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WORKING PAPER

Optimal Monetary Policy and Firm Entry

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Abstract

This paper characterises optimal short-run monetary policy in an economy with monopolistic competition, endogenous firm entry, a cash-in-advance constraint and preset wages. Firms must make profits to cover entry costs; thus the markup on goods prices is efficient. However, a distortion results from the absence of a markup on leisure. This distortion affects the investment margin due to the labour requirement in entry. In the absence of fiscal instruments such as labour income subsidies, the optimal monetary policy under sticky wages achieves higher welfare than under flexible wages. The policy maker uses the money supply instrument to raise the real wage - the cost of leisure - above its flexible-wage level, in response to expansionary shocks. This induces a rise in labour hours and more production of goods and new firms.

Key words: entry, optimal policy
JEL codes: E52, E63

1 Introduction

The creation of new firms and products[1], also referred to as extensive margin investment, propagates and amplifies shocks. See Bergin and Corsetti (2008). This paper asks whether, in the presence of nominal rigidities, stabilisation policy should be concerned with movements in the number of firms.

The analysis is based on a stylised business cycle model with firm entry as the only form of investment. There are three distortions: monopolistic competition, a cash-in-advance (CIA) constraint and preset wages. Firms have monopoly power over the goods they produce. New firms are established up to the point where monopoly profits just cover entry costs, which are modelled as labour costs. In the presence of entry costs, the monopolistic markup provides an incentive to enter the market and should not be removed. However, the absence of a markup on leisure leads to a distortion and a misallocation of resources. The available (state-contingent) policy instruments are lump sum taxes, the interest rate and the money supply. The policy

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[1] I use these two terms interchangeably throughout.

maker commits to state-contingent paths for the model variables which maximise welfare, taking as given the optimal decisions of households and firms. The main result is that optimal monetary policy achieves higher welfare than the flexible equilibrium. The reason is the misalignment of markups between leisure and consumption goods that calls for subsidies to labour income or to firm entry. Optimal monetary policy mimicks such subsidies by manipulating the real wage in response to shocks.

The mechanism is as follows. An inefficiently low real wage implies that hours are too low, and therefore too little is produced at both the intensive margin (production of goods) and at the extensive margin (production of firms). As shown by Bilbiie et al (2008), efficiency can be restored through a labour income subsidy that aligns the markup on the price of leisure with that on the price of consumption goods. Here, the required subsidy is higher than in Bilbiie et al (2007), as a result of imperfectly competitive markups in both product and labour markets. Both markups depress hours and production, such that leisure is suboptimally high and consumption is suboptimally low.

In the absence of distortionary fiscal instruments, the optimal monetary policy under sticky wages does not replicate the flexible allocation. Instead, the policy maker can use the money supply instrument to bring the real wage closer to its efficient level. A policy of raising the real wage above its flexible-wage level increases hours and expands production at the intensive and extensive margin. The money supply and the interest rate are separate instruments. The CIA restriction on consumption purchases introduces a role for money. The optimality of the Friedman Rule - a standard result in CIA models - also applies here. Setting a higher interest rate taxes consumption relative to leisure, thereby worsening the aforementioned allocative distortion.

In models with endogenous firm entry, Bergin and Corsetti (2008) and Bilbiie et al (2007) find that it is optimal to fully stabilise goods prices, i.e. to replicate the flexible-price solution, while letting the number of firms fluctuate freely. In both studies, monetary frictions are ignored and appropriate fiscal policies ensure that the flexible-price allocation is efficient. This paper instead considers monetary policy as a tool to stabilise fluctuations around a distorted steady state, i.e. in the absence of short-run fiscal policy. This paper is more closely related to Adão et al (2003), who consider optimal monetary policy in an economy with sticky prices where fiscal policy is restricted to lump sum taxation. They also find that the optimal allocation under nominal rigidities and the flexible allocation are not the same. However, entry is absent and monopolistic markups are inefficient in their model. Berentsen and Waller (2008) analyse optimal monetary policy in a model with endogenous entry and a microfounded demand for money. They find that the Friedman Rule is optimal if the entry cost is modelled as a fixed cost.[2]

2 Model

The economy is initially in a state of nature denoted by s^0. Thereafter, it is hit by a series of stochastic i.i.d. shocks to government spending, entry costs and productivity. Every variable determined at time t is indexed by the history of shocks that have occurred up to t, denoted by s^t. Let S^t be the set of possible state histories. The probability of observing a particular history is denoted by $\Pr\left(s^t\right)$.

2.1 Final Goods Sector

There is a mass $N\left(s^t\right)$ of differentiated intermediate goods, each produced by a monopolistically competitive firm. A firm is indexed by $f \in \left[0, N\left(s^t\right)\right]$. A final goods firm bundles these intermediate goods $Y\left(f, s^t\right)$, taking as given their price $P\left(f, s^t\right)$, and sells the output $Y\left(s^t\right)$ to consumers and to the government at the competitive price $P\left(s^t\right)$. The optimisation problem of the final goods firm is to choose the amount of inputs that maximise profits, i.e. it solves

$$\max_{Y\left(f,s^t\right)_{f\in\left[0,N\left(s^t\right)\right]}} \left\{ P\left(s^t\right) Y\left(s^t\right) - \int_0^{N\left(s^t\right)} Y\left(f, s^t\right) P\left(f, s^t\right) df \right\}$$

subject to the Dixit-Stiglitz (1977) production function

$$Y\left(s^t\right) = \left(\int_0^{N\left(s^t\right)} Y\left(f, s^t\right)^{\frac{\theta-1}{\theta}} df \right)^{\frac{\theta}{\theta-1}}, \theta > 1 \tag{1}$$

The first order condition gives the following input demand function

$$Y\left(f, s^t\right) = \left(\frac{P\left(f, s^t\right)}{P\left(s^t\right)} \right)^{-\theta} Y\left(s^t\right) \tag{2}$$

Substituting the input demand in the production function yields the price index

$$P\left(s^t\right) = \left(\int_0^{N\left(s^t\right)} P\left(f, s^t\right)^{1-\theta} df \right)^{\frac{1}{1-\theta}}$$

2.2 Intermediate Goods Sector

Intermediate firms use labour $L_c\left(s^t\right)$ to produce differentiated goods. They set prices to maximise profits

$$P\left(f, s^t\right) Y\left(f, s^t\right) - W\left(s^t\right) L_c\left(s^t\right)$$

[2] With an increasing entry cost due to a congestion externality in entry, deviations from the Friedman Rule are needed to reduce inefficiently high entry levels. Such congestion effects are beyond the scope of this paper.

subject to the demand function given by (2) and the production function

$$Y\left(f,s^{t}\right)=Z\left(s^{t}\right)L_{c}\left(s^{t}\right) \tag{3}$$

where $Z\left(s^{t}\right)$ is labour productivity and $W\left(s^{t}\right)$ is the wage rate. The optimal price is a constant[3] markup over marginal cost

$$P\left(f,s^{t}\right)=\frac{\theta}{\theta-1}\frac{W\left(s^{t}\right)}{Z\left(s^{t}\right)}$$

Profits are a constant fraction of firm revenue

$$D\left(f,s^{t}\right)=\frac{1}{\theta}P\left(f,s^{t}\right)Y\left(f,s^{t}\right) \tag{4}$$

Producers are perfectly symmetric.

2.3 Firm Entry

Starting up a firm requires labour services $L_f\left(s^t\right)$. Let $F\left(s^t\right)$ denote the exogenous entry cost, in the form of effective labour units $Z\left(s^t\right)L_f\left(s^t\right)$. In nominal terms, the after-tax entry cost is

$$\left[1-\mu\left(s^{t}\right)\right]\frac{W\left(s^{t}\right)F\left(s^{t}\right)}{Z\left(s^{t}\right)}$$

where $\mu\left(s^t\right)$ is the rate at which the government subsidises entry. Households finance the entry costs incurred by new firms in exchange for claims on those firms' profits. At the end of each period, the entire stock of firms depreciates.

