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# WORKING PAPER

# Marginal Cost of Indirect Taxation in the presence of a Demerit Externality with an Application to Carbon Dioxide Emissions in Belgium

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# Marginal Cost of Indirect Taxation in the presence of a Demerit Externality with an Application to Carbon Dioxide Emissions in Belgium

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**Abstract**

This paper aims to calculate marginal costs of funds (MCF) in the presence of an externality with demerit properties by using a utility scaling approach. It is an extension of a model put forward by Schroyen (2010). In the empirical section the MCF of indirect taxes in Belgium are calculated taking into account the existence of carbon dioxide emissions as demerit externality. The results reveal that scaling has a significant impact on switches in the ranking of the MCF. JEL Classification D12, H21, H23.

## 1 Introduction

Raising an indirect tax implies a cost for society, social welfare decreases. In marginal indirect tax reform analysis, the social welfare costs of raising one euro of tax money by increasing different indirect tax rates are computed. These welfare costs are called marginal costs of funds (MCF). In the optimum, the MCF of all indirect taxes must be equal, otherwise social welfare could be increased by altering tax rates in a budget neutral fashion. The literature in this field originated from the seminal paper of Ahmad and Stern (1984) who calculate MCF for indirect taxes in India. In that contribution only private utility is considered in the social welfare function and externalities are not taken into account. In subsequent studies, the model was extended to include externalities, e.g. Schöb (1996) or Mayeres and Proost (2001) who considered the effects on pollution and traffic congestion. The externality enters the MCF expression in an additive way. In all of this literature, the planner (government) endorses consumer sovereignty. Public and private goods are only valuable for society to the extent that households value them. Private goods are valued using the market price, public goods are valued using the sum of the valuations of all households. We will refer to this literature as welfaristic.

In reality, however, governments do not always accept consumer sovereignty. Private demerit goods such as tobacco, alcohol or hard drugs are valued too highly by the households. Governments aim to discourage, restrict or even forbid the consumption of these commodities[1]. The most common demerit arguments in the literature are uncertainty, irrational preferences, information deficiency (myopia, ignorance...) and the assertion that human beings are in fact split personalities: on the one hand they have 'market preferences' and on the other hand they have 'ethical preferences' (see Mazzanti (2002) and Ver Eecke (2003)) that do not necessarily coincide. When deciding upon consumption, individuals use their market preferences, but when they reflect upon their deeds, their ethical principles come up so they might disapprove of their behaviour.

[0] I would like to thank Dirk Van de gaer for constructive comments on preliminary versions of this paper and Brent Bleys and An-Sofie Cottyn, two of my colleagues, for making linguistic suggestions and corrections.

[1] Musgrave (1959) was the first to define the concept of a (de)merit good. Merit and demerit goods are goods for which the preference of the planner differs from the preference of the households. The planner attributes a higher value than the household(s) to merit goods and a lower value to demerit goods.

In these instances the social welfare function cannot be simply based on private utility alone but has to be adapted in some way. In a number of recent contributions (Schroyen 2005 and 2010) MCF expressions for indirect taxes have been derived for a situation in which there is a private commodity with demerit properties.

We aim to extend Schroyen's model to include externalities with demerit properties. In this case the planner has to make two separate corrections: he needs to correct the MCF both for the externality problem and for the demerit problem. The first one is a welfaristic correction, the second one is a non-welfaristic correction. Note that in both cases the planner disagrees with the household: regarding the valuation of a commodity causing an externality and regarding the valuation of a commodity causing an externality with demerit properties. There is however an important distinction between the two. In the case of an externality, the households suffering the externality are perfectly aware of the fact that they are suffering it. Consequently the planner intervenes only insofar as other households are affected. When the externality has demerit properties, households are insufficiently aware of the bad consequences of the externality and the planner acts as if the household is at a lower utility level than it considers itself to be at.[2] Below we show that the combination of an externality and a demerit argument poses specific challenges because the household's valuation (willingness to pay) for a commodity needs to be adapted twice. Furthermore, there is an interaction between the two corrections. This adapted willingness to pay expression is then used in the MCF formulae.

We apply the theory to carbon dioxide emissions as a demerit externality because it seems that governments disagree with households on its value. Governments might feel the ethical obligation to take measures now to avoid very bad consequences for future generations[3]. Households from their side may support this kind of policy because of the split personality argument: consumption decisions, causing emission of carbon dioxide, are based on market preferences. When the household reflects upon these decisions, its ethical principles come up[4] and it might regret its behaviour because it feels it could have emitted less. So it can be argued that observed household (market) preferences are mistaken. Households don't value the impact of their behaviour on the externality correctly because some consequences (e.g. for future generations) of their consumption decisions are not taken into account adequately.

In the next section we show formally how the planner's valuation of a commodity differs from the household's valuation. Based on this, the implications for the calculation of the MCF of indirect taxation are analysed. In the third section of the paper we apply the framework to Belgian indirect taxes in the presence of carbon dioxide emissions. The fourth section concludes.

# 2 The model

## 2.1 Notation and household behaviour

Assume there are $M$ commodities and an externality ($E$). Each household $h$, $h = 1, ..., n$, consumes a commodity vector $x^h \in \mathbb{R}_+^M$, $x^h = [x_1^h, ..., x_M^h]$. The typical element of $x^h$ is $x_j^h \in \mathbb{R}_+$ and $x_j \in \mathbb{R}_+^n$ is the vector of consumption of commodity $j$ by all households, $x_j = [x_j^1, ..., x_j^n]$. Finally, $x \in \mathbb{R}_+^{Mn}$ is the $M \times n$ dimensional matrix of commodity consumption by all households. The externality $E$ is created by total household consumption of commodities: $E(x) : \mathbb{R}_+^{Mn} \to \mathbb{R}_+$. Household $h$ has a quasi-concave utility function $u^h\left(x^h; E\right) : \mathbb{R}_+^M \times \mathbb{R}_+ \to \mathbb{R}$. The after tax price of commodity $j$ is $q_j$ and $p_j$ is the pre tax price of commodity $j$; $t_j$ is the tax on commodity $j$, so $q_j = p_j + t_j$ ($q_j, p_j \in \mathbb{R}_+^M$ and $t_j \in \mathbb{R}^M$). The vector $q = [q_1, ..., q_M]$ is the vector of commodity

[2]In the case of an externality, after the planner's intervention to solve the externality problem, society is better off (but there may be winners and losers because of the intervention). When the externality has demerit properties, after the planner's intervention to solve the demerit problem, the consequence may well be that every household's utility level decreases.

[3]A non-welfaristic uses a lower discount rate for future households' utilities than a welfarstic planner.

[4]The household might want to protect future generations, it is in favour of a low discount rate for future generations' utilities.

prices. Household $h$'s normalized price (price relative to income) of commodity $j$ is $\pi_j^h = \frac{q_j}{m^h}$ with $m^h$ the amount of income household $h$ has at its disposal. The vector $\pi^h$ is household $h$'s vector of normalized commodity prices; $\pi^h = [\pi_1^h, ..., \pi_M^h]$. Household $h$'s Marshallian demand for commodity $j$ is $x_j^h\left(q, m^h; E\right)$ or, written differently, $x_j^h\left(\pi^h, 1; E\right)$.

Each household solves the following problem

$$\text{Max } u^h\left(x^h; E\right) \text{ s.t. } \sum_{j=1}^{M} \pi_j^h x_j^h = 1. \tag{1}$$

It decides upon the amount of private commodities it consumes, taking into account the impact of its decisions on $E$. The first order conditions can be written as

$$\pi_i^h = \frac{\frac{\partial u^h\left(x^h;E\right)}{\partial x_i^h} + \frac{\partial u^h\left(x^h;E\right)}{\partial E}\frac{\partial E}{\partial x_i^h}}{\sum_{j=1}^{M}\frac{\partial u^h\left(x^h;E\right)}{\partial x_j^h}x_j^h + \sum_{j=1}^{M}\frac{\partial u^h\left(x^h;E\right)}{\partial E}\frac{\partial E}{\partial x_j^h}x_j^h}, \forall\ i = 1, ..., M. \tag{2}$$

The right hand side of this expression measures household $h$'s normalized willingness to pay for a unit of commodity $i$, it is the household's valuation of $x_i^h$ relative to its income. The numerator is the household's marginal utility of consumption of commodity $i$. The denominator is the sum of the marginal utilities of each commodity multiplied by the consumed amount of each commodity, this is the impact on household $h$'s utility of a percentage increase of all consumed commodities. It is a measure for the marginal utility of an extra percent of income[5] and in the rest of the paper we will refer to it as such. Notice that in expression (2) the household takes into account the impact of its consumption on $E$ only on its own utility, not the impact on others.

The planner does not agree with this valuation for two reasons. First of all, the planner aims to maximize social welfare, not just individual $h$'s welfare. To the extent that individual $h$'s behaviour influences other households' utilities, the formula needs to be adapted. This is the standard welfaristic externality correction (see e.g. Pigou (1947) or Cornes and Sandler (1996)). The second correction has to do with demerit externality arguments. Each of these corrections influences the willingness to pay expressions. We analyse them one by one.

First, when the planner takes into account the externality, he maximizes

$$W(x; E) = \sum_{h=1}^{n} \lambda^h u^h\left(x^h; E\right) \text{ s.t. } \sum_{j=1}^{M} \pi_j^{eh} x_j^h = 1,\ \forall\ h,$$

with $\lambda^h$ a welfare weight for household $h$; $\lambda^h > 0\ \forall\ h$. This yields first order conditions

$$\pi_i^{eh}(x; E) = \frac{\lambda^h \frac{\partial u^h}{\partial x_i^h} + \sum_{l=1}^{n}\lambda^l \frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_i^h}}{\lambda^h \sum_{j=1}^{M}\frac{\partial u^h}{\partial x_j^h}x_j^h + \sum_{j=1}^{M}\sum_{l=1}^{n}\lambda^l \frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_j^h}x_j^h},\ \forall\ h, \forall\ i. \tag{3}$$

This expression measures the social valuation of the consumption of one unit of $x_i^h$ (taking into account the externality). The numerator of the expression is the social marginal utility of the consumption of $x_i^h$, measuring how much social welfare increases when household $h$ consumes an extra unit of commodity $i$. The denominator measures the impact on social welfare $W$ if there is a percentage increase of household $h$'s income. In this sense $\pi_i^{eh}$ measures the social valuation of $x_i^h$ relative to giving individual $h$ one more percent of income. Observe that both the numerator and the denominator contain welfare weights and other households' utilities, through the impact via $E$. For this reason $\pi_i^{eh}$ differs from $\pi_i^h$ (see expression (B5) in appendix B). Notice that in expression (3) the planner only corrects for the externality, demerit arguments are not yet included.

[5]This is shown in footnote 28 in appendix C.

The second correction the planner performs has to do with public demerit arguments. In this case household welfare is not perceived in the same way by the government as by the household[6]. Assume the planner evaluates household $h$'s situation with other preferences:

$$u^{ph}\left(x^h;E\right). \tag{4}$$

For this function it holds that $u^{ph}\left(x^h;E\right):\mathbb{R}_+^M\times\mathbb{R}_+\rightarrow\mathbb{R}$. In the next section this expression will be used in the social welfare function instead of $u^h\left(x^h;E\right)$. As a result the planner corrects the social normalized willingness to pay $(\pi_i^{eh})$ for commodities a second time, this is the non-welfaristic correction. In the next section the social willingness to pay for the consumption of commodity $i$ by household $h$ (called $\pi_i^{peh}$) will be derived taking into account both externalities and demerit arguments.

## 2.2 Derivation of the social willingness to pay

In order to be able to derive the social normalized willingness to pay expression, we first postulate a formal relationship between $u^{ph}\left(x^h;E\right)$ and $u^h\left(x^h;E\right)$. One approach to take into account demerit arguments has been formalized by Schroyen (see Schroyen (2005) and Schroyen (2010)), for a private (de)merit good. In order to incorporate demerit arguments, he proposes a scaling approach to commodity consumption. He uses the distance function $d^h\left(x^h,\overline{u^h};E\right):\mathbb{R}_+^M\times\mathbb{R}\times\mathbb{R}_+\rightarrow\mathbb{R}_+$ (see Deaton and Muellbauer (1980)), which is implicitly defined by

$$u^h\left(\frac{x^h}{d^h\left(x^h,\overline{u^h};E\right)};\frac{E}{d^h\left(x^h,\overline{u^h};E\right)}\right)=\overline{u^h}. \tag{5}$$

The distance $d^h\left(x^h,\overline{u^h};E\right)$ is the amount by which private consumption and $E$ need to be scaled to reach a reference utility level $\overline{u^h}$. If $x^h$ and $E$ happen to be on the indifference curve corresponding to $\overline{u^h}$, $d^h\left(x^h,\overline{u^h};E\right)=1$. The distance $d^h\left(x^h,\overline{u^h};E\right)$ is a cardinal measure of household $h$'s utility[7]. The planner disagrees with the amount $d^h\left(x^h,\overline{u^h};E\right)$ because of demerit arguments. He believes that, in order to reach the reference utility level, commodity consumption and $E$ need to be scaled by a different amount $d^{ph}\left(x^h,\overline{u^h};E\right):\mathbb{R}_+^M\times\mathbb{R}\times\mathbb{R}_+\rightarrow\mathbb{R}_+$. As Schroyen, we assume that the planner uses the following formal relationship between $d^h\left(x^h,\overline{u^h};E\right)$ and $d^{ph}\left(x^h,\overline{u^h};E\right)$:

$$d^{ph}\left(x^h,\overline{u^h};E\right)=d^h\left(x^h,\overline{u^h};E\right)+eE, \tag{6}$$

with $e<0$ a public demerit parameter for the externality[8], it has the dimension of a normalized price (a price relative to income). It measures how much the household should value one unit of externality $E$ as a percentage of income, up and above its private valuation. Now it can be shown[9]

[6] Observe that in $\pi_i^{eh}$, the planner respects household preferences, he only solves the externality problem that the households didn't take into account when deciding upon consumption.

[7] For a public bad, the indifference curves have a positive slope. This implies that the distance need not be uniquely defined. We use the distance with respect to the point of the indifference curve closest to the origin.

[8] We assume that $e$ is a constant, meaning that the preference for the first unit of commodity $E$ is equally mistaken as the preference for the last unit. It is a negative value because $E$ has demerit properties.

[9] First define $u^{ph}$ as

$$u^{ph}\left(\frac{x^h}{d^{ph}\left(x^h,\overline{u^h};E\right)};\frac{E}{d^{ph}\left(x^h,\overline{u^h};E\right)}\right)=\overline{u^h}.$$

that the formal relationship between $u^{ph}$ and $u^h$ is

$$\begin{aligned} u^{ph}\left(x^h; E\right) &= u^h\left(\frac{x^h}{1-eE}; \frac{E}{1-eE}\right) \\ &= u^h\left(\widehat{x}^h; \widehat{E}\right) \qquad (7) \\ \text{with } \widehat{x}^h &= \frac{x^h}{1-eE} \text{ and } \widehat{E} = \frac{E}{1-eE}. \end{aligned}$$

This is the counterpart of expression (8) in Schroyen (2010) on page 46. The planner evaluates the household at a different (lower) utility level because there are demerit arguments which the household does not take into account.

