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WORKING PAPER

Estimating Europe’s Natural Rates from a forward-looking Phillips curve

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Estimating Europe's Natural Rates from a forward-looking Phillips curve *

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Abstract

This paper re-estimates potential output, the NAIRU, and the core inflation rate using aggregated euro area data. The empirical model consists of a Phillips curve linking inflation to unemployment. An Okun-type relationship is used to link the output gap to cyclical unemployment. Using recent developments in the field of New Keyenesian economics, the Phillips curve is forward-looking. The model further accounts for new developments in unobserved component models by allowing (i) for correlation between shocks to the trend and the cycle and (ii) structural breaks in the drift of potential output.

1 Introduction

Measuring equilibrium rates of key macroeconomic variables and the resulting gaps has a long-standing history in the economic literature. Since the seminal contribution of Burns and Mitchell (1946) several studies estimated business cycles using various different concepts of equilibrium rates and different statistical techniques to extract them from the data. The deviation of actual output from its long-run level, referred to as potential output, provides an estimate about the cyclical position of the economy. A positive output gap, i.e. actual output exceeds potential output, implies inflationary pressure as demand exceeds supply. A closely related concept is that of a natural rate of unemployment and has been pioneered by Friedman (1968) and Phelps (1968) who claim that unemployment is at its natural level when neither inflationary nor deflationary

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pressure emanates from the labour market. This is called the non-accelerating-inflation-rate of unemployment (NAIRU).

A popular method to estimate natural rates is the Unobserved Component (UC) model in which trend and cycle are treated as latent state variables, the former modelled as a non-stationary process while the cycle is mean reverting. After casting the model in state-space form, it can be estimated using the Kalman filter. The UC model, pioneered by Harvey (1985), Watson (1986), and Clark (1987), typically assumes zero correlation between shocks to the trend and the cycle. However, Morley et al. (2003) showed that this restriction strongly influences the resulting decomposition. They further showed that, under certain conditions, this correlation is identified and can be estimated. Moreover, when focusing on output decomposition, Perron and Wada (2005) showed the importance of properly specifying a process for potential output that is capable to account for structural breaks. Specifically they argue that modelling trend growth as either deterministic or a simple random walk yields to an estimate in which the trend is very close to actual output, leaving little to the cycle. Hence, it does not accord with what is commonly viewed as a business cycle.

The existing literature is dominated by studies that focus on the US economy. Recent contribution include Basistha and Nelson (2007) who estimate the US output gap from a bivariate system combining inflation and output through a forward-looking Phillips curve. Additionally they allow for correlated trend and cycle shocks. Domenech and Gomez (2006) estimate potential output, core inflation, and the NAIRU from a model that combines information contained in real GDP, inflation, unemployment, and investment. Their model includes a forward-looking Phillips curve and allows for some volatility breaks. The literature that focuses on the euro area is much scarcer. As an exception Ruenstler (2002) estimates the real-time output gap for the euro area and studies its reliability over different model specification. Fabiani and Mestre (2004) focus on estimating the euro area NAIRU using a backward-looking Phillips curve and do not allow for correlated trend cycle shocks.

This paper re-estimates potential output, the NAIRU, and the core inflation rate using aggregate euro area data. The empirical model, building on Apel and Jansson (1999a,b), consists of a Phillips curve linking inflation to unemployment. An Okun-type relationship is used to link the output gap to cyclical unemployment. Using recent developments in the field of New Keyenesian economics, the Phillips curve is forward-looking. This allows for a more model based interpretation of the estimated gaps. The model further accounts for the new developments in UC models by allowing (i) for correlation between shocks to the trend and the cycle and (ii) structural breaks in the drift of potential output. In order to investigate the importance of these points two alternative models are estimated and compared to the baseline specification.

The rest of the paper is organised as follows. Section 2 briefly reviews recent developments in trend-cycle decomposition with regard to UC models. Section 3 outlines and estimates the model. In section 4 the results are presented. Section 5 concludes.

2 Recent developments in trend-cycle decomposition

Natural rates such as potential output or the NAIRU are not directly observable and therefore have to be estimated using statistical methods. Various methodologies have been suggested for estimating natural rates and the corresponding gaps. They can be divided into two groups, purely statistical and economic based. The latter approach estimates a production function and obtains a measure of potential output from various factor inputs multiplied by total factor productivity. Recent contribution include Proietti et al. (2007) and Roeger (2006). However the focus here is on the statistical methods to estimate natural rates.[1]

Among the widely used methods are the filter of Hodrick and Prescott (1997) (HP), the Band-Pass (BP) filter of Baxter and King (1999), the Beveridge and Nelson (1981) decomposition, and the UC model. The HP and the BP filter obtain trend estimates by imposing smoothness a priori. In contrast the BN decomposition and UC models have the advantage of not imposing smoothness but 'let the data speak' as much as possible. The BN decomposition is based on an unconstrained ARIMA model where the trend is defined as the long-run forecast and the cycle as the difference between the trend component and the actual data. As a consequence, the same innovation drives trend and cyclical component implying that they are perfectly negatively correlated. Similar to the BN decomposition the UC trend-cycle model also requires identification of the trend, i.e. a law of motion has to be specified. However, the UC model in state space form is not based on modelling first differences but models the series as the sum of the trend and the cyclical component. Thus,

