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WORKING PAPER

Are social welfare states facing a race to the bottom? A theoretical perspective

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March 2009

2009/572

D/2009/7012/24

1 Introduction

Every step towards the completion of European integration appears to be met with renewed concern over its potential negative social side-effects, particularly as regards to protection against social risks (unemployment, sickness and invalidity, age,...) and poverty. In the 1980s and 1990s, governments of different Member States as well as the two EU Commissions headed by Jacques Delors took seriously the menace of 'beggar-thy-neighbour' policies by means of income or social security measures in an integrated market. The EMU may have provided an even greater temptation to do so because other economic policy instruments such as trade policy or monetary policy are kept under tight control. Since the end of the 1990s, there are indeed some signs that the welfare state is rolling back in core EU-member states such as Germany, the Netherlands and France. The fear of lower social protection levels and harsher labour conditions in a more integrated European Union are often cited as an explanation of the opposition against further economic integration in Europe ([20]).

An intuitively appealing way to understand social protection competition is to consider it as a particular form of the neoclassical tax competition. In this literature (for a survey see Wilson ([21]) and Cremer and Pestiau ([23])) one almost exclusively focuses on taxes on mobile capital intended to finance a public good. The possible detrimental result of tax competition in this framework depends largely on the nature of the public good. If the public good benefits those who paid for it (e.g. Tiebout ([24])), tax competition acts as a welfare improving device since it can eradicate inefficient behaviour of a government. However, the more dominant strand in this literature since the 1980s encompasses a race to the bottom result. In seminal papers like the one by Zodrow and Mieszkowski ([25]) one ends up at a suboptimal taxation level as soon as the beneficiary of the public goods (the immobile factor) is not the payer of the public good (mobile capital).

These results no longer necessarily hold in a new economic geography framework as shown for instance by Krugman and Baldwin ([27]) and Sudekum and Pfluger ([26]). On the one hand, a race to the bottom result is unlikely to occur any longer in a core-periphery pattern of economic activity where the core region can even raises its taxes to the amount of the agglomeration rents ([17], ch.15-16). So agglomeration tends to increase national tax autonomy, assuming that the mobile production factor is ab initio agglomerated. On the other hand, the same agglomerative forces tend to increase tax competition when the mobile factor is initially dispersed. Regarding social protection in the EU, this might imply that we should expect more competition from increasing market integration between core (old) member states and less between existing and new member states. This on the condition that we can extend the results on tax competition from the new economic geography literature to social security competition.

This is precisely the aim of this thesis. We determine the conditions for social security competition in a standard Dixit-Stiglitz model of international trade. We enrichen this set-up with endogenous unemployment and governments, subject to an Atkinson equity-efficiency trade-off, that want to redistribute between the unemployed and the employed. In this way we differ in three respects from a first-best case. There are product market imperfections (monopolistic competition), labour market imperfections (unemployment via efficiency wages) and governments setting taxes to raise revenue for an unemployment benefit. Finally, we introduce agglomerative forces in this framework to examine the effect a NEG-framework has on social security competition.

In a Dixit-Stiglitz monopolistic competitive framework ([2]) consumers maximize a constant elasticity of substitution utility function that is symmetric in a bundle of differentiated goods. This reflects the varietas delectat of consumers. The non-homothetic cost function associated with these differentiated goods has a fixed capital cost component (ensuring increasing returns to scale) and a variable labour cost component (linear technology). The absence of economies of scope and simple parsimony creates the bijective relation between firms and varieties: each variety is produced by only one firm and one firm only produces one variety. In maximizing their profit, firms are considered to act atomistically by neglecting the impact their decision has on the overall market conditions. This Chamberlinian large group assumption is one of the main reasons for the tractability of the Dixit-Stiglitz framework. Trade between the regions is, in a non-autarkic case, inhibited by iceberg trade costs. This means that a certain fraction of the transported good 'melts' away during transport (hence 'iceberg'). Under free entry and exit of firms on the market, these assumptions will lead to a demand system where the equilibrium prices are a constant mark-up over marginal costs. In most forms of imperfect competition the optimal price-marginal cost mark-up depends upon the degree of competition which on its turn often depends on the prices itself. As a consequence these forms of modeling imperfect competition become analytically highly complex. A second reason for the widespread use of Dixit-Stiglitz models -and again an outcome of the invariance of the mark-up- is the mill pricing by firms. Firms fully pass on the transportation costs to the consumers. A firm's producer price is the same for sales to all markets.

The above mentioned reasons explain the widespread and dominant use of the Dixit-Stiglitz model in international trade. But the analytical workability of a Dixit-Stiglitz set-up comes at a price. The price elasticities of demand are constant and identical to the elasticities of substitution and equal to each other across all varieties. This entanglement of demand and supply parameters makes it difficult to assess the impact of demand or supply separately on the equilibrium. The constant elasticity of substitution also means that people have the same substitution behaviour independent from the amount consumed of the goods. Besides the lack of identification in comparative static analysis, the modelling set-up also leads to prices that are independent of the spatial distribution of firms and consumers which conflicts results in spatial pricing theory ([3]). Finally, the iceberg assumption implies that trade costs increase as the price of the transported good increases which is highly unlikely. Sometimes one also finds it more convenient to ignore the income effects present in a Dixit-Stiglitz setting.

The second deviation from a first-best case in this model is the introduction of unemployment via efficiency wages ([7]). The main idea behind the efficiency wage hypothesis is that the net productivity of a worker depends positively on the worker's net real income albeit at a decreasing rate. We use the formulation of Summers ([5]) of efficiency wages where the delivered effort by a worker is positively correlated with the difference between the net wage and a reference wage: $a(w) = (w(1-z) - w_R)^{\beta}$ in which z represents the tax rate set by the government on the wage. The strength of the productivity enhancing effect of higher wages is characterized by β and lies between 0 and1. The reference wage w_R represents the outside option for the worker.

As pointed out by Stiglitz ([6]) one could motivate the link between wages and workers' productivity for at least five reasons. First, firms don't want to lower wages even if there is an excess supply of labour because high wages reduce labour turnover and hence, training costs. Other theories are based on imperfect and asymmetric information. Firms could have difficulties to assess the characteristics of workers or could face problems in monitoring the labour effort of workers. In the former case, labourers get a higher wage in order to defer lower skilled persons to apply. In the latter case the increased cost of shirking induces the desired behaviour from the workers. The fourth theory stems from the development literature and states that higher wages allow for a level of nutrition above the subsistence level which promotes effort. The last justification for the efficiency wage hypothesis is called the fair wage hypothesis ([1]) or the gift exchange hypothesis ([10] and Seidel and Egger ([31])). These reciprocity-based voluntary cooperation arguments imply that, if the employee perceives the action of the employer as kind or fair, he will value the employer's payoff positively and as a consequence will deliver a higher effort level. Experiments indicate that employees indeed respond to higher wage offers, combined with higher expected effort, with higher effective effort ([9]).

As could be expected these different explanations give different purports to the reference wage. The traditional approach in choosing the reference wage w_R in efficiency wage models consists in taking the immediate alternative for the worker who may be fired, which may be the unemployment benefit or the weighted average of the wage and unemployment benefit. This approach is thus mainly based on the third motivation of efficiency wages. However, Danthine and Kuman ([8]) argue that this definition of the external wage reference is unable to explain why wage rigidities generate unemployment, since the reference wage is correlated with labour demand. As a consequence, the reference wage can be put by the government at a sufficiently low level such that the labour market clears. Hence, we propose a definition of the reference wage that is independent from the actual market wage or unemployment allowance. A reference wage based on the gift exchange hypothesis is in line with this critique: the reference wage is the wage that would apply if all the workers behaved selfishly, i.e. the market-clearing wage at which the workers provide the basic effort[1].

[1] This also implies that the tax rate set by the government equals zero as there are no unemployed people who need an unemployment benefit.

This definition of the outside option avoids the contradiction of a government able to do the first-best chooses for the second-best options. A purely redistributing government is no longer able to remedy any unemployment occurrence by setting the unemployment benefit low enough to ensure that everybody is willing to work. The use of efficiency wages to introduce the social risk of unemployment instead of the mechanism of wage bargaining between employers and trade unions (eg Lejour and Verbon ([29]) and Leite-Montero et al. ([30])), also resides in the same reason since the government (or median voter) that anticipates the behaviour of the private economic agents (sequential game), could also have restored the first best equilibrium in this case, e.g. by deciding a sufficiently low unemployment benefit in order to restore full employment.

The presence of a redistributing government constitutes the third deviation from the first-best situation. She compensates one market distortion (unemployment) by granting benefits to the unemployed. In order to this, she has to raise taxes which creates additional distortions in the economy. We assume that the government only raises taxes on labour, not on capital. In most EU countries the tax base consists primarily of immobile production factors, labour in the first place. Mobile factors are largely exempted from taxation, either because of tax competition or because of economic efficiency reasons. E.g. Lindert ([32]) argues that the difference between the welfare state in Europe and the US is not matched by differences in economic efficiency because the structure of taxation in Europe is less distortionary, considering the greater share of labour taxation and consumption taxes in the European government revenue.

The amount of taxes and redistribution is determined by maximizing an Atkinson abbreviated social welfare function $\int \frac{x^{1-e}}{1-e} f(x)dx$, when $e \neq 1$ and $\int \ln(x)f(x)dx$ when $e = 1$. $f(x)$ represents the frequency density function of (real) incomes x in society and $0 \leq e$ is inequality aversion parameter. This formulation has many desirable characteristics. The utility of each individual $\frac{x^{1-e}}{1-e}$ is symmetric (anonymity) and only depends on the income of that individual, thereby asserting the self-interest of people. This average utility expression for the social welfare also encompasses the principle of transfers (negative second derivative) and the principle of diminishing transfers (positive third derivative): a fixed income transfer from a poor person to a rich person decreases the social welfare and this decrease is stronger the lower the income of both persons is. Lastly this representation of social welfare also has the property of equiproportionate income growth neutrality as the coefficient of relative inequality aversion (the elasticity of marginal utility) is constant.

To see that the Atkinson abbreviated social welfare function includes an equity-efficiency trade-off, it suffices to rewrite it in terms of the equally distributed equivalent income ξ, which is defined as the income that, if distributed equally, would generate the same welfare as the existing income distribution ($\frac{\xi^{1-e}}{1-e} = \int \frac{x^{1-e}}{1-e} f(x)dx)^2$. Since the Atkinson index of relative inequality is defined as the fraction of income that could be sacrificed with no loss of welfare if all income was distributed equally ($I(e) = 1 - \frac{\xi}{\mu}$), the Atkinson abbrevi-

[2] If e=1, this becomes $\ln(x) = \int \ln(x)f(x)dx$

ated social welfare function can be rewritten as $\frac{1}{1-e}(\mu(1-I))^{1-e}$ if $e \neq 1$ and $\ln(\mu(1-I))$ otherwise. More efficiency (average μ) increases the social welfare as more equity does (inequality index $I(e)$). Based on this new formulation of the Atkinson abbreviated social welfare function, it is easy shown that the elasticity of social welfare with respect to equity equals the elasticity of social welfare with respect to efficiency and that both are equal to $1-e$.

Although this concept of social welfare is analytically more complex than an ad hoc social welfare function (e.g. $SW = \lambda U(\textit{unemployed}) + (1-\lambda)U(\textit{employed})$), we prefer this formulation because it has the main advantage of allowing all possible attitudes towards inequality. If $e = 0$ the government behaves benthamite and only wants to maximize total welfare without caring about redistribution. If, on the other hand, $e = \infty$ the rawslian government only cares for the income of the poorest person of society and devotes no attention at all to efficiency. It also avoids the use of more than one inequality aversion parameter as soon as there are more than two subgroups in society.

In our model there are three individual sources of (real) income: labour income w/P, unemployment benefits b/P and capital rewards CR/P. We assume that the capital rewards are evenly distributed between each individual whether he or she is employed or unemployed. This simplifies the interpretation of the governmental choice since we don't have to introduce a third class of people, namely the capital owners who lead a life of leisure and whose income solely rely on some fixed exogenous parameters on which the government has no influence[3]. As it turns out, the expression for the capital rewards that are evenly distributed among the labour force in a Dixit-Stiglitz setting with efficiency wages is also a constant. This further simplifies the model by reducing the capital rewards to a scaling factor in the indirect utility of the employed and the unemployed. Taking these income assumptions into account, we can restate the Atkinson social welfare as:

$$SW = \begin{cases} \frac{1}{1-e}\left[\sum_{i=1}^{2} f(x_i)x_i^{1-e}\right] = \frac{1}{1-e}\left[(1-u)(\frac{w(1-z)+CR}{P})^{1-e} + (u)(\frac{b+CR}{P})^{1-e}\right] & (e \neq 1) \\ \sum_{i=1}^{2} f(x_i)\ln(x_i) = (1-u)\ln(\frac{w(1-z)+CR}{P}) + (u)\ln(\frac{b+CR}{P}) & (e = 1) \end{cases} \tag{1}$$

In this definition u stands for the unemployment rate. If $e = \infty$ the government acts in a Rawslian way and wants to maximize the income of the poorest individual, in casu the income of the unemployed $SW = (b+CR)/P$. A survey of empirical methods to evaluate the inequality aversion parameter empirically (see ([15], [16])) reveal a wide range of possible values ranging between almost 0 and 10.

Besides the threefold reshaping of a first-best world into a more realistic second-best framework, we also introduce agglomerative forces. We rely for this on the footloose capital model of Martin and Rogers ([11]) which is the analytically most simple model of the new economic geography models. The main reason for its analytical workability is two folded. First, the only mobile

[3] Except for the evident influence via the price index.

factor is the fixed factor of production (capital) whereas in other models such as the core periphery model of Krugman ([9])) the variable and fixed factor of production are mobile. Secondly there is a dichotomy between the ownership of the mobile factor and the use of it. The owner does not relocate, the capital itself can relocate. This implies, contrary to the footloose entrepreneur model of Ottaviano and Forslid ([13]), that there are no circular causalities in the FC-model. Besides the analytical tractability of the footloose capital model, it also reflects best, in our opinion, the European context where full labour mobility is a very strong assumption ([28]). The third reason for choosing this set-up lies in the fact that we can simplify our model by abolishing the traditional A-sector of NEG-models without destroying the agglomerative characteristics of our set-up. In most NEG-models a second Walrasian sector is needed to ensure that in core-periphery equilibria each region preserves the possibility to consume. Since the owners of the mobile factor do not move and receive the rewards to capital irrespective of the location of the employment of the capital each region always has a certain expenditure level. Moreover, it would be difficult to introduce endogenous unemployment in a model with a second perfect Walrasian sector without making additional strong assumptions about the nature of the production factors in that sector.

As stated above, one of the drawbacks of using a non-homothetic cost function as in the footloose capital model lies in the loss of some of the core-periphery features such as circular causality, locational hysteresis and endogenous asymmetry. However, as it turns out, some of these features are restored in our asymmetric taxrate-model by introducing unemployment via efficiency wages. Another possible shortcoming lies in the transitional behaviour of the footloose capital model. Although all new economic geography models have an identical mathematical structure as Robert-Nicoud ([14]) proved, does not mean that all exhibit similar dynamical results. It can be shown that the transition time towards a core situation in a FC-setting is much slower than in the other new economic geography models ([22]). Finally by reforming the footloose capital model from a two-sector to a one-sector model, we expose the whole economy of a region to international trade. All goods produced are tradeables. As non-tradeable services form a dominant share of GDP in modern western economies[4] this assumption is quite strong. A possible way to correct for this in our model, is reducing the level of the trade freeness. We act as if the traded and non-traded goods and services are reshaped to traded goods but subject to higher transaction costs.

In the next section we will start with the derivation of the model in an autarkic situation. While this set-up is not very interesting, it enables us to form an analytical workable bench-mark for the following sections.

[4] 70% or more of GDP stems from services in EU, Japan and U.S. (http://www.ecb.int/mopo/eaec/html/index.en.html)

2 Autarkic situation

2.1 Consumers' choice

The region is endowed with a fixed number of consumers L. Each consumer j consumes an amount c_{ij} (at the price p_i) of a good i. The preferences of these consumers exhibit varietas delectat and are represented by maximizing the following CES utility function:

$$U_j = (\int_0^n c_{ij} di)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

The integral runs over the exogenously given number of produced goods (n in total) and $\sigma(> 1)$ represents the elasticity of substitution between goods which is equal for all goods. Consumers are constrained by their budget. They cannot spend more on goods as their total income which equals their expenditures e_j as there are no savings in our static model:

$$\int_0^n p_i c_{ij} di = e_j \tag{3}$$

Standard utility maximization and aggregating the individual demand of all consumers lead to the following result for the market demand of a variety i:

$$c_i = (\frac{p_i}{P})^{-\sigma}(\frac{E}{P}) \tag{4}$$

where $E = \sum_{j=0}^{L} e_j$ stands for the total expenditures of the region and $P = (\int_0^n p_i^{1-\sigma} di)^{\frac{1}{1-\sigma}}$ is the price index. The consumption of a good i decreases as the price of that good increases ($\frac{\partial c_i}{\partial p_i} < 0$) as could be expected. It also increases as the income of the region increases.

Indirect utility V_j of a consumer j is determined by substituting c_{ij} in (2) with $(\frac{p_i}{P})^{-\sigma}(\frac{e_j}{P})$:

$$V_j = \frac{e_j}{P} \tag{5}$$

Observe that P is a perfect price index in that real income defined with P is a measure of (indirect) utility.

2.2 Producers' choice

Each manufacturer i produces an amount x_i of only one good using a fixed amount of capital (k units) and a variable amount of labour l_i. The production function of a firm is given by:

$$x_i = a(w_i) l_i \tag{6}$$

Note that the productivity parameter $a(w_i)$ depends on the wage (see section (2.3.1)). Given the wage cost w_i and capital cost π_i, the total cost function TC_i is equal to

$$TC_i = k\pi_i + l_i w_i \tag{7}$$

Under the Chamberlinian large group assumption profit maximization with respect to the price leads to the typical Dixit-Stiglitz monopolistic competitive price that is a fixed mark-up over marginal labour costs

$$p_i = \frac{\sigma}{\sigma - 1}\frac{w_i}{a(w_i)} \tag{8}$$

Based on this expression, the price index can be simplified as

$$P = n^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma - 1}\frac{w_i}{a(w_i)} \tag{9}$$

Because there is free entry and exit of firms the zero-profit condition ($p_i * c_i - w_i * l_i - k * \pi_i = 0$) has to hold. Based on this expression, we can easily derive the equilibrium firm scale x_{eq} and the reward to capital. The former is given by $x_{eq} = (\sigma - 1)\frac{k\pi}{w_i/_i a(w_i)}$, while the latter is given by

$$\pi_i = \pi = \frac{E}{n\sigma k} \tag{10}$$

Firms become bigger as the fixed cost reward increases relative to the variable cost reward and become smaller when the operating profit margin decreases. The latter also holds for the reward to capital. Finally, since the right-hand side of equation (10) is independent of i, the left hand side is also independent of i: all firms pay the same capital reward.

