UNIVERSITEIT GENT

FACULTEIT ECONOMIE
EN BEDRIJFSKUNDE

TWEEKERKENSTRAAT 2
B-9000 GENT
Tel. : 32 - (0)9 – 264.34.61
Fax. : 32 - (0)9 – 264.35.92

WORKING PAPER

Pricing Decisions and Insider Trading in Horse Betting Markets

Adi Schnytzer[a], Vasiliki Makropoulou[b], Martien Lamers[c]

[a] Department of Economics, Bar-Ilan University
[b] Department of Management Science and Technology, Athens University of Economics and Business
[c] Department of Financial Economics, Ghent University

February 2012

2012 / 772

D/2012/7012/05

Pricing Decisions and Insider Trading in Horse Betting Markets☆

Adi Schnytzer[a], Vasiliki Makropoulou[b], Martien Lamers[c]

[a]*Department of Economics, Bar-Ilan University*
[b]*Department of Management Science and Technology, Athens University of Economics and Business*
[c]*Department of Financial Economics, Ghent University*

Abstract

This paper builds on a theoretical model by Schnytzer, Lamers, and Makropoulou (2010) that conceptualizes fixed odds horse betting markets as implicit call option markets. We model the decision making process of a bookmaker that sets his prices under uncertainty. We extend the paper of Schnytzer et al. (2010) by relaxing some assumptions and allowing for betting at multiple time periods. We show that when a bookmaker follows this pricing process built upon implicit options, the returns will exhibit a favorite-longshot bias. By performing Monte Carlo simulations we generate the option values and are able to measure the degree of insider trading, which we find to be around 60% in our dataset.

Keywords: Betting, Insider Trading, Contingent Pricing
JEL: D81, D82, G13

Introduction

This paper looks at the decision a bookmaker makes when setting prices under uncertainty in a fixed odds betting market. We argue that this decision can be modeled in a call option framework to measure the degree of insider trading in racetrack betting markets. Makropoulou and Markellos (2011) first developed an option-pricing framework for the pricing of bets in fixed-odds markets and in particular for the European soccer betting market. In this market, the odds are offered by bookmakers via fixed-odds coupons several days before the game and they remain largely unchanged throughout the betting period. Their model deals with expert traders who either exploit public information in a manner superior to that of bookmakers or obtain access to new public information sooner than bookmakers do. Our approach differs in that we focus on racetrack betting where odds change frequently during the half an hour betting period. In our context public information is irrelevant since it can be incorporated into new odds as soon as it hits the market. On the contrary, we deal with insiders who possess private information. Of course, the implications of trading with insiders in the racetrack betting market where the bookmaker changes his odds frequently can be quite similar to those of trading with experts in a market where the odds remain unchanged. However, the MM framework could not be readily applied to the racetrack betting market due to structural differences between the two markets. In order to fill in this gap, Schnytzer, Lamers, and Makropoulou (2010) developed a model for the pricing of bets in a market with insiders relying on the MM framework and applied it to the Australian racetrack betting market. In this paper we extend the work done by Schnytzer et al. (2010) in several aspects. First, we relax their assumption of continuous arrival of information by employing a more realistic specification in which information arrives in discrete amounts and therefore the true probability of a horse winning exhibits quantum jumps and dives. Second,

☆The computational resources (Stevin Supercomputer Infrastructure) and services used in this work were provided by Ghent University, the Hercules Foundation and the Flemish Government - department EWI.

Email addresses: schnyta@mail.biu.ac.il (Adi Schnytzer), vmakropo@aueb.gr (Vasiliki Makropoulou), martien.lamers@ugent.be (Martien Lamers)

the model is extended to allow for more periods in which betting takes place. Whereas the previous work of Schnytzer et al. (2010) assumes a betting period in which the bookmaker sets his prices once (at opening prices), we extend the model to accommodate more time periods in which a bookmaker quotes prices. More specifically, we follow the data at our disposal and allow for betting by insiders both at opening prices and middle prices, instead of only at one of those. Finally, to derive the probability of insider trading, the zero profit condition of the bookmaker does not have to hold for every single horse. This condition is necessary only for the race, allowing the bookmaker to make losses only on the horses he expects insiders to bet on and to make profit on the horses that are backed by outsiders, who bet according to subjective winning probabilities in accordance with public information as explained below.

The remainder of this paper is organized as follows. In the second section we discuss the general framework. In section 3 we will build the theoretical model and the empirical model is discussed in section 4. Finally, in section 5 we will present our findings.

1. General Framework and Model Assumptions

Our objective is to build a model of bookmaker optimal pricing, assuming that there are two populations of bettors in the market; namely, outsiders and insiders. We begin by describing the general framework and main assumptions with respect to the information possessed by the market agents and their betting criteria along with the trading process and the pricing response by bookmakers.

Assume there are N horses in a race. The problem of the bookmaker is that of determining the opening odds. We denote by $\theta_j(0)$ the odds quoted by the bookmaker at time 0 against horse j winning, where $j = 1, ..., N$. If a bet is successful then, ignoring taxes, the bettor receives back $1 + \theta_j(0)$ on a one unit bet. An opening price $OP = \phi_j(0)$ implies odds $\theta_j(0) = \frac{1-\phi_j(0)}{\phi_j(0)}$.

Suppose also that the horses' true winning probabilities at any point in time t are given by $P_j(t)$, $j = 1, ..., N$ where $\sum_{j=1}^{N} P_j(t) = 1$. These true winning probabilities are assumed to evolve according to the flow of information, both public and private, throughout the betting period until the race starts and are therefore stochastic. Moreover, we assume that the flow of information is tied to the flow of bets. In this sense new information is said to have hit the market only if bets arrive in the marketplace in a way that alters the horses' winning probabilities, as those were until then perceived by the bookmaker. The stochastic process for the true winning probabilities could be either continuous or discontinuous, i.e. a jump process, or a mixture of the two. Strictly speaking, the process that affects the true probability should be seen as discontinuous, since the flow of information from small events that may affect the outcome of the race is not continuous. Moreover, we assume that the flow of public information during the betting period is negligible, at least compared to the amount of private information that may hit the market. This assumption makes sense especially if one considers the nature of racetrack betting and the short betting period (about 30 minutes). Moreover, it implies that whenever the bets arrive in a way different from the bookmaker's expectation, it is due to trading on inside information, unknown to the bookmaker until the actual trade has taken place. The above suggest that the expected value of $P_j(t)$ at any point in time, $E\left[P_j(t)\right]$, should be equal to the initial value $P_j(0)$.

Regarding the information possessed by the two presumed populations of bettors, namely outsiders and insiders, and their betting behavior, we make the following assumptions. Firstly, nobody, not even an insider, knows in advance which horse will win the race, in contrast to Shin (1991, 1992, 1993) who assumed that insiders know which horse will win the race. Secondly, an insider knows only the true winning probability of one horse k, $\widehat{P}_k$, before this knowledge becomes public. However, she does not know how $1 - \widehat{P}_k$ is distributed among the other horses. Given the quoted opening price for horse k, $\phi_k(0)$, this true winning probability might involve a profit opportunity for the insider. A risk-neutral insider will wager on horse k only if she expects a positive return. The expected return of the insider on a one unit bet is the expected value of either $\left(-1 + \frac{\widehat{P}_k}{\phi_k(0)}\right)$ or zero, whichever is greater, since the insider bets only if $-1+\frac{\widehat{P}_k}{\phi_k(0)}>0 \Leftrightarrow \widehat{P}_k>\phi_k(0)$. This is similar to saying that bookmakers actually offer insiders (call) options on the horses. Obviously, it is in the bookmakers' interest to offer net out-of-the money options. However, when they err by underestimating a particular horse's true winning probability, they are liable to offer a net in-the-money option on this particular horse, which the insider (who knows her horse's true winning probability) will be glad to snap up.

