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WORKING PAPER

Published in Empirical Economics 43:2, 2012, 457-474 (a1).

What do we know about real exchange rate nonlinearities?

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September 2010

2010/667

D/2010/7012/38

What do we know about real exchange rate nonlinearities?

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Abstract Nonlinear modeling of adjustments to purchasing power parity has recently gained much attention. However, a huge body of the empirical literature applies ESTAR models and neglects the existence of other competing nonlinear models. Among these, the Markov Switching AR model has a strong substantiation in international finance. Our contribution to the literature is five-fold: First, we compare ESTAR and MSAR models from a unit root perspective. To this end, we propose a new unit root test against MSAR as the second contribution. Thirdly, we study the case of misspecified alternatives in a Monte Carlo setup with real world parameter constellations. The ESTAR unit root test is not indicative, while the MSAR unit test is robust. Fourthly, we consider the case of correctly specified alternatives and observe low power of the ESTAR but not for the MSAR unit root test. Fifthly, an empirical application to real exchange rates suggests that they may indeed be explained by Markov Switching dynamics rather than ESTAR.

JEL numbers: C12, C22, F31

Keywords Real exchange rates · unit root test · ESTAR · Markov Switching · PPP

Threshold models, most prominently the ESTAR model, turn out as the workhorse of empirical applications. In the ESTAR model, the real exchange rate behaves like a random walk when it is close to its equilibrium value. As soon as the price differences increase, a smooth transition process starts and arbitrage adjusts prices and thus the real exchange rate towards PPP. The rationale behind threshold models is the existence of trade barriers, such as transaction and shipping costs and tariffs (see the model by Dumas 1992). The thresholds may also reflect the sunk costs of international arbitrage and the observation that traders tend to wait until arbitrage opportunities are sufficiently large, before they exploit them (for a discussion of the rationales behind threshold models see Sarno 2005). Kilian and Taylor (2003) add another rationale for the use of ESTAR models: Internationally operating firms rely on the advice of financial analysts. The latter ones, however, will only agree on the existence of a misalignment in terms of PPP with an increasing deviation of the real exchange rate from equilibrium.

In contrast to ESTAR models, Markov switching autoregressive models (MSAR) have only been recently applied to real exchange rates (Kanas and Genius 2005, Kanas 2006, a simple Markov switching model is applied by Sarantis 1999). For a general survey on Markov Switching models see Hamilton and Raj (2002). The MSAR breaks up the relation between deviation from between regime switches and the deviation from PPP and describes long swings in the exchange rate, as documented by Engel and Hamilton (1990). These swings refer to switching from one regime to another, with each regime lasting for years. The main difference between ESTAR and MSAR models is that the regime switches in the MSAR framework are genuinely stochastic, whereas they are deterministic and depending on past deviations from PPP in the ESTAR model. Furthermore, within each regime the MSAR is conditionally linear, while the ESTAR model is nonlinear with a time-varying adjustment speed.

The idea behind both models is that real exchange rates may be driven by various forces: stabilizing forces drive the exchange rate back to PPP, whereas destabilizing forces cause ongoing deviations from equilibrium. Due to its characteristics, the ESTAR model is often - but not exclusively, see Kilian and Taylor (2003) - seen as a model of goods arbitrage. On the contrary, the Markov switching model is often regarded as a model of bubbles (Hall et al. 1997, 1999) and commonly related to frictions and heterogeneous agents on financial markets. De Grauwe and Grimaldi (2005), for instance, link the reasoning behind these views to heterogeneous agents in this market, i.e. international goods arbitrage and short-term speculation. Both views of real exchange rate behaviour have a strong substantiation in international finance research, but highlight differing aspects of the market. A major weakness of a huge body of the empirical literature on real exchange rates is that it routinely neglects the existence of other competing nonlinear models in general, and the existence of the MSAR model in particular. Although both models show some common characteristics, there have been only few attempts to incorporate both in a single model that contains elements of both, the deviation-dependent ESTAR model and the purely stochastic MSAR model. One notable exception to this tendency is Sarno and Valente (2006) who consider a generalized model in which both types of nonlinearities co-exist.

Our contribution to the literature is five-fold: First, we compare ESTAR and MSAR models for real exchange rates indirectly from a unit root perspective. To the best of our knowledge, we are the first to take this step. Alternatively, one may compare ES-

TAR and MSAR models directly, although this a very complicated task as both models are non-linear and non-nested, but such an approach would rule out the possibility of unit roots in real exchange rates. We shall not assume that PPP holds a priori by considering only stationary models. For the ESTAR model, we study the widely applied unit root test suggested by Kapetanios et al. (2003).

In order to conduct a fair comparison of the two non-linear models, we propose a new unit root test against a MSAR model with state-dependent autoregressive parameters. This is our second contribution. All other existing unit root tests against MSAR (Hall et al. 1999, Kanas 2006 and Kanas and Genius 2005) would not allow a fair comparison as they permit too many additional features like state-dependent means and error variances. Our comparison is done by means of simulating the empirical properties of the two tests under real world parameter constellations and by means of an empirical application.

Thirdly, we explicitly study the case of misspecified alternatives in the following sense: We analyze the properties of the ESTAR (MSAR) test, when the true data generating process is an MSAR (ESTAR). This analysis enables us to draw conclusions about the probability of confusing the two models in case of a rejection. Our simulation results show that the ESTAR test is prone to reject the null hypothesis if the true model is an MSAR. On the contrary, the MSAR test appears to be robust against ESTAR dynamics. We emphasize that a rejection in favor of the ESTAR model should be taken with a cautionary note, while a rejection towards the MSAR model is less problematic. An explanation for the different behaviour of the two tests is the way they deal with the problem of unidentified parameters under the null hypothesis: the ESTAR test is based on a Taylor approximation which may capture many types of non-linearity. Our simulation results show that the MSAR model indeed reproduces a type of non-linearity which can be captured by this kind of Taylor approximation. On the contrary, the test for MSAR builds upon a computationally intensive grid search method which is less affected by a different type of non-linearity.