Now we return to the social welfare function. The planner maximizes

$$\begin{aligned} W(x; E) &= \sum_{h=1}^{n} \lambda^h u^{ph}\left(x^h; E\right) \\ &= \sum_{h=1}^{n} \lambda^h u^h\left(\widehat{x}^h; \widehat{E}\right), \end{aligned}$$

which leads to the following problem

$$\text{Max} \sum_{h=1}^{n} \lambda^h u^h\left(\widehat{x}^h; \widehat{E}\right) \text{ s.t. } \sum_{j=1}^{M} \pi_i^{peh} x_j^h = 1 \; \forall h.$$

The first order conditions of this problem are:

$$\begin{aligned} \pi_i^{peh}\left(\widehat{x}(e), \widehat{E}(e)\right) &= \frac{\lambda^h \left(\sum_{k=1}^{M} \frac{\partial u^h\left(\widehat{x}^h; \widehat{E}\right)}{\partial \widehat{x}_k^h} \frac{\partial \widehat{x}_k^h}{\partial x_i^h} + \frac{\partial u^h\left(\widehat{x}^h; \widehat{E}\right)}{\partial \widehat{E}} \frac{\partial \widehat{E}}{\partial x_i^h}\right)}{\sum_{j=1}^{M} \sum_{l=1}^{n} \lambda^l \left(\sum_{k=1}^{M} \frac{\partial u^l\left(\widehat{x}^l; \widehat{E}\right)}{\partial \widehat{x}_k^l} \frac{\partial \widehat{x}_k^l}{\partial x_j^h} x_j^h + \frac{\partial u^l\left(\widehat{x}^l; \widehat{E}\right)}{\partial \widehat{E}} \frac{\partial \widehat{E}}{\partial x_i^h} x_j^h\right)} \\ &+ \frac{\sum_{\substack{l=1 \\ l \neq h}}^{n} \lambda^l \sum_{k=1}^{M} \frac{\partial u^l\left(\widehat{x}^l; \widehat{E}\right)}{\partial \widehat{x}_k^l} \frac{\partial \widehat{x}_k^l}{\partial x_i^h} + \sum_{\substack{l=1 \\ l \neq h}}^{n} \lambda^l \frac{\partial u^l\left(\widehat{x}^l; \widehat{E}\right)}{\partial \widehat{E}} \frac{\partial \widehat{E}}{\partial x_i^h}}{\sum_{j=1}^{M} \sum_{l=1}^{n} \lambda^l \left(\sum_{k=1}^{M} \frac{\partial u^l\left(\widehat{x}^l; \widehat{E}\right)}{\partial \widehat{x}_k^l} \frac{\partial \widehat{x}_k^l}{\partial x_j^h} x_j^h + \frac{\partial u^l\left(\widehat{x}^l; \widehat{E}\right)}{\partial \widehat{E}} \frac{\partial \widehat{E}}{\partial x_i^h} x_j^h\right)}, \forall h, \forall i. \quad (8) \end{aligned}$$

This is the social valuation of $x_i^h$ relative to giving household $h$ an extra percentage of income, taking into account both the externality and the demerit arguments. Formally, the expression resembles expression (3). Again the numerator measures the social marginal utility of the consumption of $x_i^h$. Observe that it has three parts: a private part, only for household $h$ (the household consuming the commodity) in the first line of the expression and two parts in the second line of the expression taking

If $d^{ph}\left(x^h, \overline{u^h}; E\right) = 1$, $u^{ph} = \overline{u^h}$. Now use expressions (5) and (6) to get

$$u^h\left(\frac{x^h}{d^{ph}\left(x^h, \overline{u^h}; E\right) - eE}; \frac{E}{d^{ph}\left(x^h, \overline{u^h}; E\right) - eE}\right) = \overline{u^h},$$

which implies that

$$u^{ph}\left(\frac{x^h}{d^{ph}\left(x^h, \overline{u^h}; E\right)}; \frac{E}{d^{ph}\left(x^h, \overline{u^h}; E\right)}\right) = u^h\left(\frac{x^h}{d^{ph}\left(x^h, \overline{u^h}; E\right) - eE}; \frac{E}{d^{ph}\left(x^h, \overline{u^h}; E\right) - eE}\right).$$

If this expression is evaluated at $d^{ph}\left(x^h, \overline{u^h}; E\right) = 1$ (this means that the planner evaluates the household at the bundle he is actually consuming) we get the required expression.

into account the impact on other households (via $E$). The first of these terms takes into account the (indirect) effect of $x_i^h$ on $\widehat{x}_k^l$, an effect via $E$ in the denominator of $\widehat{x}_k^l$. The last term takes into account both the direct effect on the utility levels of other households via $E$ and an indirect effect for all households via $E$ in the denominator of $\widehat{E}$. The denominator of the expression measures the marginal social utility of a one percent increase in all commodities consumed by household $h$, as such it is a measure of the social value of giving the household an extra percent of income. In both numerator and denominator, externalities and demerit arguments are taken into account. The externality is taken into account by the inclusion of other households $l$ in the expression (compare with expressions (2) and (3)). The demerit considerations enter because $\widehat{x}^l$ and $\widehat{E}$ are used instead of $x^l$ and $E$. If $e = 0$ we are in the welfaristic case, $\left(\widehat{x}^l; \widehat{E}\right) = \left(x^l; E\right) \; \forall l$. In this case $\pi_i^{peh}$ is equal to $\pi_i^{eh}$.

In appendix A we approximate $\pi_i^{peh}$ using a first order Taylor expansion. This yields expression (A31) for the social normalized valuation of $x_i^h$, repeated here for convenience:

$$\pi_i^{peh}\left(\widehat{x}\left(e\right), \widehat{E}\left(e\right)\right) \approx \pi_i^{eh}\left(1 + e\left(E - E^h b^h\right)\right) + e\frac{\partial E}{\partial x_i^h} b^h + s_i^{eh} e E, \tag{9}$$

an expression in which a number of new parameters come up. First remember that in $\pi_i^{eh}$ the externality is included, so the expression only shows how the planner's valuation changes when the demerit arguments are taken into account. The term $\frac{\partial E}{\partial x_i^h}$ is the impact on the externality of the consumption of $x_i^h$ and $E^h$ is the total amount of externality household $h$ causes[10]. Parameter $b^h$ is the ratio of the impact on social welfare of a percentage increase in all households' incomes and the impact on social welfare of a percentage increase in household $h$'s income (see expression (A29) in appendix A). It measures the percentage of income for household $h$ the planner is willing to give up for a percentage increase in all households' incomes.[11] The parameter $s_i^{eh}$ is a scale parameter measuring how the household's normalized willingness to pay changes when the household is put on a different utility level.

Expression (9) consists of three parts. In the first part, $\pi_i^{eh}$ is multiplied with $\left(1 + e\left(E - E^h b^h\right)\right)$ measuring the socially relevant fraction of income to be evaluated by household $h$.[12] All normalized demand prices of household $h$ have to be scaled with this amount. The part $eE$ measures how much the households should value the demerit aspect of $E$. The term $eE^h$ measures how much all households suffer the demerit impact of the amount of externality created by household $h$ (as a percentage of their incomes), so $eE^h b^h$ measures which percentage of income of household $h$ the planner is willing to give up for a percentage of income $eE^h$ for everybody. It is the value of the total demerit impact on others of household $h$'s externality, expressed in a percentage of income of household $h$.[13]

The second term in expression (9) is $e\frac{\partial E}{\partial x_i^h} b^h$, a term that takes into account the demerit impact of the consumption of $x_i^h$ for all households, translated into a percentage of income of household $h$. The part $e\frac{\partial E}{\partial x_i^h}$ measures how every household suffers the demerit effect of the consumption of unit

[10] Formally $E^h = \sum_{j=1}^{M} \frac{\partial E}{\partial x_j^h} x_j^h$, this is definition (A14) in appendix A.

[11] We are dealing with externalities, decisions of one household influence other households' utilities. In this paper, these influences are expressed as percentages of income of the households suffering them. As such $b^h$ is an important term because it translates percentages of income for all other households into a percentage of income for household $h$, in this sense $b^h$ is the link between household $h$ and all other households.

[12] The amount of demerit externality the household suffers that is not caused by itself is measured here as $E - E^h b^h$. It can be seen as a (negative) percentage of income given to the household: $e\left(E - E^h b^h\right)$. This is consistent with the theory on public good provision (Cornes and Sandler 1996) (see also appendix A above expression (A24)).

Observe that if household $h$ is the only household in the world (the externality only affects household $h$ and there is no other $E$ than the one caused by household $h$), then $b^h = 1$ and $E = E^h$; expression (9) reduces to $\pi_i^{peh}\left(\widehat{x}\left(e\right), \widehat{E}\left(e\right)\right) \approx \pi_i^{eh} + e\frac{\partial E}{\partial x_i^h} + s_i^{eh} e E$. This expression is similar to Schroyen's expression for the private (de)merit good.

[13] Notice that the part $E - E^h b^h$ is a measure for the amount of externality realized abroad.

$x_i^h$ (as a percentage of their income), so $e\frac{\partial E}{\partial x_i^h}b^h$ measures which percentage of income of household $h$ the planner wants to give up for $e\frac{\partial E}{\partial x_i^h}$ more income for everyone. In what follows, we will refer to these first two terms as the 'direct' correction.

The third term in the expression, $s_i^{eh}eE$, has to do with the scale effect[14]. The part $eE$ measures how much preferences are mistaken due to demerit aspects, how far the planner evaluates the household to be from its consumed bundle (in percent of income). Below we will refer to this term as the 'scale' correction. Remark that if $e = 0$, then $\pi_i^{peh} = \pi_i^{eh}$ and we are in the welfaristic case.

Notice that $\pi_i^{eh}$ is unobservable, so in order to use expression (9) we need a relationship between $\pi_i^h$ and $\pi_i^{eh}$. Using expression (B5) from appendix B we find:

$$\pi_i^{eh} = \pi_i^h \lambda^h \gamma^h + \left(\Pi_E^h - \pi_E^h\right) \frac{\partial E}{\partial x_i^h}.$$

A number of new parameters are introduced in this expression. First there is the parameter $\gamma^h$, the ratio of household $h$'s marginal utility of income from the household's and from the planner's point of view[15]. Second the planner takes into account $\left(\Pi_E^h - \pi_E^h\right)\frac{\partial E}{\partial x_i^h}$, the welfaristic externality correction. The parameter $\Pi_E^h$ is the ratio of the impact on social welfare of an increase in $E$ (for all households) and the impact on social welfare of a percentage increase in household $h$'s income (see expression (B3) in appendix B). As such it measures the planner's marginal rate of substitution between $E$ and household $h$'s income, it is the percentage of household $h$'s income the planner wants to give up for an extra unit of $E$. The parameter $\pi_E^h$ is the ratio of the impact only for household $h$'s utility of an increase in $E$ (multiplied by $\lambda^h$) and the impact on social welfare of a percent increase in household $h$'s income, see expression (B4) in appendix B. It holds that $\sum_{h=1}^n \pi_E^h = \Pi_E^h$.[16]

Expression (9) can now be written as (assuming $\lambda^h = 1$ for simplicity)

$$\begin{aligned}\pi_i^{peh}\left(\widehat{x}(e), \widehat{E}(e)\right) &\approx \left[\pi_i^h\gamma^h + \frac{\partial E}{\partial x_i^h}\left(\Pi_E^h - \pi_E^h\right)\right]\left(1 + e\left(E - E^h b^h\right)\right) \\ &\quad + e\frac{\partial E}{\partial x_i^h}b^h + s_i^{eh}eE. \qquad (10)\end{aligned}$$

This establishes the formal relationship between $\pi_i^{peh}$ and $\pi_i^h$.

## 2.3 Derivation of the Marginal Costs of Funds

Using the expressions derived above the marginal cost of funds formulae can be calculated. Let $W(t)$ be social welfare, depending on the planner's valuation of household welfare and $R$ the planner's revenue constraint ($\overline{R}$ is exogenous revenue, e.g. from income taxation), then

$$\begin{aligned}W(t) &= \sum_{h=1}^n \lambda^h u^{ph}\left(x^h(t); E\right) \\ &= \sum_{h=1}^n \lambda^h u^h\left(\widehat{x}^h(t); \widehat{E}\right), \qquad (11)\end{aligned}$$

and

$$R = \overline{R} + \sum_{i=1}^M \sum_{h=1}^n t_i x_i^h, \qquad (12)$$

[14]The scale effects satisfy the condition $\sum_{i=1}^M s_i^{eh}x_i^h = -1$ so it can be checked that $\sum_{i=1}^M \pi_i^{peh}x_i^h = 1$, the normalized demand prices satisfy the adding-up condition. The three corrections due to demerit arguments are interrelated, they influence the normalized willingness to pay in such a way that the adding-up condition is satisfied.

[15]This is the ratio of the denominators of expressions (2) and (3).

[16]Note that in expression (B5) $\pi_E^h$ is subtracted in order to avoid double counting. It is assumed that the household takes into account the impact of its own behaviour on itself, so it is already present in $\pi_i^h$.

where $t_i$ is the indirect tax rate on commodity $i$ and $\lambda^h$ is the welfare weight the planner attaches to household $h$. The planner maximizes $W(t)$ such that the revenue constraint is satisfied. As usual, the marginal cost of funds is

$$MCF_i = -\frac{\frac{\partial W}{\partial t_i}}{\frac{\partial R}{\partial t_i}}. \tag{13}$$

It measures the marginal impact on social welfare of raising additional revenue by increasing the tax on commodity $i$. First we calculate $\frac{\partial R}{\partial t_i}$. Note that we assume horizontal supply curves, resulting in fixed producer prices; $\frac{\partial x_j^h}{\partial t_i} = \frac{\partial x_j^h}{\partial q_i}\ \forall i, j, h$.

$$\frac{\partial R}{\partial t_i} = \sum_{h=1}^{n} x_i^h + \sum_{j=1}^{M}\sum_{h=1}^{n} t_j \frac{\partial x_j^h}{\partial q_i}.$$

Now we multiply with $q_i$ and transform derivates into elasticities.

$$\begin{aligned}
\frac{\partial R}{\partial t_i} q_i &= \sum_{h=1}^{n} x_i^h q_i + \sum_{j=1}^{M}\sum_{h=1}^{n} \frac{t_j}{q_j} q_j x_j^h \varepsilon_{ji}^h, && (14)\\
\text{where } \varepsilon_{ji}^h &= \frac{\partial x_j^h}{\partial q_i}\frac{q_i}{x_j^h}.
\end{aligned}$$

The numerator of the marginal cost of funds formula is

$$\frac{\partial W(t)}{\partial t_i} = \sum_{h=1}^{n} \lambda^h \frac{\partial u^h\left(\widehat{x}^h(t);\widehat{E}\right)}{\partial t_i}.$$

In appendix C it is shown that from this, it follows that

$$\begin{aligned}
\frac{\partial W(t)}{\partial t_i} q_i = & -\sum_{h=1}^{n} \lambda^h D^h\left(\widehat{x}^h(e),\widehat{E}(e)\right)\left(1+e\left(E-E^h b^h\right)\right)\frac{x_i^h q_i}{m^h} && (15)\\
& +\sum_{h=1}^{n} B^h\left(\widehat{x}(e),\widehat{E}(e)\right)\left(\left(1+e\left(E-E^h b^h\right)\right)\left(\Pi_E^h-\pi_E^h\right)+eb^h\right)\sum_{j=1}^{M} r_j^h \varepsilon_{ji}^h x_j^h q_j\\
& +\sum_{h=1}^{n} \lambda^h D^h\left(\widehat{x}^h(e),\widehat{E}(e)\right) eE\sum_{j=1}^{M}\sigma_j^{eh}\varepsilon_{ji}^h \pi_j^h x_j^h\\
& +\sum_{h=1}^{n} B^h\left(\widehat{x}(e),\widehat{E}(e)\right) eE\left(\Pi_E^h-\pi_E^h\right)\sum_{j=1}^{M}\sigma_j^{eh}\varepsilon_{ji}^h r_j^h q_j x_j^h. && (16)
\end{aligned}$$

The terms $D^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)$ and $B^h\left(\widehat{x}(0),\widehat{E}(0)\right)$ are the denominators of expressions (2) and (3) respectively. They measure household $h$'s individual and social marginal utility of one percent of income respectively and play an important role in expression (15). The first line of the expression contains budget share $\frac{x_i^h q_i}{m^h}$, a term that appears in every MCF analysis. This budget share is corrected by two terms. First it is weighted by $\lambda^h D^h\left(\widehat{x}^h(e),\widehat{E}(e)\right)$, a term that is the combination of individual $h$'s marginal utility of income and a welfare weight. Second it is corrected by the term $e\left(E-E^h b^h\right)$, a term taking into account the amount of externality realized abroad (see expressions (9) and (10)). The second line measures the impact via the externality, weighted by $B^h\left(\widehat{x}\left(e\right),\widehat{E}\left(e\right)\right)$. The part $eb^h$ is the demerit valuation for $E$, for all households, translated into an income share of household $h$. The part $e\left(E-E^hb^h\right)$ measures how all household $h$'s normalized demand prices need to be adapted due to the amount of externality realized abroad. The term $\sum_{j=1}^{M} r_j^h \varepsilon_{ji}^h x_j^h q_j$ measures how much $E$ increases due to a change in $t_i$. We will refer to the terms depending on $e$ in the first two lines as the 'direct' effect of the incorporation of the demerit arguments.