Outsiders have access only to public information regarding past performance and current conditions. Therefore, we would expect outsiders to support the horses in proportion to the winning probabilities implied by "public information", $P_j(0)$, which are equal to the expected values of the true winning probabilities at the closing of betting, $E\left[P_j(T)\right]$. However, in reality the winning probabilities perceived by the outsiders should also account for their attitudes towards risk as well as for the existence of any behavioral biases among them. Consequently, outsiders are assumed to support the horses in proportion to their subjective winning probabilities, denoted by $\pi_j(t)$. A favorite-longshot bias may arise if bettors are risk-loving (e.g. Quandt, 1986) or due to behavioral biases such as those considered by Kahneman and Tversky (1979). There may of course also be herding which would lead to plunge horses being overbet. The bookmakers are also assumed to know the horses' winning probabilities implied by "public information", i.e. $E\left[P_j(t)\right]$. Compared to outsiders, bookmakers are particularly skillful in gathering and processing public information and are therefore assumed to also know the marginal density function of each horse.[1] In addition, we assume that the bookmaker can accurately predict the expectations of outsiders, i.e., the outsiders' subjective probabilities are known with certainty to the bookmaker.

Trading proceeds in a number of stages. At time zero the bookmaker declares the opening prices (OP), $\phi_j(0)$, based on his perception of the true winning probabilities at this time, $P_j(0)$. At this first stage, a proportion of the outsiders bet in the market at the OP set by the bookmaker. Suppose now that a private signal revealed to a group of insiders indicates that the true winning probability of k is actually higher than the quoted OP, i.e. $\widehat{P}_k>\phi_k(0)$. The insiders will then bet on this horse, say at time t^*. Note that such signals indicating mispricing might be revealed for more than one horse. The bookmaker observes the insider betting pattern and therefore the new value of the true winning probability and adjusts his prices accordingly. At the other stages, the rest of the outsiders bet at the new updated prices. Note that insiders are faced with a timing dilemma. To understand this, suppose that there are two such groups of insiders, each wishing to plunge their own horse. Since a plunge reduces the prices of other horses, each group has an incentive to wait for the other to plunge first. Insiders must utilize any special information they have during the betting, since it loses all value once the race starts. Furthermore, since insider trading is both legal - only jockeys are forbidden to bet - and takes place at fixed prices, insiders have no incentive to hide their trading behavior from outsiders. Moreover, since the insider information concerning any given horse is likely known to more than one person, the longer insiders wait, the greater the risk that the information will leak to a third party. The recipient of the leak will then plunge the horse and the group of insiders - except perhaps the one responsible for the leak - may be left with odds at which betting is no longer worthwhile. (see also Schnytzer and Shilony, 2002)

In the option pricing framework developed in this paper to model the effect of information asymmetries on prices, we do not account for competitive interactions among insiders since this would increase significantly the complexity of the problem in hand while offering limited additional insight. For simplicity, we assume that insiders will place their bet once they receive the private signal.[2]

Price updating effectively continues until the last stage at which starting prices (SP) are determined as the equilibrium prices observed in the market at the end of betting. Since in contrast to the British market there is no legal SP betting in the Australian market, these prices may be assumed to embody all the available useful information regarding the race's outcome. Although price updating might actually

[1] As we will see in Section 4, knowing the marginal density function is equivalent to knowing the volatility of the jump size and the Poisson arrival rate.

[2] One way to capture potential value erosion of the option due to other insiders would be to incorporate a dividend yield. According to the theory of options, it is never optimal to exercise an American option before maturity in the absence of dividends. This means that, in our context, insiders would always bet at the last minute. It is the presence of other insiders (dividends) that makes it optimal to bet before maturity.

take place several times throughout the betting period, our empirical analysis considers only three stages, the first, an intermediate and the last stage, at which opening prices (OP), middle prices (MP) and starting prices (SP) are set, respectively.

The paper develops a model of bookmaker pricing that can be used to derive not only the OP but also any intermediate prices. At each point in time, the prices are modeled as the equilibrium of a perfectly competitive bookmaker market. Specifically, the bookmaker is assumed to be risk-neutral, (i.e. an expected profit maximizer) and there is free entry in the market. Thus, the long-run competitive equilibrium will be established when all bookmakers earn zero expected profits in the market corresponding to each race. Moreover, assuming perfect competition allows for the simplifying assumption of inelastic outsider demand. Note that if the bookmaker were a monopolist and demand were totally inelastic, maximizing profits would lead to unbounded prices.[3]

Insiders are assumed to have a collective wealth W_i. When bookmakers price horses according to the methodology developed in this paper, they assume that insiders bet to the full extent of their wealth W_i should the opportunity arise and that W_i is evenly distributed among insider horses. Therefore, in a race of N horses, up to $\frac{1}{N}W_i$ can be placed by insiders on each horse.

We do not make any assumptions concerning the likelihood of inside traders *vis-à-vis* either favorites or longshots. Finally, transaction costs are assumed to be negligible.

2. The Theoretical Model

2.1. Development of the Mathematical Model

The problem of the bookmaker is that of determining the opening odds $\left(1+\theta_j(0)\right)$ for each one of the N horses in a race, such that his expected profit is equal to zero. Assume for the moment that only outsiders exist in the market. Then, ignoring the time-value of money, the expected profit of the bookmaker at time zero (stage 1) is equal to the total amount of money, W_n, bet by outsiders at stage 1 on the N horses minus the amount of money that the bookmaker is expected to pay out to the winners. Assume also that $w_{n,j}$ is the amount bet on horse j, where $j = 1, 2, ..., N$ and $E_0\left[P_j(T)\right]$ is the expected value of the true winning probability of horse j at the end of the betting period (time T). Note that as explained in the previous section, the proportionate amount of money bet by outsiders on each horse, $\frac{w_{n,j}}{W_n}$, is known to the bookmaker. Regarding the true winning probabilities, these might change throughout the betting period since they evolve according to the flow of information, public and private, as this information is revealed through the flow of bets. However, in the absence of insiders and under the assumption that the flow of public information during the betting period is negligible (see section above), then $E_0\left[P_j(T)\right] = P_j(0)$. The expected profit of the bookmaker is:

$$E_0(\Pi) = W_n - \sum_{j=1}^{N} P_j(0) w_{n,j} \left(1+\theta_j(0)\right) \tag{1}$$

Setting $\phi_j(0) = \frac{1}{1+\theta_j(0)}$, where the notation $\phi_j(0)$ is used to denote opening prices (OP), we obtain:

$$E_0(\Pi) = W_n - \sum_{j=1}^{N} \frac{P_j(0)}{\phi_j(0)} w_{n,j} \tag{2}$$

Given that $\sum_{j=1}^{N} P_j(0) = 1$ then, for the bookmaker to have a zero expected profit, it is sufficient that for each j the OP satisfy the following equation:

[3] A formal proof can be found in the appendix.

$$\phi_j(0) = \frac{w_{n,j}}{W_n} \tag{3}$$

Therefore, if only outsiders exist in the market and, as assumed earlier, the bookmaker can accurately predict their expectations then, for the latter to have zero expected profit on each horse, it is sufficient that opening prices are set equal to the expectation of the bookmaker about the proportion of money bet on each horse i.e., $\phi_j(0) = \pi_j$, where $\pi_j = \frac{w_{n,j}}{W_n}$ is the winning probability of horse j as perceived by outsiders. Considering that π_j actually reflects outsiders' beliefs as those are shaped by public information, risk attitudes and behavioral biases then, under the assumption that the flow of public information is small, there is no reason for the opening prices to change during the betting period.