Fourthly, we also consider the power properties of the two tests under correctly specified alternatives. Our results for the ESTAR test are in line with the literature on low power of standard unit root tests like Sarno and Taylor (2002) in the sense that this fact also holds for the unit root test against non-linear ESTAR. We provide some statistical explanations for the the performance of the ESTAR test. In contrast, the results for the power of the MSAR test are much more promising. This test enables the practitioner to detect Markov Switching dynamics with much greater chance if the true data generating process is of MSAR-type.

Fifthly, we apply the unit root tests to real exchange rate data of G7 countries against the United States. Our results strongly suggest that the PPP adjustment mechanism is driven by Markov Switching, while no evidence is found in favor of ESTAR.

The paper is organized as follows. Section 2 introduces the ESTAR model and the considered unit root test against ESTAR models in more detail. In section 3 we propose a new unit root test against a MSAR process. Section 4 contains our Monte Carlo study, section 5 applies the unit root tests introduced above on six real exchange rates. Section 6 summarizes and concludes.

2 Unit root test against ESTAR

In this section we review the ESTAR model (see Teräsvirta 1994) and a popular test for a unit root against the ESTAR alternative (Kapetanios et al. 2003). The particular specification of the ESTAR model we consider, as used in several studies like Michael et al. (1997), Sarantis (1999), Taylor et al. (2001) and more recently, Rapach and Wohar (2006), is given by

$$y_t = \phi_t y_{t-1} + \varepsilon_t \tag{1}$$

where ε_t is assumed to be a zero mean white noise process and ϕ_t is a time-varying autoregressive parameter. Its dynamics are determined by the smooth transition function $\exp\{-\gamma(y_{t-1} - c)^2\}$ which is the source of nonlinearity in this model. The autoregressive parameter ϕ_t is bounded between zero and one and depends on a transition variable y_{t-1}, a smoothness parameter $\gamma > 0$ and a location parameter $c \in R$.

The ESTAR model behaves locally like a random walk if the lagged real exchange rate (y_{t-1}) is exactly equal to c, since the autoregressive parameter ϕ_t equals one in this case. If y_{t-1} departs from c, the process is stationary and therefore mean-reverting. Despite the local non-stationarity of y_t, the ESTAR model is globally stationary, see Kapetanios et al. (2003) for a proof. In the exponential smooth transition model, the degree of mean-reversion depends on the squared difference between y_{t-1} and c. In economic terms, if the real exchange rate was quite close to its long run equilibrium value in the last period then it behaves like a random walk. Furthermore, there are driving forces like arbitrage that lead to mean-reversion if the real exchange rate departs from its long run equilibrium. Moreover, arbitrage may not be profitable if departures are small as arbitrageurs face transaction costs. Therefore, the degree of mean-reversion is small as well. The parameter γ controls the shape of the exponential function and therefore influences the sensitivity of ϕ_t towards the deviation of y_{t-1} from c.

Kapetanios et al. (2003) suggest a modification of the Dickey-Fuller test for testing the random walk hypothesis against ESTAR in order to increase the power of the standard Dickey-Fuller test. Their test is based on the following test regression which is very similar to the original Dickey-Fuller regression:

$$\Delta y_t = \psi y_{t-1}^3 + u_t \tag{2}$$

with $\Delta y_t = y_t - y_{t-1}$. The nonlinear smooth transition function is approximated by the cubic power y_{t-1}. In this regression, the pair of hypotheses is now $H_0 : \psi = 0$ (unit root) versus $H_1 : \psi < 0$ (stationary ESTAR). Kapetanios et al. (2003) suggest a Dickey-Fuller-type test for this hypothesis given by

$$t_{\text{KSS}} \equiv \frac{\widehat{\psi}}{\sqrt{\operatorname{var}(\widehat{\psi})}} = \frac{\sum_{t=1}^{T} y_{t-1}^3 \Delta y_t}{\sqrt{\widehat{\sigma}^2 \sum_{t=1}^{T} y_{t-1}^6}}, \tag{3}$$

where $\widehat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}(\Delta y_t - \widehat{\psi} y_{t-1}^3)^2$ is the usual estimator of the error variance. The limiting distribution of the test statistic t_{KSS} is non-standard and asymptotic critical values can be found in Kapetanios et al. (2003). Still, we also provide small sample critical values in section 4.1. Deterministic components as a constant or a constant and a linear trend are removed in a first step, i.e. one applies the test to previously de-meaned or de-trended data. For further details, see Kapetanios et al. (2003).

3 Unit root test against Markov Switching

We consider a MSAR model which has similar properties to the ESTAR model presented in the previous section. For an extensive discussion of MSAR models with error correction see Psaradakis et al. (2001). The main difference between the ESTAR and the MSAR model is, at least from a statistical viewpoint, the regime switching mechanism. Within the ESTAR model, regime switches are determined by past observable values of y_{t-1}, while the MSAR model is based on an unobservable stochastic Markov process (s_t). Our specification of the MSAR model is given by

$$y_t = \phi_{s_t} y_{t-1} + \varepsilon_t \ , \tag{4}$$

where the autoregressive parameter ϕ_{s_t} depends on a first-order Markov chain (s_t) that takes the values one or two. Furthermore, it is assumed that s_t is irreducible and aperiodic, i.e., it is characterized by the transition probability matrix

$$\begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{bmatrix}$$

with $p_{ii} = P(s_t = i | s_{t-1} = i)$ for $i = 1, 2$, being the probability that the process is in regime i in period t, given that it was in the same regime in the previous period.