2.4 Households

There exists a continuum of measure 1 of households. As in Erceg et al (2000), each household, indexed by $h\in[0,1]$, supplies a differentiated labour type to a competitive labour packer, who produces a labour bundle subject to the production function $L\left(s^t\right)=\left(\int_0^1 L\left(h,s^t\right)^{\frac{\phi-1}{\phi}}dh\right)^{\frac{\phi}{\phi-1}}$, $\phi>1$, and sells it to intermediate firms and to entrants at price $W\left(s^t\right)$.[4]

Households choose paths for consumption, wages and asset holdings to maximise expected lifetime utility

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\Pr\left(s^t\right)\left\{U\left(C\left(s^t\right)\right)-V\left(L\left(s^t\right)\right)\right\}$$

subject to a sequence of budget constraints explained below, labour demand from the labour packer $L\left(h,s^t\right)=\left[W\left(h,s^t\right)/W\left(s^t\right)\right]^{-\phi}L\left(s^t\right)$, and the cash-in-advance constraint

$$P\left(s^{t}\right)C\left(s^{t}\right)\leq M\left(s^{t}\right) \tag{5}$$

[3] The markup may well be a (negative) function of the number of producers if goods become more substitutable - the product space becomes more crowded - as more firms enter the market. Then additional entry puts downward pressure on the markup. Fluctuations in the number of firms become more dampened as new entry limits itself through its negative effect on markups. Although a possibly interesting extension, I leave the analysis of markup endogeneity for future research, in order to keep the model as simple as possible. See Lewis (2009b), which takes up this issue in a non-monetary model, focussing on optimal taxation.

[4] To simplify notation, I drop the h-subscript from here on.

$U(\cdot)$ is strictly increasing and concave; $V(\cdot)$ is strictly increasing and convex. At the start of period t, households make a portfolio allocation decision in the asset market facing the constraint

$$\begin{aligned} \mathcal{W}\left(s^{t}\right) \quad &\geq \quad M\left(s^{t}\right)+B\left(s^{t}\right)+\sum_{s^{t+1} \mid s^{t}} Q\left(s^{t+1} \mid s^{t}\right) A\left(s^{t}, s^{t+1}\right) \\ &+\int_{0}^{N\left(s^{t}\right)} S\left(f, s^{t}\right)\left[1-\mu\left(s^{t}\right)\right] \frac{W\left(s^{t}\right) F\left(s^{t}\right)}{Z\left(s^{t}\right)} d f-X\left(s^{t}\right) \end{aligned} \tag{6}$$

Total wealth is denoted by $\mathcal{W}\left(s^{t}\right)$. Households receive a monetary transfer $X\left(s^{t}\right)$ from the government and buy four types of assets. Money holdings are denoted by $M\left(s^{t}\right)$. $B\left(s^{t}\right)$ are one-period nominal risk-free bonds[5] that have a price of one currency unit and a return of $R\left(s^{t}\right) \geq 1$, the gross interest rate, next period. $A\left(s^{t}, s^{t+1}\right)$ are nominal state-contingent bonds[6] that cost $Q\left(s^{t+1} \mid s^{t}\right)$ and pay a return of one currency unit in period $t+1$ if and only if the economy is in the state of nature s^{t+1}. A share is denoted by $S\left(f, s^{t}\right)$. Its price is a share of the firm entry cost and its payoff is a share of the entrant's monopoly profits earned at the end of period t and paid out as dividends at the start of period $t+1$.

After the closure of asset markets, production takes place and goods markets open. The agents work and use money to make consumption purchases. At the end of the period, they receive labour income supplemented by a proportional gross subsidy $\tau\left(s^{t}\right)$ and pay a lump sum tax $T\left(s^{t}\right)$ to the government. At the beginning of period $t+1$, the household has a stock of wealth given in nominal terms by

$$\begin{aligned} \mathcal{W}\left(s^{t+1}\right) \quad &= \quad M\left(s^{t}\right)+R\left(s^{t}\right) B\left(s^{t}\right)+A\left(s^{t}, s^{t+1}\right) \\ &+\int_{0}^{N\left(s^{t}\right)} S\left(f, s^{t}\right) D\left(f, s^{t}\right) d f \\ &+\tau\left(s^{t}\right) W\left(s^{t}\right) L\left(s^{t}\right)-P\left(s^{t}\right) C\left(s^{t}\right)-T\left(s^{t}\right) \end{aligned}$$

Asset holdings are money carried over from the previous period, interest income on bond holdings and dividends on share holdings. Initial household wealth is zero, such that $\mathcal{W}\left(s^{0}\right)=0$.[7] I rule out Ponzi schemes on asset holdings by assuming

$$\lim_{T \rightarrow \infty} Q\left(s^{T} \mid s^{0}\right)\left[B\left(s^{T}\right)+\sum_{s^{T+1} \mid s^{T}} Q\left(s^{T+1} \mid s^{T}\right) A\left(s^{T}, s^{T+1}\right)\right] \geq 0 \tag{7}$$

[5] Risk-free bonds are needed in order to define the interest rate.

[6] I introduce state-contingent bonds in order to simplify the policy problem. Under complete financial markets one can write the household budget constraint in present value form, which then becomes an implementability constraint for the policy maker.

[7] This is consistent with the result in Chamley (1986) that only *initial* wealth should be taxed, and at a rate of 100%.

The first order conditions for asset holdings imply

$$\begin{aligned} Q\left(s^{t+1}|s^t\right) &= \beta \Pr\left(s^{t+1}|s^t\right) \frac{U_C\left(s^{t+1}\right)}{U_C\left(s^t\right)} \frac{P\left(s^t\right)}{P\left(s^{t+1}\right)} && (8) \\ \frac{U_C\left(s^t\right)}{P\left(s^t\right)} &= R\left(s^t\right) \beta \sum_{s^{t+1}|s^t} \Pr\left(s^{t+1}|s^t\right) \frac{U_C\left(s^{t+1}\right)}{P\left(s^{t+1}\right)} && (9) \\ \left[1-\mu\left(s^t\right)\right] \frac{W\left(s^t\right) F\left(s^t\right)}{Z\left(s^t\right)} &= \frac{D\left(f, s^t\right)}{R\left(s^t\right)} && (10) \end{aligned}$$

Equation (8) defines the household's stochastic discount factor, the marginal utility growth of nominal wealth, given a particular state of nature in $t+1$. The period-zero value of consumption in period $t+1$ must obey $Q\left(s^{t+1}|s^0\right) = Q\left(s^t|s^0\right) Q\left(s^{t+1}|s^t\right)$. Combining (8) and (9) yields an arbitrage condition between risk-free and state-contingent bonds $\sum_{s^{t+1}|s^t} Q\left(s^{t+1}|s^t\right) = \frac{1}{R(s^t)}$. Equation (10) states that the cost of setting up a firm must equal profits discounted by the interest rate. Under flexible wages, (11) equates the after-tax real wage to the marginal rate of substitution between leisure and consumption, adjusted for the cost of holding money.

$$\tau\left(s^t\right) \frac{W\left(s^t\right)}{P\left(s^t\right)} = \frac{\phi}{\phi-1} \frac{V_L\left(s^t\right)}{U_C\left(s^t\right)} R\left(s^t\right) \tag{11}$$

Under preset wages, the first order condition for wages is

$$W\left(s^t\right) = \frac{\phi}{\phi-1} \frac{\sum_{s^t|s^{t-1}} \Pr\left(s^t|s^{t-1}\right) V_L\left(s^t\right) L\left(s^t\right)}{\sum_{s^t|s^{t-1}} \Pr\left(s^t|s^{t-1}\right) \frac{U_C(s^t)}{R(s^t)P(s^t)} \tau\left(s^t\right) L\left(s^t\right)} \tag{12}$$

There are two reasons for assuming sticky wages instead of sticky prices. First, Lewis (2009a) shows that, in a model with endogenous entry, wage stickiness helps to reconcile the model impulse responses of profits and entry to a monetary policy shock with those observed in the data. The second reason is analytical convenience. Under price flexibility, profits are a constant fraction of revenue, which simplifies considerably the optimality condition for share holdings and, as a result, the policy problem.