Suppose now that insiders also exist in the market. Obviously, the final distribution of bets will depend upon both the expectations of outsiders and insiders. The bookmaker can predict with accuracy the expectations of outsiders but not those of insiders, since the latter are shaped according to the private information they receive; moreover, this information is revealed to the bookmaker only after an inside trade has taken place.

Assume again that the bookmaker gives at time zero (opening) prices $\phi_j(0)$ for each one of the horses and that the betting period is again T periods of time. It is assumed that only part of outsiders will bet at OP, $\omega_n^{OP} = \frac{W_n^{OP}}{W_n}$, while the other part will bet at later stages after they have observed insider behavior. A risk-neutral insider will wager on horse k only if she expects a positive return. The expected return of the insider on a one unit bet is the expected value of either $\left(-1 + \frac{\widehat{P}_k}{\phi_k(0)}\right)$ or zero, whichever is greater, since the insider bets only if $-1 + \frac{\widehat{P}_k}{\phi_k(0)} > 0 \Leftrightarrow \widehat{P}_k > \phi_k(0)$.

Under the above assumptions, the bookmaker is always expected to lose from trading with insiders. In particular, the bookmaker's expected loss to an insider at time zero on a one unit bet (placed at time t^*) is:

$$E_0(\Pi) = -E_0\left[\max\left(-1 + \frac{\widehat{P}_k}{\phi_k(0)}\right), 0\right] \tag{4}$$

It holds that $\widehat{P}_k = P_k(t^*) \neq P_k(0)$, where $P_k(t^*)$ is the true winning probability of horse k at the time the insiders place their bet (which is now revealed to the bookmaker).

The expected profit of the bookmaker is therefore:

$$E_0(\Pi) = W_n^{OP} - \sum_{j=1}^{N} \frac{E_0\left[P_j(T)\right]}{\phi_j(0)} w_{n,j}^{OP} - \sum_{j=1}^{N} w_{i,j}^{OP} E_0\left\{\max\left(-1 + \frac{P_j(t^*)}{\phi_j(0)}\right), 0\right\} \tag{5}$$

where $w_{i,j}^{OP}$ is the amount of money bet by insiders at OP on horse j. Note that now that insiders also exist in the market, the bookmaker cannot know what the true wining probability will be at the end of the betting period. However, as explained in the previous section, the bookmaker is assumed to know the expected value of the true winning probability $E\left[P_j(t)\right]$ at any time t.

The expression above is complicated by the fact that t^* cannot be known *a priori* to the bookmaker and hence it should be treated as stochastic. In order to simplify this, we assume that private information that may alter the true winning probability of a given horse may arrive only once for each horse. Then, we can safely state that $\widehat{P}_k = P_k(t^*) = P_k(T)$, where $P_k(T)$ is the value of the true winning probability at the closing of betting since as we said earlier private information regarding a certain horse may arrive in the marketplace only once. Of course, one might argue that the true winning probability of horse k may be lowered if at a later time new (positive) information regarding a second horse r hits the market, implying, $\widehat{P}_r > P_r(0)$. Obviously, this would always be true in a race of two horses only. However, in a race of many horses, one could accept the supposition that this new information would reduce the true winning probabilities of all other horses (for which no inside information has hit the market) except for horse k.

Given that $w^{OP}_{i,j} = \frac{1}{N} W_i$, for the bookmaker to have zero expected profit, the following condition must be met:

$$1 - \sum_{j=1}^{N} \frac{w^{OP}_{n,j}}{W^{OP}_n} \frac{E_0\left[P_j(t)\right]}{\phi_j(0)} = \frac{1}{N} \frac{W_i}{W^{OP}_n} \sum_{j=1}^{N} E_0 \left\{ \max\left(-1 + \frac{P_j(T)}{\phi_j(0)}\right), 0 \right\} \tag{6}$$

or

$$1 = \sum_{j=1}^{N} E_0\left[P_j(T)\right] \left\{ \frac{w^{OP}_{n,j}}{W^{OP}_n} \frac{1}{\phi_j(0)} + \frac{1}{N} \frac{W_i}{W^{OP}_n} \frac{E_0\left\{\max\left(-1 + \frac{P_j(T)}{\phi_j(0)}\right), 0\right\}}{E_0\left[P_j(T)\right]} \right\} \tag{7}$$

Given that $\sum_{j=1}^{N} E_0\left[P_j(T)\right] = 1$, for the above equation to hold, it is sufficient that the opening price of each horse j satisfies the following equation:

$$\frac{w^{OP}_{n,j}}{W^{OP}_n} \frac{1}{\phi_j(0)} + \frac{1}{N} \frac{W_i}{W^{OP}_n} \frac{E_0\left\{\max\left(-1 + \frac{P_j(T)}{\phi_j(0)}\right), 0\right\}}{E_0\left[P_j(T)\right]} = 1 \tag{8}$$

or multiplying with the term $\left(\frac{W^{OP}_n}{W_n}\right)$ and rearranging we obtain:

$$\left(\frac{W^{OP}_n}{W_n}\right) - \left(\frac{W^{OP}_n}{W_n}\right) \frac{w^{OP}_{n,j}}{W^{OP}_n} \frac{1}{\phi_j(0)} = \frac{1}{N} \left(\frac{W_i}{W_n}\right) \frac{E_0\left\{\max\left(-1 + \frac{P_j(T)}{\phi_j(0)}\right), 0\right\}}{E_0\left[P_j(T)\right]} \tag{9}$$

The left hand side of the above equation is the expected bookmaker gain from trading with outsiders while the right-hand side is his expected loss to insiders. The optimal price is the one that equalizes the gain from outsiders to the loss to insiders. It can be found by solving the above equation through trial and error, given the proportion of outsiders that bet at OP, $\omega^{OP}_j = \frac{W^{OP}_n}{W_n}$, outsider's subjective probabilities, $\pi^{OP}_j = \frac{w^{OP}_{n,j}}{W^{OP}_n}$, the bookmaker's expectation about the true winning probability at the closing of betting, $E_0\left[P_j(T)\right]$, the number of runners in a race, N, and, of course, the degree of insider trading (as perceived by the bookmaker) defined as the ratio of total insider money to total outsider money, $\frac{W_i}{W_n}$.

Note that the left-hand side of this equation should be greater or equal to zero since the right-hand side is always non-negative. Therefore, if insiders exist in the market then, in order for the bookmaker to have zero expected profit, prices should be set greater than outsiders' subjective probabilities, i.e.:

$$\phi_j(0) \geq \frac{w^{OP}_{n,j}}{W^{OP}_n} = \pi^{OP}_j \tag{10}$$

To summarize, our model suggests that since any private information is conveyed to the bookmaker only after an informed trade takes place, the latter should include a premium in the OP to compensate him for this risk. Moreover, this premium is related to the cost of trading with insiders, which in turn is a function of the degree of insider trading ($\frac{W_i}{W_n}$) and the potential value of private information that may be exploited by insiders (as captured by the term $E\left\{\max\left(-1 + \frac{P_j(T)}{\phi_j(0)}\right), 0\right\}$). Under these considerations, the sum of OP would always be greater than one.