Since each firm utilizes k units of capital, the total capital reward TCR of all firms is equal to $TCR = nk\pi = E/\sigma$. Dividing the TCR by the total number of inhabitants L in the region, gives us the constant capital income CR of each citizen in terms of σ:

$$CR = \frac{E}{\sigma L} \tag{11}$$

2.3 Labour markets

2.3.1 Determination of the wage

A firm i determines the wage w employees receive by maximizing their profit:

$$\max_w (p_i * x_i - w_i * l_i - k * \pi) \tag{12}$$

With the help of the envelop theorem ($\frac{\partial x}{\partial w} = 0$), the first order condition leads to the well known Solow condition:

$$\frac{w_i \frac{\partial a(w_i)}{\partial w_i}}{a(w_i)} = 1 \tag{13}$$

This condition states that the elasticity of the efficiency function with respect to the wage equals one. The firm keeps hiring additional people as long as the wage per unit of effort is falling.

The final expression for the employee's remuneration is given by combining expression (13) and the Summers expression for the efficiency wage $a(w) = (w(1-z) - w_R)^{\beta}$. We find that the paid wage increases when the reference wage w_R or the effect of higher wages on the productivity increases (β):

$$w_i(z) = w(z) = \frac{w_R}{(1-\beta)(1-z)} \tag{14}$$

Two interesting conclusions can be formed based on this reward to labour expression. First, the labourers' remuneration is independent of firm-specific parameters. As a consequence all northern firms behave identically. They pay the same rewards to the factors of production, produce at the same price and sell the same quantities. Secondly, while the gross wage increases when the tax on labour increases, the net wage does not. Any tax rise is fully passed through in price increases as the taxation needed for the social security benefits doesn't affect the effort delivered by the labourers.

Substituting the optimal wage set by the firms (14) in the Summers' expression of efficiency wages lead to the optimal level of effort procured by the workers:

$$a^{opt} = w_R^{\beta}\left(\frac{\beta}{1-\beta}\right)^{\beta} \tag{15}$$

2.3.2 Unemployment

We are now able to determine the level of unemployment in the region. Substituting expression (14) in the zero profit condition ($p_i * x_i - w_i * l_i - k * \pi = 0$), lets us determine the amount of labour each firm employs:

$$l = \frac{(\sigma-1)k\pi}{w} \tag{16}$$

Each firm recruits less people when the wages rise. Since the capital reward (10) is constant in the autarkic case, the total amount of wages paid by all firms to their employees $\sum_{i=1}^{n} lw$ is also invariable. This means that, as could be expected, the tax set by a government does not have any influence on the tax base.

From expression (16) it is only a small step to the unemployment level $u(z)$:

$$u(z) = 1 - \frac{nl}{L} \stackrel{16}{=} 1 - \frac{n(\sigma-1)k\pi}{Lw} \stackrel{14,10}{=} = 1 - \frac{(\sigma-1)E(1-\beta)(1-z)}{Lw_R\sigma} \tag{17}$$

2.3.3 Reference wage

The reference wage is defined as the wage that would apply if all the workers behaved selfishly, i.e. the market-clearing wage at which the workers provide the basic effort. At this wage level, there is no unemployment ($nl = L$) and the government does not have to raise any taxes ($z = 0$). Using expressions (16) and (10), this definition lead to the following expression:

$$w_R = \frac{(\sigma - 1)}{\sigma}\frac{E}{L} \tag{18}$$

The reference wage increases as the GDP per person rises but decreases when people appreciate varieties less. Substituting (18) in the expression (17), greatly simplifies the unemployment level in our model:

$$u(z) = 1 - (1 - \beta)(1 - z) \tag{19}$$

A simple look at this expression reveals that the unemployment increases when taxes increase and when people are less willing to put more effort into their work given a certain wage level.

2.4 Government

2.4.1 Unemployment benefit in function of taxes

Our model is static and as a consequence, the government has to run a balanced budget. The total amount of taxes raised on the labour income ($nlwz$) must be higher or equal to the total amount of benefits handed out to the unemployed ($(L - nl)b$). We use this balanced budget to state the unemployment benefit in terms of the tax rate set by the government:

$$b(z) = \frac{1 - u(z)}{u(z)} w(z) * z \overset{19,14}{=} \frac{w_R * z}{\beta + (1 - \beta) * z} \tag{20}$$

In the simple autarkic model, the unemployment benefit always increases when the tax rate increases. The unemployment benefit is also automatically higher than the reference wage and the net wage in the autarkic case.

2.4.2 Benthamite case (e=0)

When the government only has efficiency considerations, the social welfare the government wants to maximize is

$$SW(e = 0) = (1 - u(z)) * \frac{w(z) * (1 - z) + CR}{P(z)} + u(z) * (\frac{b(z) + CR}{P(z)}) \tag{21}$$

Using the balanced budget constraint and the definition of the wage (14), this can be simplified to:

$$SW(e = 0) = \frac{CR + w_R}{P(z)} \tag{22}$$

The first derivative of (22) amounts to

$$\frac{dSW(e = 0)}{dz} = \frac{-(CR + w_R)}{(P(z))^2} \frac{n^{\frac{1}{1-\sigma}} * \sigma * w_R}{(\sigma - 1) * (1 - \beta) * a^{opt}} \frac{1}{(1 - z)^2}, \tag{23}$$

which is clearly always negative. Since the only way in which the utilitarian government can exert an influence on the social welfare is via the price index and since the effect of a tax raise on the price index is negative, a Benthamite government will always choose for the corner solution of a zero tax rate.

2.4.3 Rawslian case (e=∞)

The other extreme is the situation of a Rawslian government. Now the government is only concerned in the welfare of the poorest, in casu the unemployed:

$$SW(e = \infty) = \frac{b(z) + CR}{P(z)} \tag{24}$$

The optimal tax rate for the government in this case will depend on a comparative assessment between the positive effect a tax raise has on the unemployment benefit and the negative effect the same tax increase has on the purchasing power of the unemployed person. Both effects are easily determined via $dSW/dz = dSW/dz|_{P=cst} + dSW/dz|_{b=cst}$, where the first term on the right-hand side of the equation represents the benefit effect and the second term the price effect. Simple calculus leads to the following expressions for these effects:

$$dSW/dz|_{P=cst} = benefit\ effect = \frac{\beta w_R}{(\beta + (1-\beta)z)^2 * P(z)} \tag{25a}$$

$$dSW/dz|_{b=cst} = price\ effect = \frac{-(b(z) + CR)}{(1-z) * P(z)} \tag{26}$$

Equating both effects leads to a quadratic equation in z of which the (largest) root is given by[5]:

$$z^{opt} = \frac{-\beta * (w_R + (1-\beta) * CR) + \sqrt{\beta * (w_R + (1-\beta) * CR)}}{(1-\beta) * (w_R + (1-\beta) * CR} \tag{27}$$

Simplifying this expression by substituting the reference wage w_R by it's definition (18) and the constant capital income of each individual CR by (11), shows that as soon as $\sigma > 2$, the optimal tax rate for a Rawslian government is always positive. If the elasticity of substitution lies between 1 and 2, the corner solution of a zero tax rate becomes possible if $\beta > \sigma - 1$. Or in other words, it becomes more likely for a government to opt for a zero tax rate the higher the productivity enhancing effect of higher wages becomes or the stronger people prefer variety. The reason for this lies in a dominant price effect. For these extreme values of β and σ the price index approaches zero which makes everybody almost equally rich. As a result the government no longer sees the need to redistribute. One can understand this more intuitively by realizing that these extreme values for β and σ coincide with a very high value for the capital reward per capita ($CR > \frac{W_R}{\beta}$). This high value in effect means that all people become (in the limit) equally rich.

[5] The smallest root is always negative.

2.4.4 General case

We already know that on the one hand, a utilitarian government will choose for a zero tax rate since any tax increase will reduce the efficiency of the economy. On the other hand a Rawslian government normally chooses for a positive tax rate as she is only concerned in the welfare of the unemployed person. In this subsection we generalize these conclusions by considering the general social welfare function. An inspection of (1) reveals that besides the effect a tax increase has on the purchasing power (the price effect) and the benefits (benefit effect), there is now also a third effect, namely the unemployment effect. Any tax increase will increase the unemployment and hence reduce the efficiency of the economy. Taking the first derivative of the general social welfare function with respect to the tax rate z and keeping the relevant variables constant ($dSW/dz = dSW/dz|_{P,u=cst} + dSW/dz|_{b,u=cst} + dSW/dz|_{b,P=cst}$) gives the expressions for the three effects:

$$dSW/dz|_{b,u=cst} = price\ effect = \begin{cases} \frac{-1}{1-z} & (e=1) \\ \frac{-(P(z))^{e-1}}{1-z}\Big[(1-\beta)(1-z)(\frac{w_R}{1-\beta}+CR)^{1-e} & \\ +(\beta+z*(1-\beta))*(b(z)+CR)^{1-e}\Big] & (e\neq 1) \end{cases} \tag{28}$$

$$dSW/dz|_{P,u=cst} = benefit\ effect = \begin{cases} \frac{\beta * w_R}{(b(z)+CR)*u(z)} & (e=1) \\ \frac{w_R*\beta}{u(z)*P(z)}*(\frac{b(z)+CR}{P(z)})^{-e} & (e\neq 1) \end{cases} \tag{29}$$

$$dSW/dz|_{b,P=cst} = unemployment\ effect = \begin{cases} (1-\beta)*\log(\frac{b(z)+CR}{\frac{w_R}{1-\beta}+CR} & (e=1) \\ \frac{1-\beta}{1-e}*\frac{1}{(P(z))^{1-e}}* & \\ \Big[(b(z)+CR)^{1-e}-(\frac{w_R}{1-\beta}+CR)^{1-e}\Big] & (e\neq 1) \end{cases} \tag{30}$$

The sign of these effects in the general case are unambiguously determined. The price effect and the unemployment effect[6] are always negative, while the benefit effect is always positive. If we equate the sum of these three effects to zero, we would be able to derive an expression for the optimal tax rate. Unfortunately this is not analytically possible in the general case and therefore, we rely on simulations.

The parameters in this simulation are calibrated in such a way that comparability between the simple autarkic model and the subsequent more complicated models is facilitated. It also serves intuition. We set the non-crucial parameters $L = n = k$ equal to 1 and take the value 0.6 for β. The reason for this relatively

[6] This can be seen by realizing that the net wage $w(1-z) = \frac{w_R}{1-\beta}$ always exceeds the unemployment benefit $b(z)$.

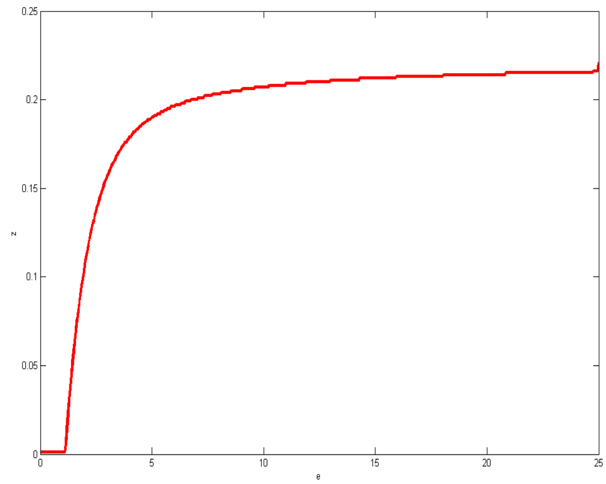

Figure 1: Optimal tax rate in function of inequality aversion

high value for the leap-frogging effect compared to the value Summers suggest ([5]) is two folded. First lower values of β (e.g. 0.1) lead in the two-country simulations to values of the unemployment benefit which were significantly higher than the net wage. We wanted to exclude these cases from our model. Secondly we also believe that the high value of β and as a consequence the high value of the unemployment rate (around 50-70%) are not that unrealistic since in our model there are no inactive persons. People either work or are unemployed. There are no inactive persons like children, pensioners, etc. in our model. For instance, according to the data of ECODATA[7] 60.2% of the Belgian population was inactive. The elasticity of substitution is taken to be equal to 2.5. An alteration of these parameters will shift the curves but will not change the nature of the solutions.

As a first step we determine the optimal tax rate in function of the inequality aversion. This is represented in graph (1). We clearly see that as governments take greater care for the worse off persons in the society, the tax rate increases. Only for low values of the inequality aversion parameter, the strongly utilitarian inclined government chooses not to levy any taxes. Table (1) gives some tax rates in function of the inequality aversion parameter.

In our set-up the government with an inequality aversion parameter larger than 10 mimics the Rawslian behaviour, formally defined at $e = \infty$. When σ is increased in the simulation, the optimal autarkic tax rates will also increase (e.g. $z_{opt}(e = \infty) = 0.346$ at $\sigma = 5$ and $z_{opt}(e = \infty) = 0.384$ at $\sigma = 8$). An increase in σ will increase the competition on the goods markets and hence, make the

[7] http://ecodata.mineco.fgov.be/mdn/bevolking.jsp

e	0	1	2	3	5	10	15	25	∞
$\mathbf{z}_{opt}$	0	0	0.107	0.17	0.19	0.207	0.212	0.215	0.221

Table 1: Optimal tax rates in function of inequality aversion

economy more efficient (a higher tax base). The income of the unemployed and the employed increases by the same amount. So the inequality index does not change. But given her fixed equity-efficiency trade-off, the government decides to raise her taxes.

We also looked at the relative strength of the three effects previously discussed in the optimum. As graph (2) shows, the unemployment effect plays a marginal role compared to the negative price and the positive benefit effect. The strength of these effects also increases exponentially for high values of the inequality aversion parameter.

The total expenditures level E and the number of inhabitants L in a region don't have any influence on the optimal tax rate. Both parameters only appear combined (as E/L) in the expression for the reference wage (18) and the capital reward (11) and hence don't influence the unemployment benefit, the net wage or the unemployment rate. They only affect the price index and the average capital reward and a change in these can only lead to equiproportional changes in the social welfare fucntion which don't affect the optimal choice of the tax rate (the SW-function only shifts up- or downwards) . On the other hand, a raise in the number of firms n and the units of capital required as a fixed cost k has a positive impact on the chosen tax rate. The reason for this lies in the combination of the decrease in the average capital reward and the increase (for n) or the status quo (for k) of the price index. The other constituents of the social welfare function, namely the unemployment benefit and the unemployment rate are invariant to a change of these two parameters as can be easily seen from (19) and (20). This causes a non-homothetic shift of the social welfare function. Otherwise said, the real increase of the indirect utility due to an increase of n or k is less strong for the unemployed than the employed and as a result of this, the government raises her taxes. Since this divergent evolution of the relative utility between the unemployed and the employed can only occur through the average capital reward and we don't focus on this consant term, we will assume in the subsequent chapters that the values of the parameters remain constant at $L = n = k = E = 1$.

3 Two-country case without capital mobility

3.1 Consumers' choice

There are two regions, called the north and the south. We assume that both regions are symmetric in terms of consumers' tastes ,technology, openness to

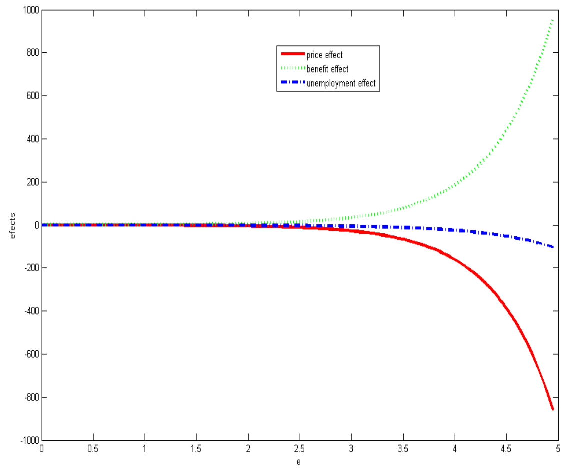

Figure 2: 3 effects in function of inequality aversion.

trade and factor supplies. The northern region is endowed with a fixed number of consumers L_N, the south has L_S inhabitants. We also assume that the inhabitants of the northern region have an endowment of K_N units of capital while the south has K_S units of capital at it's disposal. The worldwide capital endowment is denoted as $K_W = K_N + K_S$. We will often work with the capital shares instead of simple endowments: $s_K = \frac{K_N}{K_W}$ and $1 - s_K = \frac{K_S}{K_W}$. For reasons of expositional simplicity, we will limit the exposition to the northern region.

The constrained optimization problem for the northern consumer j with an expenditure level e_j who consumes an amount c_{ij} (at the price p_i) of a good i is now equal to:

$$U_j = (\int_0^{n+n^*} c_{ij} di)^{\frac{\sigma}{\sigma-1}} \tag{31}$$

$$\text{s.t.} \int_0^{n+n^*} p_i c_{ij} di = e_j \tag{32}$$

The integral runs over the exogenously given number of produced goods (n northern goods and n^* southern varieties, their sum equal to n^W). Standard utility maximization and aggregating the individual demand of all consumers leads to the following result for the northern market demand of a variety i:

$$c_i = (\frac{p_i}{P})^{-\sigma}(\frac{E_N}{P}) \tag{33}$$

where $E_N = \sum_{j=1}^{L_N} e_j$ [8]stands for the total northern expenditures and $P = (\int_0^{n+n^*} p_i^{1-\sigma} di)^{\frac{1}{1-\sigma}}$ is the northern price index[9]. The consumption of a good i decreases as the price of that good increases ($\frac{\partial c_i}{\partial p_i} < 0$[10]) as could be expected. It also increases as the income of the region increases.

Indirect utility V_j of a consumer j is determined by substituting c_{ij} in (2) with $(\frac{p_i}{P})^{-\sigma}(\frac{e_j}{P})$:

$$V_j = \frac{e_j}{P} \tag{34}$$

Observe that P is a perfect price index in that real income defined with P is a measure of (indirect) utility.

3.2 Producers' choice

3.2.1 Prices

The production function x_i and the total cost function TC_i of a northern manufacturer i are, just as was the case in autarky given, by:

$$x_i = a(w_i)l_i \tag{35}$$

$$TC_i = k\pi_i + l_i w_i \tag{36}$$

Contrary to the autarkic case, a firm now sells in two regions. The export to the southern region is inhibited by trade costs τ. The total production of a firm x_i is in equilibrium, when the markets clear, equal to the sum of the consumption of the good i in the north c_{iN} and the consumption of the good in the south c_{iS} multiplied by the trade costs:

$$x_i = c_{iN} + \tau * c_{iS} \tag{37}$$

Under the Chamberlinian large group assumption profit maximization with respect to the price the northern firm applies in the north p_{iN} and in the south p_{iS} leads to the typical Dixit-Stiglitz monopolistic competitive price that is a fixed mark-up over marginal labour costs

$$p_{iN} = \frac{\sigma}{\sigma - 1}\frac{w_i}{a(w_i)} \tag{38}$$

$$p_{iS} = \frac{\sigma}{\sigma - 1}\frac{w_i}{a(w_i)}\tau \tag{39}$$

[8]For the south we have a similar expression $E_S = \sum_{j=1}^{L_S} e_j$. The sum of the northern and southern expenditures, the world expenditures is denoted as E^W.

[9]The southern price index is denoted with an '*': $P^* = (\int_0^{n+n^*} p_i^{1-\sigma} di)^{\frac{1}{1-\sigma}}$

[10] $\frac{\partial c_{ij}}{\partial p_i} = e_j * p_i^{-\sigma-1} * P^{\sigma-1} * (-\sigma + (\sigma - 1) * (\frac{p_i}{P})^{1-\sigma} * P^{\sigma})$
This is negative iff $(\frac{p_i}{P})^{1-\sigma} < \frac{\sigma}{\sigma-1}$, which is always the case.

Comparing (38) and (39), it is clear that firms find it optimal to engage in mill pricing. The full shipping costs to the southern region are passed on to the southern consumers.