Within the MSAR framework, Francq and Zakoïan (2001) establish a necessary and sufficient condition for global stationarity of the MSAR model. It is given by the following two inequalities $c_1 < 1$ and $c_2 < 2$, where

$$\begin{aligned} c_1 &= p_{11}\phi_1^2 + p_{22}\phi_2^2 + (1 - p_{11} - p_{22})\phi_1^2\phi_2^2 \ , \\ c_2 &= p_{11}\phi_1^2 + p_{22}\phi_2^2 \ . \end{aligned}$$

Similar to the ESTAR model, the MSAR model can also be locally non-stationary while maintaining global stationarity. Suppose that one of the autoregressive parameters is equal to one, while the other autoregressive parameter satisfies the condition for local stationarity, i.e., $\phi_1 = 1$ and $0 \leq \phi_2 < 1$. In this case, global stationarity is still given regardless of what values p_{11} and p_{22} are.

As no unit root test against this specific MSAR model exists, we newly develop such a test and make use of the results obtained by Francq and Zakoïan (2001). The non-linear MSAR model becomes a random walk when $\phi_1 = \phi_2 = 1$. In this case, the condition for c_1 is violated. Like in the other applications of Markov Switching models, testing the linearity hypothesis is complicated by the presence of unidentified parameters (so-called Davies (1987) problem) in this model as well. Here, the probabilities p_{11} and p_{22} are unidentified parameters when testing the null hypothesis $H_0 : \phi_1 = \phi_2 = 1$. The treatment of unidentified parameters follows Hansen (1996) and Garcia (1998) which is explained below in detail.

Our test statistic is constructed similar to the one suggested in Caner and Hansen (2001) who construct a unit root test against a Threshold Autoregressive (TAR) model and also face the problem of unidentified parameters under H_0. For convenience, we re-write the specification of the MSAR model we use in a Dickey-Fuller style i.e.,

$$\Delta y_t = \psi_{s_t} y_{t-1} + \varepsilon_t \ ,$$

where $\psi_{s_t} = \phi_{s_t} - 1$. A one-sided Wald test statistic for $H_0 : \psi_1 = \psi_2 = 0$ (unit root) against $H_1 : \psi_1 < 0$ or $\psi_2 < 0$ (stationary MSAR) is given by

$$R = 1\left(\widehat{\psi}_1 < 0\right) t^2_{\psi_1=0} + 1\left(\widehat{\psi}_2 < 0\right) t^2_{\psi_2=0} ,$$

where $t_{\psi_i=0}$ denotes the conventional t-statistic for the null hypothesis that ψ_i equals zero. $1(\cdot)$ denotes the indicator function. Parameters are estimated jointly via maximum likelihood. As mentioned above, the transition probabilities p_{11} and p_{22} are unidentified under the validity of the null hypothesis. In order to tackle this problem, we follow Garcia (1998) and consider a sequence of test statistics $R(p_{11}, p_{22})$ where the transition probabilities take values of a bounded grid $\Pi = (0,1) \times (0,1)$. This means that we fix p_{11} and p_{22} at certain values and compute the test statistics R. We then proceed by using other values for p_{11} and p_{22} and so on until we consider all possible values in Π. As a next step, we consider the supremum of the random sequence of test statistics $R(p_{11}, p_{22})$, i.e.,

$$R^* = \sup_{p_{11}, p_{22} \in \Pi} R(p_{11}, p_{22}) .$$

In our Monte Carlo study we provide critical values for the R^* statistic. Regarding the deterministic terms, we follow the procedure suggested by Kapetanios et al. (2003), see our section 2.1. This means that data is de-meaned or de-trended before the unit root test is applied in order to cope with non-zero means or linear trends.

4 Monte Carlo study

4.1 General approach

The following Monte Carlo study is about the empirical power of the Dickey-Fuller test, the unit root test against ESTAR by Kapetanios et al. (2003) and the new unit root test against MSAR under situations which are realistic in practice when analyzing real exchange rates.

In a related study, Choi and Moh (2007) consider the behavior of various unit root tests against different non-linear alternatives. Among these tests is the Kapetanios et al. (2003) test, which are considered in this paper as well. Choi and Moh (2007) find that all unit root tests have power against various non-linear alternatives. Whether a test has power does not depend on the correct specification of the alternative but on the distance to the null of a unit root. However, Choi and Moh (2007) consider idealized parameter constellations and therefore obtain a satisfying power for each test. They do not consider real world parameter constellations which are the focus of this paper.

Our Monte Carlo study addresses the question whether this might be due to a lack of power of the developed tests under realistic situations rather than to a correct decision of the test by not rejecting the unit root hypothesis. We consider whether the unit root test against ESTAR has also power against Markov Switching processes and vice versa.

In general, unit root tests have good power properties in Monte Carlo studies relying on parameter constellations which do not appear in the analysis of real exchange rates. It is quite common to simulate processes with $N(0,1)$ innovations, but we account

Table 1 Parameter estimation results

	DEM/USD
ESTAR	$\gamma = 0.264$, $c = -0.007$, $\sigma = 0.035$
MSAR	$[\psi_1, \psi_2] = [-0.074, 0.007]$, $[p_{11}, p_{22}] = [0.917, 0.945]$, $\sigma = 0.028$
	$c_1 = 0.995$, $c_2 = 1.744$
	GBP/USD
ESTAR	$\gamma = 0.449$, $c = 0.150$, $\sigma = 0.033$
MSAR	$[\psi_1, \psi_2] = [-0.310, 0.028]$, $[p_{11}, p_{22}] = [0.300, 0.860]$, $\sigma = 0.030$
	$c_1 = 0.971$, $c_2 = 1.052$
	JPY/USD
ESTAR	$\gamma = 0.165$, $c = 0.515$, $\sigma = 0.033$
MSAR	$[\psi_1, \psi_2] = [-0.233, 0.001]$ $[p_{11}, p_{22}] = [0.235, 0.953]$, $\sigma = 0.030$
	$c_1 = 0.982$, $c_2 = 1.093$

