regressions and the residuals of the FE Fisher regression. For those series that are potentially non-stationary, we also report results for the first-differences to avoid spurious non-zero correlations. We first compute the average cross-correlation coefficient $\overline{\widehat{\rho}}$ which is the average of the country-by-country cross-correlation coefficients $\widehat{\rho}_{ij}$ (for $i \neq j$). The original data and the residuals from both the ADF-GLS regressions and the FE Fisher regression all exhibit considerable positive cross-sectional correlation. Next, we compute the cross-sectional dependence (CD) test of Pesaran (2004). This shows that the null hypothesis of no cross-sectional dependence is strongly rejected for all variables and residuals. The finding of significant cross-sectional dependence implies that the MW panel unit root and cointegration tests do not have the conventional χ^2 distribution and therefore the MW p-values reported in Table 2 should not be trusted.

Common factor structure

In the recent panel literature, cross-sectional dependence is typically assumed to stem from omitted common variables or global shocks that affect each country differently and is therefore modelled using a common factor structure with country-specific factor loadings (see e.g. Bai and Ng, 2004; Coakley et al., 2006; Pesaran, 2006). More precisely, assume that the data generating process (DGP) of a series X_{it} is given by the following prototypical common factor model

$$X_{it} = \lambda_i' F_t + e_{it}, \tag{6}$$

where F_t is an $r \times 1$ vector of common factors with country-specific factor loadings λ_i and e_{it} is an idiosyncratic error term. Cross-sectional dependence stems from the common component $\lambda_i' F_t$ which is correlated over countries. The series X_{it} is non-stationary if at least one of the common factors in F_t is non-stationary, or the idiosyncratic error e_{it} is non-stationary, or both.

Table 3: Cross-sectional dependence test

	Sample period: 1983:Q1-2010:Q4, 21 countries					
	Levels			First-differences		
	$\bar{\bar{\rho}}$	CD	p-val	$\bar{\bar{\rho}}$	CD	p-val
Original data						
i_{it}	0.79	120.99	0.00	0.23	34.45	0.00
π_{it}	0.54	83.50	0.00	0.29	44.91	0.00
r_{it}	0.58	89.19	0.00	0.14	21.86	0.00
Residuals ADF-GLS regression						
i_{it}	0.22	32.20	0.00			
π_{it}	0.25	36.65	0.00			
r_{it}	0.15	22.63	0.00			
Residuals Fisher regression						
$\widehat{\epsilon}_{it}^{FE}$	0.53	81.49	0.00	0.15	22.35	0.00

Notes: the average cross-correlation coefficient $\bar{\bar{\rho}} = (2/N(N-1)) \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \widehat{\rho}_{ij}$ is the average of the country-by-country cross-correlation coefficients $\widehat{\rho}_{ij}$ (for $i \neq j$). CD is the Pesaran (2004) test defined as $\sqrt{2T/N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \widehat{\rho}_{ij}$, which is asymptotically standard normal under the null of cross-sectional independence.

Table 4 reports results for estimating the total number of relevant common factors r in the data for i_{it}, π_{it} and r_{it} and in the residuals of the FE Fisher regression using the panel information criteria suggested by Bai and Ng (2002). As consistency of these criteria requires stationary data, we take first-differences of all series (also see Bai and Ng, 2004, p. 1144). The top panel of Table 4 reports the $IC_{1,2,3}$, $PC_{1,2,3}$, AIC_3 and BIC_3 criteria with the maximum number of factors (r_{max}) ranging from 2 to 6. Using the $PC_{1,2,3}$ and AIC_3 criteria, the optimal number of factors is found to increase with r_{max} for all series. The results of the $IC_{1,2,3}$ and BIC_3 criteria are more stable over alternative choices of r_{max} and point to a single common factor in all series, except when using the IC_1 and IC_3 criteria on i_{it} for which the number of common factors increases with r_{max} and when using the BIC_3 criterion on r_{it} and $\widehat{\epsilon}_{it}^{FE}$ for which no factors are found for lower values of r_{max}. Note that the contradictory results over the various information criteria are in line with the Monte Carlo simulations in Bai and Ng (2002) which show that in samples of moderate size, i.e. $\min\{N,T\} < 40$, the IC criteria tend to underparameterize (especially for larger values of r) while the PC criteria tend to overparameterize (estimated number of components is found to increase with r_{max}), with the problem being even more severe for the AIC and BIC criteria. Taking this into account, the information criteria suggest that there is at least 1 common factor in i_{it}, π_{it} and r_{it} and in the residuals of the FE Fisher regression.

Bai (2004) has proposed a set of information criteria that are closely related to those of Bai and Ng (2002) but that can be applied to the levels of the series to determine the number of non-stationary factors.[3] The results for the $IPC_{1,2,3}$ criteria are reported in the bottom panel of Table 4, again with the maximum number of factors r_{max} ranging from 2 to 6. The results suggest a single non-stationary common factor in r_{it} and $\widehat{\epsilon}_{it}^{FE}$ and at least one non-stationary common factor in i_{it} and π_{it}.

To visualize the importance of the common factors, Figure 1 plots the data for i_{it}, π_{it} and r_{it} and the FE residuals $\widehat{\epsilon}_{it}^{FE}$ together with the first 3 factors estimated using the differencing and recumulating

[3]Note that consistency of the information criteria in Bai (2004) requires the idiosyncratic component to be stationary. Evidence that this is indeed the case is presented in Section 3.5 below.

Table 4: Estimating the number of common factors r

Sample period: 1983:Q1-2010:Q4, 21 countries

	$r_{max}=2$				$r_{max}=3$				$r_{max}=4$				$r_{max}=5$				$r_{max}=6$			
	i_{it}	π_{it}	r_{it}	$\widehat{\epsilon}_{it}^{FE}$	i_{it}	π_{it}	r_{it}	$\widehat{\epsilon}_{it}^{FE}$	i_{it}	π_{it}	r_{it}	$\widehat{\epsilon}_{it}^{FE}$	i_{it}	π_{it}	r_{it}	$\widehat{\epsilon}_{it}^{FE}$	i_{it}	π_{it}	r_{it}	$\widehat{\epsilon}_{it}^{FE}$
Data in first-differences: estimating the total number of factors																				
IC_1	1	1	1	1	1	1	1	1	4	1	1	1	5	1	1	1	5	1	1	1
IC_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
IC_3	2	1	1	1	3	1	1	1	4	1	1	1	5	1	1	1	6	1	1	1
PC_1	2	1	1	1	3	2	1	1	4	3	2	2	5	4	3	3	6	5	5	5
PC_2	1	1	1	1	2	1	1	1	4	2	1	1	5	3	3	3	6	4	4	4
PC_3	2	2	1	1	3	3	2	2	4	3	3	3	5	5	4	4	6	6	6	6
AIC_3	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5	6	6	6	6
BIC_3	1	1	0	0	1	1	0	0	1	1	0	0	1	1	1	1	1	1	1	1
Data in levels: estimating the number of non-stationary factors																				
IPC_1	1	1	1	1	2	1	1	1	2	2	1	1	2	2	1	1	3	2	1	1
IPC_2	1	1	1	1	2	1	1	1	2	2	1	1	2	2	1	1	3	2	1	1
IPC_3	1	0	0	0	1	1	1	0	1	1	1	1	1	1	1	1	2	1	1	1

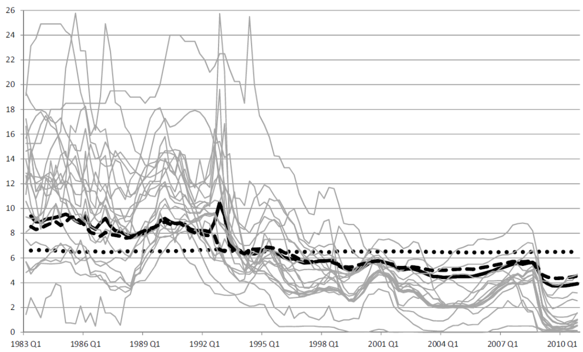
Notes: Prior to computation of the eigenvectors, each first-differenced series is demeaned and standardized to have unit variance (see Bai and Ng, 2002, p. 203).

approach outlined in Bai and Ng (2004). Because the true factors can only be identified up to scale, the factors are rotated such that (i) the average of the factor loadings on each factor equals 1 and (ii) the average of each factor coincides with the panel wide average of the plotted data. First, the graph for i_{it} in panel (a) of Figure 1 shows that the fist two factors are important, while the third is clearly unimportant. Looking more closely at the first two factors shows that they are virtually the same apart from a short period in 1992-1993. This is the period of the EMS crisis during which a lot of European countries sharply raised their short-term interest rates to defend their currencies. To visualize more clearly how this is picked up by the common factors, panel (b) plots an alternative representation by combining the first two factors leaving the full effect $\lambda_i' F_t$ unchanged for each country.[4] The first factor seems to be non-stationary, decreasing from about 11% in the early 1980s to just below 2% in the late 2000s. The second factor now shows up as a stationary component capturing the temporary increase in many European nominal interest rates during the EMS crisis. Second, from the graph for π_{it} in panel (c) of Figure 1 it is clear that only the first factor is an important global driver of inflation. It exhibits non-stationary behaviour, starting around 8% in the early 1980s to stabilize around 2% in the late 1990s and 2000s. This factor captures the disinflation proces all OECD countries went through in the 1980s and the early 1990s and relative stable inflation around 2% from the mid 1990s onwards. The second and the third factor are of no overall importance at all. The graphs for r_{it} and $\widehat{\epsilon}_{it}^{FE}$ in panels (d) and (e) of Figure 1 are highly similar. Only the first and to a lesser extent also the second factor seem important. In line with the results for i_{it} the EMS crisis shows up as a clear spike. However, the EMS crisis is a very specific event, common to only a part of the countries in the sample over a limited period of time. Moreover, it implied higher nominal interest rates mainly for reasons other than inflation expectations. Therefore, instead of trying to capture it using the common factor structure, in the remainder we will control for