We will see in paragraph (3.3.1) that, just as was the case in case in autarky, the labourers' remuneration is independent of firm-specific parameters. This means that prices (a fixed mark-up over marginal labour costs) and the consumption of a certain variety c_i are non-specific to firm characteristics (albeit region-specific). All northern varieties are hence produced in the same amounts and also sold in the same amounts on each market. This allows us in the subsequent elaboration of the model to introduce four 'kinds' of goods: a 'northern' variety sold in the north, a northern variety sold in the south, a southern variety sold in the south and a southern variety sold in the north. The prices and amounts consumed of these types of goods are respectively given by:

$$p_{NN} = \frac{\sigma}{\sigma - 1} * \frac{w}{a}, \qquad c_{NN} = (\frac{p_{NN}}{P})^{-\sigma} * \frac{E_N}{P} \tag{40}$$

$$p_{NS} = \frac{\sigma}{\sigma - 1} * \frac{w}{a} * \tau, \qquad c_{NN} = (\frac{p_{NS}}{P^*})^{-\sigma} * \frac{E_S}{P^*} \tag{41}$$

$$p_{SS} = \frac{\sigma}{\sigma - 1} * \frac{w^*}{a^*}, \qquad c_{SS} = (\frac{p_{SS}}{P^*})^{-\sigma} * \frac{E_S}{P^*} \tag{42}$$

$$p_{SN} = \frac{\sigma}{\sigma - 1} * \frac{w^*}{a^*} * \tau, \qquad c_{SN} = (\frac{p_{SS}}{P})^{-\sigma} * \frac{E_N}{P} \tag{43}$$

3.2.2 Price indices

Based on the previous four expressions for the prices, we can work out the northern and southern price index as follows:

$$P = \alpha * (\frac{w^*}{a^*}) * \Delta^{\frac{1}{1-\sigma}} = \alpha * (\frac{w^*}{a^*}) * (\epsilon * s_n + (1 - s_n) * \phi)^{\frac{1}{1-\sigma}} \tag{44}$$

$$P^* = \alpha * (\frac{w^*}{a^*}) * (\Delta^*)^{\frac{1}{1-\sigma}} = \alpha * (\frac{w^*}{a^*}) * ((1 - s_n) + s_n * \epsilon * \phi)^{\frac{1}{1-\sigma}} \tag{45}$$

We grouped the constant parameters in $\alpha = \frac{\sigma}{\sigma-1} * (n^W)^{\frac{1}{1-\sigma}}$. We also redefined the number of firms in each region (n and n*) in terms of shares: $s_n = \frac{n}{n^W}$ is the share of northern firms, $1 - s_n$ the share of the southern firms. Note that in a two-country model without capital mobility the share of capital employed in the northern region, s_n ,is per definition equal to the initial endowment of capital s_K. $\phi = \tau^{1-\sigma}$ represents the well-known freeness of trade which can also be described as the economic distance between the two regions. That is, the freeness of trade rises from $\phi = 0$, with infinite trade costs, to $\phi = 1$, with zero trade costs. In this way we defined two important variables in our model in a compact space. This is handy for inspection of the expressions and also makes the numerical simulations later on more reliable. The last, yet to explain variable is ϵ, which is equal to the northern relative production costs $(\frac{w/a(w)}{w^*/a^*(w^*)})^{1-\sigma}$. When the north has lower (higher) production costs as the south, ϵ is larger (smaller) than 1. So ϵ can serve as a measure of the relative competitiveness of the northern region versus the southern region and varies in principle between 0 and ∞.

By writing the price indices in this way, we can see that each price index is composed of a part stemming from the sales of the domestic firms and a part stemming from imports, weighted by the economic distance between the two regions ϕ, the relative competitiveness ϵ or both.

3.2.3 Sales and the equilibrium firm scale

As a next step we determine the sales of a firm in function of the share of expenditures s_E. We will use this later on when we focus on the capital reward. The total sales of a northern firm S equal the sum of his sales in the north ($p_{NN} * c_{NN}$) and the sales in the south ($p_{NS} * c_{NS}$). This can under the market clearing condition be written as:

$$S = p_{NN} * c_{NN} + p_{NS} * c_{NS} = p_{NN} * (c_{NN} + \tau c_{NS}) = p_{NN} * x \tag{46}$$

Using the definition of the consumption and prices of northern goods sold in the north (40) and the south (41), we can rewrite the northern sales as

$$S = \frac{\sigma}{\sigma - 1} * \frac{w}{a} * E^W * (\frac{(\frac{\sigma}{\sigma-1} * \frac{w}{a})^{-\sigma} * s_E}{P^{1-\sigma}} + \tau * (1 - s_E)\frac{(\tau * \frac{\sigma}{\sigma-1} * \frac{w}{a})^{-\sigma} * E^S}{(P^*)^{1-\sigma}})$$

Substituting the northern and southern price indices with the appropriate expressions (44) and (45) allows for the following simplification of the sales in terms of the share of expenditures:

$$S = \frac{E^W}{n^W} * \epsilon * \left[\frac{s_E}{\Delta} + \frac{\phi * (1 - s_E)}{\Delta^*}\right] = \frac{E^W}{n^W} * B \tag{47}$$

The southern sales S^* are similarly derived:

$$S^* = \frac{E^W}{n^W} * \left[\frac{\phi * s_E}{\Delta} + \frac{(1 - s_E)}{\Delta^*}\right] = \frac{E^W}{n^W} * B^* \tag{48}$$

By realizing that $s_n * B + (1 - s_n) * B^* = 1$, it is easy to interpret the B's as the biases in sales, as the extent to which the sales of a variety exceeds the world average per variety sales. This is a familiar way of writing sales in a NEG-framework. A closer look at (47) and (48) shows that a price increase always reduces the sales of a firm ($\partial S/\partial p_{NN} < 0$) which should come as no surprise since the elasticity of substitution $\sigma(> 1)$ equals the elasticity of demand in a Dixit-Stiglitz framework. Secondly, it is easy to derive that an increase of the share of expenditures in the home country of the firm always increases the sales of the firm ($\partial S/\partial s_E > 0 \Leftrightarrow 1 > \phi^2$, which always holds[11]).

To determine the equilibrium firm scale, we apply the zero-profit condition which equates the operating profit to the total capital reward. The operating profit of a northern firm equals $S - w * l$, which can be written as $p_{NN} * x - \frac{w}{a} * x = p_{NN} * x/\sigma$ using (35) and (40). As a consequence the equilibrium firm scale of a (northern) firm is equal to

$$x^{eq} = \frac{(\sigma - 1) * k * \pi}{w/a} \tag{49}$$

Firms become bigger as the fixed cost reward increases relative to the variable cost reward and becomes smaller when the operating profit margin decreases. This mimics the autarkic case result.

3.2.4 Capital reward

Since physical capital is used only in the fixed cost component of industrial production, the reward to capital is the Ricardian surplus of a typical variety, that its, the operating profit of a typical variety[12]. We get the following expressions for the capital reward of the north π and the south π^*:

$$\pi = \gamma * \epsilon * \left[\frac{s_E}{\Delta} + \frac{\phi * (1 - s_E)}{\Delta^*}\right] = \gamma * B \tag{50}$$

$$\pi^* = \gamma * \left[\frac{\phi * s_E}{\Delta} + \frac{(1 - s_E)}{\Delta^*}\right] = \gamma * B^* \tag{51}$$

Due to the symmetry of our model ($k = k^*$), the regroupment of a string of constant parameters, $\gamma = \frac{E_W}{k * \sigma * n_W}$, is the same for both regions. Since $0 < s_n < 1$ and $\epsilon > 0$, Δ and Δ^* will always be positive. This lets us conclude that, if s_E lies between 0 and 1 (as it will), both the profit in the north and in the south are positive. The positivity of both expressions is important for the numerical simulations later on.

3.3 Labour markets

3.3.1 Determination of the wage

A firm i determines the wage w_i employees receive by maximizing their profit:

$$\max_{w_i}(p_{iN} * c_{iN} + p_{iS} * c_{iS} - w_i * l_i - k * \pi_i) \tag{52}$$

[11] $\frac{\partial S}{\partial s_E} = \frac{E^W}{n^W}\left[\frac{1}{\Delta} - \frac{\phi}{\Delta^*}\right]$ is positive if and only if $\frac{1}{\Delta} > \frac{\phi}{\Delta^*}$, which can be rewritten using the definition of Δ and Δ^* as $1 - s_n > (1 - s_n)\phi^2$

[12] The reward to capital would be bid up to the point where it equalled operating profit as noticed by Baldwin et. al.([17]).

With the help of the envelop theorem ($\frac{\partial x}{\partial w} = 0$), the first order condition leads to the well known Solow condition that is exactly equal to the autarkic case:

$$\frac{w_i \frac{\partial a(w_i)}{\partial w_i}}{a(w_i)} = 1 \tag{53}$$

This condition states that the elasticity of the efficiency function with respect to the wage equals one. The firm keeps hiring additional people as long as the wage per unit of effort is falling. Just as in the autarkic case, we find the final expression for the employee's remuneration by substituting the Summers expression for the efficiency wage $a(w) = (w(1-z) - w_R)^\beta$ in expression (53):

$$w_i(z) = w(z) = \frac{w_R}{(1-\beta)(1-z)} \tag{54}$$

This proves our previous statement that the wages are set firm-independently. As a consequence all northern firms behave identically and sell the same amount of goods to each market. An inspection of (54) also reveals that, once again, the gross wage increases when the tax on labour increases but the net wage does not. It also shows that the wage setting is independent from the foreign tax setting. The expression for the optimal level of effort procured by the workers is the same as in the autarkic case:

$$a^{opt} = w_R^\beta (\frac{\beta}{1-\beta})^\beta \tag{55}$$

The symmetry between the two regions entails the identity between the northern efficiency parameter β and the southern one β^*. Owing to this, the southern equivalents for (54) and (55) are easily derived to be equal to:

$$w^*(z^*) = \frac{w_R^*}{(1-\beta)(1-z^*)}, \qquad a^{*,opt} = (w_R^*)^\beta (\frac{\beta}{1-\beta})^\beta \tag{56}$$

3.3.2 Unemployment

We are now able to determine the level of unemployment in the region. Substituting expression (40) and (35) in the zero profit condition ($p_{NN} * x - w * l - k * \pi = 0$), lets us determine the amount of labour each firm employs:

$$l = \frac{(\sigma-1)k\pi}{w} \tag{57}$$

Each firm recruits less people when the wages rise. Although this expression is identical to the autarkic one, there is a big difference between the two frameworks. While in the autarkic case the total amount of wages paid by all firms to their employees $\sum_{i=1}^{n} lw$ is constant, this is no longer the case in the two-country case. Tax changes will change the capital reward and as a consequence influence the tax base

From expression (57) it is only a small step to the unemployment level $u(z)$:

$$u(z,z^*) = 1 - \frac{nl}{L_N} \overset{16}{=} 1 - \frac{n(\sigma-1)k\pi(z,z^*)}{L_N * w} \overset{54,??}{=} 1 - \delta * s_n * (1-z) * \pi(z,z^*) \quad (58)$$

We grouped all the constant parameters in a new parameter δ which is equal to $\frac{(\sigma-1)*(1-\beta)*n_W*k}{L_N*w_R}$. Note that this expression for the unemployment (58) does not guarantee that the unemployment rate is always equal to or larger than zero. We will have to impose an extra restriction on the social welfare optimization later on in order to ensure meaningful results.

3.3.3 Share of expenditures

The capital reward of a region depends on the expenditure shares of the regions (see (50) and (51)) and the expenditure shares of a region hinge on the capital and labour rewards earned in the same region. This means that we have to solve this circularity before we can introduce a government choosing a tax rate. We do this by finding a closed expression for the share of expenditures in terms of the tax rates. We start by looking at the total reward TLR stemming from labour income (employed and unemployed):

$$TLR = n * l * w * (1-z) + (L_N - n * l) * b \quad (59)$$

Using the same budget balance of the government as in the autarkic case ($n * l * w * z = (L_N - n * l) * b$) and expression (57), expression (59) can be written in terms of the capital reward:

$$TLR = n * l * w = (\sigma - 1) * k * n_W * s_n * \pi \quad (60)$$

People not only earn income from working (or receiving an unemployment benefit) but also from having a certain amount of capital. Just as before we assume that each individual has an equal share of the total capital income of a region, TCR. However, determining the total capital reward of a region is a lot trickier in comparison to the autarkic case, since we normally would have to know where the capital owned by the northern residents is working. We can overcome the introduction of a supplementary variable[13] by assuming, as Martin and Rogers did in their footloose capital model ([11]), that half of the capital used in each region belongs to the northern capital owners regardless of s_n. This implies at the same time that each region owns half of the worldwide

[13] Namely, the amount of the northern-owned capital that is working in the north. The amount of the northern-owned capital working in the south would be equal to 1 minus the previous amount.

capital ($s_K = 1/2$). As a consequence each unit of capital, independent of the ownership of it, earns the world average reward to capital ACR . This is given by:

$$\begin{aligned} ACR &= \frac{TCR + TCR^*}{K_W} = \frac{n * k * \pi + n^* * k * \pi^*}{K_W} \overset{50,51}{=} \\ & \frac{k * n^W * \gamma * (s_n * B + (1 - s_n) * B^*)}{K_W} \overset{s_n * B + (1 - s_n) * B^* = 1}{=} \frac{E_W}{K_W * \sigma} \end{aligned} \quad (61)$$

Multiplying the average capital reward ACR with the total number of units of capital owned by the north K gives us the total northern capital reward $TCR = s_K \frac{E_W}{\sigma}$.

Given both components of income (or expenditures) in a region, the share of expenditure is easily derived as:

$$s_E(z, z^*) = \frac{TCR + TLR}{E_W} = \frac{s_K}{\sigma} + \frac{\sigma - 1}{\sigma} * s_n * B \quad (62)$$

Substituting B by (47) and solving for s_E gives a closed-form expression for the northern share of expenditures:

$$s_E(z, z^*) = \frac{(\sigma - 1) * s_n * \epsilon * \phi * \Delta + s_K * \Delta * \Delta^*}{\sigma * \Delta * \Delta^* - (\sigma - 1) * s_n * \epsilon * (\Delta^* - \phi * \Delta)} \quad (63)$$

Note that although we started from a classical footloose capital set-up, we didn't obtain a share of expenditures that is independent from s_n. This result even holds when capital is immobile. Due to the introduction of endogenous unemployment via efficiency wages we have that production shifting (Δs_n) leads to expenditure shifting (Δs_E). This linkage will put the demand-linked circular causality back on-line as we will discuss in the next section (4). As a final remark, it is important to realize that the share of expenditures in a region depends on the tax of the home country but also on the tax rate of the foreign country. Since the capital reward depends on s_E, all the variables depending on the capital reward such as the unemployment and the unemployment benefit also depend on the tax rates of both countries.

Before we go on and determine the reference wage, we assess the conditions under which the share of expenditures given by formula (63) always lies between 0 and 1. This in order to make sure that the numerical simulations we will rely on later on don't give unrealistic results. As it turns out the share of expenditures automatically fulfills this restriction. For we can rewrite the condition $se < 1$ as $s_n * \epsilon * (s_K - 1) < \phi * (1 - s_n) * (\sigma - s_K)$, which always holds because the exogenous parameters s_K, s_n, and σ in here have the right sign restrictions: $0 < s_K < 1, 0 < s_n < 1$ and $\sigma > 1$. The second restriction $s_E > 0$ can be rewritten as $\sigma * \epsilon^2 * \phi * sn^2 + \sigma * (1 - s_n)^2 * \phi + \quad \sigma * \epsilon * \phi^2 * (1 - sn) + (1 - s_n) * sn * \epsilon * (1 - \phi^2) + \sigma * \epsilon * \phi^2 * sn * (1 - s_n) > 0$, which -evidently given the same restriction on the exogenous parameters- always hold.

3.3.4 Reference wage

The reference wage is defined as the wage that would apply if all the workers behaved selfishly, i.e. the market-clearing wage at which the workers provide the basic effort. At this wage level, there is no unemployment ($nl = L_N$, $n^* l^* = L_S$) and the government does not have to raise any taxes ($z = 0$, $z^* = 0$). Given the symmetric nature of our regions, this means that the share of firms in each region also equals 1/2. Using expressions (57), the northern definition of the reference wage becomes:

$$w_R = \frac{n^R * (\sigma - 1) * k * \pi^R}{L^N}, \tag{64}$$

with n^R equal to $\frac{1}{n^W * 2}$. The capital reward in this benchmark case is given by (50). Given the fact that $s_n = s_K = \frac{1}{2}$, the share of expenditures s_E in the region is equal to 1/2 which allows us to rewrite the profit π^R as γ. As a result, we can write the reference wage as

$$w_R = \frac{1}{2}\frac{n^W}{L_N}(\sigma - 1) * k * \gamma = \frac{\sigma - 1}{2 * \sigma}\frac{E_W}{L_N} \tag{65}$$

Under the additional assumption that $L_N = L_S$, the reference wages of both regions are equal. Just as in the autarkic case, the reference wage increases when the GDP per person rises.

3.4 Government

3.4.1 Unemployment benefit in function of taxes

The balanced budget constraint of the government allows us to determine the unemployment benefit in function of the tax rate:

$$b(z, z^*) = \frac{1 - u(z, z^*)}{u(z, z^*)} w(z) * z \overset{58,54}{=} \frac{\delta * s_n * w_R * \pi(z, z^*) * z}{(1 - \beta) * (1 - \delta * s_n * \pi(z, z^*) * (1 - z))} \tag{66}$$

Replacing the capital reward in the denominator by $\gamma * B(z, z^*)$ (see (50)) and using the definitions of the parameters δ and γ, we can simplify this expression further as follows:

$$b(z, z^*) = \frac{(\sigma - 1) * E_W * w_R * s_n * B(z, z^*) * z}{L_N * w_R * \sigma - s_n * B(z, z^*) * (1 - z) * (\sigma - 1) * (1 - \beta) * E_W} \tag{67}$$

In the simple autarkic model, the unemployment benefit always increases when the tax rate increases. The (positive) unemployment benefit is also always automatically higher than the reference wage. Contrary to the autarkic case, the complexity of formula (67) it is impossible to analytically guarantee that the unemployment benefit is always lower as the net wage people receive. This means that we have to impose a second extra restriction on the social welfare optimization by the government.