The third and the fourth line in the expression measure the scale effect, taking into account how much the valuation of commodities changes when the households are evaluated at a different utility level. The amount by which preferences are mistaken is measured by $eE$. The part $\sum_{j=1}^{M}\sigma_j^{eh}\varepsilon_{ji}^h\pi_j^h x_j^h$ measures how the tax increase influences the normalized demand prices of all commodities. This type of scaling term also appears in the work of Schroyen. We will refer to the terms depending on $\sigma_j^{eh}$ as the 'scale' effect of the incorporation of the demerit arguments.

Remark that, if $e=0$ we are in the welfaristic case. The numerator of the $MCF_i$ expression becomes

$$\frac{\partial W\left(t\right)}{\partial t_i}q_i=-\sum_{h=1}^{n}\lambda^h D^h\left(\widehat{x}^h\left(0\right),\widehat{E}\left(0\right)\right)\frac{x_i^h q_i}{m^h}+\sum_{h=1}^{n}B^h\left(\widehat{x}\left(0\right),\widehat{E}\left(0\right)\right)\left(\Pi_E^h-\pi_E^h\right)\sum_{j=1}^{M}r_j^h\varepsilon_{ji}^h x_j^h q_j, \tag{17}$$

a combination of a private part $\lambda^h D^h\left(\widehat{x}^h\left(0\right),\widehat{E}\left(0\right)\right)\frac{x_i^h q_i}{m^h}$ and an externality part depending on $\left(\Pi_E^h-\pi_E^h\right)$. The only difference with the marginal cost of funds expressions in Schöb (1996) and Mayeres and Proost (2001) is the way household utilities are weighted; in our work the weight for the private part, $\lambda^h D^h\left(\widehat{x}^h\left(0\right),\widehat{E}\left(0\right)\right)$, differs from the weights in the externality part, $B^h\left(\widehat{x}\left(0\right),\widehat{E}\left(0\right)\right)$.

Using expressions (13), (14) and (16), the marginal cost of funds can be calculated.

# 3 Empirical application

## 3.1 The data

We calculate the marginal costs of funds for the indirect taxes on 13 commodities (in Belgium) in order to illustrate how the model works. We rely on data from the 2004 budget survey. There are 10 household income categories and there are on average 2,33 individuals per household, which means that every income decile in Belgium consists of roughly 450.000 households. In appendix D some information on the data is provided. The household budget survey provides information on budget shares ($\frac{x_i^h q_i}{m^h}$) spent on each of the 13 commodities. Information on $\Pi_E^h$ and $\pi_E^h$ is based on the value of one tonne of carbon dioxide on the global level, i.e. 20 euro per tonne. The willingness to pay for Belgian households is calculated based on the assumption that the income elasticity of this valuation equals 1. This amounts to an average value of 0,5 eurocents per decile. The information on indirect tax rates is based on the COICOP classification, for 974 commodities aggregated into 13 commodity categories.

The information on price and scale elasticities is derived from the estimation of an almost ideal demand system[17] (see e.g. Deaton and Muellbauer (1980) and Decoster and Schokkaert (1990)) while the welfare weights are determined using the formula put forward by Ahmad & Stern : $\lambda^h=\left(\frac{m^1}{m^h}\right)^{\upsilon}$, with $\upsilon\geq 0$ a measure of inequality aversion. For simplicity, we calculate $D^h\left(\widehat{x}^h\left(e\right),\widehat{E}\left(e\right)\right)$ as $D^h\left(\widehat{x}^h\left(0\right),\widehat{E}\left(0\right)\right)$ and $B^h\left(\widehat{x}\left(e\right),\widehat{E}\left(e\right)\right)$ as $B^h\left(\widehat{x}\left(0\right),\widehat{E}\left(0\right)\right)$ in appendix C

[17] We use national accounts data and price data for 53 years (1954 - 2006). When estimating the demand system, the compensated own price elasticities of 3 commodity categories (commodities clothing & shoes, gas and fuels) appeared to be positive. This is inconsistent with demand theory, so 3 extra constraints have been added to the demand system, setting these three compensated demand elasticities equal to 0.

under the assumption that the marginal utility of one percent of income equals one.[18] This implies that $D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right) = m^h$ and $B^h\left(\widehat{x}(0), \widehat{E}(e)\right) = \lambda^h m^h + E^h \sum_{\substack{l=1 \\ l\neq h}}^n m^l \pi_E^l$ (including the impact on other households' utilities). It follows that $b^h = \frac{\sum_{l=1}^n B^l\left(\widehat{x}(0),\widehat{E}(0)\right)}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)} + \frac{\sum_{t=1}^n m^t \pi_E^t \left(E - \sum_{l=1}^n E^l\right)}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}$. Data on the impact of commodity consumption on the externality $(r_j^h)$ is calculated from input-output analysis based on data from Belgostat (National Bank of Belgium). Based on this and the amount of carbon dioxide per sector we can calculate emissions of $CO_2$ per household $(E^h)$. For total emissions $E$ we use world emissions of $CO_2$ instead of Belgian emissions[19].

We will perform sensitivity analysis using different values of $e$. The IPCC calculations are based on damage estimates for a tonne of $CO_2$ between 6 euro and 400 euro, so this will be the range for our simulations. We make two simplifying assumptions: first we assume that there is no difference between the price and cross price elasticities over households, so $\varepsilon_{ji}^h = \varepsilon_{ji}$ $\forall h, i, j$ and second we assume that $r_j^h = r_j$ $\forall j, h$, the externality impact of the consumption of a commodity is the same for all households.

### 3.2 Results

The results are shown in tables 1a and 1b below in which the contribution of each component of the MCF is shown. The parameter of inequality aversion $\upsilon$ is taken to be equal to one[20]. The parameter $e$ is varied so as to reflect the differences in valuations, simulations are performed with values of $e$ as a multiple of the average value of a tonne of carbon dioxide in Belgium $(\pi_E)$, between 0 and 20 times $\pi_E$.[21] This entails a number of rank switches that we will focus on.

When $e = 0$ (on the left side of the table), we are in the welfaristic case (base case scenario). The second column in table 1a provides the MCF as calculated based on expression (16). Numerically only the part due to Ahmad and Stern plays a role because all terms incorporating the welfaristic externality (the second part in expression (17)) are close to 0. This implies that the externality does not matter much for a planner with welfaristic principles. Moving to the right in the table, demerit arguments start to play a role. For each value of $e$ there are four columns: each first and second column provide the ranking of commodities and their marginal costs of funds (columns 'rank' and 'MCF' respectively). Each third and fourth column (labelled 'direct' and 'scale' respectively) provides information on the impact of the demerit arguments. The column MCF is the sum of the MCF due to Ahmad and Stern and the numbers in the columns 'direct' and 'scale'. Each third column contains information on the direct impact on the MCF because of demerit externality arguments. These terms are mainly due to the term $e\left(E - E^h b^h\right)$ in the first two lines of expression (15). It measures how the amount of externality caused abroad influences all normalized demand prices, $E - E^h b^h$ is a measure for amount of externality that is caused abroad, multiplied with $e$ it becomes a (negative) budget share. Consequently the column 'direct' could be split up in two parts: one part due to the fact that the planner takes into account this amount of negative income and one part $eb^h$, as can be seen in expression (9). The latter part is numerically small compared with the former[22]. The terms in each third column are negative due to the fact that $e\left(E - E^h b^h\right)$

[18] Alternatively one could approximate $D^h\left(\widehat{x}^h(e), \widehat{E}(e)\right)$ and $B^h\left(\widehat{x}(e), \widehat{E}(e)\right)$ using a first order Taylor expansion around $D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)$ and $B^h\left(\widehat{x}(0), \widehat{E}(0)\right)$. This yields $D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right) + e\left(Es^{D^h} + E^h\left(D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right) + \frac{\partial u^h}{\partial E}\left(E - E^h\right)\right)\right)$ and $B^h\left(\widehat{x}(e), \widehat{E}(e)\right) \simeq B^h\left(\widehat{x}(0), \widehat{E}(0)\right) + e\left(Es^{B^h} + E^h B\left(\widehat{x}(0), \widehat{E}(0)\right)\right)$ respectively with $s^{D^h}$ and $s^{B^h}$ the scale elasticities of marginal utility of income and $B\left(\widehat{x}(0), \widehat{E}(0)\right)$ as defined in appendix A.

[19] $E > \sum_{h=1}^n E^h$.

[20] This implies weights between 1 (for the poorest household) and 0.25 (for the richest household). We performed sensitivity analysis with values of $\upsilon$ between 0 and 2 but the impact on the results was not significant.

[21] When $e$ is in the range we use, it implies that $eE$ is between 0 and $-0.205$, so $1 - eE$ is between 1 and 1.205. The planner considers the household then to be 20.5% worse off.

[22] This observation is due to two reasons, first of all our choice of carbon dioxide emissions as externality. In this case each household is small compared to the total amount of externality. Things would be different if there was one household realizing all externality and the others suffering it (or, more in general for an externality where there

is negative. Each fourth column provides information on the scaling part. The numbers in this column depend on the sign of the scale elasticities (see appendix D for numerical values). These terms are due to the fact that the households are evaluated at a different (lower) utility level than they consider themselves to be at.

Each step to the right in the table brings switches in ranking with it. The numbers for the MCF in each second column are the sum of the MCF of the base case (on the left side of the table) and the numbers in each third and fourth column. The rank correlation between the Ahmad and Stern terms and the MCF incorporating demerit arguments decrease in each step[23].

Table 1a:

| $\mathbf{e = 0}$ | | $\mathbf{e = 4\pi_E}$ | | | | $\mathbf{e = 8\pi_E}$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| comm. | MCF | rank | MCF | direct | scale | comm. | MCF | direct | scale |
| FU | 0.776 | FU | 0.743 | -0.032 | 0.000 | FU | 0.711 | -0.063 | -0.001 |
| CP | 0.718 | CP | 0.712 | -0.029 | 0.023 | CP | 0.706 | -0.059 | 0.047 |
| AT | 0.696 | AT | 0.685 | -0.028 | 0.018 | MF | 0.703 | -0.051 | 0.125 |
| CU | 0.634 | MF | 0.666 | -0.026 | 0.063 | AT | 0.675 | -0.057 | 0.036 |
| MF | 0.629 | RW | 0.596 | -0.023 | 0.057 | RW | 0.630 | -0.046 | 0.113 |
| EL | 0.627 | CU | 0.566 | -0.026 | -0.042 | CL | 0.574 | -0.043 | 0.088 |
| DU | 0.578 | GA | 0.560 | -0.023 | 0.030 | GA | 0.568 | -0.045 | 0.060 |
| RW | 0.563 | DU | 0.559 | -0.024 | 0.005 | DU | 0.541 | -0.047 | 0.010 |
| GA | 0.552 | CL | 0.552 | -0.022 | 0.044 | SE | 0.516 | -0.042 | 0.040 |
| CL | 0.529 | EL | 0.550 | -0.026 | -0.052 | CU | 0.498 | -0.052 | -0.084 |
| SE | 0.518 | SE | 0.517 | -0.021 | 0.020 | EL | 0.472 | -0.051 | -0.104 |
| FB | 0.408 | PT | 0.407 | -0.014 | 0.075 | PT | 0.468 | -0.028 | 0.150 |
| PT | 0.346 | FB | 0.383 | -0.017 | -0.009 | FB | 0.357 | -0.033 | -0.018 |

Table 1b:

| $\mathbf{e = 12\pi_E}$ | | | | $\mathbf{e = 20\pi_E}$ | | | |
|---|---|---|---|---|---|---|---|
| comm. | MCF | direct | scale | comm. | MCF | direct | scale |
| MF | 0.740 | -0.077 | 0.188 | MF | 0.814 | -0.128 | 0.313 |
| CP | 0.700 | -0.088 | 0.070 | RW | 0.731 | -0.115 | 0.283 |
| FU | 0.679 | -0.095 | -0.001 | CP | 0.688 | -0.146 | 0.117 |
| AT | 0.664 | -0.085 | 0.054 | PT | 0.650 | -0.071 | 0.375 |
| RW | 0.664 | -0.069 | 0.170 | AT | 0.643 | -0.142 | 0.089 |
| CL | 0.596 | -0.065 | 0.131 | CL | 0.640 | -0.108 | 0.219 |
| GA | 0.575 | -0.068 | 0.091 | FU | 0.615 | -0.158 | -0.002 |
| PT | 0.529 | -0.042 | 0.225 | GA | 0.591 | -0.113 | 0.151 |
| DU | 0.522 | -0.071 | 0.015 | SE | 0.513 | -0.106 | 0.100 |
| SE | 0.515 | -0.063 | 0.060 | DU | 0.485 | -0.118 | 0.025 |
| CU | 0.430 | -0.078 | -0.126 | CU | 0.295 | -0.129 | -0.209 |
| EL | 0.394 | -0.077 | -0.157 | FB | 0.279 | -0.083 | -0.046 |
| FB | 0.331 | -0.050 | -0.027 | EL | 0.238 | -0.128 | -0.261 |

Now we take a closer look at the commodities for which there are rank switches. Basically there are three types of commodities. First of all there are 5 commodities for which the rankings are not influenced much by the incorporation of demerit externality arguments: car purchase (CP), alcohol & tobacco (AT), gas (GA), services (SE) and food and beverage (FB).[24] Second, there are four commodities the rankings of which decrease when $e$ goes up: fuels (FU), car use (CU), electricity

are 'emitting' and 'suffering' households). The second reason is the fact that only households living in the planner's jurisdiction are taken into account in the social welfare function. If also households living abroad are taken into account, the term $b^h$ increases because more households are affected.

[23]For a value of $e = 4\pi_E$ the rank correlation is 0.90, for $e = 8\pi_E$ the rank correlation is 0.72, for $e = 12\pi_E$ the rank correlation is 0.54, for $e = 20\pi_E$ the rank correlation is 0.17.

[24]Notice that for all these commodities (except food & beverage) the values in the columns 'direct' and 'scale' cancel out more or less.

(EL) and to a lesser extent durables (DU). Third there are 4 commodities the ranking of which increases: public transport (PT), clothing & shoes (CL), meat & fish (MF) and rent & water (RW).

These rank switches are due to a combination of the direct impact of the demerit externality (column 'direct') and the part due to scaling (column 'scale'). In general the numbers in these columns are about the same size in absolute value, so the incorporation of the direct effect of the demerit externality is numerically equally important as the impact of scaling. The real reason for the rank switches, however, is scaling[25].

In the four right-most columns (where $e = 20\pi_E^h$) it can be seen that the commodities with a negative value in the column 'scale' rank lowest, the commodities with a (large) positive value in this column rank highest. The rank correlation between the marginal cost of funds in this case and the numbers in the fourth column is 84%. One exception on this observation is the commodity fuels, for which the scaling part is numerically close to 0. All rank switches for this commodity are due to the direct effect of the incorporation of demerit externality arguments. The rank correlation between the MCF and the numbers in the third column is considerably lower (10%). Consider for example the commodities with the highest numbers in the column 'direct' (alcohol & tobacco, car purchase and fuels). Even for the highest values of $e$, these commodities' rankings are quite high, stimulating a decrease in tax rates, not an increase. On the other hand, commodities with low numbers in the column 'direct' (e.g. food & beverage) do not necessarily rank high.