Remarks: Estimated parameter values for DEM/USD, GBP/USD and JPY/USD are taken from Rapach and Wohar (2006) for ESTAR models. Markov Switching models are estimated via conditional maximum likelihood in Gauss.

for small standard deviations that are often found empirically, see Rapach and Wohar (2006). Another issue is that the location parameter c in ESTAR models is usually assumed to be equal to zero which is not correct in many practical situations either. Kruse (2010) shows that this assumption may lead to a substantial loss in power if it is wrong. In order to obtain realistic parameter settings, estimations are carried out using data from the International Financial Statistics database from 1973:02 to 1996:12 for the DEM/USD, FRF/USD, GBP/USD and JPY/USD real exchange rates as done in Rapach and Wohar (2006) for ESTAR models. Their reported estimates are very close to those reported in Taylor et al. (2001). Due to the fact that the estimation results for the DEM/USD and the FRF/USD are quite similar, we do not consider the latter currency in our study. Since Markov Switching models are neither considered in Rapach and Wohar (2006) nor in Taylor et al. (2001), we fit the Markov Switching model described in section 3 to the same data set in order to achieve the highest degree of comparability.

The exact parameter constellations are given in Table 1 for the three considered pairs of currencies (JPY/USD, DEM/USD, GBP/USD). In each case we use first–order autoregressive models. An application of standard diagnostic tests[5] suggest that these models are correctly specified. Starting with the ESTAR specification, we observe that the smoothness parameter γ takes quite different values ranging from 0.165 (JPY/USD) to 0.449 (GBP/USD). Note that it is difficult to distinguish an ESTAR process that exhibits a small value of γ from a unit root process as $y_t = y_{t-1} + \varepsilon_t$ for $\gamma \to 0$. Therefore, the expected power is low for the JPY/USD parameter constellation and somewhat higher for the GBP/USD parameters. However, one should also bear in mind that small changes of γ near zero do change the behavior of the process significantly. Therefore, we expect to find clear differences in the behavior of the tests for the different parameter constellations. The location parameter c varies also across currencies, while the estimated standard deviation of the error term σ is very low and far away

[5] Available upon request from the authors.

Table 2 Small sample critical values

de-meaning							
$T = 250$	t_{DF}	t_{KSS}	R^*	$T = 500$	t_{DF}	t_{KSS}	R^*
1%	-3.46	-3.46	27.42	1%	-3.44	-3.51	29.26
5%	-2.88	-2.91	18.65	5%	-2.87	-2.94	20.03
10%	-2.57	-2.63	15.02	10%	-2.57	-2.67	16.20
de-trending							
$T = 250$	t_{DF}	t_{KSS}	R^*	$T = 500$	t_{DF}	t_{KSS}	R^*
1%	-3.99	-3.99	31.53	1%	-3.98	-4.01	32.85
5%	-3.43	-3.49	22.62	5%	-3.42	-3.40	22.98
10%	-3.13	-3.12	18.40	10%	-3.13	-3.12	19.00

Remarks: t_{DF} labels the Dickey-Fuller test, t_{KSS} is the Kapetanios et al. (2003) test against ESTAR and R^* is the unit root test against MSAR.

from unity for each currency. The location parameter c is significantly different from zero in each case although it seems to be rather small for some currencies.

Regarding the Markov Switching model, we always find a stationary regime characterized by $\widehat{\psi}_1 < 0$ and a second regime with an autoregressive parameter which is very close to zero. The latter implies a random walk regime. The MSAR model is globally stationary for all pairs of currencies because the two conditions (c_1 and c_2 in Table 1) derived in Francq and Zakoïan (2001) are not violated, see Table 1. Therefore, the behavior of real exchange rates can be reproduced. The state probabilities are also close to one for the DEM/USD exchange rate. Consequently, the estimated MSAR model for the DEM/USD exchange rate is close to a random walk in both regimes which means that the expected power of the Markov Switching unit root test is low for this currency pair. This can also be seen by considering the values for c_1 and c_2. They imply that we can expect that the Markov Switching test has higher power when the estimated model for the British Pound is considered instead of the one for the German Mark. The estimated standard deviation is similar to that of the ESTAR models and thus again far away from unity for each currency.

We simulate 2,000 replications of each process and apply them to the standard Dickey-Fuller unit root test (denoted by DF) as a benchmark test, the unit root test versus ESTAR suggested by Kapetanios et al. (denoted by KSS) and the Markov Switching test proposed in section 3. The power is considered at the 5% level by using size adjusted small sample critical values obtained from 20,000 replications for sample sizes of $T = 250$ and $T = 500$ which corresponds approximately to 20 and 40 years of monthly data, respectively. Note, that we simulate processes of length $T + 100$ and delete the first hundred observations in order to reduce the effect of the starting value. We use simulated small sample critical values for all tests and not just for the Markov Switching test in order to obtain comparability of the results. The size-adjusted critical values for all tests are given in Table 2.