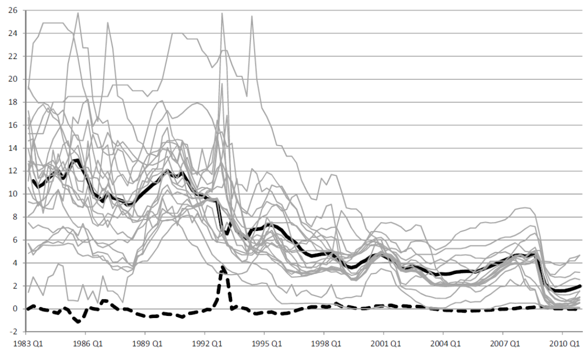
[4] We first set $F_{1t}^* = F_{1t} - \lambda F_{2t}$ and $F_{2t}^* = F_{2t}$ and recalculate the factor loadings as $\lambda_{i1}^* = \lambda_{i1}$ and $\lambda_{i2}^* = \lambda_{i2} + \lambda\lambda_{i1}$, with λ set equal to 1.3. Next, F_{1t}^* and F_{2t}^* and their factor loadings are again rotated such that the average of the factor loadings on each factor equals 1, the average of the first factor coincides with the panel wide average of the plotted data and the average of the second factor is zero.

the EMS crisis using dummy variables[5] when estimating the Fisher equation. Using the FE estimator including the EMS dummies the point estimate of β in (4) is virtually unchanged (estimation results are reported in Table 9 below), but the residuals $\hat{\epsilon}_{it}^{FEd}$ and the common factors in panel (f) of Figure 1 are now purged of the EMS crisis. Only the first factor is important now. It seems to be non-stationary, increasing from around 0% in the early 1980s to over 2% in the mid 1980s and then decreasing slowly to around -2% at the end of the sample.

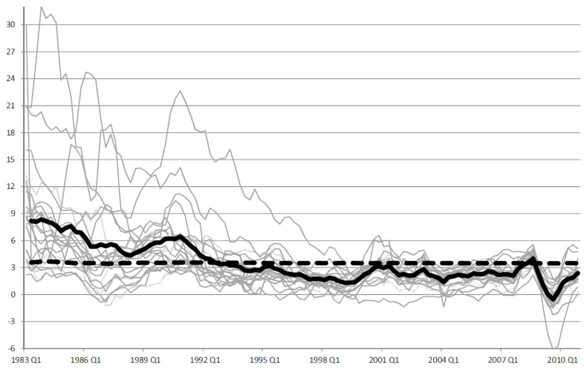
Figure 1: Time plots of the data and the FE residuals together with the first 3 estimated factors



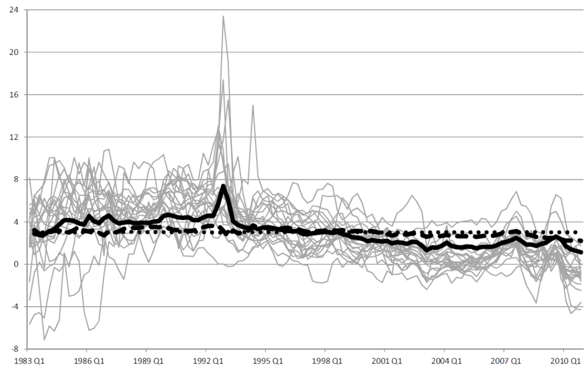
(a) i_{it}



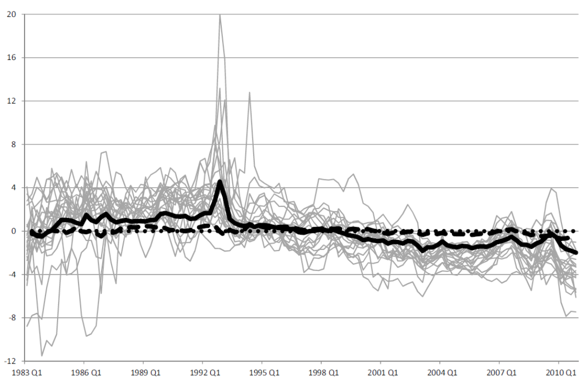
(b) i_{it} (combining first two factors)



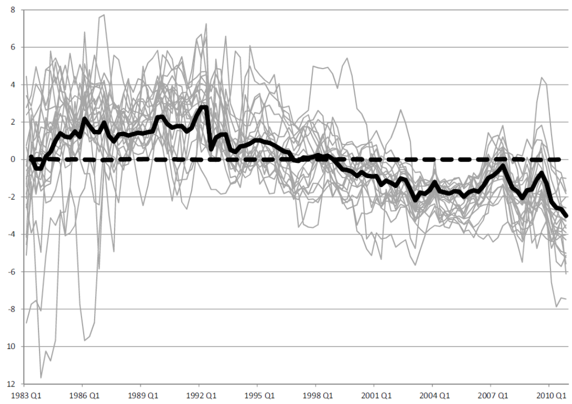
(c) π_{it}



(d) r_{it}



(e) $\hat{\epsilon}_{it}^{FE}$



(f) $\hat{\epsilon}_{it}^{FEd}$

Notes: Country-specific data: thin solid gray lines
Factor 1: bold solid line, Factor 2: bold dashed line, Factor 3: bold dotted line

[5] After careful studying the evolution of short-term interest rates during the EMS crisis, country- and time-specific intervention dummies were constructed for the following quarters: Belgium 1993Q3-1993Q4; Denmark 1992Q2-1993Q1 and 1993Q3; Finland 1992Q3; France 1993Q1; Greece 1994Q2; Ireland 1992Q3-1993Q1; Italy 1992Q3-1992Q4; Norway 1992Q3-1992Q4; Sweden 1992Q3-1992Q4.

As a final check, Table 5 reports the cross-sectional correlation in the idiosyncratic part of the data, i.e. e_{it} in (6), after taking out the contribution of r common factors with r ranging from 0 to 3. In line with the picture emerging from Figure 1, one factor seems to be sufficient to remove the cross-sectional dependence from π_{it}. For i_{it}, r_{it} and $\hat{\epsilon}_{it}^{FE}$, at least two factors seem to be necessary. However, after including the EMS dummies one factor seems to be sufficient to remove the cross-sectional dependence from $\hat{\epsilon}_{it}^{FEd}$.

Table 5: Cross-sectional correlation $\bar{\bar{\rho}}$ after taking out r common factors

Sample period: 1983:Q1-2010:Q4, 21 countries

	Levels					First-differences				
	i_{it}	π_{it}	r_{it}	$\hat{\epsilon}_{it}^{FE}$	$\hat{\epsilon}_{it}^{FEd}$	i_{it}	π_{it}	r_{it}	$\hat{\epsilon}_{it}^{FE}$	$\hat{\epsilon}_{it}^{FEd}$
$r = 0$	0.79	0.54	0.58	0.53	0.52	0.23	0.29	0.14	0.15	0.13
$r = 1$	0.38	0.00	0.23	0.17	0.05	0.09	−0.01	0.03	0.03	−0.01
$r = 2$	0.02	−0.01	0.09	0.08	0.04	0.00	−0.02	0.00	−0.00	−0.01
$r = 3$	−0.01	−0.01	0.08	0.08	0.04	−0.01	−0.03	0.00	0.00	−0.01

Note: see Table 3 for definition of $\bar{\bar{\rho}}$.

The tentative conclusion from Tables 4 and 5 and Figure 1 is that, after correcting for the EMS crisis, the cross-sectional correlation observed in the data and in the residuals of the FE estimator is due to a single non-stationary common factor in each of these series. In the next section, we more formally test the time series properties of the data using unit root tests that allow for cross-sectional dependence induced by unobserved common factors.

3.5 Second generation panel unit root tests

Unit root tests allowing for cross-sectional dependence have been proposed by, most notably, Pesaran (2007), Moon and Perron (2004) and Bai and Ng (2004). These tests are similar in that they assume an observed data series to be, in the spirit of the representation in equation (6), the sum of an unobserved idiosyncratic component and a number of unobserved common factors to which each individual can react differently. The tests differ in the allowed number and order of integration of the unobserved common factors and in the way these factors are eliminated.