The four commodities the ranking of which decreases all have a link with energy consumption (except durables). The rank switches are due to the fact that households' valuations of energy related consumption increase when they get richer (see appendix D). The scale elasticity of e.g. car use is positive, when the household is evaluated at a lower utility level, it values car use less. A tax increase on car use has, in the eyes of the planner, a lower impact on social welfare so its MCF decreases as $e$ increases. The same holds for the commodities for which the ranking increases. These are commodities for which the households' willingness to pay decreases when they get richer. Take for example meat and fish which has a positive scale elasticity. When the household is evaluated at a lower utility level, it values meat and fish higher. Consequently a tax increase on this commodity has, from the point of view of the planner, a bigger impact on social welfare so the MCF increases as $e$ increases. This implies that the incorporation of $CO_2$ as demerit externality stimulates the planner to put a higher tax on commodities for which the normalized willingness to pay increases when households get richer, and a lower tax on commodities for which the (normalized) willingness to pay decreases when individuals get richer[26].

A revenue neutral welfare increasing tax reform could consist of a decrease of the tax rates of meat & fish, rent & water and car purchase combined with an increase of the tax rates on electricity, car use and food & beverage. Remark that this type of tax reform does not necessarily imply a decrease in carbon dioxide emissions because the rankings are mostly based on scaling, not on the impact on $E$. One could think of a reform that combines an increase in social welfare and a decrease in carbon dioxide emissions. Then one would increase the indirect tax on commodities with a low MCF and a big impact on $CO_2$ and decrease the indirect tax on commodities with a high MCF and a small impact on $CO_2$.

# 4 Conclusion

In this paper we have shown how the existence of externalities with demerit properties might influence the planner's indirect tax decisions. The problem is more complex than the incorporation of private demerit considerations because the planner's valuation of household consumption needs to be adapted both for the externality and for demerit reasons. The demerit correction in turn can be split in two parts: a 'direct' correction of the incorporation of demerit arguments and a 'scale' correction because the planner evaluates the households at a different utility level than they themselves considers to be at. The complicating feature of the model is the fact that one household's

[25] Due to the fact that households are evaluated at a different utility level, their marginal rate of substitution between commodities changes.

[26] In our empirical application the scale elasticities of energy related commodities are positive. This might be more a coincidence than a rule.

consumption influences other households' utilities via the externality, so a number of parameters that link utilities of different households enter the expressions.

In the empirical section we used carbon dioxide emissions as externality. When this commodity is considered only as an externality, no rank switches of the marginal cost of funds are realized. Only when demerit considerations are taken into account the MCF rankings switch. These rank switches are not so much due to the fact that the consumption of some commodities has a bigger impact on the externality, but to the fact that, due to demerit arguments, the planner evaluates the households at a different (lower) utility level. The results suggest that the planner should put a higher (lower) tax on commodities for which the households' willingness to pay increases (decreases) when income rises. More specifically, as the planner evaluates the households further away from the utility level they consider themselves to be at, the MCF rankings of car use, durables, fuels and electricity decrease, and the MCF rankings of meat & fish, clothing & shoes and rent & water and public transport increase.

A weakness of the current empirical application is the fact that rather broadly defined commodities are used. It may be an interesting exercise to disaggregate commodity categories based on carbon intensity and to differentiate tax rates between them. For example if fruit and vegetables are transported over long distances, this has considerable impact on the $CO_2$ intensity of these foods. Due to limited data availability we are not able to perform this exercise, this might be a subject for future research. The model could also be applied to different types of externalities.

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**APPENDIX A: derivation of the planner's marginal willingness to pay for private goods and the externality.**

**Derivation of WTP expressions**

We start from the following identity (see expression (7)):

$$\begin{aligned} u^{ph}\left(x^h;E\right) &= u^h\left(\underbrace{\frac{x_1^h}{1-eE}}_{\widehat{x}_1^h},\dots,\underbrace{\frac{x_i^h}{1-eE}}_{\widehat{x}_i^h},\dots;\underbrace{\frac{E}{1-eE}}_{\widehat{E}}\right) \\ &= u^h\left(\widehat{x}^h;\widehat{E}\right). \end{aligned} \tag{A1}$$

The normalized marginal social willingness to pay for individual $h$'s consumption of commodity $i$, including externalities and demerit externality arguments, is given in expression (8) (observe that the expression is slightly rewritten):

$$\pi_i^{peh}\left(\widehat{x}(e),\widehat{E}(e)\right)=\frac{\sum_{l=1}^n\lambda^l\sum_{k=1}^M\frac{\partial u^l\left(\widehat{x}^l;\widehat{E}\right)}{\partial\widehat{x}_k^l}\frac{\partial\widehat{x}_k^l}{\partial x_i^h}+\sum_{l=1}^n\lambda^l\frac{\partial u^l\left(\widehat{x}^l;\widehat{E}\right)}{\partial\widehat{E}}\frac{\partial\widehat{E}}{\partial x_i^h}}{\sum_{j=1}^M\sum_{l=1}^n\lambda^l\left(\sum_{k=1}^M\frac{\partial u^l\left(\widehat{x}^l;\widehat{E}\right)}{\partial\widehat{x}_k^l}\frac{\partial\widehat{x}_k^l}{\partial x_j^h}x_j^h+\frac{\partial u^l\left(\widehat{x}^l;\widehat{E}\right)}{\partial\widehat{E}}\frac{\partial\widehat{E}}{\partial x_i^h}x_j^h\right)}. \tag{A2}$$

This expression consists of a part that has to do with household consumption and a part that has to do with the externality. The numerator is

$$A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)=\sum_{l=1}^n\lambda^l\left(\sum_{k=1}^M\frac{\partial u^l\left(\widehat{x}^l;\widehat{E}\right)}{\partial\widehat{x}_k^l}\frac{\partial\widehat{x}_k^l}{\partial x_i^h}+\frac{\partial u^l\left(\widehat{x}^l;\widehat{E}\right)}{\partial\widehat{E}}\frac{\partial\widehat{E}}{\partial x_i^h}\right) \tag{A3}$$

and the denominator is

$$B^h\left(\widehat{x}(e),\widehat{E}(e)\right)=\sum_{j=1}^M\sum_{l=1}^n\lambda^l\left(\sum_{k=1}^M\frac{\partial u^l\left(\widehat{x}^l;\widehat{E}\right)}{\partial\widehat{x}_k^l}\frac{\partial\widehat{x}_k^l}{\partial x_j^h}x_j^h+\frac{\partial u^l\left(\widehat{x}^l;\widehat{E}\right)}{\partial\widehat{E}}\frac{\partial\widehat{E}}{\partial x_i^h}x_j^h\right), \tag{A4}$$

so expression (A2) can be written as

$$\pi_i^{peh}\left(\widehat{x}(e),\widehat{E}(e)\right)=\frac{A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)}{B^h\left(\widehat{x}(e),\widehat{E}(e)\right)}. \tag{A5}$$

We want to know how this expression behaves for different values of $e$, so we linearize expression (A5) around $\pi_i^{peh}\left(\widehat{x}(0),\widehat{E}(0)\right)$ to get expression (A6):

$$\begin{aligned} &\pi_i^{peh}\left(\widehat{x}(e),\widehat{E}(e)\right)\approx \\ &\frac{A_i^h\left(\widehat{x}(0),\widehat{E}(0)\right)}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)} \\ &\underbrace{+e\left[\frac{1}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}\left.\frac{\partial A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)}{\partial e}\right|_{e=0}-\frac{A_i^h\left(\widehat{x}(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}(0),\widehat{E}(0)\right)\right)^2}\left.\frac{\partial B^h\left(\widehat{x}(e),\widehat{E}(e)\right)}{\partial e}\right|_{e=0}\right]}_{T^h\left(\widehat{x}(0),\widehat{E}(0)\right)} \end{aligned} \tag{A6}$$

First we derive expressions for $A_i^h\left(\widehat{x}(e), \widehat{E}(e)\right)$ and $B^h\left(\widehat{x}(e), \widehat{E}(e)\right)$ in order to calculate $\left.\frac{\partial A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)}{\partial e}\right|_{e=0}$ and $\left.\frac{\partial B^h\left(\widehat{x}(e),\widehat{E}(e)\right)}{\partial e}\right|_{e=0}$.

**Derivation of $\mathbf{A}_i^h\left(\widehat{x}(e), \widehat{E}(e)\right)$ and $\mathbf{B}^h\left(\widehat{x}(e), \widehat{E}(e)\right)$**

To derive $A_i^h\left(\widehat{x}(e), \widehat{E}(e)\right)$ we need an expression for each of the $n$ terms between brackets in expression (A3),

$$\frac{\partial u^l\left(\widehat{x}^l; \widehat{E}\right)}{\partial x_i^h} = \sum_{k=1}^{M} \frac{\partial u^l\left(\widehat{x}^l; \widehat{E}\right)}{\partial \widehat{x}_k^l} \frac{\partial \widehat{x}_k^l}{\partial x_i^h} + \frac{\partial u^l\left(\widehat{x}^l; \widehat{E}\right)}{\partial \widehat{E}} \frac{\partial \widehat{E}}{\partial x_i^h}, \ \ \forall l. \tag{A7}$$

In this expression it is clear that there is an effect on utility through the consumption of private commodities and through $E$. First of all remember that $\widehat{x}_k^l = x_k^l(1-eE)^{-1}$ and $\widehat{E} = E(1-eE)^{-1}$. Based on this, we calculate $\frac{\partial \widehat{x}_k^l}{\partial x_i^h}$ $\forall k, l$ and $\frac{\partial \widehat{E}}{\partial x_i^h}$. Observe that the expression for $\frac{\partial \widehat{x}_i^h}{\partial x_i^h}$ will look a bit different because $x_i^h$ influences $\widehat{x}_i^h$ both directly and indirectly via the effect on $E$:

$$\frac{\partial \widehat{x}_i^h}{\partial x_i^h} = \frac{1}{1-eE} + \frac{e x_i^h}{(1-eE)^2} \frac{\partial E}{\partial x_i^h},$$

$$\frac{\partial \widehat{x}_k^l}{\partial x_i^h} = \frac{e x_k^l}{(1-eE)^2} \frac{\partial E}{\partial x_i^h},$$

and

$$\begin{aligned} \frac{\partial \widehat{E}}{\partial x_i^h} &= \left[\frac{1}{1-eE} + \frac{eE}{(1-eE)^2}\right] \frac{\partial E}{\partial x_i^h} \\ &= \frac{1}{(1-eE)^2} \frac{\partial E}{\partial x_i^h}. \end{aligned}$$

This implies that we can rewrite expression (A7). For household $h$ (the household consuming the commodity) we collect terms to get

$$\frac{\partial u^h\left(\widehat{x}^h; \widehat{E}\right)}{\partial x_i^h} = \frac{\partial u^h}{\partial \widehat{x}_i^h}\left(\frac{1}{1-eE} + \frac{e x_i^h}{(1-eE)^2} \frac{\partial E}{\partial x_i^h}\right)$$
$$+ \sum_{\substack{k=1 \\ k \neq i}}^{M} \frac{\partial u^h}{\partial \widehat{x}_k^h} \frac{e x_k^h}{(1-eE)^2} \frac{\partial E}{\partial x_i^h} + \frac{\partial u^h}{\partial \widehat{E}} \frac{1}{(1-eE)^2} \frac{\partial E}{\partial x_i^h}$$

which can be rewritten as

$$\frac{\partial u^h}{\partial \widehat{x}_i^h} \frac{1}{1-eE} + \sum_{k=1}^{M} \frac{\partial u^h}{\partial \widehat{x}_k^h} \frac{e x_k^h}{(1-eE)^2} \frac{\partial E}{\partial x_i^h} + \frac{\partial u^h}{\partial \widehat{E}} \frac{1}{(1-eE)^2} \frac{\partial E}{\partial x_i^h}. \tag{A8}$$

The first term is the direct utility effect of the consumption of commodity $i$ by the household, the second term is the effect of $x_i^h$ on $E$, and thereby on the denominator of the first $M$ arguments of $u^h$, and the last term is the direct utility effect for household $h$ of the consequences of $x_i^h$ for $E$. For the other households' utilities ($l \neq h$) we get

$$\frac{\partial u^l\left(\widehat{x}^l;\widehat{E}\right)}{\partial x_i^h}=\sum_{k=1}^{M}\frac{\partial u^l}{\partial \widehat{x}_k^l}\frac{e x_k^l}{(1-eE)^2}\frac{\partial E}{\partial x_i^h}+\frac{\partial u^l}{\partial \widehat{E}}\frac{1}{(1-eE)^2}\frac{\partial E}{\partial x_i^h}. \tag{A9}$$

Observe that this expression consists of indirect effects via the effect on the scaling of private commodities consumed by household $l$ (the effect of $x_i^h$ on $E$, and thereby on the denominator of the first $M$ arguments of $u^l$), and a direct utility effect of $E$.

Consequently we get for expression (A3)

$$\begin{aligned}
A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right) &= \lambda^h\left(\frac{\partial u^h}{\partial \widehat{x}_i^h}\frac{1}{1-eE}+\sum_{k=1}^{M}\frac{\partial u^h}{\partial \widehat{x}_k^h}\frac{e x_k^h}{(1-eE)^2}\frac{\partial E}{\partial x_i^h}+\frac{\partial u^h}{\partial \widehat{E}}\frac{1}{(1-eE)^2}\frac{\partial E}{\partial x_i^h}\right)\\
&\quad+\sum_{\substack{l=1\\ l\neq h}}^{n}\lambda^l\left(\sum_{k=1}^{M}\frac{\partial u^l}{\partial \widehat{x}_k^l}\frac{e x_k^l}{(1-eE)^2}\frac{\partial E}{\partial x_i^h}+\frac{\partial u^l}{\partial \widehat{E}}\frac{1}{(1-eE)^2}\frac{\partial E}{\partial x_i^h}\right)\\
A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right) &= \lambda^h\frac{\partial u^h}{\partial \widehat{x}_i^h}\frac{1}{1-eE}+\sum_{l=1}^{n}\lambda^l\sum_{k=1}^{M}\frac{\partial u^l}{\partial \widehat{x}_k^l}\frac{e x_k^l}{(1-eE)^2}\frac{\partial E}{\partial x_i^h}\\
&\quad+\sum_{l=1}^{n}\lambda^l\frac{\partial u^l}{\partial \widehat{E}}\frac{1}{(1-eE)^2}\frac{\partial E}{\partial x_i^h}.
\end{aligned} \tag{A10}$$

Now we can calculate $B^h\left(\widehat{x}(e),\widehat{E}(e)\right)$ (expression (A4)) as the sum of $A_j^h\left(\widehat{x}(e),\widehat{E}(e)\right)x_j^h$ over all commodities. First we use expression (A10) to calculate $A_j^h\left(\widehat{x}(e),\widehat{E}(e)\right)$ and multiply it with $x_j^h$ to get

$$\begin{aligned}
&\lambda^h\frac{1}{1-eE}\frac{\partial u^h}{\partial \widehat{x}_j^h}x_j^h+\sum_{l=1}^{n}\lambda^l\sum_{k=1}^{M}\frac{e x_k^l}{(1-eE)^2}\frac{\partial u^l}{\partial \widehat{x}_k^l}\frac{\partial E}{\partial x_j^h}x_j^h\\
&+\sum_{l=1}^{n}\lambda^l\frac{1}{(1-eE)^2}\frac{\partial u^l}{\partial \widehat{E}}\frac{\partial E}{\partial x_j^h}x_j^h.
\end{aligned}$$

Now we take the sum over all $j$ to get

$$\begin{aligned}
B^h\left(\widehat{x}(e),\widehat{E}(e)\right) &= \lambda^h\sum_{j=1}^{M}\frac{1}{1-eE}\frac{\partial u^h}{\partial \widehat{x}_j^h}x_j^h+\sum_{j=1}^{M}\sum_{l=1}^{n}\lambda^l\sum_{k=1}^{M}\frac{\partial u^l}{\partial \widehat{x}_k^l}\frac{e x_k^l}{(1-eE)^2}\frac{\partial E}{\partial x_j^h}x_j^h\\
&\quad+\sum_{j=1}^{M}\sum_{l=1}^{n}\lambda^l\frac{1}{(1-eE)^2}\frac{\partial u^l}{\partial \widehat{E}}\frac{\partial E}{\partial x_j^h}x_j^h,
\end{aligned} \tag{A11}$$

this is the denominator of expression (A2). Observe that, from (A10),

$$A_i^h\left(\widehat{x}(0),\widehat{E}(0)\right)=\lambda^h\frac{\partial u^h}{\partial x_i^h}+\sum_{l=1}^{n}\lambda^l\frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_i^h} \tag{A12}$$

and, from (A11),

$$B^h\left(\widehat{x}(0),\widehat{E}(0)\right)=\lambda^h\sum_{j=1}^{M}\frac{\partial u^h}{\partial x_j^h}x_j^h+\sum_{l=1}^{n}\sum_{j=1}^{M}\lambda^l\frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_j^h}x_j^h. \tag{A13}$$

Take into account the following definition of $E^h$

$$E^h=\sum_{j=1}^{M}\frac{\partial E}{\partial x_j^h}x_j^h. \tag{A14}$$

Now expression (A13) can be written as $B^h\left(\widehat{x}(0),\widehat{E}(0)\right)=\lambda^h\sum_{j=1}^{M}\frac{\partial u^h}{\partial x_j^h}x_j^h+\sum_{l=1}^{n}\lambda^l\frac{\partial u^l}{\partial E}E^h$.