We conduct some size experiments for the proposed unit root test against MSAR at the nominal significance levels of one, five and ten percent. The data generating process is

Table 3 Size experiments for the R^* test statistic

de-meaning							
$T = 250$	Exp1	Exp2	Exp3	$T = 500$	Exp1	Exp2	Exp3
$\alpha = 1\%$	0.7	0.9	1.2	$\alpha = 1\%$	0.8	0.9	1.1
$\alpha = 5\%$	4.8	5.3	5.5	$\alpha = 5\%$	4.8	5.1	5.3
$\alpha = 10\%$	9.5	10.2	10.2	$\alpha = 10\%$	9.6	10.3	10.4
de-trending							
$T = 250$	Exp1	Exp2	Exp3	$T = 500$	Exp1	Exp2	Exp3
$\alpha = 1\%$	0.6	1.1	1.4	$\alpha = 1\%$	0.8	1.2	1.3
$\alpha = 5\%$	4.7	5.2	5.6	$\alpha = 5\%$	4.9	4.9	5.4
$\alpha = 10\%$	9.7	10.4	10.1	$\alpha = 10\%$	9.6	9.8	10.3

Remarks: R^* is the unit root test against MSAR. The DGP is given by $y_t = y_{t-1} + u_t$ with $u_t = \rho u_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim N(0,1)$ and $\rho = \{-0.5, 0, 0.5\}$ in Exp1, Exp2 and Exp3, respectively.

given by $y_t = y_{t-1} + u_t$ with $u_t = \rho u_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim N(0,1)$. The three different experiments (Exp1, Exp2 and Exp3) cover the cases $\rho = \{-0.5, 0, 0.5\}$. Results are reported in Table 3. The results show that the new test has accurate nominal size and that the distortions are small.

Regarding our power experiments, we consider both, correctly and misspecified models. To this end, we simulate MSAR and ESTAR models corresponding to the parameters we found for real data, see Table 1. It should be noted, that the alternative is misspecified for the standard Dickey-Fuller test for all considered models.

4.2 Main results

In this subsection, we discuss the power results for the non-linear unit root tests. Table 4 gives the power for a sample size of $T = 250$ observations. We consider all non-linear unit root tests after de-meaning as well as after de-trending as both type of deterministic terms can be reasonable for real exchange rate data. Note, that we include a constant or a constant and a linear trend term in the Dickey-Fuller test regression. However, the results are rather similar in both cases. The ESTAR test has no remarkable power against any of our models. Interestingly enough, the standard Dickey-Fuller test has higher power against ESTAR than the ESTAR test for the JPY/USD and the DEM/USD in the de-trended case.

However, the power of all tests is extremely low when the true DGP is an ESTAR model in any case. This also holds for the Markov Switching test. When the true DGP is ESTAR, the Markov Switching test proves to be conservative. In opposition to the ESTAR test, this is a rather convincing test property as a non-rejection of the test is the desired property for a correct model selection. Unfortunately, the ESTAR test has power against the Markov Switching model. In each case it is at least in the same region as the power against ESTAR models. For the GBP/USD it is far higher for the Markov Switching alternative than for the ESTAR alternative. Only for the DEM/USD exchange rate, the power of the tests is quite low. This is expected as the

Table 4 Empirical power, $T = 250$

de-meaning	t_{DF}	t_{KSS}	R^*	de-trending	t_{DF}	t_{KSS}	R^*
JPY–ESTAR	10.5	10.1	2.6	JPY–ESTAR	8.2	7.0	9.2
JPY–MSAR	7.6	10.3	39.5	JPY–MSAR	6.1	6.7	37.7
DEM–ESTAR	11.2	12.9	2.7	DEM–ESTAR	9.1	7.8	9.3
DEM–MSAR	12.0	8.9	16.6	DEM–MSAR	7.5	5.3	13.4
GBP–ESTAR	14.3	15.7	4.8	GBP–ESTAR	10.2	10.5	10.5
GBP–MSAR	20.1	35.8	87.1	GBP–MSAR	11.5	24.5	79.7

Remarks: t_{DF} labels the Dickey-Fuller test, t_{KSS} is the Kapetanios et al. (2003) test against ESTAR and R^* is the unit root test against MSAR. JPY-ESTAR is the simulated ESTAR model with parameters according to JPY/USD real exchange rate, see Table 1. The other entries are analogous.

Table 5 Empirical power, $T = 500$

de-meaning	t_{DF}	t_{KSS}	R^*	de-trending	t_{DF}	t_{KSS}	R^*
JPY–ESTAR	16.1	18.8	2.0	JPY–ESTAR	11.4	10.8	20.1
JPY–MSAR	16.4	19.6	74.6	JPY–MSAR	9.3	11.2	70.9
DEM–ESTAR	22.3	29.7	2.2	DEM–ESTAR	13.4	14.2	21.7
DEM–MSAR	22.8	11.6	42.3	DEM–MSAR	13.1	7.5	30.8
GBP–ESTAR	30.5	49.1	1.9	GBP–ESTAR	22.4	23.9	23.7
GBP–MSAR	50.1	63.5	92.3	GBP–MSAR	31.6	49.1	93.9

Remarks: t_{DF} labels the Dickey-Fuller test, t_{KSS} is the Kapetanios et al. (2003) test against ESTAR and R^* is the unit root test against MSAR. JPY-ESTAR is the simulated ESTAR model with parameters according to JPY/USD real exchange rate, see Table 1. The other entries are analogous.

Markov Switching model is close to a unit root in this case. The Markov Switching test has satisfying power properties. Its power is quite high against a Markov Switching DGP except for the DEM/USD exchange rate where a low power is expected because of the near unit root structure of the DGP. On the other hand it has low power against ESTAR models. The DF test has similar power properties to the ESTAR test.

Similar results can be observed for $T = 500$ (see Table 5). As expected, the power is generally higher compared to $T = 250$ but the results are qualitatively the same as before. The results for the de-trending case are qualitatively similar to those of the de-meaning case although all tests have less power under de-trending. This is expected as another deterministic parameter has to be fitted under de-trending. Unfortunately, the Markov Switching test is no longer conservative under de-trending when the true DGP is ESTAR. However, its power is still low and within the range of the ESTAR test.