Pesaran (2007) allows for a single stationary common factor and suggests to eliminate it by augmenting the standard ADF regression with the cross-sectional averages of the lagged levels and first-differences of the individual series. This cross-sectionally augmented ADF statistic (denoted CADF), or its rejection probabilities, can then be used to construct a modified version of the t-bar test proposed by Im et al. (2003) or of the MW test used above. Moon and Perron (2004) propose test statistics based on pooled estimates of the first-order autoregressive parameter, akin to the original Levin et al. (2002) test, but that are calculated from an orthogonal projection of the data on the common factors identified using principal component analysis. This setting can account for multiple common factors but, as in Pesaran (2007), these are restricted to be stationary such that any non-stationarity in the observed series must be due to the presence of a unit root in the idiosyncratic component. The most general approach is the PANIC of Bai and Ng (2004), which allows for non-stationarity in either the common factors, or in the

Table 6: Second generation panel unit root tests (model with constant)

Sample period: 1983:Q1-2010:Q4, 21 countries

Pesaran (2007)[(a)]

	i_{it}			π_{it}			r_{it}	
k	CIPS		k	CIPS		k	CIPS	
3	-3.02	(0.01)	5	-2.94	(0.01)	5	-3.55	(0.01)

Moon and Perron (2004)[(b)]

	i_{it}				π_{it}				r_{it}			
r	t_a^*		t_b^*		t_a^*		t_b^*		t_a^*		t_b^*	
1	-11.08	(0.00)	-5.56	(0.00)	-15.08	(0.00)	-6.18	(0.00)	-22.42	(0.00)	-7.80	(0.00)
2	-16.82	(0.00)	-6.86	(0.00)	-13.55	(0.00)	-5.72	(0.00)	-25.22	(0.00)	-8.65	(0.00)

Bai and Ng (2004)[(c)]

	i_{it}				π_{it}				r_{it}			
	$\widehat{F}_t$		$\widehat{e}_{it}$		$\widehat{F}_t$		$\widehat{e}_{it}$		$\widehat{F}_t$		$\widehat{e}_{it}$	
r	ADF-GLS		MW		ADF-GLS		MW		ADF-GLS		MW	
1	0.32	(0.77)	45.89	(0.31)	0.43	(0.81)	66.21	(0.01)	-0.84	(0.35)	81.85	(0.00)
	m	MQ_c			m	MQ_c			m	MQ_c		
2	1	-1.77	62.21	(0.02)	1	-5.78	47.48	(0.26)	1	-3.18	107.24	(0.00)
	2	-38.71^{***}			2	-18.30			2	-30.83^{**}		

	$\widehat{\epsilon}_{it}^{FE}$				$\widehat{\epsilon}_{it}^{FEd}$			
	$\widehat{F}_t$		$\widehat{e}_{it}$		$\widehat{F}_t$		$\widehat{e}_{it}$	
r	ADF-GLS		MW		ADF-GLS		MW	
1	-0.99	(0.29)	86.61	(0.00)	-0.64	(0.44)	96.63	(0.00)
	m	MQ_c			m	MQ_c		
2	1	-4.01	106.02	(0.00)	1	-1.34	105.63	(0.00)
	2	-27.13^{**}			2	-34.84^{***}		

Notes: (a) CIPS is the mean of the individual CADF statistics with a common lag order k determined as the nearest integer of the mean of the individual lag lengths of the ADF tests in Table 2. Approximate p-values calculated from Table II(b) in Pesaran (2007) are reported in parentheses.

(b) t_a^* and t_b^* are pooled panel unit root test statistics based on de-factored data for different number of common factors $r = 1, 2, 3$. The long-run variances required for calculating these statistics are obtained using a Quadratic Spectral kernel function with Newey-West bandwidth. Corresponding p-values (from the standard normal distribution) are reported in parentheses.

(c) For $r = 1$ the unit root test on the single common factor $\widehat{F}_t$ is a ADF-GLS test for a model with constant. The corresponding (simulated) p-values are reported in parentheses. For $r > 1$, the MQ_c statistic tests the number of independent non-stationary factors (m) in the vector $\widehat{F}_t$. The critical values at the 1%, 5% and 10% level of significance are -20.151, -13.730 and -11.022 for $m = 1$ and -31.621, -23.535 and -19.923 for $m = 2$. *** indicates that the MQ_c test is significant at the 1% level, ** at the 5% and * at the 10% level. MW is a MW panel unit root test on the estimated idiosyncratic errors $\widehat{e}_{it}$ for different number of common factors $r = 1, 2, 3$. See Table 2 for more details. The corresponding p-values (taken from the χ^2_{2N} distribution) are reported in parentheses.

idiosyncratic errors or in both. Rather than testing the order of integration of the observed data, these are first decomposed in unobserved common factors and idiosyncratic errors which are then tested separately. The key to this is a 'differencing and recumulating' procedure that permits consistent estimation of the unobserved components when it is not known a priori whether they are $I(0)$ or $I(1)$.

Results are reported in Table 6. Both the Pesaran (2007) and the Moon and Perron (2004) test strongly reject the null hypothesis of a unit root in i_{it}, π_{it} and r_{it}. For the latter test, this finding is robust over alternative choices for r. In contrast to this, the results of the Bai and Ng (2004) PANIC imply that each of the three variables is non-stationary, with this non-stationarity being induced by the common factor(s) leaving the idiosyncratic error terms stationary. First consider i_{it}. The analysis in

Section 3.4 suggests that 2 common factors are necessary to capture the cross-sectional dependence in the data. Setting $r = 2$, the idiosyncratic errors $\widehat{e}_{it}$ are found to be stationary using the MW test. The MQ_c statistic shows that the space spanned by the two common factors is non-stationary but there is only 1 independent non-stationary common factor. This is consistent with the interpretation above that the second factor captures the EMS crisis. Second, for both π_{it} and r_{it} the analysis in Section 3.4 suggests 1 common factor which is found to be non-stationary using the ADF-GLS test. Setting $r = 1$, the idiosyncratic errors $\widehat{e}_{it}$ are found to be stationary using the MW test. The finding of non-stationary common factors implies that the results from the Pesaran (2007) and the Moon and Perron (2004) tests are not trustworthy as these can only deal with stationary common factors and, together with the finding of stationary idiosyncratic errors, that the (panel) unit root tests ignoring the common factor structure in the data tend to over-reject the null hypothesis of a unit root. Finally, also the residuals from the FE regressions (with or without EMS dummies) are found to be non-stationary, with a single non-stationary common factor and stationary idiosyncratic errors. Urbain and Westerlund (2011) show that the standard result in Phillips and Moon (1999) that panel regressions yield consistent results even if there is no cointegration does not longer hold when the non-stationary in the error term is induced by a common factor. This implies that the results from the FE estimators reported in Table 2 should be considered spurious.

4 The Fisher equation in the presence of an unobserved $I(1)$ common factor

In this section, we augment the standard Fisher specification (4) by allowing for an $I(1)$ unobserved common component which we interpret as representing permanent fluctuations in the world real interest rate. We discuss how this common factor-augmented specification can be estimated and how to test whether this is a cointegrating relation.

4.1 An $I(1)$ world real interest rate

The main conclusion from the PANIC in Section 3.5 is that there is an $I(1)$ common factor in both the real interest rate r_{it} and the residuals ϵ_{it} of the Fisher equation (4). This has two important implications for modelling the Fisher effect.

First, the finding that ϵ_{it} is $I(1)$ implies that i_{it} and π_{it} are not cointegrated, but does not automatically invalidate the Fisher effect. It does signal, though, that equation (4) is miss-specified, i.e. the assumption that the composite error term $\epsilon_{it} = \nu_{it} - \beta\zeta_{it}$ is stationary is wrong. As non-stationarity of the forecast error ζ_{it} would be at odds with rational expectations, the observed non-stationarity in ϵ_{it} is most probably due to ν_{it} which represents time variation in the real factors driving the ex ante real interest rate. Standard neoclassical growth models with household intertemporal utility maximization imply that the real interest rate is a function of time preference, risk aversion and the steady-state growth rate of technological change. While time preference and risk aversion are generally believed to be fairly stable, or at least changing only slowly over extended periods of time, shifts in steady-state growth, such as the 'productivity slowdown' of the early 1970s and the 'New Economy' resurgence of growth in the late 1990s, have been widely documented in the literature (see e.g. Oliner and Sichel, 2000; Roberts, 2001). Moreover, in the Diamond overlapping-generations model, a permanent increase in government spending leads to a permanently higher real interest rate. Additional determinants of real interest rates suggested in the literature are demographic changes, changes in the stance of fiscal policy and the evolution of public debt, changes in the taxation of profits, (de)regulation of financial markets, ... (see e.g. Blanchard and Summers, 1984; Chadha and Dimsdale, 1999; Ardagna, 2009). Permanent shifts in any of these factors induce a unit root in the ex ante real interest rate r^e_{it} which implies a unit root in the ex post real interest rate r_{it} and in the residuals ϵ_{it} of the Fisher equation (4). Ideally, the non-stationary determinants of real interest rates should be included as covariates in the Fisher equation. Unfortunately, there is a large variety of possible determinants which are, moreover, not directly observable or at least hard to measure.