Suppose now that at a later point in time, time τ (stage 2), after the bookmaker has observed insider trading, he will set new prices (called MP). For those horses on which insider trading has taken place, say m horses, prices will be set equal to the horses' new true winning probabilities (in the absence of any bookmaker margin). The reason is that insiders pose no further risk to the bookmaker since they can only bet at either OP or MP on a given horse but not both.[4] For the rest of the horses ($N-m$), the bookmaker will set prices as above. Therefore, at the second stage the total amount of money available by insiders is $W_i - \frac{m}{N}W_i \leq W_i$. Thus, the bookmaker will quote MP as if $\frac{1}{N-m}\left(W_i - \frac{m}{N}W_i\right) = \frac{1}{N}W_i$ would be wagered by insiders on each one of the remaining $N-m$ horses should the opportunity arise. Therefore, we have:

$$\left(\frac{W_n^{MP}}{W_n}\right) - \left(\frac{W_n^{MP}}{W_n}\right)\frac{w_{n,j}^{MP}}{W_n^{MP}}\frac{1}{\phi_j(\tau)} = \frac{1}{N}\left(\frac{W_i}{W_n}\right)\frac{E_\tau\left\{\max\left(-1+\frac{P_j(T)}{\phi_j(\tau)}\right),0\right\}}{E_\tau\left[P_j(T)\right]} \tag{11}$$

where the term $\pi_j^{MP} = \frac{w_{n,j}^{MP}}{W_n^{MP}}$ captures outsiders' new subjective probabilities as these have been shaped after observing the insider trading pattern at stage 1 and $\omega_j^{MP} = \frac{W_n^{MP}}{W_n}$ is the proportion of outsiders that bets at this second stage.

Price updating effectively continues until the last stage at which starting prices (SP) are determined as the equilibrium prices observed in the market at the end of betting. Under the assumption of zero bookmaker profit, the sum of SP would be equal to one. Then, following our model, in the presence of insiders the sum of OP should always be greater in any race than the sum of SP. In reality, the sum of OP is always greater in any race than the sum of SP even in the apparent absence of insider trading.[5] The reason is that opening prices tend to have a "cartel" level of profit built in since they are recommended to individual bookmakers by the bookmakers' association. Once betting begins, there is competition among bookmakers and thus the sum of prices will tend to decrease. This practically means that the estimates of insider trading obtained when applying our model may overestimate its true extent if the premium included in OP is largely due to this "cartel" profit rather than to the risk that bookmakers face in the presence of insiders. On the other hand, it may be that the expected profit margins built into OP are designed just to compensate the bookmakers for inside trades.

2.2. The Option Analogy

The commitment made by bookmakers to sell at fixed prices, the quoted odds, can be analyzed as a call option. Specifically, the bookmaker gives an insider a call option on horse j, i.e., the right to bet at a fixed price. Obviously, the underlying asset whose value changes stochastically is horse's j true winning probability. Apparently, only insiders are entitled to the option. The reason is that while an insider has perfect information (both public and private) and therefore knows her horse's true winning probability, outsiders form their expectations, at least partially, according to the public component of information and based on that they assign subjective probabilities. The insider will exercise her option to bet at her horse at the opening prices only if she expects a positive return, i.e. if the true probability at the time the insider places her bet, t^*, is greater than the opening price.

One could assume that insiders would be better off exercising their option at the last minute, i.e. at the closing of betting. The reason is that since a plunge reduces the prices of other horses, each group of insiders has an incentive to wait for the other groups to plunge first. However, since the insider information concerning any given horse is likely known to more than one person, the longer insiders wait, the greater the risk that the information will leak to a third party. The recipient of the leak will then plunge the horse and the group of insiders - except perhaps the one responsible for the leak - may be left with odds at which betting is no longer worthwhile. This timing dilemma is similar to the problem of the optimal exercise time

[4] If they bet at OP then prices will exhibit a plunge and therefore betting at MP would be worthless. This is is true under the assumption that information regarding a certain horse can be revealed only once.

[5] Races in which there are no plunges visible in the data (odds at no point fall for any horse during the betting) are races in which inside trades are not observed. Of course, it could be that an insider has place a discreet bet with a single bookmaker and that this bet cannot be discerned in the average odds that rule in the market and are published. The greater the extent of this phenomenon, the more will our estimates of insider trading underestimate its true extent.

faced by the holder of an American option on a dividend-paying stock. In the betting market, the dividend equivalent is the potential value leakage as a result of competition among insiders. However, for simplicity we ignore competitive interactions among insiders. We assume instead that insiders place their bet once they observe mispricing.

Assuming that the bookmaker is risk-neutral then today's option price (time zero) can be determined by discounting the expected value of the terminal option price by the riskless rate of interest. Therefore, neglecting the time-value of money, the value of the call option is:

$$C_j(0) = C_j^{OP} = E_0 \left\{ \max \left(-1 + \frac{P_j(T)}{\phi_j(0)} \right), 0 \right\} \tag{12}$$

Similarly:

$$C_j(\tau) = C_j^{MP} = E_\tau \left\{ \max \left(-1 + \frac{P_j(T)}{\phi_j(\tau)} \right), 0 \right\} \tag{13}$$

The value of the option can be derived by assuming a stochastic process for the true winning probability and performing Monte Carlo simulations (see Section 4).

2.3. The Favorite-Longshot Bias

In this section we show that the optimal prices set by the bookmaker using Equation 9 will exhibit the favorite-longshot bias.

Expected returns will exhibit the favorite-longshot bias if and only if $\frac{\partial E(R_j)}{\partial \left(E_0\left[P_j(T)\right]\right)} > 0$, where $E(R_j) = -1 + \frac{E_0\left[P_j(T)\right]}{\phi_j}$. This is equivalent to:

$$\frac{\partial \left(\frac{E_0\left[P_j(T)\right]}{\phi_j} \right)}{\partial E_0 \left[P_j(T)\right]} > 0 \tag{14}$$

Denoting $f_j(0) = \frac{E_0\left[P_j(T)\right]}{\phi_j}$, Equation 9 can be written:

$$\left(\frac{W_n^{OP}}{W_n} \right) E_0 \left[P_j(T)\right] - \left(\frac{W_n^{OP}}{W_n} \right) \left(\frac{W_{n,j}^{OP}}{W_n^{OP}} \right) f_j(0) = \frac{1}{N} \left(\frac{W_i}{W_n} \right) E_0 \left\{ \max\left(-1 + f_j(T)\right), 0 \right\} \tag{15}$$

where $f_j(T) = \left(\frac{E_T\left[P_j(T)\right]}{\phi_j} \right) = \frac{P_j(T)}{\phi_j}$.

Differentiating the above with respect to $E_0\left[P_j(T)\right]$ and setting $\frac{\partial E_0\left\{\max\left(-1+f_j(T)\right),0\right\}}{\partial E_0\left[P_j(T)\right]} = \frac{\partial E_0\left\{\max\left(-1+f_j(T)\right),0\right\}}{\partial f_j(0)} \frac{\partial f_j(0)}{\partial E_0\left[P_j(T)\right]}$, we obtain:

$$\frac{\partial f_j(0)}{\partial E_0\left[P_j(T)\right]} = \frac{\left(\frac{W_n^{OP}}{W_n}\right)\left(1 - f_j(0)\frac{\partial\left(w_{n,j}^{OP}/W_n^{OP}\right)}{\partial\left(E_0\left[P_j(T)\right]\right)}\right)}{\left(\frac{W_n^{OP}}{W_n}\right)\left(\frac{w_{n,j}^{OP}}{W_n^{OP}}\right) + \frac{1}{N}\frac{W_i}{W_n}\frac{\partial E_0\left\{\max\left(-1+f_j(T)\right),0\right\}}{\partial f_j(0)}} \tag{16}$$

We focus on the denominator first. The first term is always positive. The second term is positive too since the partial derivative $\frac{\partial E_0\left\{\max\left(-1+f_j(T)\right),0\right\}}{\partial f_j(0)}$ is always positive. Note that a higher level of $f_j(0) = \left(\frac{E_0\left[P_j(T)\right]}{\phi_j}\right)$

is equivalent to a lower quoted price for the same level of expected true probability. Therefore, the potential profit of insiders, as captured by the term $E_0\left\{\max\left(-1+f_j(T)\right),0\right\}$, should increase since a lower quoted price makes underpricing more likely. In the terminology of options this is equivalent to saying that a lower strike price increases the value of a call option.