2.5 Government

The government finances an exogenous stream of consumption purchases $G\left(s^t\right)$, entry subsidies and labour income subsidies with lump sum taxes collected in the goods market. In addition, it makes a monetary transfer to the household in the asset market financed by an expansion of the money stock. Thus, the government budget constraint is

$$\begin{aligned} & P\left(s^t\right) G\left(s^t\right) + \left[\tau\left(s^t\right)-1\right] W\left(s^t\right) L\left(s^t\right) + N\left(s^t\right) \mu\left(s^t\right) \frac{W\left(s^t\right) F\left(s^t\right)}{Z\left(s^t\right)} + X\left(s^t\right) \\ = \; & T\left(s^t\right) + M^s\left(s^t\right) - M^s\left(s^{t-1}\right) \end{aligned}$$

The law of motion for the money stock is $M^s\left(s^t\right) = M^s\left(s^{t-1}\right) + X\left(s^t\right)$.

2.6 Market Clearing

Labour is used to produce firms and to produce consumption goods.

$$L\left(s^t\right) = N\left(s^t\right)\left[L_f\left(s^t\right) + L_c\left(s^t\right)\right]$$

Using the respective production functions, labour market clearing requires

$$Z\left(s^t\right)L\left(s^t\right) = N\left(s^t\right)\left[Y\left(f, s^t\right) + F\left(s^t\right)\right] \tag{13}$$

The market clearing conditions for final goods, for the two types of bonds, for shares and for money are, respectively,

$$Y\left(s^t\right) = C\left(s^t\right) + G\left(s^t\right) \tag{14}$$

$$B\left(s^t\right) = A\left(s^t, s^{t+1}\right) = 0$$

$$S\left(f, s^t\right) = 1 \tag{15}$$

$$M\left(s^t\right) = M^s\left(s^t\right) \tag{16}$$

An imperfectly competitive equilibrium is set of prices, allocations and policies, such that first, the optimality conditions of the final goods firm, the intermediate goods firms, the labour packer and the household are satisfied; second, all markets clear.

2.7 Returns to Product Diversity and Marginal Rate of Transformation

Under endogenous firm entry, the Dixit-Stiglitz aggregator (1) exhibits increasing returns to variety. This has implications for how, in the aggregate, inputs are converted into final output. To understand the First Best efficiency conditions presented in the next section, I now derive the marginal rate of transformation (MRT) for this economy.

The symmetry of the intermediate firms' output levels implies that the production function of the final goods firm reduces to

$$Y\left(s^t\right) = N\left(s^t\right)^{1+\frac{1}{\theta-1}}Y\left(f, s^t\right) \tag{17}$$

From (17) we see that $1+\frac{1}{\theta-1}$ represents the degree of returns to product diversity. If $1+\frac{1}{\theta-1} > 1 \Leftrightarrow \theta > 1$, there are increasing returns to variety. As $\theta \to \infty$, i.e. as the elasticity of substitution between inputs into final good production increases, the degree of increasing returns to variety diminishes. See also Kim (2004). The symmetry of the intermediate goods prices implies that the aggregate price index is

$$P\left(s^t\right) = P\left(f, s^t\right)N\left(s^t\right)^{-\frac{1}{\theta-1}} \tag{18}$$

Because of increasing returns to product diversity, the price index is decreasing in the number of differentied goods. As the number of goods rises, it becomes less costly to produce the same amount of final output. Next, I derive an aggregate production function for this economy by combining the production function of the final goods firm (17) with the production function of the intermediate goods firms (3)

$$Y\left(s^t\right) = N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right) L_C\left(s^t\right) \tag{19}$$

where $L_C\left(s^t\right) = N\left(s^t\right) L_c\left(s^t\right)$ is total labour used in the production of goods. Differentiating (19) with respect to labour, we have

$$\frac{\partial Y\left(s^t\right)}{\partial L_C\left(s^t\right)} = N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right)$$

One additional labour unit is transformed into $N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right)$ units of the final good. Replacing total consumption output with $C\left(s^t\right) + G\left(s^t\right)$ using the market clearing condition for final goods, we can derive MRT of labour into private consumption as

$$\frac{\partial C\left(s^t\right)}{\partial L_C\left(s^t\right)} = \frac{N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right)}{1+\Gamma\left(s^t\right)} \tag{20}$$

where we have introduced the variable $\Gamma\left(s^t\right) = G\left(s^t\right)/C\left(s^t\right)$.

In the standard model with an exogenous *level* of government consumption and a constant number of firms, the MRT is simply equal to productivity $Z\left(s^t\right)$. Then the social and private marginal rates of substitution are the same. Here, due to the multiplicative nature of the government spending shock, $\Gamma\left(s^t\right)$ is like a negative productivity shock and enters the MRT, too. In a model without entry, Teles (2009) shows that if $\Gamma\left(s^t\right)$ is exogenous, the First Best can only be implemented if the government can use proportionate taxes. More importantly though, the social MRT contains the endogenous term $N\left(s^t\right)^{\frac{1}{\theta-1}}$ relating to the increasing returns to product diversity. Raising the number of firms by one unit gives rise to a positive externality on total output.

We can rewrite the aggregate production function (19) to express the economy-wide effective labour requirement as

$$Z\left(s^t\right) L_C\left(s^t\right) = N\left(s^t\right)^{-\frac{1}{\theta-1}} C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]$$

The reduction in the labour requirement from increasing the number of firms is then obtained by differentiating this expression with respect to $N\left(s^t\right)$,

$$\frac{\partial Z\left(s^t\right) L_C\left(s^t\right)}{\partial N\left(s^t\right)} = -\frac{1}{\theta-1} N\left(s^t\right)^{-\frac{1}{\theta-1}-1} C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right] \tag{21}$$

3 First Best Equilibrium

The First Best equilibrium is a useful benchmark with which one can compare any constrained-efficient allocation. The First Best problem is as follows

$$\max_{\left\{(C(s^t),L(s^t),N(s^t))_{s^t\in S^t}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\sum_{s^t}\beta^t \Pr\left(s^t\right)\left\{U\left(C\left(s^t\right)\right)-V\left(L\left(s^t\right)\right)\right\}$$

subject to the resource constraint (25). The First Best allocation satisfies

$$\frac{V_L\left(s^t\right)}{U_C\left(s^t\right)} = \frac{N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right)}{1+\Gamma\left(s^t\right)} \tag{22}$$

$$F\left(s^t\right) = \frac{1}{\theta-1}N\left(s^t\right)^{-\frac{1}{\theta-1}-1} C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right] \tag{23}$$

Equation (22) is an intrasectoral efficiency condition. It states that the marginal rate of substitution between labour and consumption, $MRS\left(s^t\right)=V_L\left(s^t\right)/U_C\left(s^t\right)$, must equal the marginal rate of transformation of labour into (private) consumption. See (20). Equation (23) is an intersectoral efficiency condition. It states that the cost (in effective labour units) of producing one additional firm, $F\left(s^t\right)$, must equal the reduction in the number of effective labour units required in the production of goods, i.e. the efficiency gain, brought about by this extra firm. See (21). Let's define two wedges, an intrasectoral wedge and an intersectoral wedge. The intrasectoral wedge is the ratio of $MRS\left(s^t\right)$ to $MRT\left(s^t\right)$, minus 1. The intersectoral wedge $ISW\left(s^t\right)$ is defined as

$$ISW\left(s^t\right)=\frac{1}{\theta-1}\frac{N\left(s^t\right)^{-\frac{1}{\theta-1}} C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{Z\left(s^t\right)}-\frac{N\left(s^t\right)F\left(s^t\right)}{Z\left(s^t\right)}$$

In the First Best equilibrium, $\frac{MRS\left(s^t\right)}{MRT\left(s^t\right)}-1=0$ and $ISW\left(s^t\right)=0$.