For future reference, $D^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)$ is the marginal utility of one percent of income from the household's point of view:

$$D^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)=\sum_{j=1}^{M}\frac{\partial u^h}{\partial x_j^h}x_j^h+\sum_{j=1}^{M}\frac{\partial u^h}{\partial E}\frac{\partial E}{\partial x_j^h}x_j^h. \tag{A15}$$

Remark that

$$\begin{aligned}\pi_i^{peh}\left(\widehat{x}(0),\widehat{E}(0)\right) &= \frac{A_i^h\left(\widehat{x}(0),\widehat{E}(0)\right)}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}\\ &= \frac{\lambda^h\frac{\partial u^h}{\partial x_i^h}+\sum_{l=1}^{n}\lambda^l\frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_i^h}}{\lambda^h\sum_{j=1}^{M}\frac{\partial u^h}{\partial x_j^h}x_j^h+\sum_{j=1}^{M}\sum_{l=1}^{n}\lambda^l\frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_j^h}x_j^h}\\ &= \pi_i^{eh}\left(x^h,E\right). \qquad \text{(A16)}\end{aligned}$$

This is the social valuation of commodity $i$ consumed by household $h$, taking into account only externalities; it is the valuation of the welfaristic planner.

**The linearization**

In order to facilitate the derivation, we start by multiplying the nominator and the denominator of expression (A5) with $(1-eE)^2$ and take into account expressions (A10) and (A11) to get

$$(1-eE)^2A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)=(1-eE)\lambda^h\frac{\partial u^h}{\partial \widehat{x}_i^h}+\sum_{l=1}^{n}\lambda^l\sum_{k=1}^{M}ex_k^l\frac{\partial u^l}{\partial \widehat{x}_k^l}\frac{\partial E}{\partial x_i^h}+\sum_{l=1}^{n}\lambda^l\frac{\partial u^l}{\partial \widehat{E}}\frac{\partial E}{\partial x_i^h} \tag{A17}$$

and

$$\begin{aligned}(1-eE)^2B^h\left(\widehat{x}(e),\widehat{E}(e)\right) &= \lambda^h\sum_{j=1}^{M}(1-eE)\frac{\partial u^h}{\partial \widehat{x}_j^h}x_j^h+\sum_{j=1}^{M}\sum_{l=1}^{n}\lambda^l\sum_{k=1}^{M}\frac{\partial u^l}{\partial \widehat{x}_k^l}ex_k^l\frac{\partial E}{\partial x_j^h}x_j^h\\ &\quad+\sum_{j=1}^{M}\sum_{l=1}^{n}\lambda^l\frac{\partial u^l}{\partial \widehat{E}}\frac{\partial E}{\partial x_j^h}x_j^h. \qquad \text{(A18)}\end{aligned}$$

Observe that the derivative of the left hand side of expression (A17) with respect to $e$ is

$$\begin{aligned}\frac{\partial\left[(1-eE)^2A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e} &= 2(1-eE)(-E)A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)\\ &\quad+(1-eE)^2\frac{\partial A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)}{\partial e},\end{aligned}$$

evaluated at $e = 0$, we get

$$\left.\frac{\partial\left[(1-eE)^2 A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e}\right|_{e=0} = -2EA_i^h\left(\widehat{x}(0),\widehat{E}(0)\right) + \left.\frac{\partial A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)}{\partial e}\right|_{e=0},$$

so

$$\left.\frac{\partial A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)}{\partial e}\right|_{e=0} = \left.\frac{\partial\left[(1-eE)^2 A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e}\right|_{e=0} + 2EA_i^h\left(\widehat{x}(0),\widehat{E}(0)\right). \tag{A19}$$

Similarly, from the derivative of the left hand side of expression (A18) with respect to $e$ we can derive

$$\left.\frac{\partial B^h\left(\widehat{x}(e),\widehat{E}(e)\right)}{\partial e}\right|_{e=0} = \left.\frac{\partial\left[(1-eE)^2 B^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e}\right|_{e=0} + 2EB^h\left(\widehat{x}(0),\widehat{E}(0)\right). \tag{A20}$$

Now we turn back to the term $T^h\left(\widehat{x}(0),\widehat{E}(0)\right)$ (the part between brackets in expression (A6)) and fill in expressions (A19) and (A20):

$$\begin{aligned}
&T^h\left(\widehat{x}(0),\widehat{E}(0)\right)\\
=\;& \frac{1}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}\left(\left.\frac{\partial\left[(1-eE)^2 A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e}\right|_{e=0} + 2EA_i^h\left(\widehat{x}(0),\widehat{E}(0)\right)\right)\\
&-\frac{A_i^h\left(\widehat{x}(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}(0),\widehat{E}(0)\right)\right)^2}\left(\left.\frac{\partial\left[(1-eE)^2 B^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e}\right|_{e=0} + 2EB^h\left(\widehat{x}(0),\widehat{E}(0)\right)\right).
\end{aligned}$$

This simplifies into

$$\begin{aligned}
T^h\left(\widehat{x}(0),\widehat{E}(0)\right) =\;& \frac{1}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}\left.\frac{\partial\left[(1-eE)^2 A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e}\right|_{e=0}\\
&-\frac{A_i^h\left(\widehat{x}(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}(0),\widehat{E}(0)\right)\right)^2}\left.\frac{\partial\left[(1-eE)^2 B^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e}\right|_{e=0}.
\end{aligned} \tag{A21}$$

Now we need expressions for $\left.\frac{\partial\left[(1-eE)^2 A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e}\right|_{e=0}$ and $\left.\frac{\partial\left[(1-eE)^2 B^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e}\right|_{e=0}$, the derivatives of the right hand sides of expressions (A17) and (A18) with respect to $e$. The derivative of expression (A17) evaluated at $e = 0$ is (remember that the marginal utilities depend on the parameter $e$)

$$\begin{aligned}
\left.\frac{\partial\left[(1-eE)^2 A_i^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e}\right|_{e=0} =\;& -\lambda^h E\frac{\partial u^h}{\partial x_i^h} + \sum_{l=1}^{n}\lambda^l\sum_{k=1}^{M}x_k^l\frac{\partial u^l}{\partial x_k^l}\frac{\partial E}{\partial x_i^h}\\
&+\lambda^h\sum_{j=1}^{M}\frac{\partial^2 u^h}{\partial x_i^h\partial x_j^h}x_j^h E + \lambda^h\frac{\partial^2 u^h}{\partial x_i^h\partial E}EE\\
&+\sum_{l=1}^{n}\lambda^l\sum_{j=1}^{M}\frac{\partial^2 u^l}{\partial E\partial x_j^l}x_j^l E\frac{\partial E}{\partial x_i^h} + \sum_{l=1}^{n}\lambda^l\frac{\partial^2 u^l}{(\partial E)^2}EE\frac{\partial E}{\partial x_i^h}.
\end{aligned} \tag{A22}$$

The derivative of expression (A18) evaluated at $e = 0$ is

$$\left.\frac{\partial\left[(1-eE)^2 B^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e}\right|_{e=0} = -\lambda^h E\sum_{j=1}^{M}\frac{\partial u^h}{\partial x_j^h}x_j^h + \sum_{j=1}^{M}\sum_{k=1}^{M}\sum_{l=1}^{n}\lambda^l\frac{\partial u^l}{\partial x_k^l}x_k^l\frac{\partial E}{\partial x_j^h}x_j^h$$
$$+\lambda^h\sum_{j=1}^{M}\sum_{r=1}^{M}\frac{\partial^2 u^h}{\partial x_j^h\partial x_r^h}x_r^h E x_j^h + \lambda^h\sum_{j=1}^{M}\frac{\partial^2 u^h}{\partial x_j^h\partial E}x_j^h EE$$
$$+\sum_{r=1}^{M}\sum_{l=1}^{n}\lambda^l\sum_{j=1}^{M}\frac{\partial^2 u^l}{\partial E\partial x_r^l}x_r^l E\frac{\partial E}{\partial x_j^h}x_j^h + \sum_{l=1}^{n}\lambda^l\sum_{j=1}^{M}\frac{\partial^2 u^l}{(\partial E)^2}EE\frac{\partial E}{\partial x_j^h}x_j^h.$$

Rearrange terms and take into account definition (A14), to get

$$\left.\frac{\partial\left[(1-eE)^2 B^h\left(\widehat{x}(e),\widehat{E}(e)\right)\right]}{\partial e}\right|_{e=0} = -\lambda^h\sum_{j=1}^{M}x_j^h\frac{\partial u^h}{\partial x_j^h}E + \sum_{l=1}^{n}\lambda^l\sum_{k=1}^{M}\frac{\partial u^l}{\partial x_k^l}x_k^l E^h$$
$$+\lambda^h\sum_{j=1}^{M}\sum_{r=1}^{M}\frac{\partial^2 u^h}{\partial x_j^h\partial x_r^h}x_r^h E x_j^h + \sum_{l=1}^{n}\lambda^l\sum_{r=1}^{M}\frac{\partial^2 u^l}{\partial E\partial x_r^l}x_r^l E E^h$$
$$+\lambda^h\sum_{j=1}^{M}\frac{\partial^2 u^h}{\partial x_j^h\partial E}x_j^h EE + \sum_{l=1}^{n}\lambda^l\frac{\partial^2 u^l}{(\partial E)^2}EEE^h. \tag{A23}$$

Now we can calculate $T^h\left(\widehat{x}(0),\widehat{E}(0)\right)$ using expressions (A22) and (A23) into (A21):

$$\begin{aligned}
T^h\left(\widehat{x}(0),\widehat{E}(0)\right) \quad = \quad & \frac{1}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}\left(\lambda^h\sum_{j=1}^{M}\frac{\partial^2 u^h}{\partial x_i^h\partial x_j^h}x_j^h E + \sum_{l=1}^{n}\lambda^l\sum_{j=1}^{M}\frac{\partial^2 u^l}{\partial E\partial x_j^l}x_j^l\frac{\partial E}{\partial x_i^h}E\right)\\
&+\frac{1}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}\left(\lambda^h\frac{\partial^2 u^h}{\partial x_i^h\partial E}EE + \sum_{l=1}^{n}\lambda^l\frac{\partial^2 u^l}{(\partial E)^2}EE\frac{\partial E}{\partial x_i^h}\right)\\
&+\frac{1}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}\left(-\lambda^h E\frac{\partial u^h}{\partial x_i^h} + \sum_{l=1}^{n}\lambda^l\sum_{k=1}^{M}x_k^l\frac{\partial u^l}{\partial x_k^l}\frac{\partial E}{\partial x_i^h}\right)\\
&-\frac{A_i^h\left(\widehat{x}(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}(0),\widehat{E}(0)\right)\right)^2}\left(\lambda^h\sum_{j=1}^{M}\sum_{r=1}^{M}\frac{\partial^2 u^h}{\partial x_j^h\partial x_r^h}x_r^h E x_j^h + \sum_{l=1}^{n}\lambda^l\sum_{r=1}^{M}\frac{\partial^2 u^l}{\partial E\partial x_r^l}x_r^l E E^h\right)\\
&-\frac{A_i^h\left(\widehat{x}(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}(0),\widehat{E}(0)\right)\right)^2}\left(\lambda^h\sum_{j=1}^{M}\frac{\partial^2 u^h}{\partial x_j^h\partial E}x_j^h EE + \sum_{l=1}^{n}\lambda^l\frac{\partial^2 u^l}{(\partial E)^2}EEE^h\right)\\
&-\frac{A_i^h\left(\widehat{x}(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}(0),\widehat{E}(0)\right)\right)^2}\left(-\lambda^h\sum_{j=1}^{M}x_j^h\frac{\partial u^h}{\partial x_j^h}E + \sum_{l=1}^{n}\lambda^l\sum_{k=1}^{M}\frac{\partial u^l}{\partial x_k^l}x_k^l E^h\right).
\end{aligned}$$

Using the results (B8) from appendix B to replace the first and the fourth line and (B9) from appendix B to replace the second and the fifth line in the expression, this can be written as

$$
\begin{aligned}
T^h\left(\widehat{x}(0),\widehat{E}(0)\right) &= \frac{1}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}\left(-E\lambda^h\frac{\partial u^h}{\partial x_i^h}+\sum_{l=1}^n\lambda^l\sum_{k=1}^M x_k^l\frac{\partial u^l}{\partial x_k^l}\frac{\partial E}{\partial x_i^h}\right)\\
&\quad-\frac{A_i^h\left(\widehat{x}(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}(0),\widehat{E}(0)\right)\right)^2}\left(-\lambda^h\sum_{j=1}^M x_j^h\frac{\partial u^h}{\partial x_j^h}E+\sum_{l=1}^n\lambda^l\sum_{k=1}^M\frac{\partial u^l}{\partial x_k^l}x_k^lE^h\right)\\
&\quad+\left(\sum_{r=1}^M\sum_{l=1}^n\frac{\partial \pi_i^{eh}}{\partial x_r^l}x_r^l+\pi_i^{eh}\right)E+\frac{\partial \pi_i^{eh}}{\partial E}EE.
\end{aligned}
$$