3.4.2 Benthamite case (e=0)

The government chooses a tax rate by maximizing the social welfare function given by (1). The only analytical solvable case is the Benthamite case. For all other attitudes towards inequality, we have to rely on numerical simulations. Just as in the autarkic model, the social welfare function for a government with only efficiency considerations is given by

$$SW(e=0) = (1-u(z,z^*)) * \frac{w(z)*(1-z)+CR}{P(z,z^*)} + u(z,z^*) * (\frac{b(z,z^*)+CR}{P(z,z^*)}) \tag{68}$$

Using the balanced budget constraint and the definition of the gross wage (54), expression (68) can be simplified to:

$$SW(e=0) = \frac{(1-u(z,z^*)) * w(z) + w_R}{P(z,z^*)} \tag{69}$$

Substituting the unemployment rate by its definition (58) and taking the first derivative leads to the following first order derivative

$$\frac{\partial SW(e=0)}{\partial z} = \frac{\delta * s_n * w_R}{(1-\beta) * P(z,z^*)} \frac{\partial \pi(z,z^*)}{\partial \epsilon} \frac{\partial \epsilon(z,z^*)}{\partial z} - \frac{\delta * s_n * w_R * \pi(z,z^*) + CR * (1-\beta)}{(1-\beta) * (P(z,z^*))^2} \frac{\partial P(z,z^*)}{\partial z} \tag{70}$$

The derivative of the capital reward π (given by (50)) with respect to the indicator of the competitive effect ϵ, the derivative of $\epsilon = (\frac{w/a(w)}{w^*/a^*(w^*)})^{1-\sigma}$ to the tax rate z and the derivative of the price index $P(z,z^*)$ (see (44)) are respectively given by

$$\frac{\partial \pi(z,z^*)}{\partial \epsilon} = \frac{(\sigma - s_K)s_n^2(1+\epsilon^2) + 2s_n(1-s_n)\epsilon\sigma\phi + \phi^2\sigma(1-s_n)^2 + s_K(1-\phi^2) + s_K(1+\epsilon^2)\phi^2 s_n^2}{(\sigma\phi + s_n((s_n-2)\sigma\phi + s_n\epsilon^2\sigma\phi + (1-s_n)\epsilon(1+(2\sigma-1)\phi^2)))^2} \tag{71}$$

$$\frac{\partial \epsilon(z,z^*)}{\partial z} = \frac{1-\sigma}{1-z}\epsilon \tag{72}$$

$$\frac{\partial P(z,z^*)}{\partial z} = \frac{s_n \epsilon P(z,z^*)}{(1-z)\Delta} \tag{73}$$

Expressions (71) and (73) are always positive, while the second expression (72) is unambiguously negative. As a consequence, the first order derivative of the Benthamite social welfare function (70) is always negative. The only possible solution in this case is the corner solution of a zero tax rate as could be expected given the result in the autarkic model.

Note that the fact that (71) is positive, also means that in the two-country model without capital mobility, higher tax rates always lead to lower capital rewards and since the taxable base $n * l * w$ can be written as $\pi * k * n * (\sigma - 1)$ (see 58), it also follows that any tax reduction automatically leads to higher tax revenues.

3.4.3 General case

Only for the specific case of a utilitarian government we were able to explicitly derive the optimal tax rate chosen by the government. For all the other cases we rely on simulations. We will discuss the results of the two-country model without capital mobility in three steps. Firstly we will look at what effects determine the optimal tax rate set by a government given the foreign tax rate. Secondly we will identify the channels through which a southern tax rate affects the northern social welfare and vice versa. This gives intuition to the observed Nash equilibrium between the two countries. Finally we will shortly discuss the influence the choice of the parameter σ, s_n and ϕ have on the Nash equilibrium.

Benefit, unemployment and price effect A government wants to maximize social welfare by choosing an appropriate tax rate given its inequality aversion. In the first order condition of this optimization problem, the tax rate, just as we observed in the autarkic model, works through three channels. Firstly, higher taxes induce higher unemployment benefits. This positive effect is offset by two negative effects in equilibrium. Higher taxes lead to a reduced purchasing power of the economic agents and they also induce higher unemployment rates, thereby gnawing the tax base. The absence of the benefit effect evidently explains the corner solution of the zero tax rate in the Benthamite case.

The strength of these effects are found by taking the first derivative of the social welfare function with respect to the tax rate z and keeping the relevant variables constant ($dSW/dz = dSW/dz|_{P,u=cst} + dSW/dz|_{b,u=cst} + dSW/dz|_{b,P=cst}$). After simple but cumbersome calculations these three effects are equal to:

$$dSW/dz|_{b,u=cst} = \begin{cases} \frac{-s_n\epsilon(z,z^*)}{(1-z)\Delta(z,z^*)} & (e=1) \\ \frac{-s_n\epsilon(z,z^*)}{(1-z)\Delta(z,z^*)} * (1-e) * SW(z,z^*) & (e \neq 1) \\ \frac{-s_n\epsilon(z,z^*)*(b(z,z^*)+CR)}{(1-z)\Delta(z,z^*)P(z,z^*)} & (e=\infty) \end{cases} \tag{74}$$

$$dSW/dz|_{P,u=cst} = \begin{cases} \frac{\delta * s_n * w_R}{(b(z,z^*)+CR)*u(z,z^*)*(1-\beta)} * \\ \left[\pi(z,z^*)*(1-\delta s_n \pi(z,z^*)) + z * \frac{\partial\pi(z,z^*)}{\partial\epsilon}\frac{\partial\epsilon(z,z^*)}{\partial z}\right] & (e=1) \\ \frac{\delta w_R s_n}{(1-\beta)*u(z,z^*)}(\pi(z,z^*)*(1-\delta s_n\pi(z,z^*)+z*\frac{\partial\pi(z,z^*)}{\partial\epsilon}\frac{\partial\epsilon(z,z^*)}{\partial z})* \\ (b(z,z^*)+CR)^{-e}*(P(z,z^*))^{e-1} & (e\neq 1) \\ \frac{\delta w_R s_n}{(1-\beta)(u(z,z^*))^2 P(z,z^*)}(\pi(z,z^*)*(1-\delta s_n\pi(z,z^*))+ \\ z*\frac{\partial\pi(z,z^*)}{\partial\epsilon}\frac{\partial\epsilon(z,z^*)}{\partial z}) & (e=\infty) \end{cases} \tag{75}$$

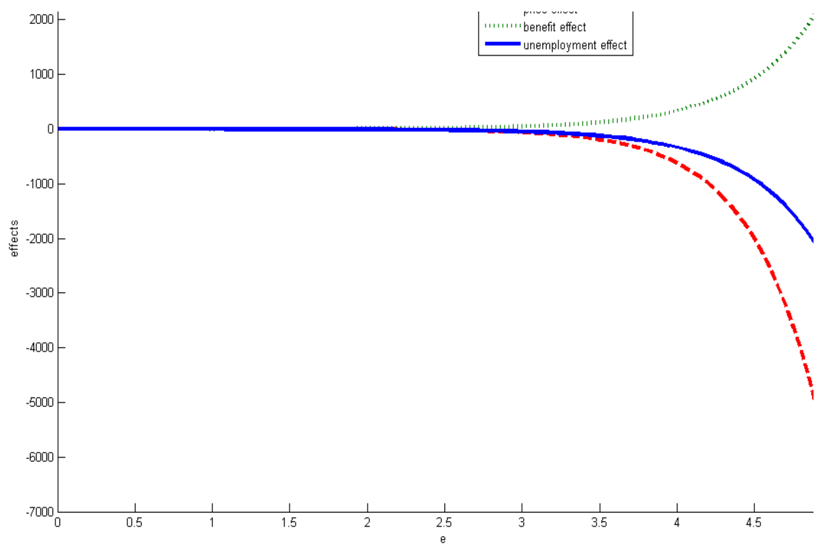

Figure 3: 3 effects in function of northern inequality aversion

$$dSW/dz|_{b,P=cst} = \begin{cases} \delta s_n * \log(\frac{b(z,z^*)+CR}{\frac{w_R}{1-\beta}+CR}) * (\pi(z,z^*) \\ \quad -(1-z)\frac{\partial\pi(z,z^*)}{\partial\epsilon}\frac{\partial\epsilon(z,z^*)}{\partial z}) & (e=1) \\ \frac{\delta s_n}{1-e} * \frac{1}{(P(z,z^*))^{1-e}} * (\pi(z,z^*) - (1-z)\frac{\partial\pi(z,z^*)}{\partial\epsilon}\frac{\partial\epsilon(z,z^*)}{\partial z}) \\ \quad * \left[(b(z,z^*)+CR)^{1-e} - (\frac{w_R}{1-\beta}+CR)^{1-e}\right] & (e\neq 1) \\ 0 & (e=\infty) \end{cases} \tag{76}$$

The derivatives $\frac{\partial\pi(z,z^*)}{\partial\epsilon}$ and $\frac{\partial\epsilon(z,z^*)}{\partial z}$ in these three definitions are given by the expressions (71) and (72) respectively. The plot of these three effects is given in figure (3) for a value of the trade freeness equal to 0.5. The evolution is similar to the one seen in the autarkic case.

Competition and real income effect In the previous paragraph we analyzed the effect the home tax rate has on the home social welfare. In this paragraph we look at the externalities that the foreign tax rate (in casu the southern) has on the own (northern) social welfare. We do this by considering the first order derivative of the (northern) social welfare with respect to the southern tax rate. We can distinguish two channels. The first one is analogous to the previous paragraph, namely the price index. A tax drop in the south will decrease the southern gross wage which shifts the marginal costs downwards. Due to mill pricing, the prices for the southern goods will also decrease while the northern goods won't change in price. The result is a lower price index in both regions. Since the Atkinson index of relative inequality is independent of equiproportional income changes, the northern inequality does not change. However, the real average income (the measurement of efficiency) increases. Given the constant inequality-efficiency trade-off of the government, this means that she has to raise taxes to increase the equality between the working and unemployed people in its society. The real income effect leads to tax rates that are substitutes between each other.

The second mechanism through which the southern tax rates affect the northern region is called the competition effect. The price drop of southern goods from the decrease of the southern tax rate will increase the southern sales and decrease the northern sales since the northern firms asked the same prices as before and because the elasticity of demand, equal to the elasticity of substitution between goods, is larger than 1. This also implies that the northern reward to capital decreases while the southern one increases. The reason for this lies in the zero-profit condition. But a lower (northern) reward to capital has as a consequence that the tax base in the north decreases because the tax base is equal to a fraction of the capital reward (again zero profit condition!). So we can conclude that a southern tax decrease creates a negative externality on the northern region: the northern tax base shrinks because of the increased competitiveness of the southern region. The northern government is thus faced with higher unemployment rates and a smaller tax base to collect taxes to pay for the unemployment benefits. This not only lowers the efficiency of their economy (lower average income) but also negatively affects the equity (more unemployed). The government with a given equity-efficiency trade-off tries to counter this by also decreasing its tax rate. As a result, the competition effect leads to tax rates that are complements between each other.

We plotted both effects in a characteristic case. We calculated the first order derivative of the northern social welfare with respect to the southern tax rate and plotted this value against the southern tax rate. The inequality aversion of both regions is assumed to be equal to 3. Each region has also the same number of firms and the trade freeness is set to be equal to 0.5. For the northern tax rate value we took the Nash equilibrium value corresponding to these values of the parameters. The other parametric values are $E_W = 1$, $L_N = 1 = L_S$, $s_K = 1/2$ and $k = 1$.

Figure (4) clearly shows that the competition effect from an increase in the foreign tax increases the social welfare of the north, while the real income effect decreases the social welfare of the north. We also see that the competition effect outweighs the real income effect although both forces become less strong for increasing values of the foreign tax rate. This can also be seen by representing the reaction curves of both regions. We represented the reaction curves for a given value of trade freeness ($\phi = 0.5$) and with the same parametric values as before ($\sigma = 3$, $\beta = 0.6$, $E_W = 1$, $L_N = 1 = L_S$, $s_n = s_K = 1/2$, $k = 1$).

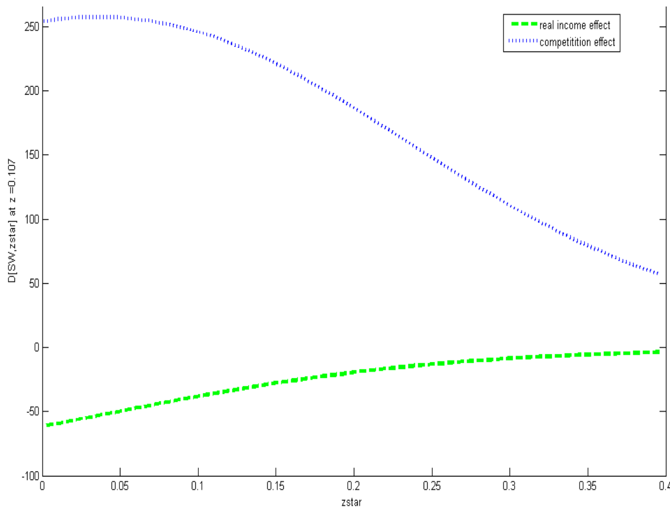

Figure 4: Effects of z^* on SW at e $= 3$,s_n =0.5 ,z =0.106 and $\phi = 0.5$

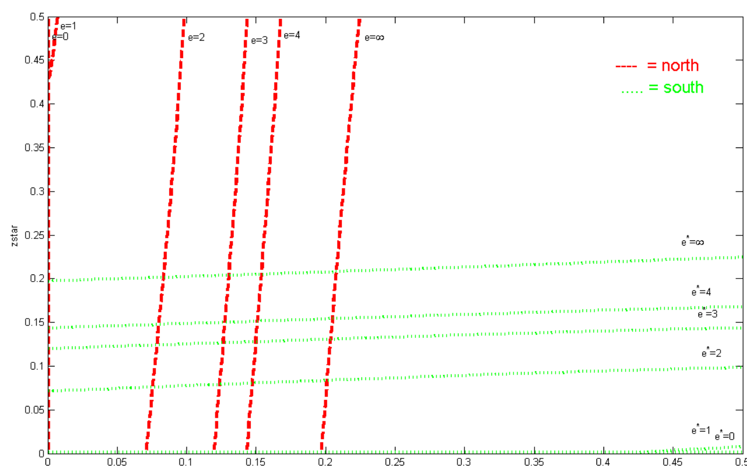

Figure 5: Reaction curves in function of e at$\phi = 0.5$ and $\beta = 0.6$

The vertical (red) lines in figure (5) represent the northern optimal tax rate given a southern tax rate (on Y-axis). The horizontal (green) lines are the reaction curves of the south given a northern tax rate (on the X-axis). The intersection of both curves is the Nash equilibrium. As illustrated on the graph this Nash equilibrium shifts towards higher tax rates when the inequality aversion of the regions increases. This is intuitively quite clear since a higher inequality aversion means that governments care more about the unemployment, hence want higher unemployment benefits and, as a result, set higher tax rates.

Based on this graph we can conclude that the tax rates set by each government act as strategic complements: any tax reduction of the foreign region will lead to a tax reduction in the home country. The negative externality imposed by a foreign tax reduction on the home tax base outweighs the positive real income effect and as a result of this, the home region also lowers its tax rate. There is, in other words, social security competition between the two regions.

The severity of the social security competition can also be seen by looking at the optimal tax rate a social planner would choose compared to the separate regions. We define the social planner as the government that sets a single tax rate in both regions to maximize the sum of the indirect utilities over the inhabitants of both regions[14]. In the following table, the second row represents the optimal tax rate set by two regions with the same inequality aversion (allows us to compare it with the social planner), the third row represents the optimal tax rate set by the social planner. We used the same parameter values as in the previous simulations (σ= 2.5). It is clear that the social planner opts for higher tax rates because he or she will internalize the effects we discussed before.

	$e=e^*=0$	$e=e^*=1$	$e=e^*=2$	$e=e^*=3$	$e=e^*=4$	$e=e^*=\infty$
$z=z^*$	0	0	0.075	0.126	0.151	0.208
z_{SP}	0	0	0.107	0.157	0.179	0.221

Table 2: Optimal tax rates of social planner and two equally inequality averse regions for several values of the inequality aversion

.

Note also that the choice of the social planner nearly mimics the choice of the autarkic region. This should come as no surprise as both optimizations are identical except for some minor parametric values (e.g. L=1 versus $L_N = L_S = 1$). This again illustrates the social security competition in our model.

Influence of σ, ϕ and s_n As a final step we discuss the impact of some parameters on the Nash equilibrium. We start with the elasticity of substitution σ. The effect is mixed. The elasticity of substitution characterizes the fierceness of the competition on the goods market and as a consequence will lead to a

[14] This means that we exclude a priori a social planner that chooses a different tax rate in each region. This is a not unlogical assumption in our symmetric model.

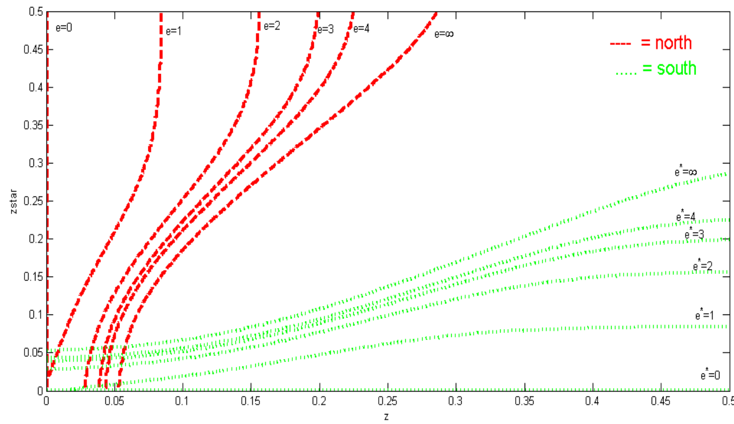

Figure 6: Reaction curves in function of e for a high value of $\sigma(= 8)$

stronger negative externality, a stronger competition effect. That's why higher values of σ are coupled with lower tax rates set by individual governments. The loss of taxable base associated with a tax rate increases when the elasticity of substitution increases. At the same time a social planner that internalizes the externalities associated with trade will opt for higher taxes since the average income of and the inequality between the people increased (see autarkic case). The Nash rates chosen by the social planner are depicted in table (3) for three different values of the elasticity of substitution and in function of the inequality aversion.

	e=e*=0	e=e*=1	e=e*=2	e=e*=3	e=e*=4	e=e*=∞
$\boldsymbol{\sigma = 2.5}$	0	0	0.107	0.157	0.179	0.221
$\boldsymbol{\sigma = 5}$	0	0.151	0.261	0.301	0.321	0.351
$\boldsymbol{\sigma = 8}$	0	0.206	0.311	0.344	0.357	0.383

Table 3: Influence of sigma

To show the influence of σ for the regions, we plot the reaction curves of both regions for a high value of $\sigma(= 8)$. Other parameters have the same values as before ($\phi = 0.5$, $\beta = 0.6$, $E_W = 1$, $L_N = 1 = L_S$, $s_n = s_K = 1/2$, $k = 1$). The reaction curves intersect at lower values of tax rates. Note that the diminution of the concavity of the reaction curves in this case can be explained by referring to figure (4). The overall positive effect of the foreign tax rates on the northern social welfare function decreases for high values of the southern tax rate. This means that the accompanying increase in the northern tax rate also diminishes for high southern tax rates.