We turn our attention now to the nominator. For the nominator to be positive, it is necessary that the term $1-f_j(0)\frac{\partial\left(w_{n,j}^{OP}/W_n^{OP}\right)}{\partial\left(E_0\left[P_j(T)\right]\right)}$ is positive. This obviously depends on the partial derivative of the outsiders' subjective probability with respect to the expected true winning probability. Suppose that outsiders tend to overestimate the winning chances of longshots relative to those of favorites, as often argued in the literature, i.e. $\frac{\partial\left(E_0\left[P_j(T)\right]\right)/\left(w_{n,j}^{OP}/W_n^{OP}\right)}{\partial\left(E_0\left[P_j(T)\right]\right)}>0 \Leftrightarrow 1-\frac{\left(E_0\left[P_j(T)\right]\right)}{\left(w_{n,j}^{OP}/W_n^{OP}\right)}\frac{\partial\left(w_{n,j}^{OP}/W_n^{OP}\right)}{\partial\left(E_0\left[P_j(T)\right]\right)}>0$.

We know that:

$$\phi_j>\frac{w_{n,j}^{OP}}{W_n^{OP}} \Rightarrow \frac{1}{\phi_j}<\frac{1}{W_{n,j}^{OP}/W_n^{OP}} \Rightarrow \frac{E_0\left[P_j(T)\right]}{\phi_j}<\frac{E_0\left[P_j(T)\right]}{W_{n,j}^{OP}/W_n^{OP}}$$

$$\Rightarrow \frac{E_0\left[P_j(T)\right]}{\phi_j}\frac{\partial\left(w_{n,j}^{OP}/W_n^{OP}\right)}{\partial\left(E_0\left[P_j(T)\right]\right)}<\frac{\partial\left(E_0\left[P_j(T)\right]\right)}{\partial\left(w_{n,j}^{OP}/W_n^{OP}\right)}\frac{\partial\left(w_{n,j}^{OP}/W_n^{OP}\right)}{\partial\left(E_0\left[P_j(T)\right]\right)}$$

$$\Rightarrow 1-\frac{E_0\left[P_j(T)\right]}{\phi_j}\frac{\partial\left(w_{n,j}^{OP}/W_n^{OP}\right)}{\partial\left(E_0\left[P_j(T)\right]\right)}>1-\frac{\partial\left(E_0\left[P_j(T)\right]\right)}{\partial\left(w_{n,j}^{OP}/W_n^{OP}\right)}\frac{\partial\left(w_{n,j}^{OP}/W_n^{OP}\right)}{\partial\left(E_0\left[P_j(T)\right]\right)}>0$$

Therefore, we have proved that when the bookmaker sets optimal prices following our model, expected returns will exhibit the favorite-longshot bias provided that either outsiders have no biases in their expectations and therefore their subjective probabilities reflect the publicly available information or that they tend to overestimate the winning chances of longshots relative to those of favorites.

3. Empirical Model

3.1. Option-Pricing Specifications of the Model

The challenge faced here is that the assumed specification must be a realistic description of probability dynamics. In particular, we want to model the true winning probability such that the following requirements are met: Firstly, said probability is concentrated on $[0,1)$. A probability of a certain horse winning equal to one implies that the probabilities of all other horses are zero. In practice this is never the case. For this reason, we set as an upper boundary for the true winning probabilities the value $p_{\max}<1$. In particular, $p_{\max}$ could be the highest single probability in our sample, which is found to be 0.7197. Secondly, the sum of probabilities is equal to one at all times. Thirdly, it may exhibit positive and/or negative jumps throughout the betting period following the release of new private information. Finally, in the long-run it reverts to a mean equal to the reciprocal of the number of runners in a race. This assumes that over a long period of time all horses have equal chances of winning. Note that the behavior of this process in the absence of mean reversion is problematic since in this case the boundaries become absorbing.

Taking the above under consideration, the following stochastic process is assumed:

$$dP_j(t)=h\left(\mu-P_j(t)\right)dt+P_j(t)\left(p_{\max}-P_j(t)\right)Jdq \tag{17}$$

where h is the speed of mean reversion, μ is the long-run mean (equal to $\frac{1}{N}$), J is the jump size which is assumed to be normally distributed with mean zero and standard deviation σ_j and dq describes a time-homogeneous Poisson jump process such that $dq=1$ with probability λdt and $dq=0$ with probability

$(1 - \lambda dt)$. Parameter λ is known as intensity or arrival rate and is the expected number of "events" or "arrivals" that occur per unit time. The term $P_j(t)\left(p_{\max} - P_j(t)\right)$, which multiplies the jump component Jdq, is employed in order to ensure that the probability will remain inside the boundaries of zero and $p_{\max}$. Furthermore, given that the jump size has a mean of zero, it can be easily shown that the expected value of $P_j(t)$ at any $t > 0$ is given by:

$$E\left[P_j(t)\right] = P_j(0)e^{-ht} + \mu\left(1 - e^{-ht}\right) \tag{18}$$

Note that when the speed of mean reversion is very small, as assumed in this paper, the expected value of $P_j(t)$, $E\left[P_j(t)\right]$, tends to the initial value $P_j(0)$. This is important since the theoretical model described previously in this paper relied heavily on this assumption.

There is one final concern with respect to the specification for the true winning probability, which, as mentioned earlier, refers to the fact that the sum of probabilities must be equal to one at all times. Suppose that the probability of horse k follows the above stochastic process, while for all other horses $j, j = 1, 2..., N, j \neq k$, it holds that:

$$dP_j(t) = h\left(\mu - P_j(t)\right) + \epsilon_j(t) \tag{19}$$

Then, taking the sum of all probabilities, setting it equal to one and observing that $\sum_{j=1}^{N} h\left(\mu - P_j(t)\right) dt = 0$, it follows directly that:

$$\sum_{\substack{j=1 \\ j\neq k}}^{N} \epsilon_j(t) + J_k P_k(t)\left(p_{\max} - P_k(t)\right) dq = 0 \tag{20}$$

Therefore, although Equation 17 does not warrant that $\sum_{j=1}^{N} P_j(t) = 1$, we can find a condition under which this holds. Thus the above specification is indeed a realistic description of probability dynamics. We now need to estimate the parameters that appear in the stochastic process followed by the true winning probability. For the purpose of this estimation we will ignore the mean-reverting component, assuming instead that the speed of mean reversion is very close to zero. Thus, we only have to estimate the parameters of the jump process and in particular, the standard deviation σ_j of the jump size and the intensity λ of the Poisson process. The intensity parameter tells us how often the true winning probability experiences a sudden jump, while the parameter of jump volatility measures the size of these jumps. We calculate these parameters by computing the second and fourth (raw) moments. These are specified as following:

$$\mu_2 = E\left(Y^2\right) = E\left(J^2\right) E\left(dq^2\right) = \sigma_j^2 \lambda \Delta t \tag{21}$$

$$\mu_4 = E\left(Y^4\right) = E\left(J^4\right) E\left(dq^4\right) = 3\sigma_j^4 \lambda \Delta t \tag{22}$$

where $Y = \frac{\Delta P}{P(1-P)}$.