Altogether, we can say that there is no ESTAR test which dominates in terms of power. It is argued, however, that the ESTAR test has rather poor power against ESTAR with our parameter constellations which are realistic for real exchange rates. In some constellations the power of the ESTAR test is even better for the Markov Switching alternative. As a result, by not rejecting the Null, this ESTAR test do not allow us to conclude that the null hypothesis unit root is correct and therefore we cannot reject the purchasing power parity hypothesis. However, when rejecting the Null we cannot conclude that the true model is ESTAR either.

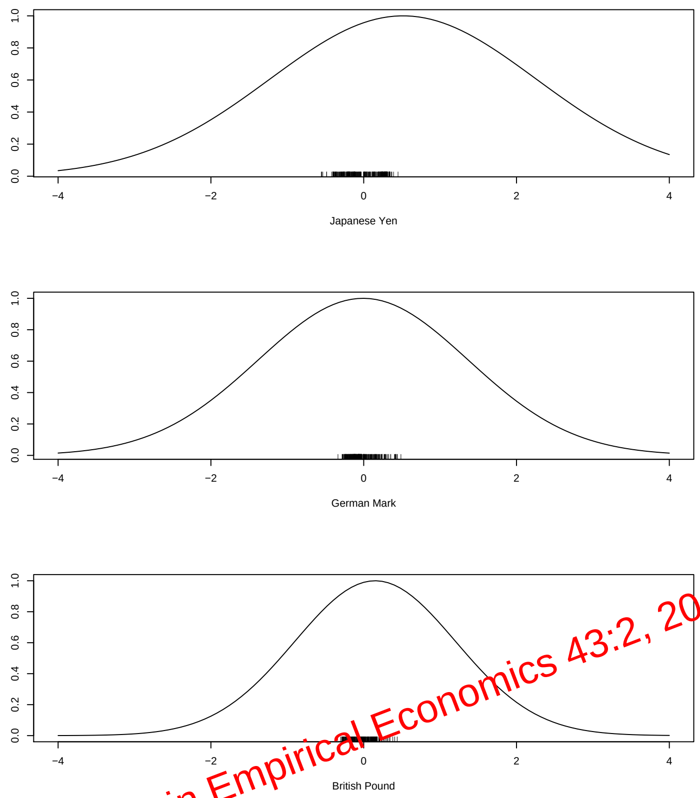

Fig. 1 Estimated transition functions and data points.

4.3 Discussion

A natural question which arises out of the results in section 4.2 is why especially the ESTAR test has so poor power properties. Figure 1 sheds light on this problem. In these graphs, the transition function of each estimated ESTAR process based on real data and parameters reported in Table 1 are depicted together with corresponding data points. Almost all data points are in the region where the transition function is close to its maximum. There are no data points at the tails of the function. Close to the maximum of the transition function the process behaves similarly to a unit root process or a highly persistent local-to-unity autoregressive process. The mean reverting property of the non-linear time series model has a strong effect only in the outer regimes away from the equilibrium. Therefore, for the vast majorities of data points, the process behaves like a linear unit root process. This makes it hard or almost impossible for the tests to detect the non-linear mean reverting behavior of the DGP, given the sample sizes we consider.

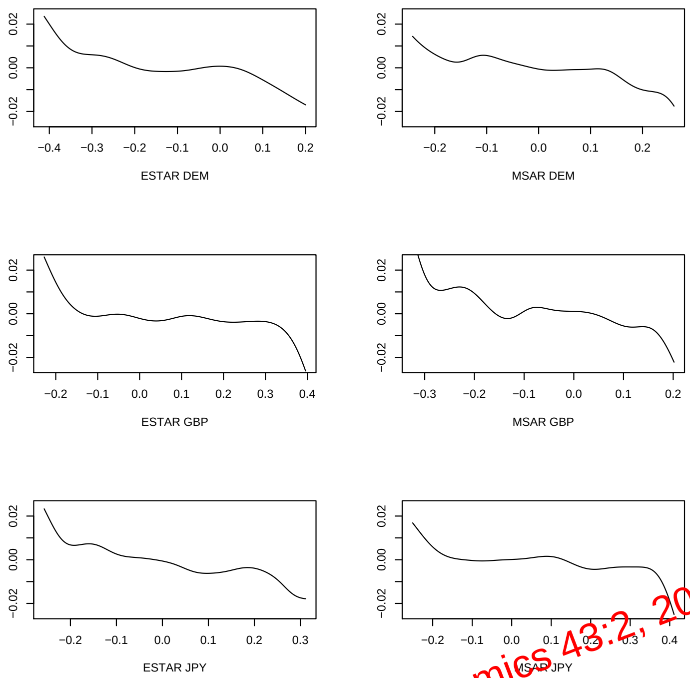

Fig. 2 Nadaraya-Watson estimates for the functional relationship of ESTAR and MSAR processes.

In addition to this, our simulation study shows that the ESTAR test has similar power properties against ESTAR as against MSAR models. To intuitively explain this finding, we generate plots of y_{t-1} against the first difference $\Delta y_t = y_t - y_{t-1}$ for ESTAR and MSAR simulated time series generated from our parameter constellations and estimate the functional relationship between Δy_t and y_{t-1} in a non-parametric way by using the Naradaya-Watson estimator, see Figure 2. If the ESTAR effect is strong, the estimated curve should be near a cubic function. If the time series process exhibits a unit root, it is flat and identical to zero. As we can see, the cubic behavior is clearly pronounced for the simulated DEM/USD and GBP/USD real exchange rate and less pronounced for the simulated JPY/USD real exchange rate which is in line with our parameter settings. Moreover, it is that both functions, the ESTAR and the MSAR function, are rather similar and quite close to each other. The MSAR process generates also a cubic shape for this function which is similar to the ESTAR model. Loosely speaking, the idea of the Kapetanios et al. test is to check whether this function has a cubic shape or not, it detects the cubic form also for the MSAR process. As both functions are close to each other, the power is similar for both models.