Secondly, we look at the influence of the share of firms in a region s_n. When

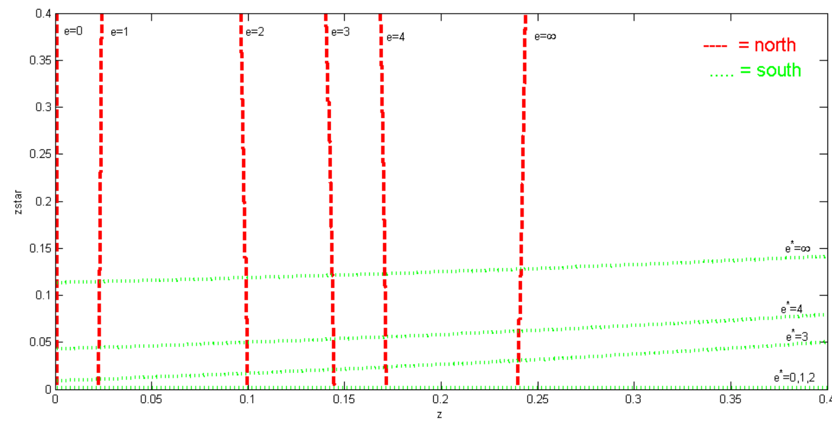

Figure 7: Reacton curves for s$_n$=0.9

the share of firms in a region increases, the price index in that region drops. Normally, the number of employed people will also increase. This means that given a fixed tax rate, the inequality will rise, besides a further increase of the average income. The only way a government can overcome this unwanted straddle is by raising the tax rate to increase the equality between the people. This effect can be seen by looking at the northern Nash rates in table (4). Due to the symmetric set-up, the southern Nash tax rate when $s_n = 0.1$ is equal to the northern Nash tax rate for $s_n = 0.9$. To illustrate this effect we plot the reaction curves for s_n=0.9 in figure (7). A higher share of firm for a region means that the reaction curves of that region are shifted to the right[15]. Note also that for the region with the high share of firms the tax rate is almost independently set from the foreign tax rate.

	e=e*=0	e=e*=1	e=e*=2	e=e*=3	e=e*=4	e=e*=∞
s$_n$= 0.1	0	0	0	0.021	0.056	0.126
s$_n$= 0.5	0	0	0.075	0.126	0.151	0.208
s$_n$= 0.9	0	0.021	0.101	0.146	0.171	0.241

Table 4: Northern Nash rates for different shares of firms

Lastly, we look at the effect of lower trade costs. We plotted the reaction

[15] The parameters in this simulation are the same as in the previous graphs:$\phi = 0.5$, $\beta = 0.6$, $E_W = 1$, $L_N = 1 = L_S$, $s_K = 1/2$, $k = 1$, $\sigma = 2.5$.

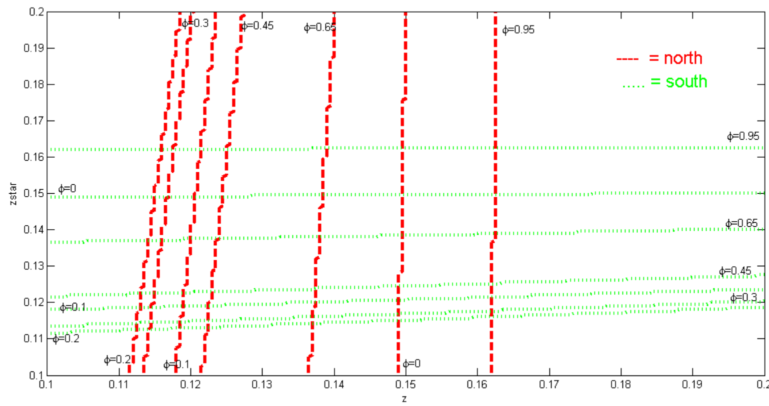

Figure 8: Reaction curves for different trade freeness values at e=e*=3.

curves in function of the trade freeness in graph (8)[16]. We can see that the Nash rates decrease until the trade freeness lies around 0.25. After that, the optimal tax rates start to increase again and even surpasses the optimal taxation level associated with high trade costs. To illustrate this phenomenon better, we depicted the optimal Nash rates together with the optimal tax rate chosen by the social planner in function of the trade freeness for the same parametric values as before. This is done in figure (9). For high values of trade costs, the Nash tax rate is always lower than the Pareto tax rate of the social planner, while, at the high end of trade freeness, the regions will want to have higher tax rates than the social planner who always chooses the same tax rate in function of ϕ. Based on this figure, one could come to the conclusion that sustained lowering of trade barriers would lead to a world with less social security competition and higher tax rates. However, as explained in the introduction (1) one could reasonable assume that the realistic range of the trade freeness lies in the lower half of possible values for ϕ since the whole economy (including services) are subject to trade in our model.

To explain the evolution of the regional Nash rates, we look at the change in the 'cost' that a tax shift of the own tax rate has on the social welfare. As seen before, the home tax rate works through three channels on the social welfare function. On the one hand there is the price index effect, on the other hand one can also distinguish the unemployment and benefit effect. Since the latter two basically cause a change in the tax base, we will group these two effects into a single effect, namely the tax base effect and we will focus, without any loss of generality, on the effect that a change in the trade freeness has on the sensititivy of the tax base to the tax rate. The price index effect will have a restraining

[16]We took the following parametric values $e = e^* = 3$, $\sigma = 2.5$, $\beta = 0.6$, $E_W = 1$, $L_N = 1 = L_S$, $s_n = s_K = 1/2$, $k = 1$

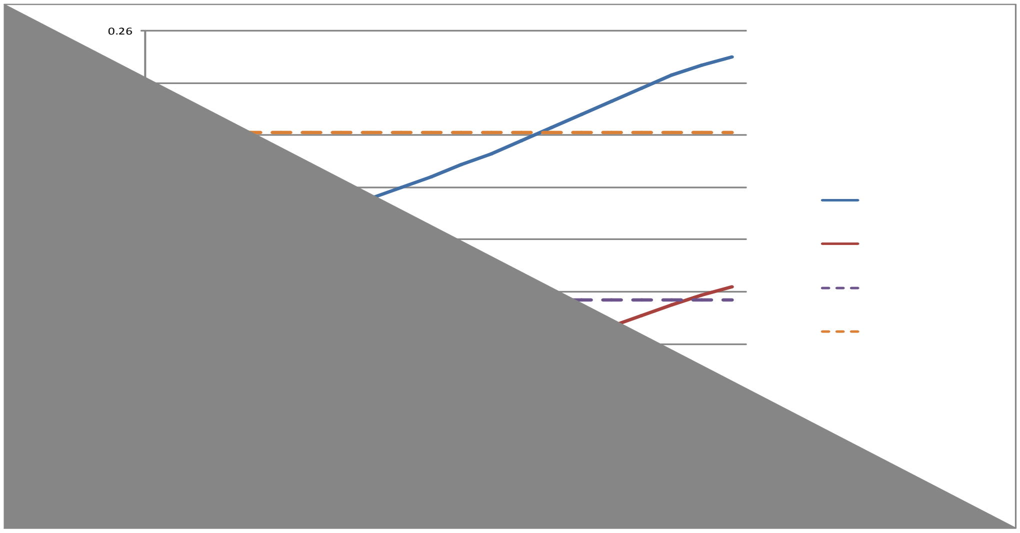

Figure 9: Nash tax rates for the region and SP in function of ϕ for two values of e

effect on the optimal tax rate since a tax raise increases the price index and hence reduces the real income of the economic agents. The decrease in the tax base caused by an increase in the tax rate will increase the social welfare since it provides the basis for an unemployment benefit. So the smaller the loss of the tax base associated with a tax increase becomes, the higher the chosen tax rate will be.

We start by looking at the evolution of the price index effect in function of the trade freeness. When trade costs decrease, all imported goods become cheaper and this will have a positive impact on the real income of active and inactive persons. As a consequence a social welfare maximizing government with a fixed equity-efficiency trade-off will choose for a higher tax rate to let the equity keep pace with the increased efficiency. Or stated otherwise, the negative effect associated with a home tax raise (namely the drop in real income) will become less powerful for higher values of the trade freeness. We captured this pheonomenon by looking at the elasticity of the price index with respect to the tax rate in function of the trade freeness. This is illustrated in graph (10) where we see that the elasticity of the price index with respect to the tax rate is positive but that it decreases when the trade costs decrease.

The evolution of the second effect is less straightforward due to the non-linearities present in the model. We now look at the elasticity of the tax base with respect to the tax rate in function of the trade freeness[17]. This elasticity is again represented in graph (10). We see that this elasticity sharply diminishes

[17] Remember that the tax base $n * l * w$ equals $n * (\sigma - 1) * k * \pi$ (57)

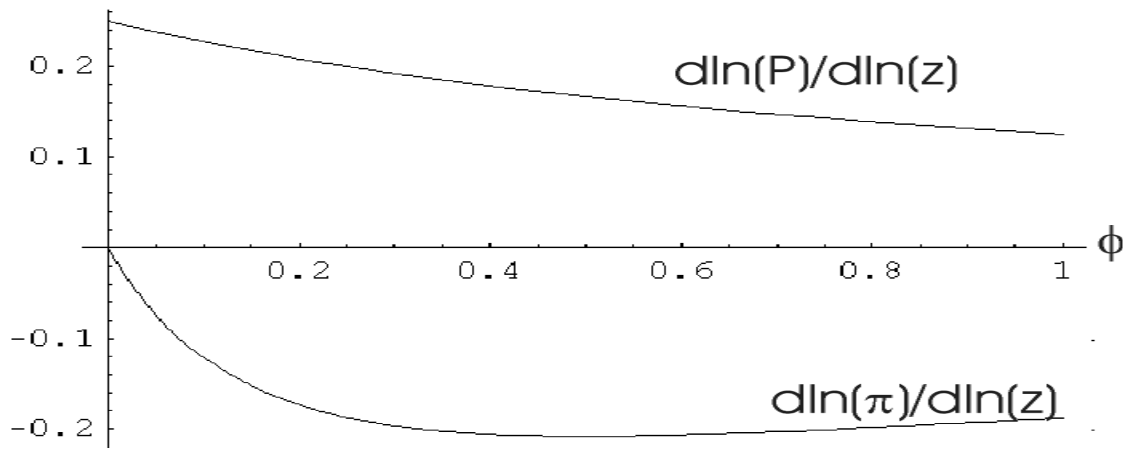

Figure 10: The unequal influence of ϕ on the tax base and P (at $z = z^* = z^{OPT}(e = e^* = 3)$).

until the trade freeness lies around 0.25 after which there is no significant change any more in the elasticity. So, the same tax rate change will induce a much larger decrease in the tax base for values of the trade costs at the high end. As a consequence, the government will decrease its chosen tax rate (if it only looks at this effect) when the ϕ evolves from 0 tot 0.25. For lower values of the trade costs, the government will, based on this tax base effect, see no reason to change the tax rate any longer.

By combining both effects, we are able to explain the evolution of the Nash tax rates. For low values of the trade freeness, the tax base effect dominates the price index effect and this will lead to a decrease of the Nash rate. After $\phi = 0.25$ the tax base effect ceases to have a noticeable impact and the only remaining effect present, namely the price index effect will cause a gradual increase of the Nash rate when the trade costs further decline.

A social planner will set its optimal tax rate invariant of the trade freeness. This is again explained by looking at the two effects as discussed just now: the first effect, the price index effect will become invariant to the trade freeness and the second effect will cease to exist. This can easily be seen by looking back at the expressions of the price index (44) and the tax base (50). These expressions reduce for a social planner ($\epsilon = 1$, $z = z^*$, $s_n = s_E = 1/2$) to $\alpha * \frac{w}{a} * (\frac{1+\phi}{2})^{1/1-\sigma}$ and γ respectively. As a consequence the price index effect $\frac{d\ln(P)}{d\ln(z)}$ is equal to $\frac{z}{1-z}$ and the tax base effect $\frac{d\ln(\pi)}{d\ln(z)}$ becomes zero. The latter can intuitively easily be understood since both (symmetric) regions are per definition always equally competitive. The first effect can graphically be restated as a homothetic social welfare function under trade freeness changes. The parabolic social welfare function in function of he tax rate z shifts upwards when the trade freeness increases without a horizontal shift. More intuitively this can be understood by realizing that a social planner internalizes the partial shifting of the welfare burden induced by a tax increase on the real income of people. Each time a government increases its taxes, a part of the burden, namely the proportion of imported goods in the foreign price index, is shifted on to the foreign country since they also become poorer in real terms. This effect no longer plays for a social planner.

We can conclude that for high values of trade costs the regional governments overestimate the positive effect of a tax reduction on the competitiveness of the own region. This is because governments neglect the fact that the foreign region will also lower its tax rates as a response. For low values of trade costs, this effect fades out and governments are now underestimating the negative effect that a tax increase has on the price index since they don't consider the side-effect of their own tax increase, namely a foreign tax increase. A social planner will internalize these effects and will as a consequence don't change its tax rate to a changing level of trade freeness.

4 Two-country case with capital mobility

4.1 Locational equilibrium

4.1.1 Determining the steady states

Previously we looked at social security competition in a context of no capital mobility. In this section we broaden the model by introducing economic geography effects. As already explained in the introduction (1), we assume that capital migrates to the other region as soon as it can get a higher nominal reward in that region. The owners of the capital don't move (a footloose capital setting) which is why we only have to look at the nominal capital reward, not the real reward of capital. In accordance with the standard NEG-models we use the following ad hoc migration equation for the interregional capital flows:

$$\frac{d(s_n)}{dt} = (\pi - \pi^*)(1 - s_n)s_n \tag{77}$$

As explained in Baldwin ([17]), this formulation encompasses two desired characteristics of (capital) migration. Not only is the rate of migration proportional to the (nominal) capital reward gap, the last two terms on the right-hand side also indicate that the capital migration will not happen at once although all capital is identical. By modeling the capital flows in this way, we neglect (and simplify) the possible forward-looking behaviour of capital (owners).

Equation (77) shows that there are two types of long-run equilibria[18]. Interior equilibria are characterized by equal capital rewards in the north and the

[18]In the NEG-literature one makes a distinction between short-run equilibria where s_n is fixed and long-run equilibria which are the steady states of migration equation (77).

south ($\pi = \pi^*$). The second kind of equilibria are the core-periphery equilibria when all the capital is located in either region ($s_n = 0$ or $s_n = 1$). Note that the concept of equilibrium used here does not mean that all agents can't gain by unilaterally deviating from the equilibrium. It is merely a concept of a steady state, the only relevant long-run equilibria are the stable long-run equilibria.

We can rewrite the profit in each region by substituting (63) in the expressions (50) and (51) as follows:

$$\begin{aligned} \pi &= \gamma \frac{\epsilon(\phi\sigma\Delta + s_K(\Delta^* - \phi\Delta)}{\sigma\Delta\Delta^* - (\sigma-1)\epsilon s_n(\Delta^* - \phi\Delta)} \\ \pi^* &= \gamma \frac{(\sigma-1)\epsilon s_n(\phi^2-1) + \sigma\Delta + s_K(\phi\Delta^* - \Delta)}{\sigma\Delta\Delta^* - (\sigma-1)\epsilon s_n(\Delta^* - \phi\Delta)} \end{aligned} \tag{78}$$

Equating both expressions of (78) and substituting Δ by (44) and Δ^* by (45), we obtain a closed-form expression for the share of capital or firms in the north:

$$s_n = \frac{\sigma\phi(1-\epsilon\phi) + s_K\epsilon(\phi^2-1)}{\sigma(\epsilon-\phi)(\epsilon\phi-1) - (\sigma-1)\epsilon(\phi^2-1)} \tag{79}$$

This means that the model has the desired characteristic of having a closed-form expression of the share of firms while at the same time, does not lose the circular causality as was the case in the footloose capital model.

In a next step we assess the locational choice in function of ϵ and ϕ where this expression (79) is valid. To simplify subsequent derivations, we already assume that the share of capital owned by a region is equal to 1/2. By equating expression (79) to 0 and 1 respectively and solving to ϵ, it is easy to establish that internal solutions are only possible if and only if

$$0 < s_n < 1 \Leftrightarrow \left[\sigma < \frac{1+\phi}{2\phi} \wedge \epsilon_0 < \epsilon < \epsilon_1\right] \quad \vee \quad \left[\sigma > \frac{1+\phi}{2\phi} \wedge \epsilon_1 < \epsilon < \epsilon_0\right] \tag{80}$$

In this expression, ϵ_0 and ϵ_1 stand for

$$\epsilon_0 = \frac{2\sigma\phi}{1+(2\sigma-1)\phi^2}, \qquad \epsilon_1 = \frac{1}{\epsilon_0} \tag{81}$$

To sharpen the intuition, we calculate the maximal allowed tax gap between the two regions in order to make an internal solution possible, whether it will be stable or not. The tax gap $\frac{1-z^*}{1-z}$ is given by $\epsilon^{1/(1-\sigma)}$ under the assumption of equal reference wages and is given in table (5). We also gave the border values for the competitiveness in brackets. We see that in general the regional tax rates can't diverge too much if an internal solution has to occur. Only for very small values of the trade freeness and the elasticity of substitution the allowable tax gap in an internal equilibrium can be quite considerable. This is an illustration of the strong agglomerative forces present in our model.

Analogously we investigate the range of trade freeness wherein the share of firms lies between 0 and 1. We have to make a distinction between cases where the northern country has a competitive disadvantage ($\epsilon < 1$) and where it has a competitive advantage ($\epsilon > 1$). In both cases there are two zones of internal equilibria albeit they are not the same:

$$0 < s_n < 1 \Leftrightarrow 0 < \epsilon < 1 \wedge \left[\phi < \phi_{0A} \vee \{\phi_{1A} < \phi < \phi_{1B} \quad \wedge \quad \sigma > \frac{1+\sqrt{1-\epsilon^2}}{\epsilon^2}\} \right] \tag{82}$$

$$0 < s_n < 1 \Leftrightarrow \epsilon > 1 \wedge \left[\phi < \phi_{1A} \quad \vee \quad \{\phi_{0A} < \phi < \phi_{0B} \wedge \sigma > \epsilon^2(1+\sqrt{\epsilon^2-1})\} \right] \tag{83}$$

In these expressions we defined the following parameters:

$$\begin{array}{ll} \phi_{0A} = \frac{\sigma-\sqrt{\epsilon^2-2\epsilon^2\sigma+\sigma^2}}{2\epsilon\sigma-\epsilon}, & \phi_{0B} = \frac{\sigma+\sqrt{\epsilon^2-2\epsilon^2\sigma+\sigma^2}}{2\epsilon\sigma-\epsilon} \\ \phi_{1A} = \frac{\epsilon\sigma-\sqrt{1-2\sigma+\epsilon^2\sigma^2}}{2\sigma-1}, & \phi_{1B} = \frac{\epsilon\sigma-\sqrt{1-2\sigma+\epsilon^2\sigma^2}}{2\sigma-1} \end{array} \tag{84}$$

Note that for $\epsilon = 1$, $\phi_{0A} = \phi_{1A}$ and $\phi_{0B} = \phi_{1B}$.

Again we give some numerical values for the introduced parameters. These are depicted in table (6). Note that the zones of internal equilibrium are again very limited. This is once more an illustration of the strong agglomerative forces present in this model.

4.1.2 Stability of the steady states

After having found the zones of internal equilibrium, it remains to study the stability of the found steady states. This is checked using Krugman's informal stability test ([12]) which is, as proven by Baldwin ([18]), equal to the formal standard mathematical stability tests. An internal equilibrium is stable when a northward migration reduces the northern capital reward gap ($\pi - \pi^*$) since the migrated capital would be better off if it had stayed in the original region. A core-periphery pattern is stable as soon as the level of the capital reward in the core exceeds the capital reward in the periphery. Mathematically this means that we should check the negativity of $\left.\frac{d(\pi-\pi^*)}{ds_n}\right|_{int}$ for internal equilibria and the sign of $\pi - \pi^*|_{s_n=1}$ for stable northern core solutions. The point where the former equals zero is called the break point, the point where the latter becomes negative is denoted as the sustain point.