Those two moments completely identify the jump components. Moreover, they can be derived from the bookmakers' odds as following: As the dataset includes prices at three points in time (OP, MP and SP), prices are available roughly every 15 minutes. The fifteen-minute moments may thus be calculated for each race:

$$2M : m_2 = \frac{1}{s-1}\left(u_1 - u_2\right)^2 \tag{23}$$

$$4M : m_4 = \frac{1}{s-1}\left(u_1 - u_2\right)^4 \tag{24}$$

where $s = 2$ and $u1, u2$ can be calculated as following:

$$u1 = \frac{\phi^{MP} - \phi^{OP}}{\phi^{OP}(1 - \phi^{OP}} \tag{25}$$

$$u2 = \frac{\phi^{SP} - \phi^{MP}}{\phi^{MP}(1 - \phi^{MP}} \tag{26}$$

Obviously, the one-minute moments can be calculated from the fifteen-minute moments by dividing with fifteen. Using Equations 21 and 22 for $\Delta t = 1$ minute and the estimated values for the one-minute moments, we determine the jump components σ_j and λ for all the horses in each race:

$$\lambda = \frac{3\mu_2^2}{\mu_4} \tag{27}$$

$$\sigma_j = \sqrt{\frac{\mu_4}{3\mu_2}} \tag{28}$$

Finally, we calculate the average values of σ_j and λ for our sample, which then are used in the options calculations. Note that these are "one-minute" values. For example $\lambda = 0.1$ implies that we have a jump every 10 minutes. The results are presented below.

3.2. A Measure of Insider Trading

We focus now on the task of estimating the degree of insider trading, i.e. the parameter $\frac{W_i}{W_n}$. To this end, we assume that in practice bookmakers set their prices according to the methodology described above. Thus, using the actual prices, we can infer the degree of insider trading by using Equation 6 to directly solve for $\frac{W_i}{W_n}$. However, the theoretical model was built under the assumptions of zero expected profit and zero transaction costs. This may yield estimates of insider trading that are biased upward. Starting from Equation 6:

$$1 - \sum_{j=1}^{N} \frac{w_{n,j}^{OP}}{W_n^{OP}} \frac{E_0\left[P_j(t)\right]}{\phi_j(0)} = \frac{1}{N}\frac{W_i}{W_n^{OP}} \sum_{j=1}^{N} E_0\left\{\max\left(-1 + \frac{P_j(T)}{\phi_j(0)}\right), 0\right\} \tag{6}$$

By multiplying with $\frac{W_n^{OP}}{W_n}$, the part of noise trading that occurs at OP:

$$\frac{W_i}{W_n}\frac{1}{N}\sum_{j=1}^{N} C_j^{OP} = \frac{W_n^{OP}}{W_n}\left(1 - \sum_{j=1}^{N} \frac{w_{n,j}^{OP}}{W_n^{OP}} \frac{E_0\left[P_j(t)\right]}{\phi_j(0)}\right) \tag{29}$$

or that

$$q\sum_{j=1}^{N} C_j^{OP} = N\frac{W_n^{OP}}{W_n} D^{OP} \tag{30}$$

Where $D^{OP} = \left(1 - \sum_{j=1}^{N} \pi_j^{OP} \frac{E_0\left[P_j(t)\right]}{\phi_j(0)}\right)$ for each race. The superscript OP indicates that these values refer to the first stage at which the opening prices are set. This is the basic equation for our empirical analysis. Obviously this expression refers only to OP. However, as we said, we assume that trading takes place in two stages. At stage 1 ($t_1 = 0$), a proportion of the outsiders bet in the market at the OP set by the bookmaker. At any subsequent point in time, $t_1 + \Delta t \leq T$ all insiders may bet should the opportunity arise. The bookmaker observes the insider trading pattern and at time, $t_2, t_1 < t_2 \leq T$ updates his prices. At stage 2, the rest of the outsiders bet at the new set of updated prices, denoted by MP. Again, at any subsequent point in time, $t_2 + \Delta t \leq T$ all insiders may bet should the opportunity arise. The bookmaker observes the insider trading pattern and at time T sets new updated prices denoted by SP (starting prices).

Similarly, for the second stage at which MP are set, we have:

$$q \sum_{j=1}^{N} C_j^{MP} = N \frac{W_n^{MP}}{W_n} D^{MP} \tag{31}$$

where $D^{MP} = \left(1 - \sum_{j=1}^{N} \pi_j^{MP} \frac{E_\tau\left[P_j(\tau)\right]}{\phi_j(\tau)}\right)$.

We can use Equations 30 and 31 to calculate the proportion of outsiders that bet at OP and MP:

$$\omega^{OP} = \frac{W_n^{OP}}{W_n} = \frac{D^{MP} \sum_{j=1}^{N} C_j^{OP}}{D^{MP} \sum_{j=1}^{N} C_j^{OP} + D^{OP} \sum_{j=1}^{N} C_j^{MP}} \tag{32}$$

Then we can use Equations 30 and 32 to calculate q. In order to do so we still have to explain how to calculate the option values at both OP and MP, C_j^{OP} and C_j^{MP}, as well as the quantities D^{OP} and D^{MP} for each race.

We begin with betting at OP. The option values C_j^{OP} can be estimated via Monte Carlo simulation. The required inputs to perform the simulations are $P_j(0)$, OP_j, T and the specifications of the stochastic process followed by the true winning probability. OP_j is the observed opening price quoted by the bookmaker. T is assumed to be equal to 30 minutes. With respect to the specifications of the stochastic process followed by the true winning probability, we need to know the speed of mean reversion, h, the long-run mean, μ, which is set equal to $\frac{1}{N}$, the standard deviation of the jump size, σ_j, given that $J \sim N\left(0, \sigma_j\right)$ and the intensity λ of the Poisson process. A way to derive those parameters has been shown in the previous section of the paper. The speed of mean reversion is assumed to be very small since we are dealing with a betting period of no more than 30 minutes, and is therefore set at 0.001. The true winning probability, $P_j(t)$, is derived via a conditional logit regression on a dummy win, ensuring that the sum of probabilities in each race equals 1. The subjective probabilities π_j^{OP} are calculated by simply normalizing OP as suggested by Dowie (1976), although this yields estimates with a favorite-longshot bias. The true winning probability is simulated in 1000 steps using the stochastic process in Equation 17. When the simulated true winning probability after 1000 steps is larger than the true winning probability in time $t = 0$, the option value is this difference, otherwise the option value is zero. Each horse is subject to 1000 repetitions. The option value for the horse is the averaged value over all repetitions. A similar procedure is followed to calculate C_j^{MP}, using as inputs $P_j(\tau)$, MP_j and $T - \tau$, where τ is assumed to be equal to 15 minutes. We use the same specifications for the stochastic process as above.

We still need to calculate D^{OP} and D^{MP} for each race. The expected true winning probabilities at the end of the betting period, $E_0\left[P_j(T)\right]$, are assumed to be equal to the true winning probabilities at time 0, i.e. $E_0\left[P_j(T)\right] = P_j(0)$. This is derived from Equation 18 if we assume that the speed of mean reversion is very small. This way we assume that mean reversion has almost no effect on the true winning probabilities in the very short betting period of 30 minutes, while any deviations from the initial value are due to the effect of jumps that come as surprises.