Now add and subtract $\frac{1}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}E\sum_{l=1}^n\lambda^l\frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_i^h}$ to get

$$
\begin{aligned}
T^h\left(\widehat{x}(0),\widehat{E}(0)\right) &= \frac{1}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}\left(-E\lambda^h\frac{\partial u^h}{\partial x_i^h}-E\sum_{l=1}^n\lambda^l\frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_i^h}\right)\\
&\quad+\frac{1}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}\left(\sum_{l=1}^n\lambda^l\sum_{k=1}^M x_k^l\frac{\partial u^l}{\partial x_k^l}\frac{\partial E}{\partial x_i^h}+E\sum_{l=1}^n\lambda^l\frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_i^h}\right)\\
&\quad-\frac{A_i^h\left(\widehat{x}(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}(0),\widehat{E}(0)\right)\right)^2}\left(-\lambda^h\sum_{j=1}^M x_j^h\frac{\partial u^h}{\partial x_j^h}E+\sum_{l=1}^n\lambda^l\sum_{k=1}^M\frac{\partial u^l}{\partial x_k^l}x_k^lE^h\right)\\
&\quad+E\pi_i^{eh}+\sum_{r=1}^M\sum_{l=1}^n\frac{\partial \pi_i^{eh}}{\partial x_r^l}x_r^lE+\frac{\partial \pi_i^{eh}}{\partial E}EE,
\end{aligned}
$$

and take into account expressions (A12) and (A13), and rearrange to get

$$
\begin{aligned}
T^h\left(\widehat{x}(0),\widehat{E}(0)\right) &= -E\pi_i^{eh}\\
&\quad+\frac{1}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}\frac{\partial E}{\partial x_i^h}\left(\sum_{l=1}^n\lambda^l\sum_{k=1}^M x_k^l\frac{\partial u^l}{\partial x_k^l}+\sum_{t=1}^n\lambda^t\frac{\partial u^t}{\partial E}E\right)\\
&\quad-\pi_i^{eh}\frac{1}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}\left(-\lambda^h\sum_{j=1}^M x_j^h\frac{\partial u^h}{\partial x_j^h}E+E^h\sum_{l=1}^n\lambda^l\sum_{k=1}^M\frac{\partial u^l}{\partial x_k^l}x_k^l\right)\\
&\quad+E\pi_i^{eh}+\sum_{r=1}^M\sum_{l=1}^n\frac{\partial \pi_i^{eh}}{\partial x_r^l}x_r^lE+\frac{\partial \pi_i^{eh}}{\partial E}EE. \qquad \text{(A24)}
\end{aligned}
$$

Now we take a closer look at the part between brackets on the second line. It measures the total effect on all households' welfare of all consumed commodities and of $E$. The term formally resembles $\sum_{l=1}^n B^l\left(\widehat{x}(0),\widehat{E}(0)\right)$ (see expression (A13)), except for the last part, which is $E$ in expression (A24) instead of $\sum_{l=1}^n E^l$. These are not necessarily equal to eachother. This might be the case for externalities with only local consequences, but not for an externality such as carbon dioxide emissions which exists at the global level. In this case, there is a difference between $\sum_{l=1}^n E^l$ and $E$. To see this consider the following derivation (take into account $\sum_{l=1}^n E^l = E^n$ in the third step, this is the total amount of emission in the economy):

$$\begin{aligned}
\sum_{l=1}^{n}\lambda^{l}\sum_{k=1}^{M}x_{k}^{l}\frac{\partial u^{l}}{\partial x_{k}^{l}}+\sum_{t=1}^{n}\lambda^{t}\frac{\partial u^{t}}{\partial E}E &= \sum_{l=1}^{n}\lambda^{l}\sum_{k=1}^{M}x_{k}^{l}\frac{\partial u^{l}}{\partial x_{k}^{l}}+\sum_{t=1}^{n}\lambda^{t}\frac{\partial u^{t}}{\partial E}\sum_{l=1}^{n}E^{l}+\sum_{t=1}^{n}\lambda^{t}\frac{\partial u^{t}}{\partial E}\left(E-\sum_{l=1}^{n}E^{l}\right)\\
&= \sum_{l=1}^{n}B^{l}\left(\widehat{x}(0),\widehat{E}(0)\right)+\sum_{t=1}^{n}\lambda^{t}\frac{\partial u^{t}}{\partial E}\left(E-\sum_{l=1}^{n}E^{l}\right)\\
&= \sum_{l=1}^{n}\left(B^{l}\left(\widehat{x}(0),\widehat{E}(0)\right)+\lambda^{l}\frac{\partial u^{l}}{\partial E}\left(E-E^{n}\right)\right).
\end{aligned}$$

The term between brackets uses a marginal utility of income concept that is different from before, it takes into account extended income, which is income including the impact of the externality realized outside the economy. To see this, divide the term between brackets by $B^{l}\left(\widehat{x}(0),\widehat{E}(0)\right)$ to get

$$\begin{aligned}
\frac{B^{l}\left(\widehat{x}(0),\widehat{E}(0)\right)+\lambda^{l}\frac{\partial u^{l}}{\partial E}\left(E-E^{n}\right)}{B^{l}\left(\widehat{x}(0),\widehat{E}(0)\right)} &= 1+\frac{\lambda^{l}\frac{\partial u^{l}}{\partial E}}{B^{l}\left(\widehat{x}(0),\widehat{E}(0)\right)}\left(E-E^{n}\right)\\
&= 1+\pi_{E}^{l}\left(E-E^{n}\right).
\end{aligned}$$

The term $\pi_{E}^{l}\equiv\frac{\lambda^{l}\frac{\partial u^{l}}{\partial E}}{B^{l}\left(\widehat{x}(0),\widehat{E}(0)\right)}$ measures which percentage of household $l$'s income the planner is willing to give up if the household has to suffer an extra unit of $E$. The fact that there is emission other than the emission caused by the households in the economy, $(E-E^{n})$, is taken into account by the planner as if the household has less income at its disposal. Or to put it in another way: the externality is equivalent to an amount of negative income. This is consistent with the theory of public good provision in which the effort of other individuals is seen as extra income for the household(s) (Cornes and Sandler 1996). Observe that for carbon dioxide emissions, $E$ is numerically a lot bigger than $E^{n}$ which has rather big consequences for the calculation of the MCF, as is apparent in the empirical section.

Now define

$$B\left(\widehat{x}(0),\widehat{E}(0)\right)=\sum_{l=1}^{n}\lambda^{l}\left(\sum_{k=1}^{M}\frac{\partial u^{l}}{\partial x_{k}^{l}}x_{k}^{l}+\frac{\partial u^{l}}{\partial E}E\right) \tag{A25}$$

which allows us to write expression (A24) as

$$\begin{aligned}
T^{h}\left(\widehat{x}(0),\widehat{E}(0)\right) &= -E\pi_{i}^{eh}+\frac{B\left(\widehat{x}(0),\widehat{E}(0)\right)}{B^{h}\left(\widehat{x}(0),\widehat{E}(0)\right)}\frac{\partial E}{\partial x_{i}^{h}}\\
&\quad -\pi_{i}^{eh}\frac{1}{B^{h}\left(\widehat{x}(0),\widehat{E}(0)\right)}\left(-\lambda^{h}\sum_{j=1}^{M}x_{j}^{h}\frac{\partial u^{h}}{\partial x_{j}^{h}}E+E^{h}\sum_{l=1}^{n}\lambda^{l}\sum_{k=1}^{M}\frac{\partial u^{l}}{\partial x_{k}^{l}}x_{k}^{l}\right)\\
&\quad +E\pi_{i}^{eh}+\sum_{r=1}^{M}\sum_{l=1}^{n}\frac{\partial\pi_{i}^{eh}}{\partial x_{r}^{l}}x_{r}^{l}E+\frac{\partial\pi_{i}^{eh}}{\partial E}EE.
\end{aligned} \tag{A26}$$

After adding and subtracting $\pi_{i}^{eh}\frac{1}{B^{h}\left(\widehat{x}(0),\widehat{E}(0)\right)}\left(E\sum_{l=1}^{n}\lambda^{l}\frac{\partial u^{l}}{\partial E}E^{h}\right)$, the second line in this expression can be written as

$$
\begin{aligned}
& -\pi_i^{eh} \frac{1}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)} \left(-\lambda^h \sum_{j=1}^M x_j^h \frac{\partial u^h}{\partial x_j^h} E + E^h \sum_{l=1}^n \lambda^l \sum_{k=1}^M \frac{\partial u^l}{\partial x_k^l} x_k^l\right) \\
= & -\pi_i^{eh} \frac{1}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)} \left(-\lambda^h \sum_{j=1}^M x_j^h \frac{\partial u^h}{\partial x_j^h} E - E \sum_{l=1}^n \lambda^l \frac{\partial u^l}{\partial E} E^h\right) \\
& -\pi_i^{eh} \frac{1}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)} \left(E^h \sum_{l=1}^n \lambda^l \sum_{k=1}^M \frac{\partial u^l}{\partial x_k^l} x_k^l + E^h \sum_{t=1}^n \lambda^t \frac{\partial u^t}{\partial E} E\right)
\end{aligned}
$$

and take into account expressions (A13) and (A25) to get

$$
= \pi_i^{eh} E - \pi_i^{eh} E^h \frac{B\left(\widehat{x}(0), \widehat{E}(0)\right)}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)}. \tag{A27}
$$

Now fill in expression (A27) in expression (A26):

$$
\begin{aligned}
T^h\left(\widehat{x}(0), \widehat{E}(0)\right) = & -E\pi_i^{eh} + \frac{B\left(\widehat{x}(0), \widehat{E}(0)\right)}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)} \frac{\partial E}{\partial x_i^h} \\
& +\pi_i^{eh} E - \pi_i^{eh} E^h \frac{B\left(\widehat{x}(0), \widehat{E}(0)\right)}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)} \\
& +E\pi_i^{eh} + \sum_{r=1}^M \sum_{l=1}^n \frac{\partial \pi_i^{eh}}{\partial x_r^l} x_r^l E + \frac{\partial \pi_i^{eh}}{\partial E} EE.
\end{aligned}
$$

$$
\begin{aligned}
T^h\left(\widehat{x}(0), \widehat{E}(0)\right) = & \frac{B\left(\widehat{x}(0), \widehat{E}(0)\right)}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)} \left(\frac{\partial E}{\partial x_i^h} - \pi_i^{eh} E^h\right) + E\pi_i^{eh} \\
& + \sum_{r=1}^M \sum_{l=1}^n \frac{\partial \pi_i^{eh}}{\partial x_r^l} x_r^l E + \frac{\partial \pi_i^{eh}}{\partial E} EE.
\end{aligned} \tag{A28}
$$

Define

$$
b^h = \frac{B\left(\widehat{x}(0), \widehat{E}(0)\right)}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)}, \tag{A29}
$$

the ratio of the impact on social welfare of a percentage increase in all households' incomes and the impact on social welfare of a percentage increase in household $h$'s income. It measures the percentage of income of household $h$ the planner is willing to give up for a percentage increase in all households' incomes. We get for expression (A28)

$$
T^h\left(\widehat{x}(0), \widehat{E}(0)\right) = b^h \left(\frac{\partial E}{\partial x_i^h} - \pi_i^{eh} E^h\right) + E\pi_i^{eh} + \sum_{r=1}^M \sum_{l=1}^n \frac{\partial \pi_i^{eh}}{\partial x_r^l} x_r^l E + \frac{\partial \pi_i^{eh}}{\partial E} EE,
$$

and take into account that the scale effect on $\pi_i^{eh}$ of a change of all commodities and $E$ is

$$s_i^{eh} = \sum_{r=1}^{M}\sum_{l=1}^{n} \frac{\partial \pi_i^{eh}}{\partial x_r^l} x_r^l + \frac{\partial \pi_i^{eh}}{\partial E} E,$$

which measures how much $\pi_i^{eh}$ changes if the household is put on a different utility level. This implies for our derivation of $T^h\left(\widehat{x}\left(0\right), \widehat{E}\left(0\right)\right)$

$$T^h\left(\widehat{x}\left(0\right), \widehat{E}\left(0\right)\right) = b^h\left(\frac{\partial E}{\partial x_i^h} - \pi_i^{eh} E^h\right) + E\pi_i^{eh} + s_i^{eh} E. \tag{A30}$$

**Derivation of $\pi_i^{peh}\left(\widehat{x}\left(e\right), \widehat{E}\left(e\right)\right)$**

Now we are ready to turn back to the linearization (expression (A6)). Using expression (A30) and taking into account expression (A16) we have

$$\pi_i^{peh}\left(\widehat{x}\left(e\right), \widehat{E}\left(e\right)\right) \approx \pi_i^{eh} + e\left(b^h\left(\frac{\partial E}{\partial x_i^h} - \pi_i^{eh} E^h\right) + E\pi_i^{eh} + s_i^{eh} E\right)$$

$$\pi_i^{peh}\left(\widehat{x}\left(e\right), \widehat{E}\left(e\right)\right) \approx \pi_i^{eh}\left(1 + e\left(E - E^h b^h\right)\right) + e\frac{\partial E}{\partial x_i^h} b^h + s_i^{eh} eE. \tag{A31}$$

**APPENDIX B: properties of the household's marginal willingness to pay for private goods and the externality.**

**Relationship between household h's valuation for commodity i and the planner's valuation of $\mathbf{x}_i^h$.**

Remember expression (3):

$$\pi_i^{eh}\left(x, E\right) = \frac{\lambda^h \frac{\partial u^h}{\partial x_i^h} + \sum_{l=1}^{n} \lambda^l \frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_i^h}}{\lambda^h \sum_{j=1}^{M} \frac{\partial u^h}{\partial x_j^h} x_j^h + \sum_{j=1}^{M}\sum_{l=1}^{n} \lambda^l \frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_j^h} x_j^h} = \frac{A_i^h\left(\widehat{x}\left(0\right), \widehat{E}\left(0\right)\right)}{B^h\left(\widehat{x}\left(0\right), \widehat{E}\left(0\right)\right)}, \tag{B1}$$

where we used expressions (A12) and (A13) to establish the second equality. First consider the part of the numerator of expression (B1) only having to do with household $h$ and divide and multiply it with $D^h\left(\widehat{x}^h\left(0\right), \widehat{E}\left(0\right)\right)$ from expression (A15):

$$\begin{aligned}
\pi_{1i}^{eh}\left(x, E\right) &\equiv \frac{\lambda^h \frac{\partial u^h}{\partial x_i^h} + \lambda^h \frac{\partial u^h}{\partial E}\frac{\partial E}{\partial x_i^h}}{\lambda^h \sum_{j=1}^{M} \frac{\partial u^h}{\partial x_j^h} x_j^h + \sum_{j=1}^{M}\sum_{l=1}^{n} \lambda^l \frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_j^h} x_j^h} \\
&= \frac{\lambda^h\left(\frac{\partial u^h}{\partial x_i^h} + \frac{\partial u^h}{\partial E}\frac{\partial E}{\partial x_i^h}\right)}{\sum_{j=1}^{M} \frac{\partial u^h}{\partial x_j^h} x_j^h + \sum_{j=1}^{M} \frac{\partial u^h}{\partial E}\frac{\partial E}{\partial x_j^h} x_j^h} \frac{\sum_{j=1}^{M} \frac{\partial u^h}{\partial x_j^h} x_j^h + \sum_{j=1}^{M} \frac{\partial u^h}{\partial E}\frac{\partial E}{\partial x_j^h} x_j^h}{\lambda^h \sum_{j=1}^{M} \frac{\partial u^h}{\partial x_j^h} x_j^h + \sum_{j=1}^{M}\sum_{l=1}^{n} \lambda^l \frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_j^h} x_j^h} \\
&= \lambda^h \frac{\widetilde{A}_i^h\left(\widehat{x}^h\left(0\right), \widehat{E}\left(0\right)\right)}{D^h\left(\widehat{x}^h\left(0\right), \widehat{E}\left(0\right)\right)} \frac{D^h\left(\widehat{x}^h\left(0\right), \widehat{E}\left(0\right)\right)}{B^h\left(\widehat{x}\left(0\right), \widehat{E}\left(0\right)\right)}.
\end{aligned}$$

Now note that $\frac{\widetilde{A}_i^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}{D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}$ is household $h$'s normalized willingness to pay for the commodity, this is $\pi_i^h$ in expression (2). The second part, $\frac{D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)}$, is the ratio of the private and the social marginal utility of one percent of income of household $h$. Call this $\gamma^h$:

$$\gamma^h = \frac{D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)}.$$

This implies that we can write

$$\pi_{1i}^{eh}(x, E) = \lambda^h \frac{\widetilde{A}_i^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}{D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)} \frac{D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)} = \pi_i^h \lambda^h \gamma^h. \tag{B2}$$

Second, consider the part of expression (B1) for the other households, $l \neq h$ (the part of the expression not considered in expression (B2)):

$$\pi_{2i}^{eh}(x, E) \equiv \frac{\sum_{\substack{l=1 \\ l\neq h}}^n \lambda^l \frac{\partial u^l}{\partial E}}{\lambda^h \sum_{j=1}^M \frac{\partial u^h}{\partial x_j^h} x_j^h + \sum_{j=1}^M \sum_{l=1}^n \lambda^l \frac{\partial u^l}{\partial E} \frac{\partial E}{\partial x_j^h} x_j^h} \frac{\partial E}{\partial x_i^h}.$$

Now write the numerator of this expression as:

$$\sum_{\substack{l=1 \\ l\neq h}}^n \lambda^l \frac{\partial u^l}{\partial E} = \sum_{l=1}^n \lambda^l \frac{\partial u^l}{\partial E} - \lambda^h \frac{\partial u^h}{\partial E}$$

and define

$$\Pi_E^h = \frac{\sum_{l=1}^n \lambda^l \frac{\partial u^l}{\partial E}}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)}, \tag{B3}$$

the social valuation of the consequence of an increase in $E$ for all households, relative to the social value of a percent of household $h$'s income, and

$$\pi_E^h = \frac{\lambda^h \frac{\partial u^h}{\partial E}}{B^h\left(\widehat{x}(0), \widehat{E}(0)\right)}, \tag{B4}$$

the social valuation of the consequence of an increase in $E$ only for household $h$, relative to the social value of a percent of household $h$'s income. As a result

$$\pi_{2i}^{eh}(x, E) = \left(\Pi_E^h - \pi_E^h\right) \frac{\partial E}{\partial x_i^h}$$

and, using expressions (B2), (B3) and (B4), expression (B1) can now be written as

$$\pi_i^{eh} = \pi_{1i}^{eh}(x, E) + \pi_{2i}^{eh}(x, E) = \pi_i^h \lambda^h \gamma^h + \left(\Pi_E^h - \pi_E^h\right) \frac{\partial E}{\partial x_i^{eh}}. \tag{B5}$$

Remark that in expressoin (B5) $\pi_E^h \frac{\partial E}{\partial x_i^{eh}}$ has to be subtracted because the household values it already.