Assuming log consumption utility and linear labour disutility, such that $U_C\left(s^t\right)=C\left(s^t\right)^{-1}$ and $V_L\left(s^t\right)=1$, equation (22) becomes

$$1=\frac{N\left(s^t\right)^{-\frac{1}{\theta-1}} C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{Z\left(s^t\right)}$$

Thus, labour employed in goods production is constant and equal to 1. We can write this equilibrium recursively as follows

$$N^{FB}\left(s^t\right) = \frac{1}{\theta-1}\frac{Z\left(s^t\right)}{F\left(s^t\right)}$$

$$C^{FB}\left(s^t\right) = \frac{N^{FB}\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right)}{1+\Gamma\left(s^t\right)}$$

$$L^{FB}\left(s^t\right) = \frac{\theta}{\theta-1}$$

The number of firms is proportional to productivity and inversely proportional to the entry cost. Expressing consumption as a function of exogenous variables only, we have $C\left(s^t\right) = \left[(\theta-1) F\left(s^t\right)\right]^{-\frac{1}{\theta-1}}\left[1+\Gamma\left(s^t\right)\right]^{-1} Z\left(s^t\right)^{\frac{\theta}{\theta-1}}$. Thus, consumption is increasing in productivity, with elasticity $\frac{\theta}{\theta-1}$. It is decreasing in the entry cost, with elasticity $-\frac{1}{\theta-1}$ and in government spending, with elasticity -1. Labour is constant[8] at $\frac{\theta}{\theta-1}$.

4 Optimal Policy

This section derives the optimal policy following the approach in Adão et al (2003). First, I collapse all equilibrium conditions into a single equation that, together with the resource constraint, restricts the set of implementable allocations for any given policy sequences. Second, I show that under both flexible and sticky wages, the optimal interest rate policy is to follow the Friedman Rule. Third, I characterise the optimal allocations under this policy by deriving the optimal intersectoral and intrasectoral wedges under flexible wages and under sticky wages. I show that the flexible-wage optimal allocation coincides with the sticky-wage optimal allocation only if labour supply is inelastic.

4.1 Imperfectly Competitive Equilibrium: Compact Form

More compactly, we can define a (symmetric) equilibrium as a set of prices

$$\left\{\left(P\left(s^t\right), P\left(f, s^t\right), Q\left(s^{t+1}|s^t\right), Q\left(s^{t+1}|s^0\right), R\left(s^t\right), W\left(s^t\right)\right)_{s^t\in S^t}\right\}_{t=0}^{\infty}$$

allocations

$$\left\{\left(C\left(s^t\right), N\left(s^t\right), L\left(s^t\right)\right)_{s^t\in S^t}\right\}_{t=0}^{\infty}$$

and policies

$$\left\{\left(T\left(s^t\right), M^s\left(s^t\right), \tau\left(s^t\right), \mu\left(s^t\right)\right)_{s^t\in S^t}\right\}_{t=0}^{\infty}$$

such that:

1. the household present-value budget constraint is satisfied,

$$0 \geq \sum_{t=0}^{\infty}\sum_{s^t}\frac{Q\left(s^t|s^0\right)}{R\left(s^t\right)}\left[R\left(s^t\right)P\left(s^t\right)C\left(s^t\right)-\tau\left(s^t\right)W\left(s^t\right)L\left(s^t\right)+T\left(s^t\right)\right] \tag{24}$$
$$+\sum_{t=1}^{\infty}\sum_{s^t}Q\left(s^t|s^0\right)\left[N\left(s^t\right)\left[1-\mu\left(s^t\right)\right]\frac{W\left(s^t\right)F\left(s^t\right)}{Z\left(s^t\right)}-\frac{1}{\theta}P\left(s^{t-1}\right)C\left(s^{t-1}\right)\left[1+\Gamma\left(s^{t-1}\right)\right]\right]$$

2. the resource constraint is satisfied[9],

$$Z\left(s^t\right)L\left(s^t\right) = N\left(s^t\right)^{-\frac{1}{\theta-1}}C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]+N\left(s^t\right)F\left(s^t\right) \tag{25}$$

[8] Note, however, that labour is constant only in the log utility case.

[9] If the resource constraint and the household budget constraint are satisfied, the government budget constraint is satisfied by Walras' Law.

3. the following equilibrium conditions are satisfied

$$\begin{aligned}
P\left(f,s^t\right) &= \frac{\theta}{\theta-1}\frac{W\left(s^t\right)}{Z\left(s^t\right)} \\
Q\left(s^{t+1}|s^0\right) &= Q\left(s^t|s^0\right)Q\left(s^{t+1}|s^t\right) \\
\sum_{s^{t+1}|s^t} Q\left(s^{t+1}|s^t\right) &= \frac{1}{R\left(s^t\right)} \\
P\left(s^t\right) &= P\left(f,s^t\right)N\left(s^t\right)^{-\frac{1}{\theta-1}} \\
M^s\left(s^t\right) &= P\left(s^t\right)C\left(s^t\right) \\
\left[1-\mu\left(s^t\right)\right]\frac{W\left(s^t\right)F\left(s^t\right)}{Z\left(s^t\right)} &= \frac{P\left(s^t\right)C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta R\left(s^t\right)N\left(s^t\right)} \qquad (26)
\end{aligned}$$

as well as

$$\tau\left(s^t\right)\frac{W\left(s^t\right)}{P\left(s^t\right)} = \frac{\phi}{\phi-1}\frac{V_L\left(s^t\right)}{U_C\left(s^t\right)}R\left(s^t\right)$$

under flexible wages or

$$W\left(s^t\right) = \frac{\phi}{\phi-1}\frac{\sum_{s^t|s^{t-1}}\Pr\left(s^t|s^{t-1}\right)V_L\left(s^t\right)L\left(s^t\right)}{\sum_{s^t|s^{t-1}}\Pr\left(s^t|s^{t-1}\right)\frac{U_C\left(s^t\right)}{R\left(s^t\right)P\left(s^t\right)}\tau\left(s^t\right)L\left(s^t\right)}$$

under sticky wages.

The resource constraint (25) is derived by substituting the final goods production function under symmetry (17) in the labour market clearing condition (13). Under the assumption of complete contingent claims markets, one can write the consumer budget constraint in present value form. First, weight each equation (6) by the period-0 value of wealth in state s^t, $Q\left(s^t|s^0\right)$. Second, sum the resulting equations across states and dates, using the no-Ponzi game condition (7). Doing so eliminates bond holdings from the budget constraint. Finally, substitute the cash-in-advance constraint (5), holding with equality, to eliminate money holdings. Using (4), (18), (17) and (14), firm profits can be expressed as a fraction of total consumption expenditure divided by the number of active firms:

$$D\left(f,s^t\right) = \frac{1}{\theta}\frac{P\left(s^t\right)C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{N\left(s^t\right)} \qquad (27)$$

We derive (24) using the market clearing condition for shares (15) and the expression for firm profits (27). Substituting (27) in the first order condition for shares yields the free entry condition (26).

The planner is free to set a path for lump-sum taxes $T\left(s^t\right)$ to satisfy (24), while the variables $P\left(f,s^t\right)$, $Q\left(s^{t+1}|s^0\right)$, $Q\left(s^{t+1}|s^t\right)$, $P\left(s^t\right)$ and $M\left(s^t\right)$ adjust to satisfy the first five equilibrium conditions under 3. The remaining equilibrium conditions restricting the planner problem are the resource constraint (25), the free entry condition (26) and the relevant wage setting equation: (11) under flexible wages or (12) under sticky wages.

4.2 Implementability Condition and Planner Problem

Under *flexible* wages, the set of implementable allocations for $\left\{\left(C\left(s^t\right), L\left(s^t\right), N\left(s^t\right)\right)_{s^t \in S^t}\right\}_{t=0}^{\infty}$ is restricted by the implementability condition (IC)

$$\frac{Z\left(s^t\right) U_C\left(s^t\right) \tau\left(s^t\right) C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta\left[1-\mu\left(s^t\right)\right] F\left(s^t\right) R\left(s^t\right)^2 N\left(s^t\right)}=\frac{\phi}{\phi-1} V_L\left(s^t\right) \tag{28}$$

and the resource constraints (25) for any path of the interest rate $R\left(s^t\right)$, the labour income tax $\tau\left(s^t\right)$ and the entry subsidy $\mu\left(s^t\right)$. Equation (28) is derived by combining the first order condition for shares (10) with the wage setting equation (11) to eliminate $W\left(s^t\right)$.