This shows that the present tests are not able to detect ESTAR non-linearities as they are found in real exchange rates. Although the tests have convincing properties in many situations, they prove to have a lack of power under the very special parameter conditions which can be found in real exchange rates. It is rather difficult to draw any conclusion from the outcome of an ESTAR test under these conditions. Neither does a non-rejection of the Null mean that the true DGP which drives real exchange rates, is a linear unit root process nor does a rejection of the Null mean that the true DGP is actually an ESTAR process.

Finally, we discuss our results in the light of the generalized nonlinear model used in Sarno and Valente (2006). The authors apply a hybrid model which exhibits a complex nonlinear dependence structure: the switch between regimes is driven by a Markov switching process and not only autoregressive parameters are regime-dependent but also intercepts and variances are. Moreover, within each regime, an ESTAR-type of nonlinear adjustment takes place. This model is able to take many more features of the data into account than the restricted and much simpler models we consider in this paper. As the ESTAR process does not influence the regime switches, as opposed to the Markov Switching process, the nonlinear dynamics are mainly driven by the latter one. Therefore, it may be expected that the nonlinear unit root tests behave not too differently from what is observed for MSAR processes. Our conjecture is that the MSAR unit root test has satisfying power against data generated from this general nonlinear process, while the ESTAR unit root test has less power.

5 Application

This section applies the unit root tests studied in the Monte Carlo simulations to the G7 exchange rates. Thus, we examine non-linearities in the real exchange rates of the US Dollar against the Canadian Dollar (CAD), Swiss Franc (CHF), German Mark (DEM), British Pound (GBP), Italian Lira (ITL) and Japanese Yen (JPY). Data is taken from the IMF International Financial Statistics database. Price levels are measured by the consumer price index (CPI). The sample covers the post-Bretton Woods period from 1973.01 to the Euro introduction 1998.12 implying a sample size of $T = 312$. This data set is chosen to achieve comparability to other studies and has the advantage that potential structural breaks that might have occurred due to the introduction of the Euro are excluded and thus not biasing our analysis. All time series seem to be persistent and locally trending, see Figure 3. The estimated partial autocorrelation functions (graphs are available upon request) indicate that all time series can be modelled by first-order processes.

Next, we apply the standard Dickey-Fuller regression including a constant, i.e.,

$$\Delta y_t = c + \rho y_{t-1} + u_t$$

and test for linearity in the residuals $\widehat{u}_t$. Linearity is tested by (i) the neural network test proposed by Lee et al. (1993), (ii) Ramsey's RESET test (1969) and (iii) the BDS test for independence by Broock et al. (1996). These tests assume stationarity which is crucial when applied to real exchange rates themselves but not when applied to residuals. Results can be found in Table 6. They show that the linearity hypothesis has to be rejected in many cases. This also means that the Dickey-Fuller test regression neglects important non-linearities and is therefore misspecified for testing PPP.

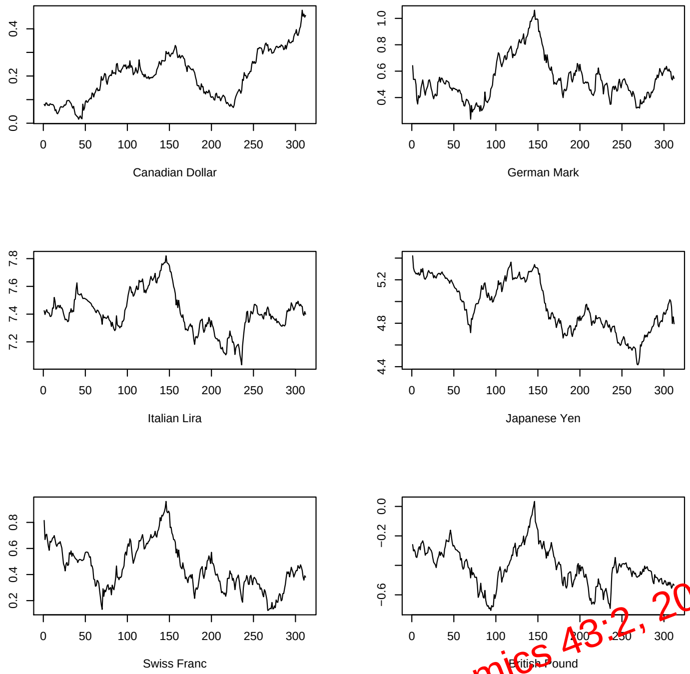

Fig. 3 Logarithm of CPI-based real exchange rates against US Dollar.

Table 6 Linearity and unit root test results

Test	CAD	CHF	DEM	GBP	ITL	JPY
Linearity tests (p-values)						
NN	0.007	0.008	0.028	0.535	0.745	0.038
RESET(2)	0.885	0.204	0.118	0.185	0.243	0.038
RESET(3)	0.727	0.356	0.053	0.147	0.288	0.228
RESET(4)	NA	0.344	0.025	0.065	0.014	0.005
BDS(2)	0.135	0.006	0.054	0.008	0.012	0.408
BDS(3)	0.033	0.016	0.178	0.013	0.001	0.216
BDS(4)	0.045	0.007	0.270	0.009	0.000	0.210
Unit root tests (test statistics)						
t_{DF}	-0.10	-2.47	-1.92	-2.17	-1.84	-1.94
t_{KSS}	0.08	-2.54	-1.36	-2.46	-2.10	-2.45
R^*	2.47	15.83	14.55	29.24	18.36	59.45

Notes: NN denotes the neural network test statistic by Lee et al. (1993). Hochberg's improved Bonferroni bound is used with one hundred draws to obtain reliable p-values for the neural network test, see Lee et al. (1993). RESET(m) is Ramsey's (1969) test statistic with terms up to power $m+1$. BDS(n) is the Broock et al. (1996) test statistic for independence with embedding dimension n. For unit root tests, see Table 2.