	ϕ=0.1	ϕ=0.5	ϕ=0.9
$\boldsymbol{\sigma = 2.5}$	63% ($0.48 < \epsilon < 2.08$)	16% ($1.25 > \epsilon > 0.8$)	4.0% ($1.06 > \epsilon > 0.94$)
$\boldsymbol{\sigma = 5}$	2.2% ($0.91 < \epsilon < 1.09$)	11% ($1.53 > \epsilon > 0.65$)	2.0% ($1.08 > \epsilon > 0.92$)
$\boldsymbol{\sigma = 7.5}$	4.2% ($1.31 > \epsilon > 0.76$)	8.2% ($1.6 > \epsilon > 0.6$)	1.4% ($1.09 > \epsilon > 0.91$)

Table 5: IMaximal allowed tax gap for internal solutions

In function of trade freeness

We start with the stability analysis in function of the trade freeness. The before mentioned first derivative is equal to

$$\left.\frac{d(\pi-\pi^*)}{ds_n}\right|_{int,sk=1/2} = \frac{4\gamma(\sigma\phi+\epsilon(-1+\phi(\epsilon\sigma+\phi-2\sigma\phi)))}{\epsilon^2(1-\phi^2)(1-(1-2\sigma)^2\phi^2)} \tag{85}$$

Equation (85) is a quadratic equation in trade freeness and has two roots as possible break points:

$$\phi_{B1} = \frac{\sigma(1+\epsilon^2)-\sqrt{4\epsilon^2(1-2\sigma)+\sigma^2(1+\epsilon^2)^2}}{2\epsilon(2\sigma-1)} \tag{86}$$

$$\phi_{B2} = \frac{\sigma(1+\epsilon^2)+\sqrt{4\epsilon^2(1-2\sigma)+\sigma^2(1+\epsilon^2)^2}}{2\epsilon(2\sigma-1)} \tag{87a}$$

Under the valid restrictions of $\epsilon > 0$ and $\sigma > 1$ it is easy to establish that $0 < \phi_{B1} < 1$ and that $\phi_{B2} > 1$. Since the sign of the quadratic term $(\epsilon\phi^2(1-2\sigma))$ in (85) is negative , stable internal equilibria only occur for values of trade freeness below ϕ_{B1}. This also means that we don't have to impose a no-black hole condition as is required normally in NEG-models.

The discussion of the sustain points in function of the trade freeness is more intricate since not only we have to make a distinction in function of the competitiveness ϵ, but also because we have to consider the core in the north and in the south separately. The difference between the capital rewards if $s_n = 1$ and $s_n = 0$ is respectively given by

$$\pi-\pi^*|_{s_n=1} = \frac{\gamma(-1+\phi(2\epsilon\sigma+\phi(1-2\sigma)))}{2\epsilon\sigma\phi} \tag{88}$$

$$\pi-\pi^*|_{s_n=0} = \frac{\gamma(\epsilon-2\phi\sigma+\epsilon(2\sigma-1)\phi^2)}{2\sigma\phi} \tag{89}$$

		ϕ_{0A}	ϕ_{0B}	ϕ_{1A}	ϕ_{1A}
	$\boldsymbol{\sigma = 2.5}$	0.10	>1	/	/
$\epsilon = \frac{1}{2}$	$\boldsymbol{\sigma = 5}$	0.05	>1	/	/
	$\boldsymbol{\sigma = 7.5}$	0.03	>1	0.25	0.28
	$\boldsymbol{\sigma = 2.5}$	0.25	0.25	1	1
$\epsilon = 1$	$\boldsymbol{\sigma = 5}$	0.11	0.11	1	1
	$\boldsymbol{\sigma = 7.5}$	0.07	0.07	1	1
	$\boldsymbol{\sigma = 2.5}$	/	/	0.14	>1
$\epsilon = \frac{3}{2}$	$\boldsymbol{\sigma = 5}$	0.21	0.53	0.06	>1
	$\boldsymbol{\sigma = 7.5}$	0.12	0.59	0.05	>1

Table 6: Values for ϕ_{0A}, ϕ_{0B}, ϕ_{1A}, and ϕ_{1B}

Equating both expressions to zero gives us four sustain points. We denote the sustain points for the northern core with ϕ_{SN1} and ϕ_{SN2} , the southern core sustain points are depicted by ϕ_{SS1} and ϕ_{SS2} :

$$\phi_{SN1} = \frac{1}{\epsilon\sigma + \sqrt{1+\sigma(\epsilon^2\sigma - 2))}}, \qquad \phi_{SN2} = \frac{1}{\epsilon\sigma - \sqrt{1+\sigma(\epsilon^2\sigma - 2))}} \tag{90}$$

$$\phi_{SS1} = \frac{\epsilon}{\sigma + \sqrt{\epsilon^2(1-2\sigma)+\sigma^2}}, \qquad \phi_{SS2} = \frac{\sigma + \sqrt{\epsilon^2(1-2\sigma)+\sigma^2}}{\epsilon(2\sigma-1)} \tag{91}$$

For equally competitive regions we see that ϕ_{SN1} equals ϕ_{SS1} and that ϕ_{SN2} coincides with ϕ_{SS2} and that both are equal to 1. For this situation, the core in the north and in the south becomes stable as soon as the trade freeness exceeds $\phi_{SN1} = \phi_{SS1}$.

For cases where the north has a competitive advantage over the south ($\epsilon > 1$), the northern core is stable for values lying between ϕ_{SN1} and ϕ_{SN2}. However, it is easily checked that the second northern sustain point ϕ_{SN2} always exceeds one, so we can simplify matters by stating that the northern core is stable as soon as the trade freeness becomes larger than ϕ_{SN1}. The square root in the denominator of (91) only gives cause to real solutions for values of the elasticity of substitution higher than $\epsilon^2(1+\sqrt{\epsilon^2-1})$[19]. It should come as no surprise that this condition on the elasticity of substitution coincides with the condition (83) for having a core in the south solution for values of $\epsilon > 1$. The higher the elasticity of substitution becomes, the larger the area (delimited by ϕ_{SS1} and ϕ_{SS2}) where the core in the south is stable becomes.

Lastly, if the south has a competitive advantage, we find that (89) is negative for values of trade freeness between ϕ_{SS1} and ϕ_{SS2} . Similar to the previous case, the second sustain point always exceeds one, as a result of what stable southern cores always occur for values of trade freeness exceeding ϕ_{SS1}. Expression (88) is only positive and real for values of trade freeness between ϕ_{SN1} and ϕ_{SN2} and for values of the elasticity of substitution higher than $\frac{1+\sqrt{1-\epsilon^2}}{\epsilon^2}$[20]. We conclude this discussion of the stability of the steady states in function of the trade freeness by noting that the stable core in the south (north) solution for values of $\epsilon > 1$ ($\epsilon < 1$) never can span the whole trade freeness range. Even for very high values of the substitution elasticity and almost evenly competitive regions, this stable region stays relatively small.

In function of the competitiveness In order to determine the break points in terms of the competitiveness, we equate expression (85) to zero and solve for the competitiveness ϵ. There are, just as before, two possible solutions for the break points:

[19] The square root is also real if $\sigma < \epsilon^2(1-\sqrt{\epsilon^2-1})$, but this values is always smaller than 1. Hence, we omitted this.

[20] The square root in the denominator of 90 is also real if $\sigma < \frac{1+\sqrt{1-\epsilon^2)}}{\epsilon^2}$, but this value is always smaller than 1. Hence, we omitted this.

$$\epsilon_{B1} = \frac{1 + (2\sigma - 1)\phi^2 - \sqrt{1-\phi^2}\sqrt{1-(1-2\sigma)^2\phi^2}}{2\sigma\phi} \tag{92}$$

$$\epsilon_{B2} = \frac{1 + (2\sigma - 1)\phi^2 + \sqrt{1-\phi^2}\sqrt{1-(1-2\sigma)^2\phi^2}}{2\sigma\phi} \tag{93a}$$

The second root in the numerator of both expressions is positive if and only if the substitution elasticity lies between $\frac{-1+\phi}{2\phi}$ and $\frac{1+\phi}{2\phi}$. Since $\frac{-1+\phi}{2\phi} < 0$, σ has to lie between 1 and $\frac{1+\phi}{2\phi}$ in order to have non-imaginary values for ϵ_{B1} and ϵ_{B2}. Under this restriction, it is also easy to show that the first break point always lies between 0 and 1 and that the second break point lies in the region where the north has a competitive advantage over the south ($\epsilon > 1$). The sign of the quadratic factor in ϵ of expression (85) is positive with the result that the internal steady states are always stable between the two break points as long as $\sigma < \frac{1+\phi}{2\phi}$.

Contrary to the previous section (4.1.2) the discussion of the sustain points is a lot easier. The solution in terms of the competitiveness of equating (88) and (89) to zero, leads to the following two sustain points for the core in the north and the core in the south solution respectively:

$$\epsilon_{SN} = \frac{1 + (2\sigma - 1)\phi^2}{2\sigma\phi}, \qquad \epsilon_{SS} = \frac{1}{\epsilon_{SN}} \tag{94}$$

For values of $\epsilon > \epsilon_{SN}$ the northern core becomes stable, for values of $\epsilon < \epsilon_{SS}$ the south becomes a stable core. Both sustain points coincide when $\sigma = \frac{1+\phi}{2\phi}$ as could be expected.

4.1.3 The tomahawk diagram revisited

We now know how the locational equilibrium shifts when the trade costs shift or when the competitiveness changes between the two regions. We also know where which equilibrium is stable. By combining both results, we are able to draw two 'tomahawk' diagrams, one in function of ϵ, one in terms of ϕ.

We start with the tomahawk diagram in function of the competitiveness. Under the assumption that $\sigma < \frac{1+\phi}{2\phi}$ it is possible to show[21] that the first break point ϵ_{B1} comes before the start of the internal zone at ϵ_0 and that the second break point always lies at values of the competitiveness higher than ϵ_1. For values of $\sigma > \frac{1+\phi}{2\phi}$, the internal zone is always unstable. Since ϵ_{SN} coincides with ϵ_1 and ϵ_{SS} with ϵ_0 we conclude that the core solutions are always stable. We can summarize the locational behaviour of the model in function of the competitiveness in diagram (11). In this diagram the solid lines represent the stable steady states, the dotted lines the unstable equilibria.

[21] Analytical proofs are somewhat tedious but not difficult. In order to not to burden the text too much, we omitted therefore the proofs of the different statements done in subsequent paragraphs. The proofs are available upon request.

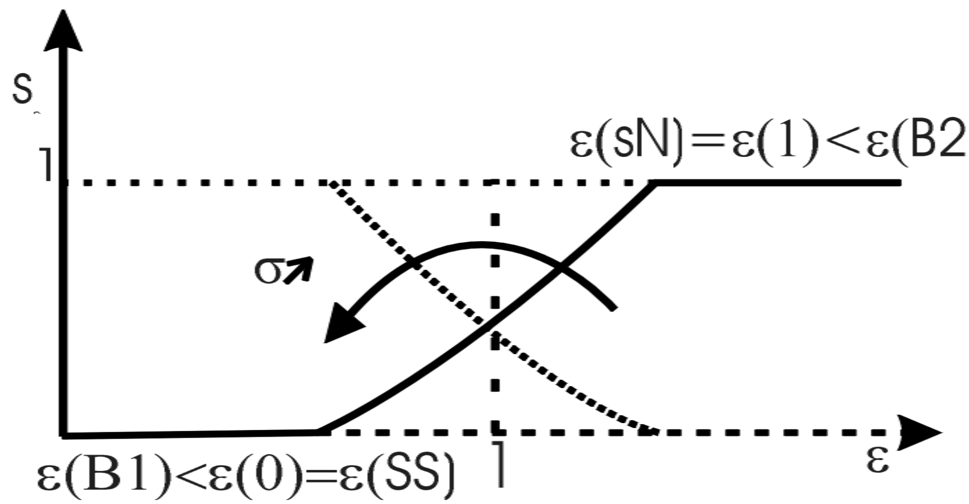

Figure 11: Tomahawk diagram in function of competitiveness.

The integration of the results of the two previous sections is a little bit more complex for the behaviour in terms of the trade freeness since we have to make a distinction whether ϵ exceeds, is equal or is smaller than 1. It is possible to show that ϕ_{B1} exceeds ϕ_{OA} when $\epsilon > 1$ and that $\phi_{1A} < \phi_{B1}$ when $\epsilon < 1$. When both region have the same competitiveness, the first break point always coincides with $\phi_{OA} = \phi_{1A}$. The first zone of internal equilibria is as a consequence, regardless of the value of the competitiveness, always stable. The second zone of internal equilibria only occurs for values of σ that are high enough[22]. When the north is more competitive than the south, the second zone of internal steady states is always unstable since $\phi_{OA} > \phi_{B1}$ and $\phi_{OB} < \phi_{B2}$. The locational equilibrium is stable for values of trade freeness that lie between ϕ_{B1} and between ϕ_{B2}. When the south has a competitive advantage, it is again easy to show that the second zone of interior equilibria is always unstable ($\phi_{1A} > \phi_{B1}$, $\phi_{1B} < \phi_{B2}$). We now only have to look into the concurrence between the core-periphery zones and their stability. By realizing that, $\phi_{1A} = \phi_{SN1}$, $\phi_{1B} = \phi_{SN2}$, $\phi_{0A} = \phi_{SS1}$ and that $\phi_{0B} = \phi_{SS2}$, we can safely conclude that the core-in-the-south solution for $\epsilon \leqslant 1$ and $\phi > \phi_{0A}$ is always stable. The same holds for the core-in-the-north solution when $\epsilon \geqslant 1$ and $\phi > \phi_{1A}$. These results are again elucidated by plotting the tomahawk diagram. The first diagram gives the situation when both regions are equally competitive, the second one gives the case when the north has a competitive disadvantage and the last figure represents the possibility where the north has a competitive advantage.

[22]For $\epsilon < 1$, $\sigma > \frac{1+\sqrt{1-\epsilon^2}}{\epsilon^2}$, and for $\epsilon > 1$, $\sigma > \epsilon^2(1 + \sqrt{1-\epsilon^2})$.

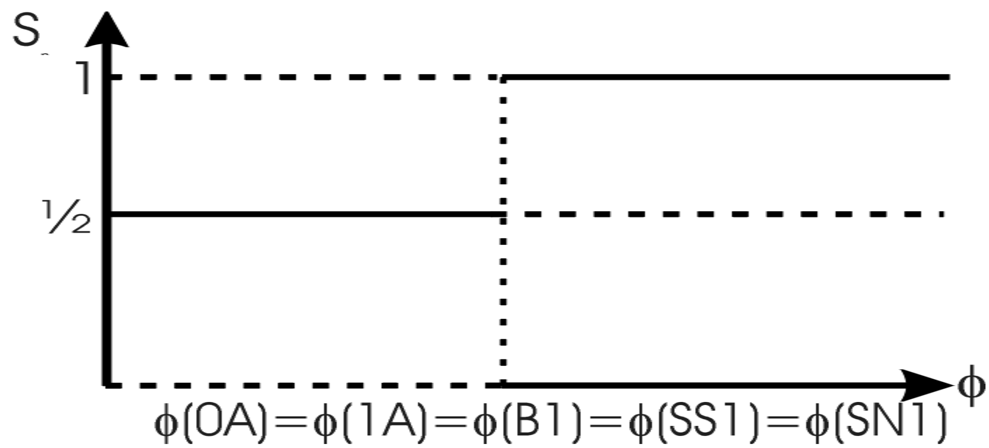

Figure 12: Tomahawk diagram in function of ϕ for $\epsilon = 1$.

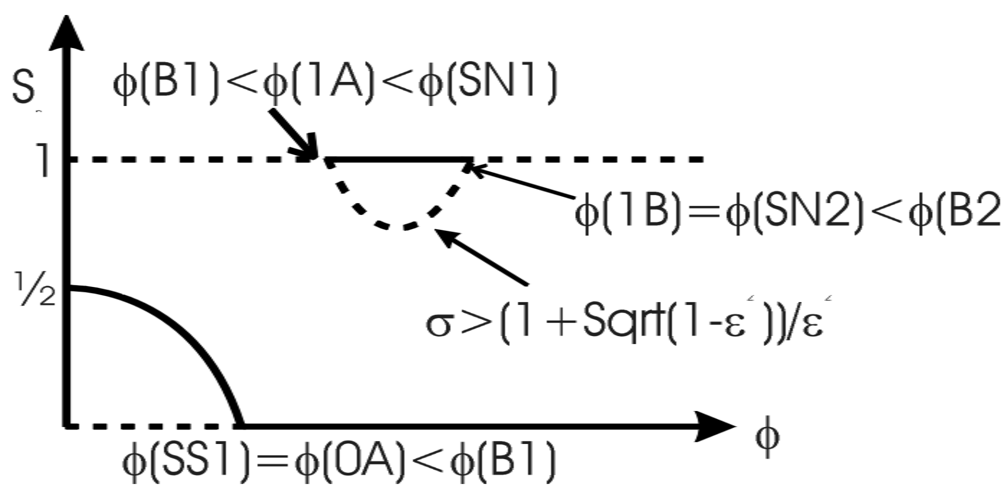

Figure 13: Tomahawk diagram in function of phi for $\epsilon < 1$

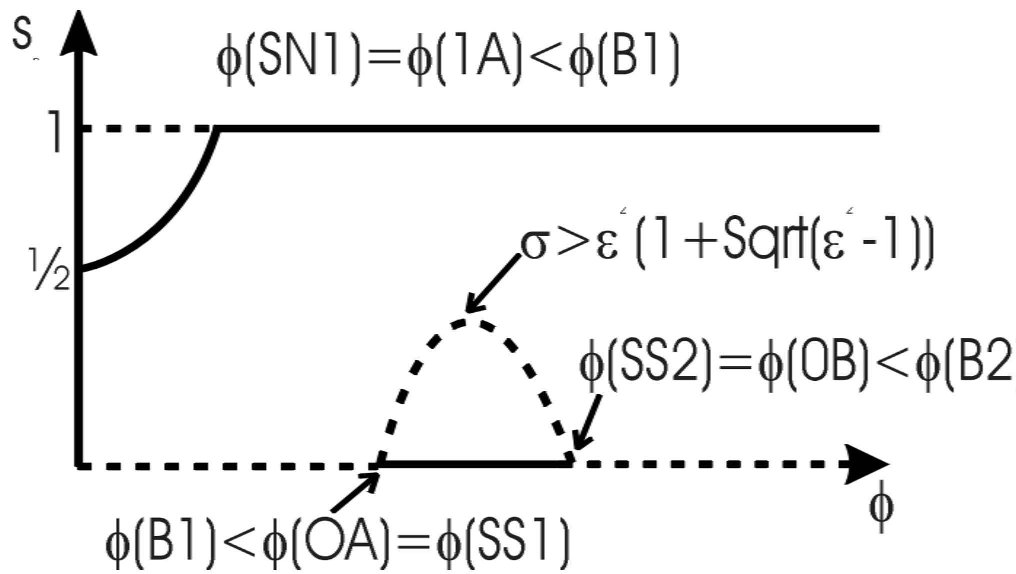

Figure 14: Tomahawk in function of ϕ for $\epsilon > 1$

4.1.4 Discussion of agglomeration forces and properties of the model

Agglomerative and dispersion forces There are two driving forces in this agglomeration model. The first one is the 'market access effect'. It describes the tendency of monopolistic firms to locate their production in the big market and export to the small markets. When the share of expenditures in a region increases, the sales of the firms located in that region also increases. As a consequence the operating profit of those firms also increases since the total labour cost to a firm didn't change. Under the zero-profit condition this higher operating profit leads to a higher capital reward which will attract firms to locate in this region. The second force is not an agglomerative force, but a dispersive one. It reflects the fact that imperfect competitive firms have a tendency to locate in regions with relatively few competitors. A small movement of firms from the south to the north raises s_n. As a result Δ in (47) will rise while Δ^* will decrease. Under a constant share of expenditures and degree of competitiveness, this will lead to lower sales for a northern firm[23]. Owing to the simultaneous reduction of the operating profit, the northern capital reward has to decrease under the zero-profit condition. The resulting dispersion force is called the local competition effect or the market crowding effect. In many NEG-models a third force is also present, namely the cost-of-living effect. Since the driving force

[23] e.g. Starting from symmetry ($s_n = s_e = 1/2, \epsilon = 1$) the derivative of the sales with respect to s_n equals -$\frac{2(1-\phi^2)}{(1+\phi^2)}$, which is clearly always negative.