Second, the finding that only the common factor in ϵ_{it} is $I(1)$ while the idiosyncratic part is $I(0)$ suggests that the permanent shifts in the real interest rate are common to all countries in the sample. This is in line with the results in e.g. Gagnon and Unferth (1995), Pain and Thomas (1997) and Lee (2002) who show that country-specific deviations from an $I(1)$ world real interest rate are stationary. Note that Blanchard and Summers (1984) already argued that increasing economic integration leads to a substantial degree of linkage between real interest rates of different countries such that national real interest rates should be related to international rather than to domestic events. The main advantage of this $I(1)$ world real interest rate is that it can be identified by exploiting the strong cross-section correlation observed over countries.

4.2 Common factor-augmented Fisher equation

To allow for an $I(1)$ world real interest rate, the DGP of ex ante real interest rates in equation (2) is rewritten to

$$r^e_{it} = \alpha_i + \gamma_i r^w_t + \mu_{it}, \tag{7}$$

where r^w_t is a single non-stationary common factor with idiosyncratic factor loadings γ_i and μ_{it} a stationary idiosyncratic component. Inserting (7) and (3) in (1) yields

$$i_{it} = \alpha_i + \beta\pi_{it} + \epsilon_{it}, \tag{8}$$

$$\epsilon_{it} = \gamma_i r^w_t + \varepsilon_{it}, \tag{9}$$

with $\varepsilon_{it} = \mu_{it} - \beta\zeta_{it}$. The Fisher equation in (8) is the basic specification in (4) augmented with a unobserved non-stationary common factor in the residuals ϵ_{it} modelled in equation (9).

The model in equations (8)-(9) in vector notation is

$$i_i = \alpha_i + \beta\pi_i + \gamma_i r^w + \varepsilon_i, \tag{10}$$

where $i_i = (i_{i1}, \ldots, i_{iT})'$, $\pi_i = (\pi_{i1}, \ldots, \pi_{iT})'$, $r^w = (r_1^w, \ldots, r_T^w)'$, $\varepsilon_i = (\varepsilon_{i1}, \ldots, \varepsilon_{iT})'$.

4.3 Estimation in the presence of unobserved $I(1)$ common factors

4.3.1 Principal Component Estimators

Bai et al. (2009) suggest a 'continuously-updated' (Cup) procedure that jointly estimates the slope coefficient β and the unobserved common factor r^w in equation (10). More specifically, the solution $(\widehat{\beta}_{Cup}, \widehat{r}^w_{Cup})$ is obtained by iteratively estimating (i) $\widehat{\beta}$ as the FE estimator for β in equation (10) conditional on $\widehat{r}^w$

$$\widehat{\beta} = \left(\sum_{i=1}^{N} \pi_i' M_{\widehat{r}^w} \pi_i \right)^{-1} \sum_{i=1}^{N} \pi_i' M_{\widehat{r}^w} i_i, \tag{11}$$

where $M_{\widehat{r}^w} = I_T - \widehat{r}^w \left(\widehat{r}^{w\prime} \widehat{r}^{w\prime} \right)^{-1} \widehat{r}^{w\prime}$ and (ii) $\widehat{r}^w$ as the first r eigenvectors (multiplied by T) of the matrix $\frac{1}{NT^2} \sum_{i=1}^{N} \left(i_i - \widehat{\beta}\pi_i \right) \left(i_i - \widehat{\beta}\pi_i \right)'$ conditional on $\widehat{\beta}$. Bai et al. (2009) show that $\widehat{\beta}_{Cup}$ is T consistent for β but has an asymptotic bias (for $N \to \infty$) arising from endogeneity of π_{it} and serial correlation in ε_{it}. They therefore suggest a bias-corrected (CupBC) and a fully modified (CupFM) version of the Cup estimator. The first estimates the asymptotic bias directly while the second modifies the data so that the limiting distribution does not depend on nuisance parameters. Both are $\sqrt{NT}$ consistent for the common slope coefficient β and are robust to mixed $I(1)/I(0)$ factors and regressors. Moreover, the estimators enable the use of standard test statistics for inference. This approach requires specifying the number of common factors r.

4.3.2 Common Correlated Effects Pooled (CCEP) estimator

Pesaran (2006) proposes to eliminate the cross-sectional dependence in ϵ_{it} by projecting out the common factor r_t^w using the cross-sectional averages of i_{it} and π_{it}. For a model with a single factor[6], inserting (9) in (8) and taking cross-sectional averages yields

$$\bar{i}_t = \overline{\alpha} + \beta \overline{\pi}_t + \overline{\gamma} r_t^w + \overline{\varepsilon}_t, \tag{12}$$

where $\bar{i}_t = N^{-1} \sum_{i=1}^{N} i_{it}$ and similarly for the other variables. Solving (12) for r_t^w

$$r_t^w = \frac{1}{\overline{\gamma}} \left(\bar{i}_t - \overline{\alpha} - \beta \overline{\pi}_t - \overline{\varepsilon}_t \right), \tag{13}$$

and inserting (13) in (8)-(9) yields

$$\begin{aligned} i_{it} &= \alpha_i + \beta \pi_{it} + \frac{\gamma_i}{\overline{\gamma}} \left(\bar{i}_t - \overline{\alpha} - \beta \overline{\pi}_t - \overline{\varepsilon}_t \right) + \varepsilon_{it}, \\ &= \widetilde{\alpha}_i + \beta \pi_{it} + c_{1i} \bar{i}_t + c_{2i} \overline{\pi}_t + \widetilde{\varepsilon}_{it}, \end{aligned} \tag{14}$$

with $\widetilde{\alpha}_i = \alpha_i - (\gamma_i / \overline{\gamma}) \overline{\alpha}$, $c_{1i} = (\gamma_i / \overline{\gamma})$, $c_{2i} = -\beta (\gamma_i / \overline{\gamma})$ and $\widetilde{\varepsilon}_{it} = \varepsilon_{it} - (\gamma_i / \overline{\gamma}) \overline{\varepsilon}_t$.

[6]Multiple factors can be treated in the same way (see Phillips and Sul, 2007), and yield the same (unrestricted) model as the one presented in (14), but are not presented here for notational convenience.

The CCEP estimator proposed by Pesaran (2006) is the FE estimator applied to the augmented regression in (14), ignoring the non-linear coefficient restrictions, given by

$$\widehat{\beta}_{CCEP} = \left(\sum_{i=1}^{N} \pi_i' M_H \pi_i \right)^{-1} \sum_{i=1}^{N} \pi_i' M_H i_i, \tag{15}$$

where $M_H = I_T - H\left(H'H\right)^{-1} H'$ with $H = \left(\bar{i}, \bar{\pi}\right)$, $\bar{i} = \left(\bar{i}_1, \ldots, \bar{i}_T\right)'$ and $\bar{\pi} = \left(\bar{\pi}_1, \ldots, \bar{\pi}_T\right)'$.

As the assumption that ε_{it} is cross-sectionally independent implies that $\operatorname*{plim}_{N\to\infty} \bar{\varepsilon}_t = 0$, the error made when approximating r_t^w by $\bar{i}_t$ and $\bar{\pi}_t$ in (13) becomes negligibly small for $N \to \infty$ such that $\widetilde{\varepsilon}_{it} \xrightarrow{p} \varepsilon_{it}$ in (14). This is the basic result in Pesaran (2006) that the inclusion of cross-sectional averages asymptotically eliminates the error cross-sectional dependence induced by the unobserved common factors such that the CCEP estimator is $\sqrt{N}$ consistent regardless of whether T is fixed or $T \to \infty$. These results hold for any fixed number of unobserved factors r, which implies that there is no need to estimate or specify r. Kapetanios et al. (2011) further shows that these results continue to hold regardless of whether the common factors are stationary or non-stationary.

An important restriction is that consistency of the CCEP estimator requires that the idiosyncratic errors ε_{it} are distributed independently of the explanatory variable π_{it}. To see why, note that the CCEP estimator in equation (15) is equivalent to the least squares estimator for β after projecting out the individual effects and the cross-sectional means from the model in equation (14)

$$\breve{i}_{it} = \beta \breve{\pi}_{it} + \breve{\varepsilon}_{it}, \tag{16}$$

where $\breve{i}_i = \left(\breve{i}_{i1}, \ldots, \breve{i}_{iT}\right)' = M_H i_i$ and $\breve{\pi}_i = \left(\breve{\pi}_{i1}, \ldots, \breve{\pi}_{iT}\right)' = M_H \pi_i$ are the residuals from country-by-country regressions of i_{it} and π_{it} on a constant, $\bar{i}_t$ and $\bar{\pi}_t$. Pesaran (2006) and Kapetanios et al. (2011) show that this orthogonalisation on the cross-sectional averages $\bar{i}_t$ and $\bar{\pi}_t$ removes all common factor(s) from both i_{it} and π_{it}. As such, $\breve{i}_{it}$ and $\breve{\pi}_{it}$ are estimates of the idiosyncratic part in i_{it} and π_{it} respectively. As these idiosyncratic parts are found to be stationary by the PANIC in Section 3.5, which is also the working assumption in Kapetanios et al. (2011), equation (16) is a regression model including stationary variables. This implies that, in contrast to the Cup estimator, the CCEP estimator is not super consistent such that endogeneity cannot be ignored asymptotically.