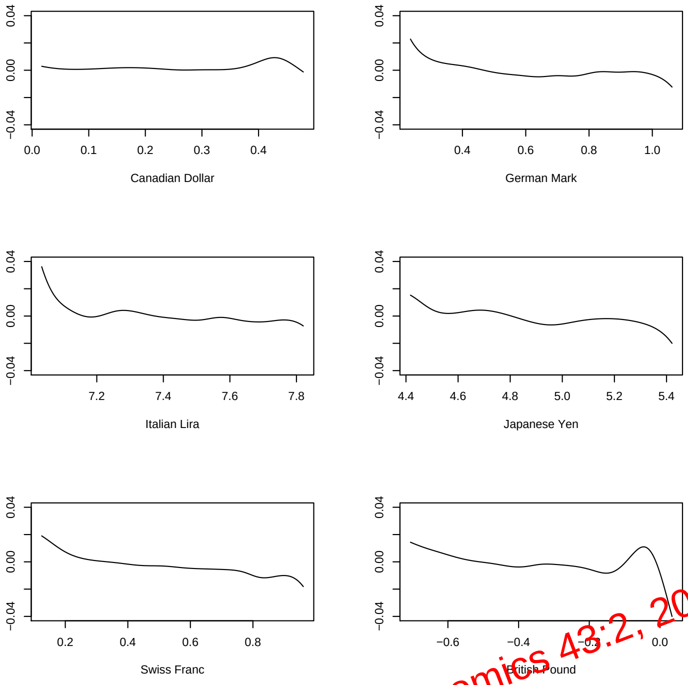

Fig. 4 Nadaraya-Watson estimates for the functional relationship.

Moreover, we investigate the non-linearities by estimating the functional relationship between Δy_t and y_{t-1} in a non-parametric way by employing the Nadaraya-Watson estimator. Figure 4 shows these estimates. Only for the CAD/USD the estimated curve is very flat suggesting that there is no relationship between Δy_t and y_{t-1} which hints at a unit root. All other plots suggest mean-reversion and the functional relationship appears to be non-linear.

As the last step, we apply the previously analyzed unit root tests to the six real exchange rate series in order to test empirically for the validity of PPP. Since all time series appear to be first-order processes, we do not include any lagged differences in the test regressions. The Dickey-Fuller regression contains a constant, while de-meaned data is used for the non-linear unit root tests.

The resulting test statistics are reported in the lower panel of Table 6. Neither the linear unit root test by Dickey and Fuller (1979) nor the non-linear unit root test by Kapetanios et al. (2003) are able to reject the null hypothesis of a unit root at the ten percent level of significance. These results contradict the validity of PPP since there is no mean-reversion when a unit root is present. On the contrary, the new test against MSAR rejects the Null in favor of a stationary MSAR model in four out of six cases. When having the outcomes of our preliminary analysis in mind (see Figure 4), it is not surprising that the unit root hypothesis cannot be rejected in the case of CAD/USD. In addition, we note that the R^* statistic for the DEM/USD is quite close to the critical value of 15.02 which means that the test decision is borderline. Due to the fact that the Markov Switching unit root test does not have substantial power against ESTAR, especially in the case of de-meaned data, it is legitimate to conclude that there is no evidence for ESTAR dynamics in the data. Markov Switching seems to be a more plausible model for explaining the dynamics of real exchange rates.

6 Conclusions

This paper provides a thorough comparison of two competing nonlinear models for deviations from PPP. Our focus is on the ESTAR model which is the prominent workhorse in empirical applications and a Markov Switching model. The latter model has been applied to real exchange rates only recently and gained less attention in the related literature so far. Both models of real exchange rate behavior have a strong foundation in international finance, but they are based on different regime transition processes. While the transition is stochastic and independent from the deviation from PPP for the MSAR model, it is deterministic and depending on the deviations from PPP for the ESTAR model.

Our comparison is conducted from an indirect unit root perspective as we shall not assume stationarity a priori. This framework enables us compare the two models with frequently applied unit root tests for real exchange rates. As there is no simple unit root test against MSAR which is available in the form needed here, we propose a new test that builds upon inference techniques developed by Hansen (1996) and refined by Garcia (1998). This brings us to the core of this research, which is to compare ESTAR and MSAR tests in a broad simulation study showing that the ESTAR test has low power, whereas the MSAR test seems to be much more powerful. Moreover, we observe that ESTAR tests are not robust with respect to Markov Switching dynamics, i.e. misspecification of the alternative, while the opposite holds for our newly developed test. This means that a rejection of the unit root hypothesis by an ESTAR test, if any occurs, does not necessarily contain information about the *type* of non-linear adjustment to equilibrium.

Finally, when applying these tests to major real exchange rates, we find that the ESTAR test does not reject the unit root hypothesis for any currency pair. This indicates that either PPP does not hold - which is not very plausible from an economic point of view - or that is due to the low power of the ESTAR test under realistic parameter settings. The question arises whether the exchange rates are well described by an ESTAR process. Our results cast some doubt on this.

Acknowledgements We would like to thank an anonymous referee and an associate editor for helpful suggestions and improvements. Moreover, we thank the participants at the Econometric Society European Meeting (ESEM) 2007 in Budapest, at the Macroeconomic Research Meeting 2008 in Bonn, at the Econometrics and Finance Seminar CORE 2008 in Louvain-La-Neuve and at several other university seminars. We are particularly grateful to Jörg Breitung and Timo Teräsvirta for very helpful comments. Technical assistance by Jürgen Wohlers and his team is also gratefully acknowledged.

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