Under *sticky* wages, the wage setting condition is given by (12). Solving the first order condition for shares (10) for the price level and substituting the result in the wage setting equation to eliminate $P\left(s^t\right)$ gives

$$1=\frac{\phi}{\phi-1} \frac{\sum_{s^t \mid s^{t-1}} \operatorname{Pr}\left(s^t \mid s^{t-1}\right) V_L\left(s^t\right) L\left(s^t\right)}{\sum_{s^t \mid s^{t-1}} \operatorname{Pr}\left(s^t \mid s^{t-1}\right) \frac{U_C\left(s^t\right) Z\left(s^t\right) C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta\left[1-\mu\left(s^t\right)\right] F\left(s^t\right) R^2\left(s^t\right) N\left(s^t\right)} \tau\left(s^t\right) L\left(s^t\right)}$$

Note that we have cancelled $W\left(s^t\right)$, which is known in $t-1$. Rearranging and using the law of iterated expectations yields the implementability condition under sticky wages

$$\sum_{s^t \mid s^{t-1}} \operatorname{Pr}\left(s^t \mid s^{t-1}\right)\left\{\frac{U_C\left(s^t\right) Z\left(s^t\right) C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta\left[1-\mu\left(s^t\right)\right] F\left(s^t\right) R^2\left(s^t\right) N\left(s^t\right)} \tau\left(s^t\right) L\left(s^t\right)-\frac{\phi}{\phi-1} V_L\left(s^t\right) L\left(s^t\right)\right\}=0 \tag{29}$$

The constraints of the policy problem are the implementability constraint (29) and the resource constraint (25). Because the IC under sticky wages is the expected value of the IC under flexible wages, and the resource constraint is binding in both cases, it follows that the Optimal Policy allocation under flexible wages is contained in the set of implementable allocations under sticky wages. Whether this allocation is optimal under sticky wages is analysed below.

Let $\beta^t \varphi\left(s^{t-1}\right) \operatorname{Pr}\left(s^t\right)$ be the Lagrange multiplier on (29). Then the planner problem under sticky wages is as follows

$$\max_{\left\{\left(R\left(s^t\right), C\left(s^t\right), L\left(s^t\right), N\left(s^t\right)\right)_{s^t \in S^t}\right\}_{t=0}^{\infty}} \min_{\left\{\left(\lambda\left(s^t\right), \varphi\left(s^{t-1}\right)\right)_{s^t \in S^t}\right\}_{t=0}^{\infty}} \mathcal{L}$$

where the Lagrangian is

$$\begin{aligned}
\mathcal{L} = & \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \operatorname{Pr}\left(s^t\right)\left\{U\left(C\left(s^t\right)\right)-V\left(L\left(s^t\right)\right)\right. \\
& \left.+\lambda\left(s^t\right)\left[Z\left(s^t\right) L\left(s^t\right)-N\left(s^t\right)^{-\frac{1}{\theta-1}} C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]-N\left(s^t\right) F\left(s^t\right)\right]\right\} \\
& +\sum_{t=0}^{\infty} \sum_{s^{t-1}} \beta^t \varphi\left(s^{t-1}\right) \sum_{s^t} \operatorname{Pr}\left(s^t\right)\left[\begin{array}{c}
\frac{U_C\left(s^t\right) Z\left(s^t\right) C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta\left[1-\mu\left(s^t\right)\right] F\left(s^t\right) R^2\left(s^t\right) N\left(s^t\right)} \tau\left(s^t\right) L\left(s^t\right) \\
-\frac{\phi}{\phi-1} V_L\left(s^t\right) L\left(s^t\right)
\end{array}\right]
\end{aligned}$$

with $\varphi_0\left(s^{-1}\right)=0$. Under flexible wages, $\varphi\left(s^{t-1}\right)$ is replaced with $\varphi\left(s^t\right)$.

Following Adão et al (2003), I first derive the optimal interest rate policy before solving for the optimal allocations under this policy. It is straightforward to show that the Friedman Rule is optimal irrespective of nominal rigidities. This method differs from the Ramsey approach to optimal policy of first solving the primal problem for the optimal allocations and then backing out the policies that support these allocations.

4.3 Optimal Interest Rate Policy

The interest rate policy problem under sticky wages is to choose a path for the interest rate $\{(R(s^t) \geq 1)_{s^t \in S^t}\}_{t=0}^{\infty}$ to maximise $\mathcal{L}$. The first order condition is

$$\frac{\partial \mathcal{L}}{\partial R(s^t)} = -\beta^t \Pr(s^t) \varphi(s^{t-1}) \frac{Z(s^t) U_C(s^t) \tau(s^t) C(s^t) [1+\Gamma(s^t)]}{\theta [1-\mu(s^t)] F(s^t) R(s^t)^3 N(s^t)} L(s^t)$$

We have $\frac{\partial \mathcal{L}}{\partial R(s^t)} < 0$. Welfare, as summarised by $\mathcal{L}$, decreases as the interest rate increases. It follows that $R(s^t)$ should be as low as possible. Given the lower bound of unity on the gross interest rate, this implies that the Friedman Rule, $R(s^t) = 1$, is optimal. As can be seen from (11), the money distortion affects the intratemporal consumption-leisure tradeoff decision. It drives a wedge between the marginal rate of substitution between consumption and labour and the after-tax real wage. The higher the interest rate, the greater is this wedge. The optimality of the Friedman Rule is a standard result in the literature following Ireland (1996) and is shown to hold under more general conditions in Correia et al (2008). Notice that under the Friedman Rule, the cash-in-advance constraint is no longer binding and hence the level of real money holdings is indeterminate. To avoid this indeterminacy, we consider equilibria in which the interest rate approaches 1.

4.4 Optimal Allocations under the Friedman Rule

Under the Friedman Rule, the planner problem is written as before, with $R(s^t)$ set equal to 1 in $\mathcal{L}$. Again, under flexible wages, $\varphi(s^{t-1})$ is replaced with $\varphi(s^t)$.

4.4.1 Flexible Wages

The first order conditions for the policy problem under flexible wages imply

$$\frac{MRS(s^t)}{MRT(s^t)} \Psi_f(s^t) - 1 = \varphi(s^t) \frac{\phi}{\phi-1} \frac{V_L(s^t)}{U_C(s^t) C(s^t)} \left[\frac{U_{CC}(s^t)}{U_C(s^t)} C(s^t) + 1 \right] \tag{30}$$

$$ISW(s^t) = \frac{\varphi(s^t)}{\Psi_f(s^t)} \frac{\phi}{\phi-1} \tag{31}$$

where

$$\Psi_f(s^t) = 1 + \varphi(s^t) \frac{\phi}{\phi-1} \frac{V_{LL}(s^t)}{V_L(s^t)}$$

Assuming log consumption utility and linear labour disutility, $\Psi_f\left(s^t\right) = 1$ and the Optimal Policy equilibrium can be written recursively as follows.

$$
\begin{aligned}
N\left(s^t\right) &= \frac{\phi-1}{\phi\theta}\frac{\tau\left(s^t\right)}{\left[1-\mu\left(s^t\right)\right]}\frac{Z\left(s^t\right)}{F\left(s^t\right)}\left[1+\Gamma\left(s^t\right)\right] &(32)\\
C\left(s^t\right) &= \frac{N\left(s^t\right)^{\frac{1}{\theta-1}}Z\left(s^t\right)}{1+\Gamma\left(s^t\right)} &(33)\\
L\left(s^t\right) &= 1+\frac{\phi-1}{\phi\theta}\frac{\tau\left(s^t\right)}{\left[1-\mu\left(s^t\right)\right]}\left[1+\Gamma\left(s^t\right)\right] &(34)
\end{aligned}
$$

Under flexible wages, the Friedman Rule is optimal and implements the unique allocation given by (32) to (34). The optimal allocation is unaffected by the money supply policy. The size of the money stock affects only (and pins down) the nominal variables $P\left(s^t\right)$, $P\left(f,s^t\right)$, $W\left(s^t\right)$ and $D\left(f,s^t\right)$. This is the nominal indeterminacy under flexible wages as explained in Ad$\widetilde{a}$o et al (2003).