As the forecast error ζ_{it} implies that π_{it} and ε_{it} are correlated, equation (16) is estimated using GMM. Valid moment conditions are

$$E\left(\breve{\pi}_{i,t-l} \breve{\varepsilon}_{it}\right) = 0 \quad \text{for each} \quad t = l+1, \ldots, T \text{ and } l \geq q+1, \tag{17}$$

with q being the order of the MA process in ζ_{it}. Equation (17) defines a relatively large set of moment conditions. Using more instruments from deeper lags of $\breve{\pi}_{it}$ improves the efficiency of the GMM estimator. However, it also reduces the sample size as observations for which lagged observations are unavailable are dropped. To avoid this trade-off between instrument lag depth and sample depth, we construct instruments by zeroing out missing observations of lags as in Holtz-Eakin et al. (1988). Furthermore, in order to avoid problems related to using too many instruments, we truncate the set of available instruments at the first L available lags. This results in the following reduced set of moment conditions

$$E\left(\breve{\pi}_{i,t-l}\breve{\varepsilon}_{it}\right) = 0 \quad \text{for each} \quad q+1 \leq l \leq L+q, \tag{18}$$

The CCEP-GMM estimator for β is obtained by minimizing the empirical moments $\sum_i \sum_t \breve{\pi}_{i,t-l}\breve{\varepsilon}_{it}$ using a Newey-West type optimal weighting matrix.

4.4 Common factor-augmented panel cointegration

Cointegration in panels with unobserved non-stationary common factors has been considered by Gengenbach et al. (2006) and Banerjee and Carrion-i Silvestre (2006). Both studies would define panel cointegration in our case as a situation where the interest rate i_{it} and the inflation rate π_{it} cointegrate with vector $(1, -\beta)$. Equations (8)-(9) show that this concept of panel cointegration requires both the common factor r_t^w and the idiosyncratic error term ε_{it} to be $I(0)$. Especially the former is highly restrictive as it requires that any non-stationary common factors in i_{it} and π_{it} should cointegrate leaving the common factor in the error term ϵ_{it} stationary. However, when our interest is in estimation and inference on β, the estimation procedures of Kapetanios et al. (2011) and Bai et al. (2009) outlined above only require $\varepsilon_{it} = (i_{it} - \beta\pi_{it} - \gamma_i r_t^w)$ to be $I(0)$. Intuitively, r_t^w is a vector of $I(1)$ variables that should be included in the model for this to be cointegrating regression. We label this common factor-augmented panel cointegration.

The most obvious approach to test whether i_{it}, π_{it} and r_t^w are cointegrated or not would be to first estimate the model in (8)-(9), using either the CCEP or the Cup estimation approach, and then test for the null hypothesis of no cointegration using e.g. a MW panel cointegration test on the estimated idiosyncratic error terms $\widehat{\varepsilon}_{it}$. This direct approach is problematic for two reasons, though. First, the country-specific orthogonalisation, either on $\widehat{r}_t^w$ in (11) or on the cross-sectional averages $\bar{i}_t$ and $\overline{\pi}_t$ in (15), implies that the distribution of a country-by-country cointegration test on $\widehat{\varepsilon}_{it}$ depends on the number of $I(1)$ factors in r_t^w. This is problematic as the CCEP estimator does not require specifying the number of factors while the Cup estimator does only require a decision on the number of factors but not on the number of $I(1)$ factors. Second, the fact that the orthogonalisation is on the same variable(s) in each country implies that the country-by-country cointegration tests are not independent and therefore the MW panel cointegration test does not have the standard χ^2 distribution.

A natural alternative approach is to use $\widehat{\epsilon}_{it} = \left(i_{it} - \widehat{\beta}\pi_{it}\right)$ instead of $\widehat{\varepsilon}_{it}$ and apply a principal component analysis as in Bai and Ng (2004) to split $\widehat{\epsilon}_{it}$ in a number of common factors and an idiosyncratic error term and then test whether the idiosyncratic error is stationary or not. The advantage of this approach is that, as shown by Bai and Ng (2004), the test whether the idiosyncratic errors are stationary does not depend on the presence or absence of common stochastic trends and/or their integration properties and thus can be tested using standard panel unit root tests. It only requires specifying the number of common factors.

5 Monte Carlo simulation

In this section, we conduct a small-scaled Monte Carlo experiment to assess (i) the finite sample properties in terms of estimation and inference of the CCEP and Cup estimators outlined in Section 4.3 and (ii) the size and power of the cointegration tests on $\widehat{\epsilon}_{it}$ and $\widehat{\varepsilon}_{it}$ outlined in Section 4.4. Under the hypothesis that the full Fisher effect holds ($\beta = 1$), the data-generating process is chosen such that the properties of the simulated data match with those of the observed data for i_{it} and π_{it} as much as possible. Although we are mainly interested in the setting $T = 112$ and $N = 21$, we also present results for a range of alternative dimensions of T to illustrate the more general finite sample properties of the estimators.[7]

5.1 Design

Data are generated based on the following design

$$
\begin{aligned}
i_{it} &= r_{it}^e + \beta\pi_{it}^e, \\
\pi_{it} &= \pi_{it}^e + \zeta_{it}, \\
r_{it}^e &= \alpha_i + \gamma_i r_t^w + \mu_{it}, \quad \alpha_i \sim i.i.d.N(\alpha, \sigma_\alpha^2), \quad \gamma_i \sim i.i.d.N(1, \sigma_\gamma^2) \\
\pi_{it}^e &= \tau_i + \lambda_i \pi_t^w + \eta_{it}, \quad \tau_i \sim i.i.d.N(\tau, \sigma_\tau^2), \quad \lambda_i \sim i.i.d.N(1, \sigma_\lambda^2)
\end{aligned}
$$

In line with the results of the PANIC in Section 3.5, the common factors r_t^w and π_t^w are generated as random walks

$$
\begin{aligned}
r_t^w &= r_{t-1}^w + \psi_{it}, \quad \psi_{it} \sim i.i.d.N(0, \sigma_\psi^2), \\
\pi_t^w &= \pi_{t-1}^w + \xi_{it}, \quad \xi_{it} \sim i.i.d.N(0, \sigma_\xi^2),
\end{aligned}
$$

while the idiosyncratic components μ_{it} and η_{it} are generated as AR(1) processes

$$
\begin{aligned}
\mu_{it} &= \theta\mu_{i,t-1} + \chi_{it}, \quad \chi_{it} \sim i.i.d.N(0, \sigma_\chi^2) \\
\eta_{it} &= \phi\eta_{i,t-1} + \omega_{it}, \quad \omega_{it} \sim i.i.d.N(0, \sigma_\omega^2).
\end{aligned}
$$

In order to obtain realistic parameter values, we calibrate the DGP outlined above to our observed sample of OECD data. As π_{it}^e and r_{it}^e are not observed we start by making the strong assumption of perfect foresight, i.e. $\zeta_{it} = 0$, such that π_{it}^e equals ex post observed inflation π_{it} and r_{it}^e equals the ex post observed real interest rate r_{it}. The observed data for both π_{it} and r_{it} are then split up into a fixed effect, a common component and an idiosyncratic component using the PANIC of Bai and Ng (2004)[8]. Parameter values are estimated from the various estimated components. This is the experiment 1:

- Experiment 1: $\sigma_\zeta = 0$, $\beta = 1$, $\alpha = 3.03$, $\sigma_\alpha = 1.05$, $\tau = 3.44$, $\sigma_\tau = 1.93$, $\sigma_\gamma = 1.09$, $\sigma_\lambda = 0.36$, $\sigma_\psi = 0.41$, $\sigma_\xi = 0.37$, $\phi = 0.77$, $\sigma_\omega = 1.21$, $\theta = 0.67$ and $\sigma_\chi = 1.54$.

[7] The results are highly robust over alternative dimensions of N. These results are available on request.
[8] γ_i and λ_i are normalized to have mean 1 and r_t^w and π_t^w to have mean 0

Next, we consider two cases with non-zero forecasting errors. From rational expectations, the forecast error ζ_{it} is assumed be white noise

$$\zeta_{it} \sim i.i.d.N\left(0, \sigma_\zeta^2\right). \tag{19}$$

Experiment 2 sets $\sigma_\zeta = 0.75$ which implies that 95% of the quarterly forecasting errors lies between -1.5 and $+1.5$ %points while for experiment 3 which sets $\sigma_\zeta = 1.25$ this is between -2.5 and $+2.5$ %points. These experiments are in line with the results in Mankiw et al. (2004) who find that the RMSE of forecasting inflation from survey data on inflation expectations from several sources ranges from 1.07% up to 1.29% over the period 1982Q3-2002Q1. Note that simply adding ζ_{it} to π_{it}^e from experiment 1 would increase the variance of the simaulted π_{it}. In order to ensure comparability of the simulation results over the experiments, σ_ω^2 is therefore lowered such that the variance of the idiosyncratic component in π_{it}, i.e. $\eta_{it} + \zeta_{it}$, is constant when varying σ_ζ^2. As a result, also σ_χ^2 is adjusted to ensure that the variance of the idiosyncratic component in i_{it}, i.e. $\mu_{it} + \beta\eta_{it}$, remains constant over the experiments. Parameter values that differ compared to experiment 1 are given by

- Experiment 2: $\sigma_\zeta = 0.75$, $\sigma_\omega = 1.11$, $\sigma_\chi = 1.64$.
- Experiment 3: $\sigma_\zeta = 1.25$, $\sigma_\omega = 0.91$, $\sigma_\chi = 1.80$.