**Derivation of the scale effects**

The scale effects provide information on how the normalized valuation of commodity $i$ by household $h$ (taking into account the externality), $\pi_i^{eh}$, changes when all commodity consumption levels (of all households) and $E$ change; we need an expression for $\sum_{l=1}^n \sum_{r=1}^M \frac{\partial \pi_i^{eh}}{\partial x_r^l} x_r^l$ and $\frac{\partial \pi_i^{eh}}{\partial E} E$. In order to derive the former, we need expressions for $\frac{\partial \pi_i^{eh}}{\partial x_r^h}$ and $\frac{\partial \pi_i^{eh}}{\partial x_r^t}$. From expression (B1) and assuming only linear effects of commodity consumption on $E$, so $\frac{\partial^2 E}{\partial x_i^h \partial x_r^h} = 0 \; \forall r$:

$$\begin{aligned}
\frac{\partial \pi_i^{eh}}{\partial x_r^h} &= \frac{1}{B^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}\left(\lambda^h \frac{\partial^2 u^h}{\partial x_i^h \partial x_r^h} + \lambda^h \frac{\partial^2 u^h}{\partial E \partial x_r^h}\frac{\partial E}{\partial x_i^h}\right) \\
&\quad - \frac{A_i^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)\right)^2}\left(\sum_{j=1}^{M} \lambda^h \frac{\partial^2 u^h}{\partial x_j^h \partial x_r^h} x_j^h + \lambda^h \frac{\partial^2 u^h}{\partial E \partial x_r^h} E^h\right) \\
&\quad - \frac{A_i^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)\right)^2}\left(\lambda^h \frac{\partial u^h}{\partial x_r^h} + \sum_{l=1}^{n} \lambda^l \frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_r^h}\right)
\end{aligned} \tag{B6}$$

and, for $t \neq h$,

$$\begin{aligned}
\frac{\partial \pi_i^{eh}}{\partial x_r^t} &= \frac{1}{B^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}\left(\lambda^t \frac{\partial^2 u^t}{\partial E \partial x_r^t}\frac{\partial E}{\partial x_i^h}\right) \\
&\quad - \frac{A_i^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)\right)^2}\left(\lambda^t \frac{\partial^2 u^t}{\partial E \partial x_r^t} E^h\right)
\end{aligned} \tag{B7}$$

Now multiply expression (B6) with $x_r^h$ to get

$$\begin{aligned}
\frac{\partial \pi_i^{eh}}{\partial x_r^h} x_r^h &= \frac{1}{B^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}\left(\lambda^h \frac{\partial^2 u^h}{\partial x_i^h \partial x_r^h} x_r^h + \lambda^h \frac{\partial^2 u^h}{\partial E \partial x_r^h}\frac{\partial E}{\partial x_i^h} x_r^h\right) \\
&\quad - \frac{A_i^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)\right)^2}\left(\sum_{j=1}^{M} \lambda^h \frac{\partial^2 u^h}{\partial x_j^h \partial x_r^h} x_j^h x_r^h + \lambda^h \frac{\partial^2 u^h}{\partial E \partial x_r^h} E^h x_r^h\right) \\
&\quad - \frac{A_i^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)\right)^2}\left(\lambda^h \frac{\partial u^h}{\partial x_r^h} x_r^h + \sum_{l=1}^{n} \lambda^l \frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_r^h} x_r^h\right)
\end{aligned}$$

and expression (B7) with $x_r^t$ to get

$$\begin{aligned}
\frac{\partial \pi_i^{eh}}{\partial x_r^t} x_r^t &= \frac{1}{B^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}\left(\lambda^t \frac{\partial^2 u^t}{\partial E \partial x_r^t}\frac{\partial E}{\partial x_i^h} x_r^t\right) \\
&\quad - \frac{A_i^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)\right)^2}\left(\lambda^t \frac{\partial^2 u^t}{\partial E \partial x_r^t} E^h x_r^t\right).
\end{aligned}$$

Now take the sum over all households to get an expression for the impact on $\pi_i^{eh}$ of a percentage increase of the consumption of commodity $r$ by all households:

$$\begin{aligned}\sum_{l=1}^{n}\frac{\partial\pi_i^{eh}}{\partial x_r^l}x_r^l &= \frac{1}{B^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)}\left(\lambda^h\frac{\partial^2 u^h}{\partial x_i^h\partial x_r^h}x_r^h+\sum_{l=1}^{n}\lambda^l\frac{\partial^2 u^l}{\partial E\partial x_r^l}\frac{\partial E}{\partial x_i^h}x_r^l\right)\\ &-\frac{A_i^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)\right)^2}\left(\sum_{j=1}^{M}\lambda^h\frac{\partial^2 u^h}{\partial x_j^h\partial x_r^h}x_j^h x_r^h+\sum_{l=1}^{n}\lambda^l\frac{\partial^2 u^l}{\partial E\partial x_r^l}E^h x_r^l\right)\\ &-\frac{A_i^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)\right)^2}\left(\lambda^h\frac{\partial u^h}{\partial x_r^h}x_r^h+\sum_{l=1}^{n}\lambda^l\frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_r^h}x_r^h\right).\end{aligned}$$

Finally take the sum over all commodities:

$$\begin{aligned}\sum_{r=1}^{M}\sum_{l=1}^{n}\frac{\partial\pi_i^{eh}}{\partial x_r^l}x_r^l &= \frac{1}{B^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)}\left(\lambda^h\sum_{r=1}^{M}\frac{\partial^2 u^h}{\partial x_i^h\partial x_r^h}x_r^h+\sum_{r=1}^{M}\sum_{l=1}^{n}\lambda^l\frac{\partial^2 u^l}{\partial E\partial x_r^l}x_r^l\frac{\partial E}{\partial x_i^h}\right)\\ &-\frac{A_i^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)\right)^2}\left(\lambda^h\sum_{r=1}^{M}\sum_{j=1}^{M}\frac{\partial^2 u^h}{\partial x_j^h\partial x_r^h}x_j^h x_r^h+\sum_{l=1}^{n}\lambda^l\sum_{r=1}^{M}\frac{\partial^2 u^l}{\partial E\partial x_r^l}x_r^l E^h\right)\\ &-\frac{A_i^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)\right)^2}\left(\lambda^h\sum_{r=1}^{M}\frac{\partial u^h}{\partial x_r^h}x_r^h+\sum_{l=1}^{n}\lambda^l\sum_{r=1}^{M}\frac{\partial u^l}{\partial E}\frac{\partial E}{\partial x_r^h}x_r^h\right).\end{aligned}$$

The part in brackets in the last line is equal to $B^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)$ (expression (A13)), and take into account expression (A16) to get

$$\begin{aligned}&\sum_{r=1}^{M}\sum_{l=1}^{n}\frac{\partial\pi_i^{eh}}{\partial x_r^l}x_r^l+\pi_i^{eh}\\ =&\ \frac{1}{B^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)}\left(\lambda^h\sum_{r=1}^{M}\frac{\partial^2 u^h}{\partial x_i^h\partial x_r^h}x_r^h+\sum_{r=1}^{M}\sum_{l=1}^{n}\lambda^l\frac{\partial^2 u^l}{\partial E\partial x_r^l}x_r^l\frac{\partial E}{\partial x_i^h}\right)\\ &-\frac{A_i^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)\right)^2}\left(\lambda^h\sum_{r=1}^{M}\sum_{j=1}^{M}\frac{\partial^2 u^h}{\partial x_j^h\partial x_r^h}x_j^h x_r^h+\sum_{l=1}^{n}\lambda^l\sum_{r=1}^{M}\frac{\partial^2 u^l}{\partial E\partial x_r^l}x_r^l E^h\right).\end{aligned}\tag{B8}$$

For the part in $E$ we have

$$\begin{aligned}\frac{\partial\pi_i^{eh}}{\partial E}E &= \frac{1}{B^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)}\left(\lambda^h\frac{\partial^2 u^h}{\partial x_j^h\partial E}E+\sum_{l=1}^{n}\lambda^l\frac{\partial^2 u^l}{(\partial E)^2}\frac{\partial E}{\partial x_i^h}E\right)\\ &-\frac{A_i^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)}{\left(B^h\left(\widehat{x}^h(0),\widehat{E}(0)\right)\right)^2}\left(\lambda^h\sum_{j=1}^{M}\frac{\partial^2 u^h}{\partial x_j^h\partial E}x_j^h E+\sum_{l=1}^{n}\lambda^l\frac{\partial^2 u^l}{(\partial E)^2}E^h E\right).\end{aligned}\tag{B9}$$

Expressions (B8) and (B9) are used in the derivation of appendix A.

**APPENDIX C: derivation of the MCF$_i$ formulae.**

**Derivation of the numerator of the MCF$_i$ formulae**
Social welfare is defined as (see expression (11))

$$\begin{aligned} W(t) &= \sum_{l=1}^{n} \lambda^l u^{pl}\left(x^l(t); E\right) \\ &= \sum_{l=1}^{n} \lambda^l u^l\left(\widehat{x}^l(t); \widehat{E}\right). \end{aligned}$$

The numerator of the $MCF_i$ formula is the derivative of $W(t)$ with respect to $t_i$:

$$\begin{aligned} \frac{\partial W(t)}{\partial t_i} &= \sum_{l=1}^{n} \lambda^l \frac{\partial u^l\left(\widehat{x}^l(t); \widehat{E}\right)}{\partial t_i} \\ &= \sum_{l=1}^{n}\sum_{h=1}^{n}\sum_{j=1}^{M} \lambda^l \frac{\partial u^l\left(\widehat{x}^l(t); \widehat{E}\right)}{\partial x_j^h}\frac{\partial x_j^h}{\partial t_i} \\ &= \sum_{h=1}^{n}\sum_{j=1}^{M}\left[\sum_{l=1}^{n}\lambda^l\left(\sum_{k=1}^{M}\frac{\partial u^l\left(\widehat{x}^l(t); \widehat{E}\right)}{\partial \widehat{x}_k^l}\frac{\partial \widehat{x}_k^l}{\partial x_j^h} + \frac{\partial u^l\left(\widehat{x}^l(t); \widehat{E}\right)}{\partial \widehat{E}}\frac{\partial \widehat{E}}{\partial x_j^h}\right)\right]\frac{\partial x_j^h}{\partial q_i}. \end{aligned}$$

In the last step we use the assumption that $\frac{\partial x_j^h}{\partial t_i} = \frac{\partial x_j^h}{\partial q_i}$. Observe that the term between square brackets is $A_j^h\left(\widehat{x}(e), \widehat{E}(e)\right)$ (see expression (A3) of appendix A). Now divide and multiply this expression with $B^h\left(\widehat{x}(e), \widehat{E}(e)\right)$ (expression (A4) of appendix A) and rearrange terms to get (taking into account expression (A5))

$$\begin{aligned} \frac{\partial W(t)}{\partial t_i} &= \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\sum_{j=1}^{M}\left[\frac{A_j^h\left(\widehat{x}(e), \widehat{E}(e)\right)}{B^h\left(\widehat{x}(e), \widehat{E}(e)\right)}\right]\frac{\partial x_j^h}{\partial q_i} \\ &= \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\sum_{j=1}^{M}\pi_j^{peh}\left(\widehat{x}(e), \widehat{E}(e)\right)\frac{\partial x_j^h}{\partial q_i}. \end{aligned} \tag{C1}$$

Now we take into account expression (A31) derived in appendix A:

$$\begin{aligned} \frac{\partial W(t)}{\partial t_i} &= \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\sum_{j=1}^{M}\left[\pi_j^{eh}\left(1+e\left(E-E^h b^h\right)\right) + e\frac{\partial E}{\partial x_j^h}b^h + s_j^{eh}eE\right]\frac{\partial x_j^h}{\partial q_i} \\ \frac{\partial W(t)}{\partial t_i} &= \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\sum_{j=1}^{M}\left(1+e\left(E-E^h b^h\right)\right)\pi_j^{eh}\frac{\partial x_j^h}{\partial q_i} \\ &\quad + \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\sum_{j=1}^{M}\left[e\frac{\partial E}{\partial x_j^h}b^h + s_j^{eh}eE\right]\frac{\partial x_j^h}{\partial q_i}. \end{aligned}$$

Taking into account expression (B5) from appendix B, we get

$$\begin{aligned}\frac{\partial W(t)}{\partial t_i} &= \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right) \sum_{j=1}^{M} \left(1+e\left(E-E^h b^h\right)\right) \left(\pi_j^h \lambda^h \gamma^h + \left(\Pi_E^h - \pi_E^h\right) \frac{\partial E}{\partial x_j^h}\right) \frac{\partial x_j^h}{\partial q_i} \\ &+ \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right) \sum_{j=1}^{M} \left[e \frac{\partial E}{\partial x_j^h} b^h + s_j^{eh} e E\right] \frac{\partial x_j^h}{\partial q_i} \\ \frac{\partial W(t)}{\partial t_i} &= \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right) \lambda^h \gamma^h \left(1+e\left(E-E^h b^h\right)\right) \sum_{j=1}^{M} \pi_j^h \frac{\partial x_j^h}{\partial q_i} \\ &+ \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right) \left(\left(1+e\left(E-E^h b^h\right)\right)\left(\Pi_E^h - \pi_E^h\right) + e b^h\right) \sum_{j=1}^{M} \frac{\partial E}{\partial x_j^h} \frac{\partial x_j^h}{\partial q_i} \\ &+ \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right) e E \sum_{j=1}^{M} s_j^{eh} \frac{\partial x_j^h}{\partial q_i}.\end{aligned}$$