The number of firms moves one-to-one with the labour income subsidy, with the entry subsidy, with productivity, and with government spending. It is a negative function of the wage markup $\frac{\phi}{\phi-1}$, the elasticity of substitution between goods θ and the entry cost. A higher product market distortion (a lower price elasticity of demand for goods θ) increases the firms' profit share $\frac{1}{\theta}$ and therefore creates an incentive to enter the market. Labour is increasing in government spending, in the entry subsidy and in the labour income subsidy. It is decreasing in the wage markup $\frac{\phi}{\phi-1}$, in the elasticity of substitution θ. Writing consumption in terms of exogenous variables we have

$$
C\left(s^t\right) = \left(\frac{\phi-1}{\phi\theta}\frac{\tau\left(s^t\right)}{\left[1-\mu\left(s^t\right)\right]F\left(s^t\right)}\right)^{\frac{1}{\theta-1}} Z\left(s^t\right)^{\frac{\theta}{\theta-1}}\left[1+\Gamma\left(s^t\right)\right]^{\frac{2-\theta}{\theta-1}}
$$

Consumption is decreasing in the entry cost, with elasticity $-\frac{1}{\theta-1}$, and in government spending, with elasticity $\frac{2-\theta}{\theta-1}$, which is negative if we assume $\theta > 2$. It is increasing in the labour income subsidy, with elasticity $\frac{1}{\theta-1}$, and in productivity, with elasticity $\frac{\theta}{\theta-1}$. Given the cash-in-advance constraint, real money balances are equal to consumption. From (11), we can back out the real wage as

$$
\frac{W\left(s^t\right)}{P\left(s^t\right)} = \frac{C\left(s^t\right)}{\tau\left(s^t\right)}
$$

The impulse responses of the number of firms, consumption and labour to a productivity shock and to an entry cost shock in the First Best allocation are identical to those in the Optimal Policy allocation under flexible wages; the difference lies purely in the steady states. The government spending shock, however, does induce different responses. In the First Best allocation it reduces consumption one-for-one and leaves the number of firms and labour hours unchanged. In the Optimal Policy allocation, the number of firms and labour rise one-for-one with government spending, while consumption decreases by $\frac{2-\theta}{\theta-1}$.

In the absence of government spending, i.e. if $\Gamma\left(s^t\right)=0$, the wedge between the Optimal Policy equilibrium and the First Best is constant and the optimal gross labour income subsidy equals the joint markup $\frac{\theta}{\theta-1}\frac{\phi}{\phi-1}$. Note that the wage markup has a similar effect as the goods markup: it makes leisure cheaper relative to consumption. The optimal entry subsidy is related to the optimal labour income subsidy through $\mu\left(s^t\right)=1-\frac{1}{\tau(s^t)}$.

With government spending, the wedge between the First Best and the Optimal Policy allocation depends positively on $\tau\left(s^t\right)$ and on $\Gamma\left(s^t\right)$, which enter the equilibrium conditions (32) and (34) in the same way. Thus, regarding the number of firms and labour, the effects of government spending and the labour income subsidy are equivalent. From (33) we see that private consumption is crowded out by government spending. Under the tax policy $\tau\left(s^t\right)=\frac{\phi}{\phi-1}\frac{\theta}{\theta-1}\frac{1}{1+\Gamma(s^t)}$, the Optimal Policy equilibrium under flexible wages coincides with the First Best equilibrium. The optimal labour income subsidy is inversely related to the government spending share in consumption. If this share is sufficiently small, more precisely if $1+\Gamma\left(s^t\right)<\frac{\phi}{\phi-1}\frac{\theta}{\theta-1}$, labour income should be subsidised, i.e. $\tau\left(s^t\right)>1$. In that case, hours are suboptimally low. Labour income should be taxed if the share of government consumption is large, in particular if $1+\Gamma\left(s^t\right)>\frac{\phi}{\phi-1}\frac{\theta}{\theta-1}$. The extra demand coming from government consumption raises hours above their efficient level.

Following Bilbiie et al's calibration $\theta=3.8$, and assuming $\phi=10$, the above implies that labour income should be taxed in those states of nature where $\Gamma\left(s^t\right)>0.508$, i.e. when exogenous government spending amounts to more than 50.8% of private consumption. Setting $\Gamma=1/3$ for the US ($\frac{C}{GDP}=0.6$ and $\frac{G}{GDP}=0.2$), the optimal gross labour income subsidy in steady state is $\tau=1.13$. Alternatively, the First Best steady state can be attained with an optimal entry subsidy of $\mu=1-\frac{1}{1.13}=0.115$, i.e. 11.5% of the entry cost should be financed by the government.

4.4.2 Sticky Wages

The first order conditions of the policy problem under sticky wages imply the optimal wedges

$$
\begin{aligned}
\frac{MRS\left(s^t\right)}{MRT\left(s^t\right)}\Psi_s\left(s^t\right)-1 &= \varphi\left(s^{t-1}\right)\frac{Z\left(s^t\right)\tau\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta\left[1-\mu\left(s^t\right)\right]F\left(s^t\right)N\left(s^t\right)}L\left(s^t\right)\left[\frac{U_{CC}\left(s^t\right)C\left(s^t\right)}{U_C\left(s^t\right)}+1\right] \\
ISW\left(s^t\right) &= \frac{\varphi\left(s^{t-1}\right)}{V_L\left(s^t\right)\Psi_s\left(s^t\right)}\frac{U_C\left(s^t\right)Z\left(s^t\right)C\left(s^t\right)\tau\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta\left[1-\mu\left(s^t\right)\right]F\left(s^t\right)N\left(s^t\right)}L\left(s^t\right)
\end{aligned}
$$

where

$$\Psi_s\left(s^t\right) = 1 + \varphi\left(s^{t-1}\right)\left\{\begin{array}{c} \frac{\phi}{\phi-1}\frac{V_{LL}\left(s^t\right)L\left(s^t\right)}{V_L\left(s^t\right)} \\ +\left[\frac{\phi}{\phi-1} - \frac{U_C\left(s^t\right)Z\left(s^t\right)C\left(s^t\right)\tau\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta\left[1-\mu\left(s^t\right)\right]F\left(s^t\right)N\left(s^t\right)}\frac{1}{V_L\left(s^t\right)}\right] \end{array}\right\}$$

Under sticky wages, the Friedman Rule is again optimal. At the Friedman Rule, there are multiple implementable allocations associated with different money supplies, which all satisfy the implementability condition. Within the set implementable allocations, the policy maker picks the optimal one, which in general does not coincide with the flexible-wage allocation. Comparing these sticky-wage optimal wedges with the flexible-wage optimal wedges (30) and (31), we see that only if $L\left(s^t\right) = 1$ does the flexible-wage allocation satisfy the first order conditions of the sticky-wage equilibrium. This is the case of inelastic labour supply studied by Bilbiie et al (2008). Then $\Psi_f\left(s^t\right) = \Psi_s\left(s^t\right)$ and the optimal wedges coincide.