We conduct two versions of experiments 2 and 3. In the first version, denoted 2a and 3a, ζ_{it} is generated as a white noise process as specified in (19). The second version, denoted 2b and 3b, takes into account that in our dataset we measure inflation as the year-on-year percent change in the consumer price index which implies that the white noise forecast error builds into an MA(3) process. Therefore, ζ_{it} is assumed to be generated as

$$\zeta_{it} = \frac{\sigma_\zeta}{\sqrt{4}} \sum_{j=0}^{3} e_{i,t-j}, \tag{20}$$

where $e_{it} \sim i.i.d.N(0,1)$. Note that the unconditional variance of ζ_{it} is σ_ζ^2 for both the white noise process in (19) and the MA(3) process in (20).

For each experiment we compute the FE, CCEP, CCEP_GMM, Cup and CupBC estimator. The CCEP_GMM estimator uses the first $L = 8$ available lags with q being adjusted according to the MA structure in ζ_{it}. Reported are two-step GMM results with optimal weighting matrix constructed from a Newey-West type of estimator with lag truncation set to 3. The CupBC estimator is calculated from a long-run covariance matrix estimated using the Bartlett kernel with bandwidth set to 5. For each estimator we report the mean bias (*bias*) of $\widehat{\beta}$, the standard deviation (*sd*) of the Monte Carlo distribution of $\widehat{\beta}$, the root mean squared error (*rmse*), the mean of the estimated standard error (*se*) and the size (*size*) of a t-test for the null hypothesis that $\beta = 1$. The analytical standard errors (se^a) are robust to heteroscedasticity and serial correlation in the error terms. As these robust standard errors only have asymptotic validity, we also report bootstrap standard errors (se^b). Bootstrap samples are obtained by resampling cross-sections as a whole as suggested by Kapetanios (2008)[9]. The bootstrap standard errors are calculated as the standard deviation of the bootstrap distribution of $\widehat{\beta}$. Each experiment was replicated 5000 times with bootstrap standard errors calculated from 1000 bootstrap replications.

Note that in all of the above experiments the setting $\theta = 0.67$ implies that there is cointegration between i_{it}, π_{it} and r_t^w. As next to the power we also want to analyze the size of the cointegration tests discussed in Section 4.4, we will first simulate data for an experiment where μ_{it} is non-stationary such that there is no cointegration between i_{it}, π_{it} and r_t^w. Experiment 0 therefore differs from experiment 1 in the following parameter value:

- Experiment 0: $\theta = 1.00$.

We preform 3 different cointegration tests: (i) a naive MW unit root test on $\widehat{\epsilon}_{it}$ which is non-stationary in all experiments, (ii) a MW cointegration test on the defactored residuals $\widehat{\varepsilon}_{it}$ (using either the CCEP or the Cup approach) and (iii) a PANIC which first decomposing $\widehat{\epsilon}_{it}$ in a single common factor $\widehat{F}_t$ and an idiosyncratic component $\widehat{e}_{it}$ and next performing an ADF-GLS unit root test on $\widehat{F}_t$ and a MW unit root test on $\widehat{e}_{it}$. For each of these tests, p-values are calculated from simulated finite-sample distributions (for details, see the notes to Tables 1, 2 and 6).

5.2 Simulation results

The simulation results for a sample size of $N = 21$ and $T = 112$ are summarized in Table 7. Table 8 reports additional results on the bias of the various estimators when varying T from 50 to 500.

First look at the cointegration tests in Table 7. In Experiment 0 ϵ_{it} and ε_{it} are non-stationary as the common factor r_t^w and the idiosyncratic error μ_{it} are both $I(1)$. However, the MW test on the composite error term $\widehat{\epsilon}_{it}$ and on the defactored error term $\widehat{\varepsilon}_{it}$ are strongly oversized. This implies that these tests should not be trusted as the null of no cointegration is wrongly rejected in far too many cases. In contrast, the PANIC has the correct size both for a unit root test on $\widehat{F}_t$ and on $\widehat{e}_{it}$. In Experiments 1-3 there is cointegration between i_{it}, π_{it} and r_t^w. The MW test on the composite error term $\widehat{\epsilon}_{it}$ rejects the null of no cointegration between i_{it} and π_{it} in almost all cases though. This shows that the $I(1)$ common factor r_t^w is not detected by a standard panel unit root test ignoring the factor structure. As such, the finding in Table 2 that there is cointegration between i_{it} and π_{it} should not be trusted. The PANIC has good size for the unit root test on the non-stationary factor $\widehat{F}_t$ while having power close to 1 for the unit root test on the stationary idiosyncratic errors $\widehat{e}_{it}$ in all cases. This shows that a PANIC on the composite error term $\widehat{\epsilon}_{it}$ is an appropriate approach to test for common factor-augmented panel cointegration.

Looking at the estimation results, first note that the FE estimator is spurious in all experiments which results in an unacceptably high size using either the analytic or the bootstrap inference. This is in line with Urbain and Westerlund (2011) who show that neglecting $I(1)$ common factors in the residuals of a panel regression implies spurious results. Second, the CCEP(_GMM) and Cup(BC) estimators yield unbiased estimates for β in experiments 0 and 1. Although the analytical standard errors underestimate the true standard deviation of $\widehat{\beta}$ resulting in oversized inference, the bootstrap inference is more or less correctly

[9]Note that block bootstrapping is not valid for non-stationary data.

Table 7: Monte Carlo simulation results: estimation and inference for $N = 21$ and $T = 112$

	Estimation precision			Inference $H_0: \beta = 1$ analytical		bootstrap		Rejection frequency cointegration tests MW		PANIC $\widehat{\epsilon}_{it}$	
	bias	*sd*	*rmse*	se^a	$size^a$	se^b	$size^b$	$\widehat{\epsilon}_{it}$	$\widehat{\varepsilon}_{it}$	$\widehat{F}_t$	$\widehat{e}_{it}$
	Experiment 0: $\theta = 1, \zeta_{it} = 0.00$										
FE	−0.00	0.36	0.36	0.07	0.69	0.21	0.22	0.24	-	0.04	0.06
CCEP	−0.00	0.18	0.18	0.06	0.46	0.18	0.04	0.20	0.64	0.04	0.05
CCEP_GMM	−0.00	0.19	0.19	0.08	0.42	0.20	0.04	0.20	0.65	0.04	0.05
Cup	−0.00	0.19	0.19	0.04	0.71	0.16	0.10	0.21	0.27	0.04	0.05
CupBC	−0.01	1.70	1.70	0.16	0.42	1.96	0.06	0.25	0.34	0.04	0.08
	Experiment 1: $\theta = 0.67, \zeta_{it} = 0.00$										
FE	0.00	0.20	0.20	0.02	0.82	0.06	0.60	0.94	-	0.04	1.00
CCEP	0.00	0.08	0.08	0.04	0.38	0.09	0.06	0.91	1.00	0.03	1.00
CCEP_GMM	0.00	0.09	0.09	0.05	0.29	0.09	0.07	0.90	1.00	0.03	1.00
Cup	−0.00	0.05	0.05	0.01	0.59	0.05	0.09	0.91	1.00	0.03	1.00
CupBC	−0.00	0.07	0.07	0.04	0.30	0.07	0.06	0.91	1.00	0.03	1.00
	Experiment 2a: $\theta = 0.67, \zeta_{it}$ is white noise with $\sigma_\zeta = 0.75$										
FE	−0.03	0.20	0.20	0.02	0.85	0.05	0.63	0.96	-	0.05	1.00
CCEP	−0.26	0.07	0.27	0.03	1.00	0.07	0.93	0.87	1.00	0.04	1.00
CCEP_GMM	0.01	0.11	0.11	0.07	0.21	0.11	0.05	0.91	1.00	0.04	1.00
Cup	−0.07	0.06	0.09	0.01	0.81	0.05	0.28	0.92	1.00	0.03	1.00
CupBC	0.04	0.07	0.08	0.03	0.46	0.06	0.13	0.92	1.00	0.04	1.00
	Experiment 2b: $\theta = 0.67, \zeta_{it}$ is MA(3) with $\sigma_\zeta = 0.75$										
FE	−0.03	0.20	0.20	0.02	0.84	0.05	0.62	0.97	-	0.05	1.00
CCEP	−0.25	0.08	0.26	0.04	0.99	0.08	0.86	0.89	1.00	0.03	1.00
CCEP_GMM	0.05	0.19	0.20	0.17	0.09	0.20	0.05	0.93	1.00	0.04	1.00
Cup	−0.07	0.06	0.09	0.02	0.80	0.05	0.26	0.94	1.00	0.03	1.00
CupBC	−0.01	0.08	0.08	0.04	0.30	0.07	0.07	0.95	1.00	0.03	1.00
	Experiment 3a: $\theta = 0.67, \zeta_{it}$ is white noise with $\sigma_\zeta = 1.25$										
FE	−0.08	0.19	0.21	0.02	0.87	0.05	0.71	0.96	-	0.05	0.99
CCEP	−0.63	0.06	0.63	0.03	1.00	0.06	1.00	0.67	1.00	0.04	0.97
CCEP_GMM	0.02	0.17	0.17	0.14	0.10	0.17	0.06	0.92	1.00	0.04	0.99
Cup	−0.21	0.16	0.26	0.02	0.98	0.09	0.61	0.87	1.00	0.03	1.00
CupBC	0.02	0.09	0.10	0.03	0.50	0.08	0.09	0.94	1.00	0.04	1.00
	Experiment 3b: $\theta = 0.67, \zeta_{it}$ is MA(3) with $\sigma_\zeta = 1.25$										
FE	−0.08	0.19	0.21	0.02	0.87	0.05	0.68	0.98	-	0.06	1.00
CCEP	−0.61	0.07	0.62	0.04	1.00	0.07	1.00	0.67	1.00	0.04	0.98
CCEP_GMM	0.16	0.42	0.45	0.42	0.02	0.41	0.05	0.89	1.00	0.05	0.99
Cup	−0.20	0.15	0.25	0.02	0.98	0.10	0.98	0.91	1.00	0.03	1.00
CupBC	−0.08	0.14	0.16	0.04	0.44	0.10	0.13	0.94	1.00	0.03	1.00