Next, we use Equation 32 to calculate $\omega^{OP} = \frac{W_n^{OP}}{W_n}$. The extent of insider trading for each race is then:

$$q = \frac{N\omega^{OP} D^{OP}}{\sum_{j=1}^{N} C_j^{OP}}$$

The probability of insider trading is simply:

$$a = \frac{q}{1+q}$$

As we said, our model is built from the viewpoint of the bookmaker and the approach we have followed so far effectively supposes that bookmakers know the probability of insider trading in advance. Or, more reasonably, such a measure is of the bookmakers' expectations regarding insider trading. However, we have access to *ex-post* plunging information which the bookmaker cannot know until after insider activity has taken place. We will use this (*ex-post*) plunging information in order to get closer to the true probability of insider trading for a given horse that got plunged.

In order to calculate the probability of insider trading per horse, we use both an unweighted and a weighted average of q. The weight is derived as the absolute size of the plunge called PW: $\max(MP - OP, 0) + \max(SP - MP, 0)$. Using the unweighted and weighted average, the probability of insider trading for each horse in a given race in the sample is calculated. Note that when we weight absolute plunges sizes, we are weighting on those horses where insiders were observed to have bet more heavily in accordance with plunge size. Using these weights, the weighted average probability of insider trading for each of the races in the sample is calculated. The simple average of these values is the probability of insider trading in the dataset.

4. Results

We use the above model to derive a measure of the extent of insider trading. Our measure is applied to a dataset of the 1998 Australian Horse Racing season, covering 4017 races with 45296 runners.[6] The dataset includes for each horse prices at three moments (OP, MP and SP). The time period during which betting takes place is set at 30 minutes, meaning that prices are available roughly every 15 minutes. Given that there were some cases in which the sum of OP, MP or SP in a race is less than one, these races are dropped from the sample. This leaves us with 3995 races out of the initial sample of 4017 races.

The true winning probabilities at OP and MP, necessary for the measure, are derived by running a conditional logit regression, the results of which are displayed in Table1.

Table 2 displays descriptive statistics for OP, MP, SP; the subjective winning probabilities at OP and MP; the true winning probabilities flowing from Table 1; and the sum of OP, MP and SP per race.

The table shows clearly that the average sum of prices decreases between OP and SP. At the opening of betting this margin is 43 percent, but by the start of the race the margin has decreased to 24 percent. The decrease in the margin indicates competition among bookmakers, forcing them to decrease prices and leading to lower profits. Since the OP are above the competitive level, this could deter insiders from trading at these prices, leading to a lower degree of insider trading.

The option values are generated via Monte Carlo simulation as explained in the previous section. The average 1 minute values for λ and σ in the dataset are 0.37 and 0.11 respectively. On average, there seems to be a jump every 3 minutes, or 10 times per a 30 minute betting period, indicating there is quite some inflow of private information into the prices. Table 3 shows the values of the non-zero options generated at OP and MP.

[6]The data were obtained from the CD-Rom, *Australasian Racing Encyclopedia '98*, presented by John Russell.

Table 1: Conditional Logit Regression

	Win	Win
OP	6.713***	
	(0.133)	
MP		7.155***
		(0.141)
N	45266	45266
Log Likelihood	-8259.18	-8238.81

Standard errors in parentheses. * significant at 10%; ** significant at 5%; *** significant at 1%

Table 2: Descriptive Statistics

Variable	Mean	Min	Max	St. Dev
OP	0.1255	0.0019	0.8889	0.1017
MP	0.1107	0.0013	0.867	0.0946
SP	0.1092	0.001	0.8462	0.0972
$\pi_j(OP)$	0.0883	0.0014	0.7197	0.0731
$\pi_j(OM)$	0.0883	0.0011	0.7165	0.0768
$P_j(OP)$	0.0883	0.0026	0.9657	0.097
$P_j(MP)$	0.0883	0.002	0.9723	0.0984
$\sum_{j=1}^{N} OP$	1.4339	1.0225	2.0631	0.1117
$\sum_{j=1}^{N} MP$	1.266	1.0003	1.8508	0.1008
$\sum_{j=1}^{N} SP$	1.2487	1.0122	1.7646	0.0921

There were 5184 horses for which a zero option was generated at OP and 6806 horses for which the option value was zero at MP. Moreover, we can see from Table 3 that the options generated at MP have a higher value, indicating more profitable trading opportunities for insiders. This should not come as a surprise as we already saw in Table 2 that the MP are lower, leading to a lower strike price for the insiders and a higher profit.

One last thing is required to calculate the degree of insider trading, namely ω^{OP}, the part of outsiders that bet at opening prices. Using the data from Tables 2 and 3, the average ω^{OP} in the dataset is calculated to be 0.41. On average 41% of outsider trading occurs at OP and 59% at MP, although there are races in which almost no outsider betting is found to occur at OP. The ratio of insider betting to outsider betting,

Table 3: Option Statistics

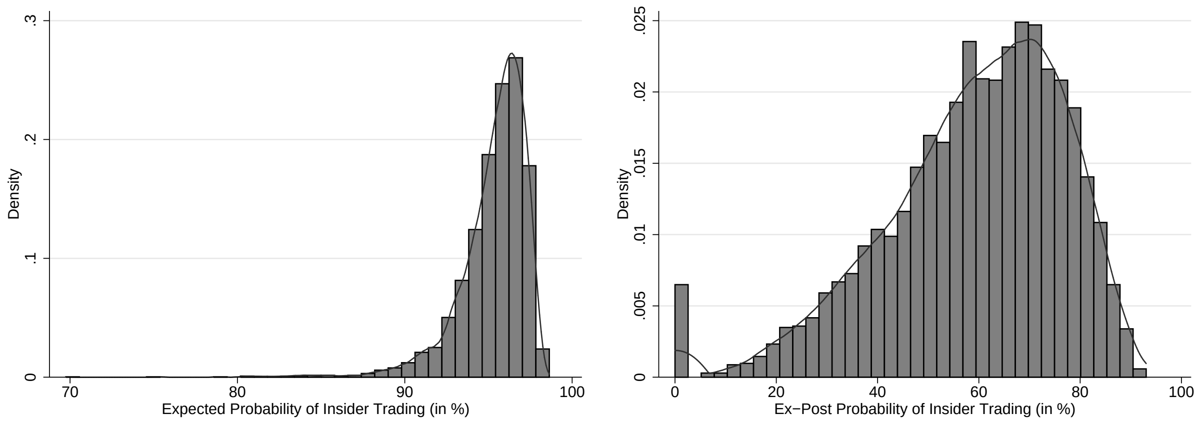
Variable	N	Mean	Max	St. Dev
C_j^{OP}	40082	0.00454	0.20610	0.01385
C_j^{MP}	38460	0.00723	0.25498	0.02087

which is expected by the zero profit bookmaker is found to have a mean of roughly 25, or an average probability of insider trading of around 95%. The density plot is shown in Figure 1a.