Under log consumption utility and linear labour disutility, the Optimal Policy equilibrium under sticky wages satisfies

$$\begin{aligned} \frac{MRS\left(s^t\right)}{MRT\left(s^t\right)}\Psi_s\left(s^t\right) - 1 &= 0 \\ ISW\left(s^t\right) &= \frac{\varphi\left(s^{t-1}\right)}{\Psi_s\left(s^t\right)}\frac{Z\left(s^t\right)\tau\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta\left[1-\mu\left(s^t\right)\right]F\left(s^t\right)N\left(s^t\right)}L\left(s^t\right) \end{aligned}$$

where

$$\Psi_s\left(s^t\right) = 1 + \varphi\left(s^{t-1}\right)\left\{\frac{\phi}{\phi-1} - \frac{Z\left(s^t\right)\tau\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta\left[1-\mu\left(s^t\right)\right]F\left(s^t\right)N\left(s^t\right)}\right\}$$

together with the resource constraint (25) and the implementability constraint

$$\sum_{s^t|s^{t-1}} \Pr\left(s^t|s^{t-1}\right)\left\{\frac{Z\left(s^t\right)\tau\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta\left[1-\mu\left(s^t\right)\right]F\left(s^t\right)N\left(s^t\right)}L\left(s^t\right) - \frac{\phi}{\phi-1}L\left(s^t\right)\right\} = 0$$

for $C\left(s^t\right)$, $L\left(s^t\right)$, $N\left(s^t\right)$, $\varphi\left(s^{t-1}\right)$.

For the sticky-wage case, I linearise the model and derive the impulse responses to shocks numerically under the following calibration (see Table 1).[10]

[insert Table 1 here]

Table 2 shows the elasticities in the shock period of the number of firms, consumption and labour to shocks to productivity, entry costs and government spending. It compares the impulse responses of the First Best (FB) allocation to the Optimal Policy allocation under flexible wages (Oflex) and under sticky wages (Os).

[insert Table 2 here]

[10] A closed-form solution does not exist even for the two-period two-state case.

Under sticky wages, the number of firms, consumption and labour increase more in response to a productivity shock than under flexible wages. In particular, the expansion in labour under sticky wages allows for a larger increase in the production of both firms and goods.

A decrease in the entry cost induces the optimal policy under sticky wages to raise labour somewhat, such that the number of firms increases more than one-for-one and consumption also increases more than under flexible wages.

The Optimal Policy responses to a government spending shock under sticky wages are fairly close to the First Best responses, with consumption falling by around 1 and the number of firms and labour falling only very slightly. Thus, shocks to government spending result in a reduction in output along the intensive margin rather than an expansion of hours and product diversity as in the flexible-wage model.

The policy maker exploits the degree of freedom given by the wage rigidity to address the undersupply of labour and the underproduction of goods and firms. As labour rises beyond its steady state level, both consumption and the number of firms expand more than in the flexible-wage allocation. The impulse responses of the real wage to the shocks are identical to those of consumption. The real wage increases more and thus leisure becomes more expensive in the sticky-wage allocation than in the flexible-wage allocation. The wage rigidity, combined with the money supply instrument, allows the policy maker to manipulate the real wage in the face of shocks, affecting directly the consumption-leisure tradeoff decision. Optimal policy chooses a different allocation than the flexible-wage allocation by introducing a markup on leisure that is absent under flexible wages. In the short run, monetary policy can mimick the labour supply subsidy, which has the same effect to make leisure more expensive relative to consumption.

The assumption of a labour requirement for firm startups is important for the results of this paper. Since the wage rate is part of the entry cost, wage stickiness affects the entry decision and through this effect monetary policy can influence the investment margin. In the Appendix, I show a variant of the model in which entry costs are specified in terms of final output. In that model, wage stickiness does not alter the set of implementable allocations that the policy maker faces. This is because wages are not part of entry costs and any wage setting restriction therefore does not distort the entry decision. Therefore, the optimal allocation is the same under sticky wages as under flexible wages. Lewis (2009a) compares impulse responses to a monetary policy shock to their empirical counterparts, for different variants of the endogenous entry model. Qualitatively, the best-performing model is one in which entry costs are in labour units, rather than in terms of final output, and wages are sticky. This evidence leads me to prefer the benchmark model over the modified version.

5 Conclusion

This paper investigates the implications of firm entry for optimal stabilisation policy. The economy has three distortions: product and labour markets are imperfectly competitive, wages are set in advance, consumption purchases must be made with money. The cash-in-advance restriction is undone via the Friedman Rule, which aligns the returns on bonds and money. The markup in the goods market is efficient, because profits are needed to cover the entry cost. However, the absence of a markup on leisure implies that leisure is too cheap relative to consumption goods. Therefore, labour is suboptimally low. Due to the labour requirement for producing new firms, this has a negative effect on entry rates. Even though implementing the flexible allocation, i.e. removing the sticky wage distortion, is feasible, it is not welfare-maximising. In response to expansionary shocks, the optimal policy implies a larger increase in hours, more consumption and higher entry than is observed in the flexible economy. The wage rigidity, combined with the money supply instrument, provides the policy maker with a tool to increase the real wage, moving it closer to its efficient level, in response to such shocks. As a result, more labour is employed at both margins: at the intensive margin (production of goods) and at the extensive margin (production of firms).

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Table 1: **Calibration**

θ	3.8	elasticity of substitution (goods)
ϕ	10	elasticity of substitution (labour)
F	0.02	steady state entry cost
Γ	$1+1/3$	steady state government spending
Z	1	steady state productivity

Appendix: Entry Cost in Terms of Final Output

I now assume that the exogenous entry cost is given in terms of final output instead of effective labour units as in the benchmark model. The new entry cost is denoted by $F_{fo}\left(s^t\right)$. The

Table 2: **Impulse Responses**

	$Z(s^t)$			$-F(s^t)$			$1+\Gamma(s^t)$		
	FB	Oflex	Os	FB	Oflex	Os	FB	Oflex	Os
$N(s^t)$	1	1	4.832	1	1	1.531	0	1	-0.088
$C(s^t)$	1.357	1.357	3.832	0.357	0.357	0.531	-1	-0.6429	-1.088
$L(s^t)$	0	0	2.28	0	0	0.121	0	1	-0.063

Note: The figures show short-run elasticities. FB stands for First Best allocation, Oflex for Optimal Policy allocation under flexible wages and Os for Optimal Policy allocation under sticky wages.

household budget constraint becomes

$$\begin{aligned} \mathcal{W}\left(s^t\right) &\geq M\left(s^t\right)+B\left(s^t\right)+\sum_{s^{t+1}|s^t} Q\left(s^{t+1}|s^t\right) A\left(s^t,s^{t+1}\right) \\ &\quad+\int_0^{N\left(s^t\right)} S\left(f,s^t\right)\left[1-\mu\left(s^t\right)\right] P\left(s^t\right) F_{fo}\left(s^t\right) df - X\left(s^t\right) \end{aligned}$$

The first order condition for shares becomes $\left[1-\mu\left(s^t\right)\right] P\left(s^t\right) F_{fo}\left(s^t\right) = D\left(f,s^t\right)/R\left(s^t\right)$. Replacing $D\left(f,s^t\right)$, we get the free entry condition

$$\left[1-\mu\left(s^t\right)\right] F_{fo}\left(s^t\right)=\frac{C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta N\left(s^t\right) R\left(s^t\right)} \tag{35}$$

The government budget constraint reads

$$\begin{aligned} & M^s\left(s^t\right)+\mu\left(s^t\right) P\left(s^t\right) N\left(s^t\right) F_{fo}\left(s^t\right)+\left[\tau\left(s^t\right)-1\right] W\left(s^t\right) L\left(s^t\right)+P\left(s^t\right) G\left(s^t\right) \\ = & M^s\left(s^{t-1}\right)+T\left(s^t\right)+X\left(s^t\right) \end{aligned}$$

The final goods market clearing condition becomes

$$Y\left(s^t\right)=C\left(s^t\right)+G\left(s^t\right)+N\left(s^t\right) F_{fo}\left(s^t\right) \tag{36}$$

Total output comprises consumption purchases and entry costs. Combining the symmetric final goods production function $Y\left(s^t\right) = N\left(s^t\right)^{1+\frac{1}{\theta-1}} Y\left(f,s^t\right)$ with the intermediate firms' production function $N\left(s^t\right) Y\left(f,s^t\right)=Z\left(s^t\right) L\left(s^t\right)$ we have the economy's aggregate production function,

$$Y\left(s^t\right)=N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right) L\left(s^t\right) \tag{37}$$

Differentiating (37) with respect to $N\left(s^t\right)$, we can derive the marginal product, in terms of final output, of one additional firm,

$$\frac{\partial Y\left(s^t\right)}{\partial N\left(s^t\right)}=\frac{1}{\theta-1} N\left(s^t\right)^{\frac{1}{\theta-1}-1} Z\left(s^t\right) L\left(s^t\right) \tag{38}$$

Combining equations (36) and (37) yields the aggregate resource constraint

$$N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right) L\left(s^t\right)=C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]+N\left(s^t\right) F_{fo}\left(s^t\right) \tag{39}$$

The remaining equilibrium conditions are unchanged.