Notes: Results based on 5000 Monte Carlo replications. 'Bias' is the mean bias, 'sd' is the standard deviation of the Monte Carlo distribution of $\widehat{\beta}$ and 'rmse' is its root mean squared error. The standard error is the mean of either the appropriate analytical estimate 'se^a' or the bootstrap estimate 'se^b' for the standard deviation of $\widehat{\beta}$. The reported sizes 'size^a' and 'size^b' are computed at the 5% nominal level for a double-sided t-test for the null hypothesis that $\beta = 1$ using 'se^a' and 'se^b' respectively.

sized. Especially for experiment 0 these are remarkable results as non-stationarity of the idiosyncratic error μ_{it} implies that there is no cointegration between i_{it}, π_{it} and r_t^w. So taking into account the $I(1)$ common factor seems to reastablish the result in Phillips and Moon (1999) that in a panel consistent estimation and valid inference is possible regardless of whether there is cointegration or not (as long the non-stationary of the error terms is not induced by a common factor). Third, introducing endogeneity in experiments 2 and 3 results in a downward bias for both the CCEP and the Cup estimator. Especially for the CCEP estimator this bias is very strong while Table 8 shows that it does not disappear as T grows larger. This is in line with the argument in Section 4.3.2 that the CCEP estimator is inconsistent in this case. The bias of the Cup estimator is smaller, although also sizable in experiment 3, but disappears as T grows larger. The CCEP_GMM and CupBC estimators significantly improve on the performance of the CCEP and Cup estimators. Their bias is relatively small in the cases 2a, 2b and 3a. Only in case 3b the bias is somewhat bigger, especially for the CCEP_GMM estimator. Table 8 shows that the bias disappears as T grows large. Using the bootstrap inference, the size is acceptable for both estimators.

Table 8: Monte Carlo simulation results: bias for $N = 21$ and $T = 50, 100, 250, 500$

	Experiment 2a				Experiment 2b			
T	50	100	250	500	50	100	250	500
FE	−0.06	−0.04	−0.02	−0.01	−0.05	−0.04	−0.02	−0.01
CCEP	−0.32	−0.27	−0.22	−0.21	−0.29	−0.25	−0.22	−0.20
CCEP_GMM	0.04	0.01	0.01	0.00	0.12	0.06	0.02	0.01
Cup	−0.12	−0.07	−0.04	−0.02	−0.11	−0.07	−0.03	−0.02
CupBC	−0.01	0.04	0.05	0.03	−0.07	−0.02	0.01	0.01
	Experiment 3a				Experiment 3b			
T	50	100	250	500	50	100	250	500
FE	−0.14	−0.09	−0.04	−0.02	−0.14	−0.09	−0.04	−0.02
CCEP	−0.70	−0.64	−0.58	−0.55	−0.66	−0.62	−0.57	−0.55
CCEP_GMM	0.02	0.02	0.01	0.01	−0.59	0.17	0.09	0.04
Cup	−0.38	−0.23	−0.10	−0.05	−0.34	−0.21	−0.09	−0.05
CupBC	−0.18	0.00	0.06	0.04	−0.26	−0.10	0.01	0.01

6 Estimation results

The estimation results for the common factor-augmented Fisher equation are reported in Table 9. All estimators are obtained by including the EMS dummies as outlined in Section 3.4. Consistent with the results for the FE estimator reported in Section 3.5, the PANIC shows that there is an $I(1)$ common factor and an $I(0)$ idiosyncratic component in the estimated composite residuals $\widehat{\epsilon}_{it}$ of the CCEP(_GMM) and Cup(BC) regressions. The FE estimator is spurious in this case such that inference should not be trusted. Note that the non-stationarity of the composite error term is not detected by the MW panel unit root test on the FE composite residuals $\widehat{\epsilon}_{it}$ but, despite the huge size distortions documented by the Monte Carlo simulation, is picked up when using CCEP(_GMM) and Cup(BC) composite residuals. PANIC points to a non-stationary common factor and a stationary idiosyncratic component for all estimators.

Table 9: Estimation results

		Sample period: 1983:Q1-2010:Q4, 21 countries									
		Inference $H_0 : \beta = 1$						p-values cointegration tests			
		analytical			bootstrap			MW		PANIC $\widehat{\epsilon}_{it}$	
	$\widehat{\beta}$	se	t-stat	p-val	se	t-stat	p-val	$\widehat{\epsilon}_{it}$	$\widehat{\varepsilon}_{it}$	$\widehat{F}_t$	$\widehat{e}_{it}$
FE	1.10	0.04	2.32	0.02	0.08	1.24	0.21	0.01	-	0.44	0.00
CCEP	0.60	0.04	-10.91	0.00	0.08	-5.35	0.00	0.98	0.00	0.83	0.00
CCEP_GMM	1.00	0.10	0.03	0.97	0.10	0.03	0.97	0.13	0.00	0.56	0.00
Cup	0.58	0.02	-24.63	0.00	0.07	-5.60	0.00	0.99	0.00	0.83	0.00
CupBC	0.83	0.04	-4.52	0.00	0.10	-1.64	0.10	0.79	0.00	0.73	0.00

Notes: All estimators are obtained by including the EMS dummies as outlined in Section 3.4. The Cup(BC) estimators are obtained setting the number of common factors $r = 1$. The CCEP_GMM estimator is obtained by setting $q = 3$ and $L = 8$.

Looking at the coefficient estimates from the various estimators, these range from low values of 0.60 and 0.58 for the CCEP and Cup estimators, over 0.83 for the CupBC estimator to 1.00 for the CCEP_GMM[10] estimator. This variation is quantitatively very much in line with the simulation results from experiment 3b in Section 5. This would imply that the CCEP_GMM and CupBC are the most accurate estimators, with the former still being somewhat upward biased and the latter being somewhat downward biased. The bootstrap inference shows that the hypothesis that $\beta = 1$ is not rejected for both the CCEP_GMM estimator and the CupBC estimator. The overall conclusion is that after taking into account a non-stationary common factor, the full Fisher hypothesis is not rejected by the data.

7 Conclusion

The Fisher effect states that inflation expectations should be reflected in nominal interest rates in a one-for-one manner to compensate for changes in the purchasing power of money. Despite its wide acceptance in theory, much of the empirical work fails to find favorable evidence. This paper examines the Fisher effect in a panel of quarterly data for 21 OECD countries over the period 1983-2010. Using a FE regression of nominal interest rates and inflation we find a slope coefficient which is not significantly different from 1 while a MW panel cointegration test finds the error terms to be stationary. These results support the full Fisher hypothesis. However, a non-stationary common factor in the error terms of this alleged cointegrating relation is detected using PANIC. This implies that the FE regression results are spurious. Our simulation results confirm that a non-stationary common factor in the error terms of the Fisher equation leads to a substantial size bias for the standard MW panel test ignoring cross-sectional dependence and to deceptive inference for the FE estimator. A possible interpretation for the non-stationary common factor is that it reflects permanent common shifts in the real interest rate induced by e.g. shifts in time preferences, risk aversion and the steady-state growth rate of technological change. We next control for an unobserved non-stationary common factor in estimating the Fisher equation using both the CCEP and the Cup estimation approach. Endogeneity of observed inflation induced by a rational expectations forecasting error is taken into account using a bias-corrected version of the Cup estimator

[10]This result is robust over alternative choices of L.

and a GMM version of the CCEP estimator. A small-scale Monte Carlo simulation shows that these two estimators perform reasonably well for the modest sample size $T = 112$, $N = 21$ that is available for our empirical analysis. From the estimation results, the hypothesis of a one-for-one relation between the nominal interest rate and inflation cannot be rejected using either the CupBC or the CCEP_GMM estimator.