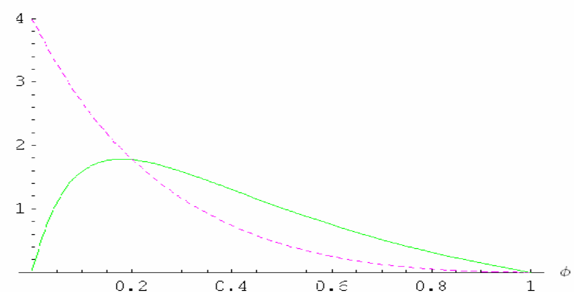

Figure 15: The agglomerative (solid) and dispersion (dotted) force ($\epsilon = 1$).

in our model is the nominal capital reward gap, not the real one, this effect is absent in our model.

Our model is analytically tractable enough to explicitly derive closed-form expressions for both forces in play. The agglomerative and dispersion force can be calculated by deriving the driving force $\pi - \pi^*$ at the internal equilibrium with respect to s_E and s_n respectively:

$$\left.\frac{\partial(\pi-\pi^*)}{\partial s_E}\right|_{s_n\ int} = \frac{8(\sigma-1)\phi((1+\epsilon^2)(1-\phi^2+2\sigma\phi^2)-4\epsilon\sigma\phi)(\sigma\phi(1+\epsilon^2-2\epsilon\phi)-\epsilon(1-\phi^2))}{\epsilon^2(\epsilon-\phi(2\sigma-1))(1-\epsilon(2\sigma-1)\phi)(1-\phi^2)^2(1-(1-2\sigma)^2\phi^2)} \tag{95}$$

$$\left.\frac{\partial(\pi-\pi^*)}{\partial s_n}\right|_{s_E,\ int} = \frac{4(\epsilon-\phi)(1-\phi\epsilon)(\phi\sigma(1+\epsilon^2-2\phi\sigma)-\epsilon(1-\epsilon\phi))}{\epsilon^2(1-\phi^2)^2(1-\epsilon\phi(2\sigma-1))((2\sigma-1)\phi-\epsilon)} \tag{96}$$

It can be proven that under the conditions (80) for an internal equilibrium the agglomerative force is always positive while the second force is always negative. To illustrate both forces more clearly, we plotted (95) and (96) when $\epsilon = 1$. Note that we plotted the inverse of the negative dispersion force in order to compare the relative strength of both forces better. For low values of trade freeness the dispersion force is still stronger than the agglomerative force and we end up in an interior steady state. For high values of the trade freeness, the core-periphery situation prevails. Since we reasoned from the interior equilibrium, the equality in strength of both forces coincides with the (first) break point. The fact that the dispersion force drops more sharply with trade freeness can be understood if one looks back at the expression of the (northern) sales (47). Lower trade costs mean that a larger share of the sales become independent of the location of the competitors while at the same time it becomes more easy for a firm to increase its market share abroad.

Circular causalities Unlike a standard footloose capital model, our model does have a cost-linked circular causality. An increase of capital (=firms) in a region will reduce the unemployment in that region and hence, increase the regions share of expenditure. This will make the region more attractive to further migrate capital to since the increased sales lead to higher capital rewards in that region. Thus, the main reason, why this model - although it has the same migrational behaviour as the model of Martin and Rogers- has circular causalities lies in the endogenous presence of unemployment via the reference wages.

Home market effect and magnification We calculate the home-market derivative ds_n/ds_E. In order to do this we equate (50) and (51) and solve for s_n.This lead to an expression for the share of firms $s_n = \frac{s_E\epsilon - \phi + (1-s_E)\epsilon\phi^2}{(\epsilon-\phi)(1-\epsilon\phi)}$ in function of the share of expenditures. Taking the derivative of this expression for s_n with respect to s_E gives us the home-market derivative:

$$\frac{ds_n}{ds_E} = \frac{\epsilon(1-\phi^2)}{(\epsilon-\phi)(1-\epsilon\phi)} \tag{97}$$

When both regions are equally competitive, this expression reduces to the standard FC-expression, namely $(1+\phi)/(1-\phi)$ from which it is clear that an exogenous change in the location of demand leads to a more than proportional relocation of industry to the enlarged region. When the south has a competitive advantage, the home market derivative is larger than 1 as long as the trade freeness remains smaller than ϵ. For values of $\epsilon > 1$, there can only be a home-market effect as long as $\phi < 1/\epsilon$. It can be easily checked that these restrictions on the trade freeness are less strict than the restrictions we derived for an internal equilibrium (84). So we conclude that the home-market effect is always active for interior equilibria.

Secondly, it is easy to show that the home-market derivative gets larger when trade costs decline. This is the home-market magnification effect of Baldwin ([19]) and can be captured by:

$$\frac{d^2s_n}{ds_E d\phi} = \epsilon(\frac{1}{(\epsilon-\phi)^2} + \frac{1}{(1-\epsilon\phi)^2}) \tag{98}$$

Freer trade makes industry become more footloose as could be expected.

Endogenous asymmetry and near-catastrophic agglomeration A gradual lowering of the trade costs, starting from prohibitive trade costs, will only have a slight locational impact, with some of the industry moving to the region with the competitive advantage. If both regions are equally competitive, the symmetricum will remain the stable steady state for a large interval of high trade costs. However, as the level of trade freeness comes into the range of the break point, the delocation will go faster and faster. This can be captured by

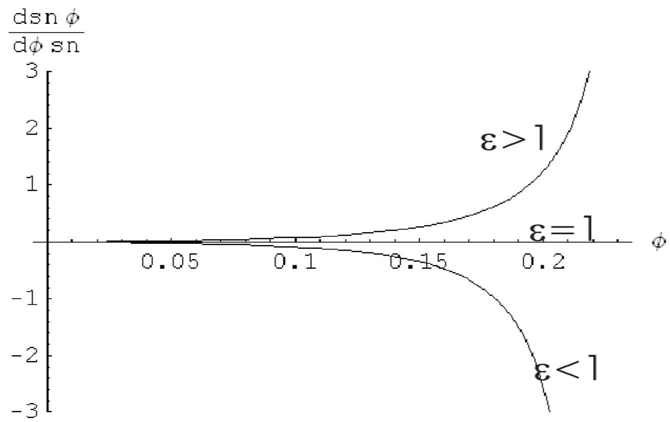

Figure 16: Delocation elasticity in function of trade freeness for three values of competitiveness.

the delocation elasticity defined as the percent change in s_n with respect to a percent change in the trade freeness. This elasticity is equal to:

$$\frac{dsn}{d\phi}\frac{\phi}{s_n} = \frac{\epsilon(1-\epsilon^2)\sigma\phi(1+(1-2\sigma)\phi^2}{(\epsilon - 2\sigma\phi + \epsilon(2\sigma-1)\phi^2)(\sigma\phi + \epsilon(-1+\phi(\sigma\epsilon + \phi(1-2\sigma)))} \tag{99}$$

For equally competitive regions the delocation elasticity equals zero. We plotted the delocation elasticity for equally competitive regions and in the case where one region has a slight competitive advantage ($\epsilon = 0.95/1.05$). We restricted the range of trade freeness to values where there is an internal equilibrium ($\phi_{SN1} = \phi_{SS1} = 0.23$ at $\sigma = 2.5$).

After the break point, all industry is agglomerated in the region with the competitive advantage. There is no full-blown catastrophic agglomeration possible as soon as one region has a competitive advantage since there is a gradual shift of stable locational equilibria in function of the trade freeness. All possible locational equilibrium states become possible between full symmetry and core-periphery. Only when both regions are equally competitive, a classic catastrophic agglomeration is possible.

Locational hysteresis For intermediate values of trade freeness and high enough elasticities of substitution multiple equilibria do exist. Both regions can sustain a core equilibrium at the same time. For instance, when the north has a competitive advantage a lowering of the trade costs makes the core-in-the-north solution stable starting from a stable interior solution. But in that range of trade costs where the agglomerative forces are the strongest (see figure (15)) the core-in-the-south solution can become stable. Of course the competitive disadvantage of the south can't be too large and the agglomerative forces have to be strong (high values of σ). The range of values for the elasticity of substitution and the competitiveness where this is possible are depicted in graph (11). So our model does display locational hysteresis

This means that there is path-dependency in our model. It matters which starting point you have in a policy analysis.

Hump-shaped agglomeration rents The agglomeration rents are defined as the loss that a capital unit would incur by relocating from the core to the periphery when full agglomeration is a stable equilibrium. These are given by

$$\pi - \pi^*|_{s_n=1} = \frac{1 + \phi((2\sigma - 1)\phi - 2\sigma\phi)}{2\sigma\epsilon\phi} \tag{100}$$

These rents are concave in trade freeness since the second derivative of the profit gap with respect to the trade freeness is negative ($d^2(\pi - \pi^*)/d\phi^2 = -1/(\epsilon\sigma\phi^3)$). It equals zero at the sustain point and reaches it maximum at $\phi = \sqrt{1/(2\sigma - 1)}$. Accordingly, the agglomerative rents first increase after the sustain point and decrease towards complete trade freeness.

4.2 Internal equilibrium

Contrary to the two-country case without capital mobility, we are no longer able to explicitly derive the Nash tax rates in any scenario. Instead, we rely on simulations to describe social security competition under capital mobility. In this section we discuss the optimal behaviour of governments when the locational equilibrium is a stable internal equilibrium. In the last section of this chapter we give the results for a stable core-periphery situation.

As before, we start with a short description of the effects through which a government's tax setting influences the social welfare. In a second step, we will discuss the reaction curves and the way the foreign tax rate influences the home tax rate. Finally we will analyze the effect of some parameters on the Nash equilibrium.

4.2.1 Benefit, unemployment and price-index effect

By focusing on internal equilibria, we have to restrict the freedom of action for a government. Each government can only choose a tax rate, given the foreign tax rate, such that (80) holds. In order not to limit the range of possibilities for each government too much, we opt for a low value of the elasticity of substitution $\sigma (= 2.5)$ and the trade freeness $\phi (= 0.05)$. This means in effect that the competitiveness may vary between 0.247 and 4.08 or otherwise said, that the north can undercut the southern tax rate by more than 150 per cent which creates more than enough space for social security competition. The fact that we are obliged to choose low values for the trade freeness is an indication that the model has very strong agglomeration forces.

A second consequence of looking at internal equilibria is the invariability of the capital reward. The expressions (50) and (51) are both equal to γ under

(79). But at the same time, a new variability, namely via the share of firms in each region, is introduced. Without this second effect a foreign tax change would only have affected the home region via the price index, no longer via the tax base. It is not that difficult to check that, under the restrictions given by (80), the sign of the derivative of s_n with respect to z is negative.

This result simplifies the interpretation of the first order derivative of the social welfare function with respect to the tax rate. As before, we can distinguish three channels through which the northern tax rate affects the northern social welfare First, the unemployment, given by (58) increases since a higher tax reduces the share of firms s_n and also the term $(1-z)$. This will exert a negative influence on the social welfare. Secondly, higher taxes will decrease the share of firms in the home region and will increase the prices charged by the own firms. As a consequence the price index will increase and the purchasing power of the people will decrease. This again leads to a negative effect on the social welfare. In equilibrium both effects are in balance with the third effect, namely the benefit effect. It turns out that the combined effect of higher taxes on a reduced tax base still allows for a higher unemployment benefit. The price, unemployment and benefit effect can be consecutively written as:

$$dSW/dz|_{b,u=cst} = \begin{cases} \frac{-1}{\Delta(z,z^*)} * \left[\frac{s_n(z,z^*)*\epsilon(z,z^*)}{1-z} - \frac{(\epsilon(z,z^*)-\phi)}{\sigma-1}\frac{\partial s_n}{\partial z}(z,z^*)\right] & (e=1) \\ -\frac{b(z,z^*)+CR}{\Delta(z,z^*)*P(z,z^*)} * \left[\frac{s_n(z,z^*)*\epsilon(z,z^*)}{1-z} - \frac{(\epsilon(z,z^*)-\phi)}{\sigma-1}\frac{\partial s_n}{\partial z}(z,z^*)\right] & (e=\infty) \\ *\frac{SW(z,z^*)*(1-e)}{\Delta(z,z^*)}\left[\frac{s_n(z,z^*)*\epsilon(z,z^*)}{1-z} - \frac{(\epsilon(z,z^*)-\phi)}{\sigma-1}\frac{\partial s_n}{\partial z}(z,z^*)\right] & (e\neq 1,\infty) \end{cases} \tag{101}$$

$$dSW/dz|_{P,b=cst} = \begin{cases} \delta*\pi*\left[s_n(z,z^*)-(1-z)*\frac{\partial s_n}{\partial z}(z,z^*)\right]*\log\left(\frac{b(z,z^*)+CR}{w_R/(1-\beta)+CR}\right) & (e=1) \\ 0 & (e=\infty) \\ \left(\frac{\delta*\pi*(P(z,z^*))^{e-1}}{1-e}\right)*\left[s_n(z,z^*)-(1-z)*\frac{\partial s_n}{\partial z}(z,z^*)\right]* & (e\neq 1,\infty) \\ (b(z,z^*)+CR)^{1-e}-\left(\frac{w_R}{1-\beta}+CR\right)^{e-1} & \end{cases} \tag{102}$$

$$dSW/dz|_{P,u=cst} = \begin{cases} \frac{w_T*\delta*\pi}{(b(z,z^*)+CR)*(1-\beta)*u(z,z^*)}* & \\ \left[s_n(z,z^*)*(1-\delta*\pi*s_n(z,z^*)+z*\frac{\partial s_n}{\partial z}(z,z^*)\right] & (e=1) \\ \frac{w_T*\delta*\pi}{P(z,z^*)*(1-\beta)*(u(z,z^*))^2}* & \\ \left[s_n(z,z^*)*(1-\delta*\pi*s_n(z,z^*)+z*\frac{\partial s_n}{\partial z}(z,z^*)\right] & (e=\infty) \\ \frac{(b(z,z^*)+CR)^{-e}*(P(z,z^*))^{e-1}*w_R*\pi*\delta}{(1-\beta)*u(z,z^*)} & \\ \left[s_n(z,z^*)*(1-\delta*\pi*s_n(z,z^*)+z*\frac{\partial s_n}{\partial z}(z,z^*)\right] & (e\neq 1,\infty) \end{cases} , \tag{103}$$

$$with \ \ \frac{\partial s_n}{\partial z}(z,z^*) = \frac{(\sigma-1)*\epsilon*(\sigma*\phi^2+(s_K-s_n)*(1-\phi^2)-2*s_n*\sigma*\phi*(\phi-\epsilon)}{(1-z)*(\sigma*(\epsilon-\phi)*(\epsilon*\phi-1)-(\sigma-1)*\epsilon*(\phi^2-1)} \tag{104}$$

To illustrate these effects we plotted them in function of the inequality aversion parameter in graph (17). The three effects become very strong for near-

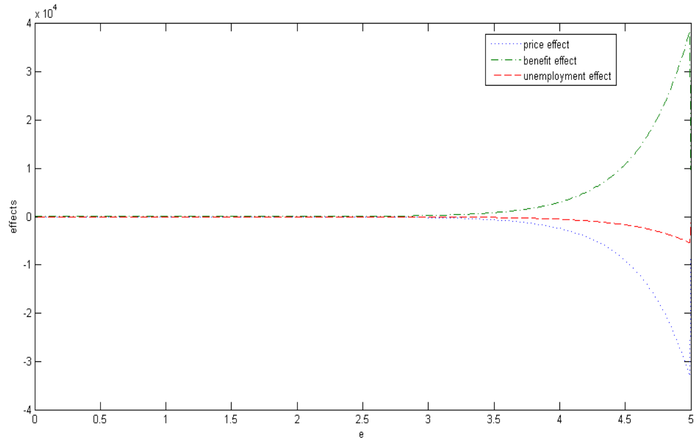

Figure 17: Benefit, unemployment and price index effect in function of northern inequality aversion

Rawslian governments which increases the accuracy requirements in that range.

4.2.2 Reaction curves

The reaction curves of both regions are given for different values of the inequality aversion e in figure (18). When governments become more Rawslian-like, they opt for higher tax rates. Comparing these curves with the result of figure (5), two differences catch the eye. Firstly the reaction curves become a straight line for foreign tax rates that are high enough (function of the inequality aversion). Secondly, the chosen tax rate changes its behaviour from a strategic substitute to a strategic complement when the foreign tax rate further reduces from the point where the reaction curve simplified into a straight line.

We know from combining condition (80) and the tomahawk diagram (11) that stable internal equilibria can only occur for values of the elasticity of substitution that are smaller than $\frac{1+\phi}{2\phi}$(=10.5 for $\phi = 0.05$) and under the condition that the competitiveness lies between ϵ_0 and ϵ_1. Under the assumption that both regions have the same number of inhabitants -and hence have the same reference wage- one could easily rewrite this condition in terms of the tax rates z and z^*: $0 < s_n < 1 \Leftrightarrow 1-(\epsilon_1)^{1/(\sigma-1)}(1-z^*) < z < 1-(\epsilon_0)^{1/(\sigma-1)}(1-z^*)$. The straight lines on the reaction curve diagram are nothing else than the representation of these limits in which the interior solutions are stable. Otherwise said, for high enough values of the foreign tax rate, a government would opt for a tax rate that is just low enough to attract all the industry within its borders.