Now use the identity[27] $\sum_{j=1}^{M} \pi_j^h \frac{\partial x_j^h}{\partial q_i} = -\frac{x_i^h}{m^h}$ to get

$$\begin{aligned}\frac{\partial W(t)}{\partial t_i} &= -\sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right) \lambda^h \gamma^h \left(1+e\left(E-E^h b^h\right)\right) \frac{x_i^h}{m^h} \\ &+ \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right) \left(\left(1+e\left(E-E^h b^h\right)\right)\left(\Pi_E^h - \pi_E^h\right) + e b^h\right) \sum_{j=1}^{M} \frac{\partial E}{\partial x_j^h} \frac{\partial x_j^h}{\partial q_i} \\ &+ \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right) e E \sum_{j=1}^{M} s_j^{eh} \frac{\partial x_j^h}{\partial q_i}.\end{aligned}$$

Multiply this expression by $q_i$ and transform derivatives into elasticities; $\varepsilon_{ji}^h = \frac{\partial x_j^h}{\partial q_i} \frac{q_i}{x_j^h}$. Take into account $\frac{\partial E}{\partial x_j^h} \frac{\partial x_j^h}{\partial q_i} q_i = r_j^h \frac{\partial\left(q_j x_j^h\right)}{\partial x_j^h} \frac{\partial x_j^h}{\partial q_i} q_i$ with $r_j^h = \frac{\partial E}{\partial\left(q_j x_j^h\right)}$. Also take into account that $\frac{\partial\left(q_j x_j^h\right)}{\partial x_j^h} \frac{\partial x_j^h}{\partial q_i} q_i = q_j \frac{\partial x_j^h}{\partial q_i} \frac{q_i}{x_j^h} x_j^h = \varepsilon_{ji}^h x_j^h q_j$ and $\sigma_j^{eh} = \frac{s_j^{eh}}{\pi_j^{eh}} = \sum_{r=1}^{M} \sum_{l=1}^{n} \frac{\partial \pi_i^h}{\partial x_r^l} \frac{x_r^l}{\pi_j^{eh}} + \frac{\partial \pi_i^h}{\partial E} \frac{E}{\pi_j^{eh}}$, implying $s_j^{eh} \frac{\partial x_j^h}{\partial q_i} q_i = \sigma_j^{eh} \pi_j^{eh} \varepsilon_{ji}^h x_j^h$. This yields

$$q_i \sum_{j=1}^{M} \frac{\partial E}{\partial x_j^h} \frac{\partial x_j^h}{\partial q_i} = \sum_{j=1}^{M} r_j^h \varepsilon_{ji}^h x_j^h q_j,$$

the impact on $E^h$ of a change in the tax on commodity $i$, and

$$q_i \sum_{j=1}^{M} s_j^{eh} \frac{\partial x_j^h}{\partial q_i} = \sum_{j=1}^{M} \sigma_j^{eh} \pi_j^{eh} \varepsilon_{ji}^h x_j^h,$$

the amount all normalized valuations of commodities by household $h$ change due to the increase in tax on commodity $i$. Use these to get

[27] Start from the identity $\sum_{j=1}^{M} q_j x_j^h = m^h$. Take the derivative with respect to $q_i$ to both sides to get $\sum_{j=1}^{M} q_j \frac{\partial x_j^h}{\partial q_i} + x_i^h = 0$. Now divide by $m^h$ and rearrange to get the identity $\sum_{j=1}^{M} \pi_j^h \frac{\partial x_j^h}{\partial q_i} = -\frac{x_i^h}{m^h}$.

$$\begin{aligned}
\frac{\partial W(t)}{\partial t_i} q_i &= -\sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\lambda^h\gamma^h\left(1+e\left(E-E^h b^h\right)\right)\frac{x_i^h q_i}{m^h} \\
&+\sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\left(\left(1+e\left(E-E^h b^h\right)\right)\left(\Pi_E^h-\pi_E^h\right)+eb^h\right)\sum_{j=1}^{M} r_j^h \varepsilon_{ji}^h x_j^h q_j \\
&+\sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right) eE\sum_{j=1}^{M}\sigma_j^{eh}\pi_j^{eh}\varepsilon_{ji}^h x_j^h.
\end{aligned}$$

Now take into account expression (B5) in the last line of this expression to get

$$\begin{aligned}
\frac{\partial W(t)}{\partial t_i} q_i &= -\sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\lambda^h\gamma^h\left(1+e\left(E-E^h b^h\right)\right)\frac{x_i^h q_i}{m^h} \\
&+\sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\left(\left(1+e\left(E-E^h b^h\right)\right)\left(\Pi_E^h-\pi_E^h\right)+eb^h\right)\sum_{j=1}^{M} r_j^h \varepsilon_{ji}^h x_j^h q_j \\
&+\sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right) eE\sum_{j=1}^{M}\sigma_j^{eh}\left(\pi_j^h\lambda^h\gamma^h+\left(\Pi_E^h-\pi_E^h\right)\frac{\partial E}{\partial x_j^h}\right)\varepsilon_{ji}^h x_j^h
\end{aligned}$$

and rearrange the last line to get

$$\begin{aligned}
\frac{\partial W(t)}{\partial t_i} q_i &= -\sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\lambda^h\gamma^h\left(1+e\left(E-E^h b^h\right)\right)\frac{x_i^h q_i}{m^h} \\
&+\sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\left(\left(1+e\left(E-E^h b^h\right)\right)\left(\Pi_E^h-\pi_E^h\right)+eb^h\right)\sum_{j=1}^{M} r_j^h \varepsilon_{ji}^h x_j^h q_j \\
&+\sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\lambda^h\gamma^h eE\sum_{j=1}^{M}\sigma_j^{eh}\pi_j^h\varepsilon_{ji}^h x_j^h \\
&+\sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right) eE\left(\Pi_E^h-\pi_E^h\right)\sum_{j=1}^{M}\sigma_j^{eh}\varepsilon_{ji}^h\frac{\partial E}{\partial x_j^h}x_j^h.
\end{aligned}$$

Now remember that

$$\begin{aligned}
B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\lambda^h\gamma^h &= B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\lambda^h\frac{D^h\left(\widehat{x}^h(e), \widehat{E}(e)\right)}{B^h\left(\widehat{x}(e), \widehat{E}(e)\right)} \\
&= \lambda^h D^h\left(\widehat{x}^h(e), \widehat{E}(e)\right),
\end{aligned}$$

this is $\lambda^h$ times household $h$'s own marginal utility of income multiplied with its income. Furthermore take into account that $\frac{\partial E}{\partial x_j^h}x_j^h = r_j^h q_j x_j^h$. This brings us to the final result

$$
\begin{aligned}
\frac{\partial W(t)}{\partial t_i} q_i &= -\sum_{h=1}^{n} \lambda^h D^h\left(\widehat{x}^h(e), \widehat{E}(e)\right)\left(1+e\left(E-E^h b^h\right)\right) \frac{x_i^h q_i}{m^h} \\
&+ \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right)\left(\left(1+e\left(E-E^h b^h\right)\right)\left(\Pi_E^h-\pi_E^h\right)+e b^h\right) \sum_{j=1}^{M} r_j^h \varepsilon_{ji}^h x_j^h q_j \\
&+ \sum_{h=1}^{n} \lambda^h D^h\left(\widehat{x}^h(e), \widehat{E}(e)\right) e E \sum_{j=1}^{M} \sigma_j^{eh} \varepsilon_{ji}^h \pi_j^h x_j^h \\
&+ \sum_{h=1}^{n} B^h\left(\widehat{x}(e), \widehat{E}(e)\right) e E\left(\Pi_E^h-\pi_E^h\right) \sum_{j=1}^{M} \sigma_j^{eh} \varepsilon_{ji}^h r_j^h q_j x_j^h . \qquad \text{(C2)}
\end{aligned}
$$

**Giving empirical content to the expressions**

Observe that the terms $\lambda^h D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)$ and $B^h\left(\widehat{x}(0), \widehat{E}(0)\right)$ (expressions (A15) and (A13) respectively) have a lot in common. We repeat them for convenience:

$$
D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)=\sum_{j=1}^{M} \frac{\partial u^h}{\partial x_j^h} x_j^h+\frac{\partial u^h}{\partial E} E^h
$$

and

$$
B^h\left(\widehat{x}(0), \widehat{E}(0)\right)=\lambda^h \sum_{j=1}^{M} \frac{\partial u^h}{\partial x_j^h} x_j^h+\sum_{l=1}^{n} \lambda^l \frac{\partial u^l}{\partial E} E^h .
$$

To see the link between the two more clearly, multiply $D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)$ with $\lambda^h$ to get

$$
\lambda^h D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)=\lambda^h \sum_{j=1}^{M} \frac{\partial u^h}{\partial x_j^h} x_j^h+\lambda^h \frac{\partial u^h}{\partial E} E^h,
$$

which implies that we can write $B^h\left(\widehat{x}(e), \widehat{E}(e)\right)$ as

$$
\begin{aligned}
B^h\left(\widehat{x}(0), \widehat{E}(0)\right) &= \lambda^h \sum_{j=1}^{M} \frac{\partial u^h}{\partial x_j^h} x_j^h+\lambda^h \frac{\partial u^h}{\partial E} E^h+\sum_{\substack{l=1 \\ l \neq h}}^{n} \lambda^l \frac{\partial u^l}{\partial E} E^h \\
&= \lambda^h D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)+\sum_{\substack{l=1 \\ l \neq h}}^{n} \lambda^l \frac{\partial u^l}{\partial E} E^h .
\end{aligned}
$$

It is clear that $B^h\left(\widehat{x}(0), \widehat{E}(0)\right)<\lambda^h D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)$ because $\frac{\partial u^l}{\partial E}<0\ \forall l$. The difference between the two depends on the valuation of $E^h$ by other households and the welfare weights. In the empirical section of the paper we assume that $D^h\left(\widehat{x}^h(0), \widehat{E}(0)\right)=m^h$ (which means that $\frac{\mu^h}{m^h}$, the marginal utility of one euro of income, equals 1)[28] and calculate $B^h\left(\widehat{x}(0), \widehat{E}(0)\right)$ as follows:

[28] To see this, remark that expression (2) in the text is the result of the maximization of lagrangian $L=u^h\left(x^h ; E\right)-$

$$B^h\left(\widehat{x}(0),\widehat{E}(0)\right)=\lambda^h m^h+\sum_{\substack{l=1\\l\neq h}}^{n}p_E^l E^h$$

with

$$p_E^l=\lambda^l\frac{\partial u^l}{\partial E}.$$

This is the social impact on $u^l$ of an increase in $E$, observe that $p_E^l$ is expressed in euro (it is not normalized). Now we can use this to calculate $b^h$ (expression (A29)). Observe first that expression (A25) can be written as

$$\begin{aligned}
B\left(\widehat{x}(0),\widehat{E}(0)\right) &= \sum_{l=1}^{n}\lambda^l\sum_{k=1}^{M}\frac{\partial u^l}{\partial x_k^l}x_k^l+\sum_{t=1}^{n}\lambda^t\frac{\partial u^t}{\partial E}E\\
&= \sum_{l=1}^{n}\lambda^l\sum_{k=1}^{M}\frac{\partial u^l}{\partial x_k^l}x_k^l+\sum_{t=1}^{n}\lambda^t\frac{\partial u^t}{\partial E}\sum_{l=1}^{n}E^l+\sum_{t=1}^{n}\lambda^t\frac{\partial u^t}{\partial E}\left(E-\sum_{l=1}^{n}E^l\right)\\
&= \sum_{l=1}^{n}B^l\left(\widehat{x}(0),\widehat{E}(0)\right)+\sum_{t=1}^{n}p_E^t\left(E-\sum_{l=1}^{n}E^l\right),
\end{aligned}$$

so we can write for $b^h$

$$\begin{aligned}
b^h &= \frac{\sum_{l=1}^{n}\left(\lambda^l m^l+\sum_{\substack{t=1\\t\neq l}}^{n}p_E^t E^l+p_E^l\left(E-\sum_{t=1}^{n}E^t\right)\right)}{\lambda^h m^h+\sum_{\substack{l=1\\l\neq h}}^{n}p_E^l E^h}\\
&= \frac{\sum_{l=1}^{n}B^l\left(\widehat{x}(0),\widehat{E}(0)\right)}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}+\frac{\sum_{t=1}^{n}p_E^t\left(E-\sum_{l=1}^{n}E^l\right)}{B^h\left(\widehat{x}(0),\widehat{E}(0)\right)}.
\end{aligned}$$

**APPENDIX D: data**

This part of the appendix contains the data we use for the MCF calculations

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$\mu^h\left(\sum_{j=1}^{M}\pi_j^h x_j^h-1\right)$. For the first order condition of commodity $i$ we get $\frac{\partial u^h\left(x^h;E\right)}{\partial x_i^h}+\frac{\partial u^h\left(x^h;E\right)}{\partial E}\frac{\partial E}{\partial x_i^h}-\mu^h\pi_i^h=0$, this can be rewritten as $\frac{\partial u^h\left(x^h;E\right)}{\partial x_i^h}+\frac{\partial u^h\left(x^h;E\right)}{\partial E}\frac{\partial E}{\partial x_i^h}-\frac{\mu^h}{m^h}q_i=0$ (If we slightly rearrange the expression, we get $\frac{\frac{\partial u^h\left(x^h;E\right)}{\partial x_i^h}+\frac{\partial u^h\left(x^h;E\right)}{\partial E}\frac{\partial E}{\partial x_i^h}}{\mu^h}=\pi_i^h$, which is expression (2)).

Now, $\mu^h$ is household $h$'s marginal utility of one percent of income and $\frac{\mu^h}{m^h}$ is household $h$'s marginal utility of one euro of income. With $v^h\left(q,m^h;E\right):\mathbb{R}_+^M\times\mathbb{R}_+\times\mathbb{R}_+\to\mathbb{R}$ household $h$'s indirect utility, marginal utility of one euro of income is $\frac{\partial v^h\left(q,m^h;E\right)}{\partial m^h}$, so $\frac{\mu^h}{m^h}=\frac{\partial v^h\left(q,m^h;E\right)}{\partial m^h}$ which implies $\mu^h=\frac{\partial v^h\left(q,m^h;E\right)}{\frac{\partial m^h}{m^h}}$. This is the term in the denominator of expression (2), it measures household $h$'s marginal utility of one euro of income times income.

| cat. | commodity | short | $\frac{t_i}{q_i}$ | $r_j$[29] | $\sigma_j$[30] | $w_j$ |
|---|---|---|---|---|---|---|
| 1 | Alcohol & Tobacco | AT | 0,431 | 0.433 | -0,648 | 0,033 |
| 2 | Food & Beverage | FB | 0,058 | 0.392 | 0,532 | 0,088 |
| 3 | Meat & Fish | MF | 0,060 | 0.923 | -2,636 | 0,040 |
| 4 | Clothing & Shoes | CL | 0,172 | 0.100 | -1,807 | 0,044 |
| 5 | Rent & Water | RW | 0,046 | 0.362 | -2,610 | 0,230 |
| 6 | Electricity | EL | 0,186 | 1.768 | 2,192 | 0,022 |
| 7 | Gas | GA | 0,181 | 1.376 | -1,736 | 0,015 |
| 8 | Other fuels (heating) | FU | 0,209 | 1.181 | 0,0179 | 0,010 |
| 9 | Durables | DU | 0,174 | 0.315 | -0,211 | 0,047 |
| 10 | Services | SE | 0,152 | 0.127 | -0,925 | 0,346 |
| 11 | Car Purchase | CP | 0,169 | 0.194 | -0,769 | 0,054 |
| 12 | Car Use | CU | 0,500 | 4.965 | 1,585 | 0,068 |
| 13 | Public Transport | PT | 0,057 | 0.202 | -5,325 | 0,006 |

[29] expressed here as tons per 1000 euro

[30] Calculations based on the procedure put forward in [**?**].