First Best Equilibrium

The First Best problem is as follows

$$\max_{\left\{(C(s^t),L(s^t),N(s^t))_{s^t\in S^t}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \Pr\left(s^t\right)\left\{U\left(C\left(s^t\right)\right) - V\left(L\left(s^t\right)\right)\right\}$$

subject to the resource constraint (39). The first order conditions satisfy

$$\frac{V_L\left(s^t\right)}{U_C\left(s^t\right)} = \frac{N\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right)}{1+\Gamma\left(s^t\right)} \tag{40}$$

$$F_{fo}\left(s^t\right) = \frac{1}{\theta-1} N\left(s^t\right)^{\frac{1}{\theta-1}-1} Z\left(s^t\right) L\left(s^t\right) \tag{41}$$

The intrasectoral efficiency condition (40) is the same as in the benchmark model: the marginal rate of substitution between labour and consumption must equal the marginal rate of transformation of labour into private consumption. Neither the MRS nor the MRT depends on the specification of the entry cost. The intersectoral efficiency condition (41) is, however, different from the benchmark. It states that the cost (in terms of consumption units) of setting up an additional firm, $F_{fo}\left(s^t\right)$, must equal the gain in consumption output that the extra firm gives rise to, i.e. the marginal product of a firm (38). Under log consumption utility and linear labour disutility, we can derive the recursive system

$$N_{fo}^{FB}\left(s^t\right) = \left(\frac{1}{\theta-2}\frac{Z\left(s^t\right)}{F_{fo}\left(s^t\right)}\right)^{\frac{\theta-1}{\theta-2}} \tag{42}$$

$$C_{fo}^{FB}\left(s^t\right) = \frac{N_{fo}^{FB}\left(s^t\right)^{\frac{1}{\theta-1}} Z\left(s^t\right)}{1+\Gamma\left(s^t\right)} \tag{43}$$

$$L_{fo}^{FB}\left(s^t\right) = \frac{\theta-1}{\theta-2} \tag{44}$$

Labour is constant in the First Best equilibrium, a consequence of the log utility assumption. Note that labour is unambiguously higher here than in the benchmark model:

$$L_{fo}^{FB}\left(s^t\right) = \frac{\theta-1}{\theta-2} > \frac{\theta}{\theta-1} = L^{FB}\left(s^t\right)$$

When entry costs are specified in units of consumption, the number of firms in the First Best responds more to productivity shocks and to entry cost shocks than in the benchmark model. The elasticities are $\frac{\theta-1}{\theta-2}$ and $-\frac{\theta-1}{\theta-2}$, which is greater (in absolute terms) than 1 and -1, respectively. In steady state, the number of firms and consumption are higher than in the benchmark model.

Implementability Condition and Planner Problem

The set of implementable allocations for $\left\{\left(C\left(s^t\right), L\left(s^t\right), N\left(s^t\right)\right)_{s^t\in S^t}\right\}_{t=0}^{\infty}$ is restricted by the free entry condition (35) and the resource constraint (39) for any path of the interest rate $R\left(s^t\right) \geq 1$.

Notice that here, the wage setting scheme does not matter for the optimal allocations. I.e. if there is wage stickiness, this does not restrict the set of implementable allocations for the policy maker. This is because the wage rate no longer enters the free entry condition and therefore does not affect the investment margin. Since wage stickiness does not matter, monetary policy cannot be used to select allocations. The labour income subsidy is also absent from the constraints in the optimal policy problem. The absence of a labour requirement to set up a firm removes the potency of two policy tools, the wage stickiness-cum monetary policy instrument, as well as the labour income subsidy, to affect the investment margin. Lump sum taxes must adjust to satisfy the household budget constraint, the money stock must adjust to satisfy the cash-in-advance constraint. The only policy variable left with which we can affect the investment margin is the entry subsidy $\mu\left(s^t\right)$.

Let $\beta^t \Pr\left(s^t\right)\varphi\left(s^t\right)$ be the Lagrange multiplier on the free entry condition. The planner problem is as follows

$$\max_{\left\{\left(R(s^t),C(s^t),L(s^t),N(s^t)\right)_{s^t\in S^t}\right\}_{t=0}^{\infty}} \min_{\left\{\left(\lambda(s^t),\varphi(s^t)\right)_{s^t\in S^t}\right\}_{t=0}^{\infty}} \mathcal{L}_{fo}$$

where

$$\begin{aligned}
\mathcal{L}_{fo} &= \sum_{t=0}^{\infty}\sum_{s^t}\beta^t \Pr\left(s^t\right)\left\{U\left(C\left(s^t\right)\right)-V\left(L\left(s^t\right)\right)\right. \\
&\quad +\lambda\left(s^t\right)\left[N\left(s^t\right)^{\frac{1}{\theta-1}}Z\left(s^t\right)L\left(s^t\right)-C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]-N\left(s^t\right)F_{fo}\left(s^t\right)\right] \\
&\quad \left.+\varphi\left(s^t\right)\left[\frac{C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta R\left(s^t\right)N\left(s^t\right)}-\left[1-\mu\left(s^t\right)\right]F_{fo}\left(s^t\right)\right]\right\}
\end{aligned}$$

Optimal Interest Rate Policy

The interest rate policy problem is to choose a path for the interest rate $\left\{\left(R\left(s^t\right)\geq 1\right)_{s^t\in S^t}\right\}_{t=0}^{\infty}$ to maximise $\mathcal{L}_{fo}$. The first order condition is

$$\frac{\partial\mathcal{L}_{fo}}{\partial R\left(s^t\right)} = -\beta^t \Pr\left(s^t\right)\varphi\left(s^t\right)\frac{C\left(s^t\right)\left[1+\Gamma\left(s^t\right)\right]}{\theta R\left(s^t\right)^2 N\left(s^t\right)}$$

Because this expression is negative, the Friedman Rule is optimal.

Optimal Allocations under the Friedman Rule

Under the Friedman Rule, we can derive and rearrange the first order conditions of the Optimal Policy problem to express the equilibrium as follows

$$\begin{aligned}
N_{fo}\left(s^t\right)^{\frac{\theta-2}{\theta-1}} &= \left(\frac{\theta-1}{\theta\left[1-\mu\left(s^t\right)\right]+1}\right)\frac{1}{\theta-2}\frac{Z\left(s^t\right)}{F_{fo}\left(s^t\right)} \\
C_{fo}\left(s^t\right) &= \theta\left[1-\mu\left(s^t\right)\right]\frac{N_{fo}\left(s^t\right)F_{fo}\left(s^t\right)}{1+\Gamma\left(s^t\right)} \\
L_{fo}\left(s^t\right) &= \frac{\theta-1}{\theta-2}
\end{aligned}$$

Labour is constant and equal to its First Best level. Suppose the government does not subsidise entry and so $\mu\left(s^t\right) = 0$. Then the number of firms in the Optimal Policy equilibrium is smaller than in the First Best. The entry subsidy that raises the number of firms to its First Best level must satisfy $\frac{\theta-1}{\theta[1-\mu(s^t)]+1} = 1$. Thus, the optimal entry subsidy is

$$\mu\left(s^t\right) = \frac{2}{\theta}$$

Suppose that the elasticity of substitution between goods varieties is calibrated as $\theta = 3.8$, see Bilbiie et al (2007). Then the government finances nearly half of the firms' entry costs. Under this subsidy, the consumption level in the Optimal Policy allocation is First Best. To conclude, when entry costs are specified in terms of final output, we can replicate the First Best using a proportional entry subsidy financed with lump sum taxes.