To understand the shape of the reaction curves we consider the first order derivative of the (northern) social welfare function with respect to the southern tax rate. As before one could distinguish two channels: a price index effect and

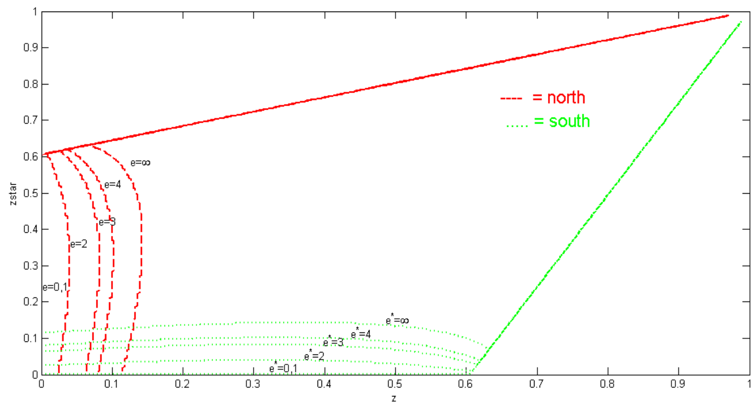

Figure 18: Reaction curves in function of e under capital mobility.

a competition effect. These are respectively given by:

$$dSW/dz^*|_{u,b=cst} = \begin{cases} \left[\frac{\epsilon(z,z^*)-\phi}{(\sigma-1)*\Delta(z,z^*)} * \frac{\partial s_n}{\partial z^*}(z,z^*) - \frac{1}{1-z^*} * (1 - \frac{s_n(z,z^*)*\epsilon(z,z^*)}{\Delta(z,z^*)}\right] & (e=1) \\ \frac{b(z,z^*)+CR}{P(z,z^*)} * dSW/dz^*|_{u,b=cst,e=1} & (e=\infty) \\ SW(z,z^*) * (1-e) * dSW/dz^*|_{u,b=cst,e=1} & (e \neq 1,\infty) \end{cases} \tag{105}$$

$$dSW/dz^*|_{P=cst} = \begin{cases} \delta * \pi * \frac{\partial s_n}{\partial z^*}(z,z^*) * \\ \left[\frac{w_R * z}{(b(z,z^*)+CR)*(1-\beta)*u(z,z^*)} - (1-z) * \log(\frac{(b(z,z^*)+CR)}{w_R/(1-\beta)+CR}\right] & (e=1) \\ \frac{w_R*\delta*\pi*z}{(1-\beta)*(u(z,z^*))^2*P(z,z^*)} * \frac{\partial s_n}{\partial z^*}(z,z^*) & (e=\infty) \\ \frac{\delta*\pi*(P(z,z^*))^{e-1}}{1-e} * \frac{\partial s_n}{\partial z^*}(z,z^*) * \left[(1-e) * \frac{(b(z,z^*)+CR)^{-e}*w_R*z}{(1-\beta)*u(z,z^*)} - \right. \\ \left. (1-z) * \left[(b(z,z^*)+CR)^{1-e} - (\frac{w_R}{1-\beta}+CR)^{1-e}\right]\right] & (e \neq 1,\infty) \end{cases} , \tag{106}$$

$$with \quad \frac{\partial s_n}{\partial z^*}(z,z^*) = -(\frac{1-z}{1-z^*})\frac{\partial s_n}{\partial z}(z,z^*) \tag{107}$$

A tax increase abroad increases the import prices for the northern region (the term $\frac{1}{1-z^*} * (1 - \frac{s_n(z,z^*)*\epsilon(z,z^*)}{\Delta(z,z^*)})$ and, at the same time, will increase the share of home goods in the consumption basket of northern inhabitants (the term $\frac{\epsilon(z,z^*)-\phi}{(\sigma-1)*\Delta(z,z^*)} * \frac{\partial s_n}{\partial z^*}(z,z^*)$). Under the condition of a stable internal equilibrium (80) it is possible to ascert analytically that the effect via the increased import

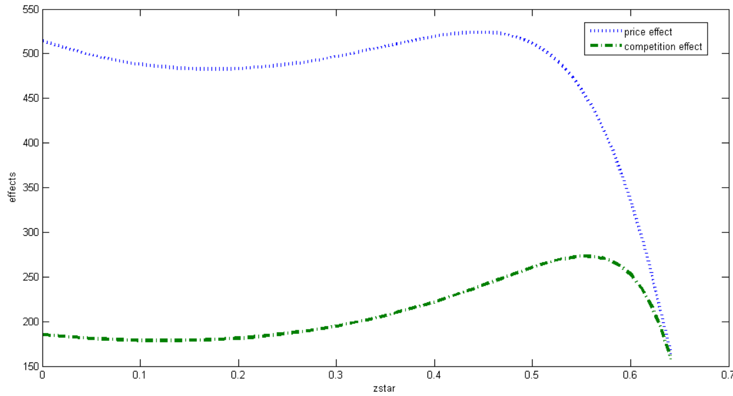

Figure 19: The effect of z^*on the northern social welfare function.

prices is always more than compensated by the effect via the increased share of domestic goods consumption on the price index. The lower price index will increase the average income of the people but will not alter the inequality index (constant under equiproportional income changes). The only way the government can react to this situation under a contant equity-efficiency trade-off is by increasing the home tax rate. In other words, as soon as one introduces capital mobility in a model of social security competition, the price effect changes sign. When there was no capital mobility a foreign tax increase would have lead to a decrease in the home tax rate.

The second effect through which the southern tax rate affects the northern region is called the competition effect. As can be seen in expression (106) the only way the foreign tax rate could influence the northern social welfare function is via the share of firms. It no longer works via the capital reward. A southern tax decrease will reduce, as long as there is a stable internal equilibrium, the share of firms in the northern region. This will, since the capital reward is constant[24], automatically lead to a decreased tax base. There are less people at work and those who are unemployed receive a lower unemployment benefit. The net result is a deterioriation of the efficiency and equity in the society. The government will react to this by lowering her own tax rate. This and the previous effect are given in figure (19) in function of the foreign tax rate. We assumed that $e = e^* = 4$ and that the home tax rate is equal to the Nash tax rate ($z = 0.089$).

The combined effect of both forces will lead to tax rates that are complements. The above graph also illustrates that the impact of a foreign tax rate change greatly diminishes for high values of the tax rate (near the $s_n = 1$-line). So the tendency to increase the home tax rate as a response to the foreign in-

[24]Remember that the capital reward π equals $\frac{l*w}{(\sigma-1)k}$.

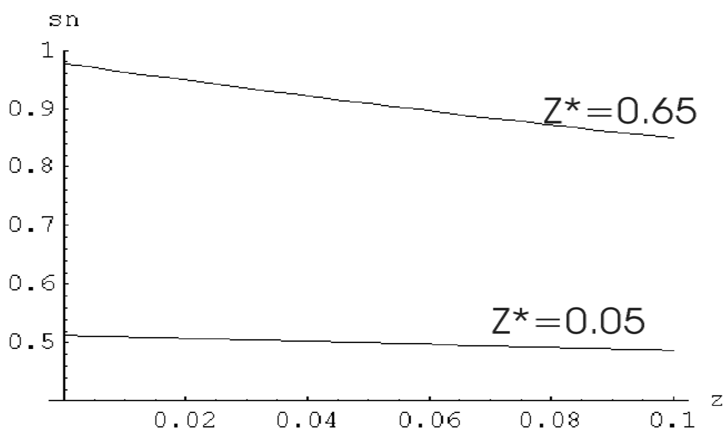

Figure 20: Effect of z on s_n for a low value of z^* and a high one.

creased rate is weakened. At the same time, the secondary effect that a raise of the northern tax rate has on the northern social welfare function via the lower share firms s_n becomes more important. The increase in the price index and the loss in the tax base (higher unemployment, loss of firms) becomes larger . This is illustrated in figure (20) where s_n is given for a low value of $z^*(= 0.05)$ and a high one ($z^* = 0.65$). The effect of the initial reacton towards a foreign tax increase is more than undone by the effect that that reaction has for high values of the foreign tax rate. More intuitively, it is more than worthwile to reduce your tax rate for high values of the foreign tax rate since the reward you get in terms of the increased share of firms more than outweighs the initial loss in equity. The dominance of the s_n-effect explains why the northern tax rate behaves like a strategic substititute for high values of the southern tax rate.

To end this section, we compare the optimal Nash tax rates for the regions and the social planner with the Nash rate found under the same circumstances ($\phi = 0.05, \beta = 0.6, \sigma = 2.5$) in the model without capital mobility, and this for different values of the inequality aversion e. This is done in table (7).

s_n		$e, e^* = 0$	$e, e^* = 1$	$e, e^* = 2$	$e, e^* = 3$	$e, e^* = 4$	$e, e^* = \infty$
endogenous	$z = z^*$	**0**	0	0.02	0.07	0.09	0.13
	z^{SP}	**0**	0	0.11	0.16	0.18	0.22
exogenous	$z = z^*$	**0**	0	0.08	0.13	0.15	0.20
	z^{SP}	**0**	0	0.11	0.16	0.18	0.22

Table 7: Nash tax rates for the regions and the social planner in a model with and without capital mobility

These figures confirm the result that the regional social security competition is reinforced by introducing capital mobility. Each region will opt for lower Nash rates under capital mobility. Secondly the social planner in both situations will choose the same tax rate. This should come as no surprise since under the restriction that each region has the same tax rate, the share of firms in the model with and without capital mobility always equals one half. This is the case for a social planner. (is het mogelijk dat een sociale planner ervoor prefereert om alle bedrijven naar een regio te krijgen? kan nooit voor een rawslian overheid, vanuit equity denken zal een SP altijd sn=1/2 kiezen, vanuit efficiency altijd sn=1 kiezen? maw kan het zinvol zijn op SP 2 verschillende tax rates te laten kiezen...)

4.2.3 Influence of σ and ϕ

As a final step we discuss the impact of two parameters: the elasticity of substitution σ and the trade freeness ϕ. Note however that in the discussion of both parameters we have to account for the severe restrictions on these parameters for having a stable internal equilibrium. We start with the impact of a change of the elasticity of substitution.

		$e, e^* = 0$	$e, e^* = 1$	$e, e^* = 2$	$e, e^* = 3$	$e, e^* = 4$	$e, e^* = \infty$
$\sigma = 2.5$	$z = z^*$	0	0	0.025	0.069	0.089	0.129
	z^{SP}	0	0	0.107	0.157	0.179	0.221
$\sigma = 5$	$z = z^*$	0	0	0.022	0.044	0.053	0.075
	z^{SP}	0	0.151	0.261	0.301	0.321	0.351
$\sigma = 8$	$z = z^*$	0	0	0	0	0	0.016
	z^{SP}	0	0.206	0.311	0.344	0.357	0.383

Table 8: Regional and social planner Nash tax rates for different values of σ and the inequality aversions

For a low value of trade freeness ($\phi = 0.05$), stable internal equilibria are possible as long as σ remains smaller than $\frac{1+\phi}{2\phi}$(=10.5 for $\phi = 0.05$). To illustrate the effect of the elasticity of substitution we give the Nash tax rates for a social planner and the regional government for different values of σ in table (8). As before, we see that the effect is mixed. Local governments will reduce their tax rates when σ increases but a social planner will increase it's optimal tax rate. The elasticity of substitution characterizes the fierceness of competition on the goods market and thus also the agglomerative forces. Otherwise said, the change in the share of firms due to a shift in the tax rates becomes stronger for higher values of σ and as a consequence, the impact on the tax base will also be strengthened. This increased strength of the agglomerative forces explains the lower Nash rates for the local governments.

On the other hand, a social planner will internalize these effects and will in the end only be faced with the same situation an autarkic social planner has faced. Higher values of the elasticity of substitution means that the economy becomes more efficient without altering the equity in the society. The social planner will amend this by increasing it's tax rate. Note also that the optimal tax rates for the social planner are the same in the model with and without capital mobility for the same reason as discussed before.

The restriction imposed on the trade freeness is more severe than the one on the elasticity of substitution, as could be seen in table (6). For small deviations from the equicompetiveness, the maximum allowed trade freeness (ϕ_{1A}/ϕ_{OA}) lies around 0.2 or smaller. The reason is again the strong nature of the agglomerative forces present in this model. As depicted in figure (21), the evolution in simple. A higher level of trade freeness means that the optimal Nash rate is lower. So we no longer have the concave effect as seen in the model without capital mobility. We can explain this by referring to the dominance of the effect of a change in the share of firms in the price effect and the competition effect. When the trade freeness increases, an identical change in the northern tax rate will lead to a much larger decrease in the northern share of firms. As a consequence, the government will restrain her optimal choice of taxation. The strength of this effect is captured by the elasticity of the share of firms with respect to the competitiveness[25]. This is depicted in graph (**??**). Since by definition the share of firms in each region equals one half for a social planner, this effect is absent in this case. As a consequence, the chosen tax rate will be constant in function of the trade freeness. More, it will be the same as the tax rate chosen by a social planner in a model without capital mobility.

4.3 Core-periphery situation

In this section we assume that the northern region is the (stable) core, the south is the periphery. This means that there is no industry left in the south and that the only income of that region stems from the transfer back of the remuneration of the soutern capital employed in the north. People can still consume (northern) goods but the government is unable to levy any taxes since there are no wages paid to employees to levy them on. On the other hand, the northern region faces a very confortable point of departure. It has all the industry, does not have to import any goods ($P(z) = \alpha\frac{w(z)}{a}$) and apparently can levy any tax it wishes. This ressembles the autarkic case which we started with[26]. As can be easily shown, the social welfare of the core region becomes independent of the trade freeness and hence, the maximizing government will choose its tax rate independent of the trade freeness.

However, the northern region is not that free as it first seems to set its tax rates. If it sets its tax rates higher than the agglomeration rents, it becomes

[25] When the competitiveness increases, the northern tax rate decreases or the southern one increases.

[26] Substitution s_n=1 in the standard expressions lead to the following formula's:$\Delta = \epsilon, \Delta^* = \epsilon\phi, P = \alpha\frac{w}{a}, P^* = \alpha\frac{w}{a}\tau,$
$s_E = 1 - \frac{1}{2\sigma}, \pi = \gamma, \pi^* = 0, u = 1 - (1-\beta)(1-z)$
,u$^* = 1, b = \frac{w_R z}{\beta + z(1-\beta)}, b^* = 0$

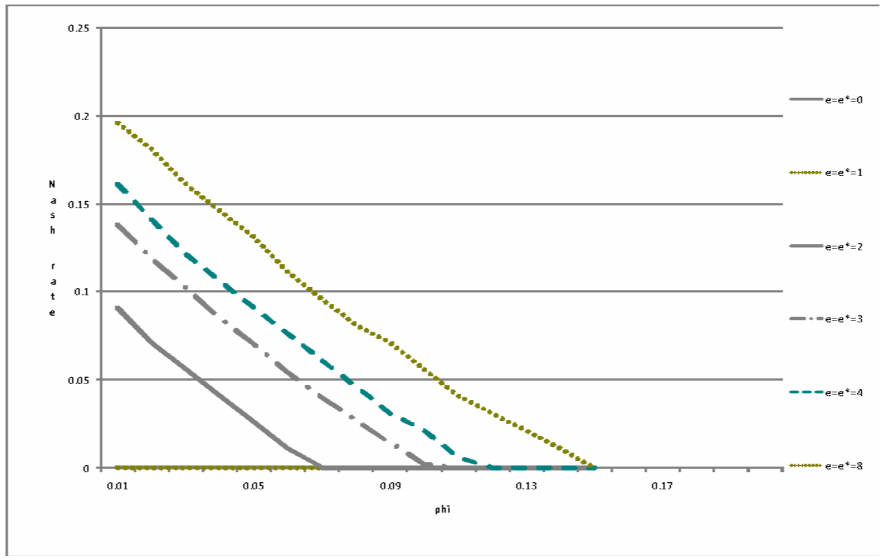

Figure 21: Influence of ϕ on Nash rate for different values of e.

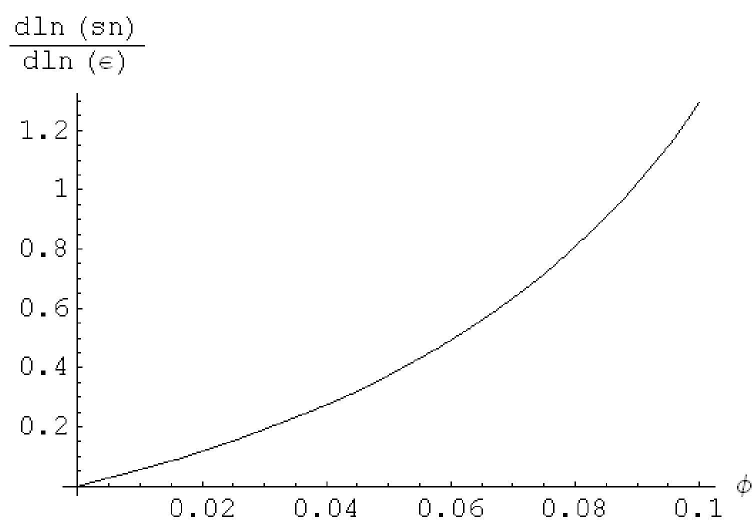

Figure 22: The elasticity of s_n with respect to ϵ in function of ϕ.

profitable for a firm to relocate all the capital to the periphery. By looking back at the tomahawk diagram in function of the competitiveness, figure (11), this means that the competitiveness of the northern region has to stay above the value of $\epsilon_{SN} = \frac{1+(2\sigma-1)\phi^2}{2\sigma\phi}$, which clearly depends on the trade freeness. However, it could happen that for low values of trade freeness and the elasticity of substitution the northern core is always unstable. The only choice the northern government has in this scenario is abiding to a zero tax rate. In that way it can keep the core since any tax increase by the south would restore a stable northern core.

The equilibrium concept we used until now, namely the static Nash equilibrium is no longer applicable in cases like this where there are severe discontinuities. The same southern tax rate could lead to two possible chosen tax rates by the north, depending on the locational equilibrium that would result from the chosen degree of competitiveness. To overcome this, we introduce, as already done by Baldwin and Krugman ([27]), a sequential tax game in which the core region moves first, followed by the southern region. In the third step of the game, migration and production occurs. In this way the south can engage in maximal fiscal competition. The best thing the south can do is choosing a zero tax rate. In this way she can make it the northern region as difficult as posible to retain the core and this is done at no cost for the southern government since her tax revenues are independent of the chosen tax rate, namely zero.

The northern optimization problem is depicted in the graph (23). The concave line on this graph represents the sustain tax rate for the north. If the northern region sets its tax rate below this threshold, it will retain a stable core. The horizontal lines represent the unrestricted optimization result for the core region for different values of the inequality aversion. One concludes, based on this graph, that the higher the inequality aversion becomes, the more strict this condition becomes.

For instance, a Rawslian government will for any value of the trade freeness be restricted, while a government with an inequality aversion of 2 will only be restricted in its optimal choice for low and high values of trade freeness.

Since the non-restricted optimalization mimics the autarkic behaviour, the (non-restricted) chosen tax rates are equal to the tax rates a social planner would choose in a model with (or without) capital mobility and at an internal equilibrium. Thus, for intermediate values of the trade freeness, the chosen tax rates by the core region largely surpasses the Nash tax rates in an internal equilibrium and one could argue that there is no social security competition at all anymore, at least from the point of view of the northern region. Evidently, the southern region but, for low values of trade freeness, also the northern region are restricted to set their tax rates substantially lower than a social planner would do.

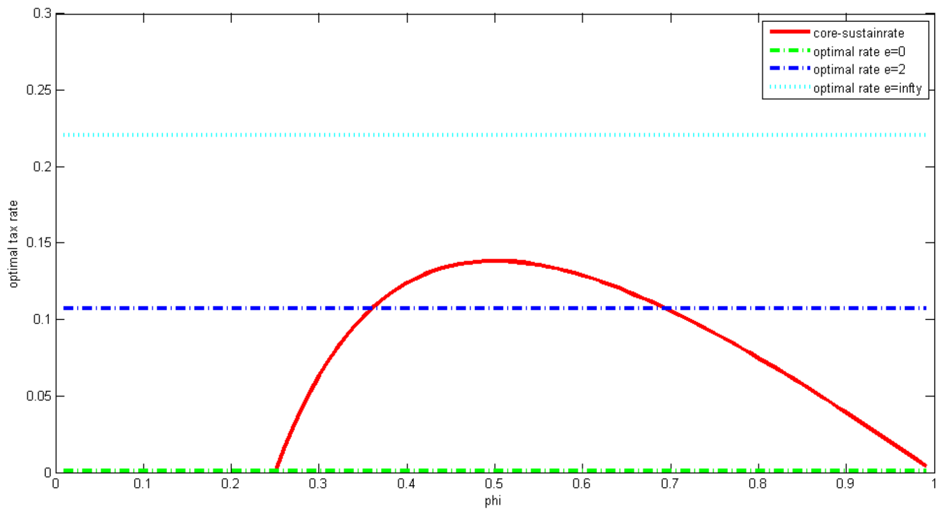

Figure 23: Unconstrained and core sustain tax rate for a core region for different values of e ($\sigma = 2.5$).

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