Figure 1: Density Plots

(a) Expected Probability of Insider Trading

(b) Ex-post Probability of Insider Trading



This seems very high but there are a few considerations to take into account. First, the insider in our model is assumed to know only the true winning probability of one horse k and does not know the winning probabilities of any other horses. When compared to the insider by (Shin, 1991, 1992, 1993), our insider does not know which horse will win but just has a better understanding of the true winning probabilities compared to the probabilities as quoted by the bookmaker. Second, the definition of insider trading is the amount of money being bet by insiders compared to outsiders. The fact that 95% of the total money is bet by insiders, does not mean that they place more bets, but could mean that they wager more money per bet. The bulk of the bets placed could still be made by outsiders, but the amount that insiders bet compared to outsiders is just much higher, i.e. for every Australian dollar bet by outsiders 25 is bet by insiders. Third, the assumption underlying the measure is that the bookmaker set his expected profit to zero, as would be the case under perfect competition. This is, of course, a very strict assumption to make and may not suit the reality all that well. A solution would be to assume that the bookmaker sets prices that guarantee him a certain level of profit. This level could be assumed to be equal to the profit the bookmaker would make in a market with no insiders. However, the prices that the bookmaker would set in this market are unobservable prices by definition. By looking at Equation 5, keeping everything else constant and assuming the bookmaker sets his prices to have a positive expected profit, it becomes obvious that we are estimating the degree of insider trading with an upward bias.

Finally, the measure that is generated is the bookmaker's expectation regarding insider trading. To have a zero expected profit, the bookmaker expects the probability of insider trading to be 95%. However, as mentioned in Section 4, we will use *ex-post* plunging information to get closer to the true probability of insider trading in the dataset. The weight that we use is based on the absolute size of the plunge: $PW = \max(MP - OP, 0) + \max(SP - MP, 0)$. We use PW to weight the extent of insider trading, q, per race. By defining a plunge as an upward movement of the price, we find that there have been 13852 plunges in the dataset, mainly occurring between MP and SP. The 13852 horses account for 30% of the total observations in the dataset. An additional benefit is that PW weights horses that have experienced a more severe plunge higher. However, a downside is that the estimate will be too high if part of the plunging is actually due to herding. The mean of the weighted extent of insider trading q_{PW} for the dataset is 2.20 and the average probability of insider trading a_{PW} is 59%. Figure 1b displays the distribution of the ex-post probability of insider trading.

We can see that there are around 611 races in which no plunges occur and hence no insider trading is observed. When insider trading does take place, the average probability is around 60%, although there is plenty of dispersion around the mean.

One final remark should be made with respect to the ex-post probability of insider trading. It should be noted that the value depends on the bookmaker's expectation of the degree of insiders compared to outsiders. If we allow for a higher than zero expected profit, his expectation will be lower and we would find values of insider trading closer to the 20%-30% range that Schnytzer et al. (2010) find.

5. Conclusions

In this paper we model a fixed odds horse betting market from a bookmaker's point of view under uncertainty. We rely upon a model by Makropoulou and Markellos (2011) and Schnytzer et al. (2010) which conceptualizes fixed odds betting markets as option markets. Starting from a profit function, we show that a bookmaker offers an implicit call option to insiders when setting prices. The insiders in this paper are assumed to know only the true winning probability of their horse, not the identity of the winning horse. Moreover, insiders only bet if their expected profit is positive. In the case in which both outsiders and insiders exist in the market, the bookmaker will set prices in such a way that his expected loss from dealing with insiders equals the expected gain from dealing with outsiders. When the bookmaker set his prices in this way, the latter will exhibit a favorite-longshot bias.

By allowing for betting in multiple time periods and making an assumption on how the insider money will arrive, the zero profit condition of Schnytzer et al. (2010) has to hold only for the race and not for each individual horse. From this model it becomes possible to measure the expectations of the bookmaker regarding the ratio of insider money to outsider money. Using Monte Carlo simulations, we generate the implicit option values as quoted by the bookmaker and find that a zero expected profit bookmaker expects 95% of the money bet to be placed by insiders. However, these estimates are biased in the sense that we do not allow the bookmaker to make a positive profit. By keeping his expected profit equal to zero, we overestimate the expected degree of insider trading. When we use ex-post plunging information, we conclude that the probability of insider trading in our dataset lies around 59%. Or to put it differently, for every Australian dollar bet by outsiders, the average amount bet by insiders is 2.20 dollars.

References

Dowie, J. (1976). On the efficiency and equity of betting markets. *Economica 43*(170), pp. 139–150.

Kahneman, D. and A. Tversky (1979). Prospect theory: An analysis of decision under risk. *Econometrica 42*(2), pp. 263–292.

Makropoulou, V. and R. N. Markellos (2011). Optimal price setting in fixed-odds betting markets under information uncertainty. *Scottish Journal of Political Economy 58*, pp. 519–536.

Quandt, R. (1986). Betting and equilibrium. *Quarterly Journal of Economics 101*, pp. 201–207.

Schnytzer, A., M. Lamers, and V. Makropoulou (2010). Measuring the extent of inside trading in horse betting markets. *The Journal of Gambling Business and Economics 4*(2), pp. 21–41.

Schnytzer, A. and Y. Shilony (2002). On the timing of inside trades in a betting market. *The European Journal of Finance 8*(2), pp. 176–186.

Shin, H. S. (1991). Optimal betting odds against insider traders. *The Economic Journal 101*, pp. 1179–1185.

Shin, H. S. (1992). Prices of state contingent claims with insider traders, and the favourite-longshot bias. *The Economic Journal 102*, pp. 426–435.

Shin, H. S. (1993). Measuring the incidence of insider trading in a market for state-contingent claims. *The Economic Journal 103*, pp.1141–1153.

Appendix

Proof

Suppose first that only outsiders exist in the market and that their demand is inelastic, i.e. $\frac{\partial W_n}{\partial OP_j} = 0$. A monopolistic bookmaker will set prices that maximize her expected profit:

$$\max E(\Pi) = W_n - \sum_{j=1}^{N} E\left[P_j(T)\right] \frac{w_j}{W_n} W_n \frac{1}{OP_j}$$

$$\frac{\partial E(\Pi)}{\partial OP_j} = 0 \Rightarrow -E\left[P_j(T)\right] \frac{w_j}{W_n} W_n \frac{-1}{OP_j^2} = 0$$

Since infinite prices do not make any sense, this leads to the conclusion that outsiders' demand should be elastic, i.e. $\frac{\partial W_n}{\partial OP_j} < 0$. In this case we have the following solution:

$$\frac{\partial E(\Pi)}{\partial OP_j} = 0 \Rightarrow \frac{\partial W_n}{\partial OP_j} - E\left[P_j(T)\right] \frac{w_j}{W_n} W_n \frac{-1}{OP_j^2} - E\left[P_j(T)\right] \frac{w_j}{W_n} \frac{1}{OP_j} \frac{\partial W_n}{\partial OP_j} = 0$$

Suppose now that insiders also exist in the market:

$$\max E(\Pi) = W_n \left(\sum_{j=1}^{N} OP_j \right) - \sum_{j=1}^{N} E\left[P_j(T)\right] \frac{w_j}{W_n} W_n \frac{1}{OP_j} - W_i \max_j C_j$$

$$\frac{\partial E(\Pi)}{\partial OP_j} = 0 \Rightarrow$$

$$\frac{\partial W_n}{\partial OP_j} - E\left[P_j(T)\right] \frac{w_j}{W_n} W_n \frac{-1}{OP_j^2} - E\left[P_j(T)\right] \frac{w_j}{W_n} \frac{1}{OP_j} \frac{\partial W_n}{\partial OP_j} - \frac{\partial W_i}{\partial OP_j} \max_j C_j - W_i \frac{\partial C_i}{\partial OP_j} = 0$$

In the above equation all terms except for the first one are positive. Specifically, $\frac{\partial W_i}{\partial OP_j}$ is negative given that insider demand drops as the price increases and $\frac{\partial C_i}{\partial OP_j}$ is negative since the option price decreases as the strike price increases (or equivalently as the level of moneyness decreases).