References

Ang, A., Bekaert, G., Wei, M., 2007. Do macro variables, asset markets, or surveys forecast inflation better? Journal of Monetary Economics 54 (4), 1163–1212.

Ardagna, S., 2009. Financial markets' behavior around episodes of large changes in the fiscal stance. European Economic Review 53 (1), 37–55.

Bai, J., 2004. Estimating cross-section common stochastic trends in nonstationary panel data. Journal of Econometrics 122 (1), 137–183.

Bai, J., Kao, C., Ng, S., 2009. Panel cointegration with global stochastic trends. Journal of Econometrics 149 (1), 82–99.

Bai, J., Ng, S., 2002. Determining the number of factors in approximate factor models. Econometrica 70 (1), 191–221.

Bai, J. S., Ng, S., 2004. A PANIC attack on unit roots and cointegration. Econometrica 72 (4), 1127–1177.

Banerjee, A., Carrion-i Silvestre, J. L., 2006. Cointegration in panel data with breaks and cross-section dependence. Economics Working Papers ECO2006/5, European University Institute.

Banerjee, A., Marcellino, M., Osbat, C., 2004. Some cautions on the use of panel methods for integrated series of macroeconomic data. Econometrics Journal 7 (2), 322–340.

Banerjee, A., Marcellino, M., Osbat, C., 2005. Testing for PPP: should we use panel methods. Empirical Economics 30, 77–91.

Barro, R., Sala-i Martin, X., 1990. World real interest rates. NBER Macroeconomics Annual 5, 15–59.

Bekaert, G., Wang, X., 2010. Inflation risk and the inflation risk premium. Economic Policy 25, 755–806.

Blanchard, O., Summers, L., 1984. Perspectives on high world real interest rates. Brookings Papers on Economic Activity (2), 273–334.

Chadha, J., Dimsdale, N., 1999. A long view of real rates. Oxford Review of Economic Policy 15 (2), 17–45.

Coakley, J., Fuertes, A., Smith, R., 2006. Unobserved heterogeneity in panel time series models. Computational Statistics & Data Analysis 50 (9), 2361–2380.

Cook, S., Manning, N., 2004. Lag optimisation and finite-sample size distortion of unit root tests. Economics Letters 84 (2), 267–274.

Crowder, W., Hoffman, D., 1996. The long-run relationship between nominal interest rates and inflation: The fisher equation revisited. Journal of Money Credit and Banking 28 (1), 102–118.

Darby, M., 1975. Financial and tax effects of monetary policy on interest rates. Economic Inquiry 13 (2), 266–276.

Dickey, D., Fuller, W., 1979. Distribution of the estimators for autoregressive time-series with a unit root. Journal of the American Statistical Association 74 (366), 427–431.

Elliott, G., Rothenberg, T. J., Stock, J. H., 1996. Efficient tests for an autoregressive unit root. Econometrica 64 (4), 813–36.

Evans, M., Lewis, K., 1995. Do expected shifts in inflation affect estimates of the long-run fisher relation.

Journal of Finance 50 (1), 225–253.

Everaert, G., 2011. Estimation and inference in time series with omitted I(1) variables. Journal of Time Series Econometrics 2 (2), Article 2.

Fisher, I., 1930. The Theory of Interest. Macmillan, New York.

Gagnon, J., Unferth, M., 1995. Is there a world real interest rate? Journal of International Money and Finance 14 (6), 845–855.

Gengenbach, C., Palm, F. C., Urbain, J.-P., 2006. Cointegration testing in panels with common factors. Oxford Bulletin of Economics and Statistics 68 (Suppl. S), 683–719.

Holtz-Eakin, D., Newey, W., Rosen, H., 1988. Estimating vector autoregressions with panel data. Econometrica 56 (6), 1371–1395.

Im, K. S., Pesaran, M. H., Shin, Y., 2003. Testing for unit roots in heterogeneous panels. Journal of Econometrics 115 (1), 53–74.

Kao, C. W., Chiang, M. H., 2000. On the estimation and inference of a cointegrated regression in panel data. In: Baltagi, B. H. (Ed.), Nonstationary panels, panel cointegration and dynamic panels. Vol. 15 of Advances in Econometrics. Elsevier, pp. 179–222.

Kapetanios, G., 2008. A bootstrap procedure for panel data sets with many cross-sectional units. Econometrics Journal 11 (2), 377–395.

Kapetanios, G., Pesaran, M. H., Yamagata, T., 2011. Panels with non-stationary multifactor error structures. Journal of Econometrics 160 (2), 326–348.

Koedijk, K., Kool, C., Kroes, T., 1994. Changes in world real interest rates and inflationary expectations. Weltwirtschaftliches Archiv-Review of World Economics 130 (4), 714–729.

Koustas, Z., Serletis, A., 1999. On the fisher effect. Journal of Monetary Economics 44 (1), 105–130.

Lai, K. S., 2008. The puzzling unit root in the real interest rate and its inconsistency with intertemporal consumption behavior. Journal of International Money and Finance 27 (1), 140–155.

Lee, J., 2002. On the characterisation of the world real interest rate. World Economy 25 (2), 247–255.

Levin, A., Lin, C., Chu, C., 2002. Unit root tests in panel data: asymptotic and finite-sample properties. Journal of Econometrics 108 (1), 1–24.

MacDonald, R., Murphy, P., 1989. Testing for the long run relationship between nominal interest rates and inflation using cointegration techniques. Applied Economics 21 (4), 439–447.

Maddala, G. S., Wu, S. W., Nov. 1999. A comparative study of unit root tests with panel data and a new simple test. Oxford Bulletin of Economics and Statistics 61, 631–652.

Mankiw, N. G., Reis, R., Wolfers, J., 2004. Disagreement about inflation expectation. NBER Macroeconomics Annual 2003 18, 209–248.

Moon, H. R., Perron, B., 2004. Testing for a unit root in panels with dynamic factors. Journal of Econometrics 122 (1), 81–126.

Mundell, R., 1963. Inflation and real interest. Journal of Political Economy 71 (3), 280–283.

Ng, S., Perron, P., 2001. Lag length selection and the construction of unit root tests with good size and power. Econometrica 69 (6), 1519–1554.

O'Connell, P. G. J., 1998. The overvaluation of purchasing power parity. Journal of International Economics 44 (1), 1–19.

Oliner, S., Sichel, D., 2000. The resurgence of growth in the late 1990s: Is information technology the story? Journal of Economic Perspectives 14 (4), 3–22.

Pain, D., Thomas, R., 1997. Real interest rate linkages: Testing for common trends and cycles. Bank of England working papers 65, Bank of England.

Pesaran, M., 2004. General diagnostic tests for cross section dependence in panels. Cambridge Working

Papers in Economics 0435, Faculty of Economics, University of Cambridge.

Pesaran, M., 2006. Estimation and Inference in Large Heterogeneous Panels with a Multifactor Error Structure. Econometrica 74 (4), 967–1012.

Pesaran, M. H., 2007. A simple panel unit root test in the presence of cross-section dependence. Journal of Applied Econometrics 22 (2), 265–312.

Phillips, P. C. B., Moon, H. R., 1999. Linear regression limit theory for nonstationary panel data. Econometrica 67 (5), 1057–1112.

Phillips, P. C. B., Sul, D., 2007. Bias in dynamic panel estimation with fixed effects, incidental trends and cross section dependence. Journal of Econometrics 137 (1), 162–188.

Rapach, D., Weber, C., 2004. Are real interest rates really nonstationary? New evidence from tests with good size and power. Journal of Macroeconomics 26 (3), 409–430.

Roberts, J. M., 2001. Estimates of the productivity trend using time-varying parameter techniques. Contributions to Macroeconomics 1 (1), Article 3.

Rose, A., 1988. Is the real interest rate stable. Journal of Finance 43 (5), 1095–1112.

Schwert, G., 1989. Tests for unit roots: a monte carlo investigation. Journal of Business and Economic Statistics 7, 147–160.

Stock, J. H., Watson, M. W., 1999. Forecasting inflation. Journal of Monetary Economics 44 (2), 293–335.

Tobin, J., 1965. Money and economic growth. Econometrica 33 (4), 671–684.

Urbain, J.-P., Westerlund, J., 2011. Least squares asymptotics in spurious and cointegrated panel regressions with common and idiosyncratic stochastic trends. Oxford Bulletin of Economics and Statistics 73 (1), 119–139.

Westerlund, J., 2008. Panel cointegration tests of the fisher effect. Journal of Applied Econometrics 23 (2), 193–233.