[1] See Canova (1998) for an overview and a discussion of the differences between statistical and economic approaches.

one must also specify a law of motion for the cycle. The two components are driven by different innovations which are typically assumed to be uncorrelated. Although the BN decomposition and the UC model are purely statistical approaches without imposing smoothness of the trend a priori they lead to very different trend cycle decomposition. The BN decomposition leads to a trend estimate which closely follows the observed data. Thus the resulting cycle is very small and noisy and can hardly be seen as a business cycle. To the contrary, the cycle obtained from a UC model is large and persistent and accords well with business cycle chronologies, such as the NBER for the US or the CEPR for the euro area. The conflicting results suggest that either the BN decomposition or the UC model is not supported by the data.

In a recent paper Morley et al. (2003) show that this puzzling feature is caused by the assumption of zero correlation between shocks to trend and cycle. They further show that the correlation is identified and can be estimated if the specified cycle bears rich enough dynamics. In a univariate trend-cycle decompositions where the trend is a random walk the cycle needs to be specified as an $ARMA(p, q)$ process with $p \geq q + 2$. When applying this correlated UC model to US GDP they found that the estimated correlation between shocks to the trend and cycle is very high and the estimated trend is virtually identical to the BN decomposition. Thus, although the assumption of zero correlation leads to reasonable business cycles, it clearly is at odds with the data. The results of Morley et al. lead to a new strand of literature which is labelled correlated unobserved components models. Contribution in this line of research include, among others, Basistha (2007), Basistha and Nelson (2007), and Sinclair (2007).

The importance of relaxing the zero correlation assumption in UC models solved one puzzle but raised another. How should one interpret the resulting components? The estimated cyclical components in Morley et al. (2003) is, by construction transitory, but noisy and small in magnitude. On the other hand the trend component follows the data closely, implying a dominant role for permanent shocks in explaining real GDP in US postwar data. At first sight this result confirms neoclassical business cycle theory which ascribes changes in real GDP to movements in production due to technology shocks. By analysing this issue in detail Perron and Wada (2005) argued that the small cycle is the result of a misspecified model. Specifically they showed that if one accounts for a single break in the slope of the trend function the delivered cycle from both, the BN decomposition and the correlated UC model, is large and persistent and accords with the

NBER chronology. Typically the drift of the trend is either treated as deterministic or modelled as a random walk (see e.g. Stock and Watson, 1998). It is important to note that even the latter approach can not adequately account for infrequent shifts in the drift of the trend function. As Perron and Wada point out the variance of an estimated random walk drift is very small implying a drift that changes only little but every period though. As a consequence, if there are infrequent but large shifts in the slope of the trend function, the random walk drift is inadequate to capture them.

The result of a small and noisy cycle seems to be more important in univariate models. Studies that estimate multivariate models typically find the cycle to be large and persistent. Intuitively this might be due to the additional information contained in e.g. the inflation rate. Nevertheless, potential output remains misspecified if there are infrequent but large shifts which are not accounted for.

3 A multivariate correlated unobserved component model

This section lays out a multivariate correlated UC model that consists of output, unemployment, and inflation. Euro area data, which are aggregate series for 12 countries[2], are taken from the area-wide model of Fagan et al. (2005) and range from 1970 until 2005. The unemployment rate, u_t, is the quarterly unemployment rate. For inflation, π_t, the first difference of the log of the seasonally adjusted quarterly GDP deflator is used. Output is the log of seasonally adjusted quarterly GDP in constant prices.

3.1 Unemployment Decomposition

The rate of unemployment is disentangled into two components, a non-stationary trend component u_t^*, and a cyclical component u_t^c

$$u_t = u_t^* + u_t^c. \tag{1}$$

[2] Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain.

The latter is modelled as a $AR(2)$ process

$$u_t^c = \phi_1 u_{t-1}^c + \phi_2 u_{t-2}^c + \eta_t^c, \tag{2}$$

where η_t^c is a Gaussian mean zero white noise error term. The $AR(2)$ specification allows cyclical unemployment to exhibit the standard hump-shaped pattern. The NAIRU, i.e. the trend component in the unemployment rate, is specified as a random walk with drift

$$u_t^* = \gamma_t + u_{t-1}^* + \eta_t^u \tag{3}$$

$$\gamma_t = \delta_1 \gamma_{t-1} + \eta_t^\gamma, \tag{4}$$

where the drift, γ_t, evolves according to a $AR(1)$ process. η_t^u and η_t^γ are Gaussian mean zero white noise error terms. From an economic perspective it would be sufficient to model long-run unemployment as a simple random walk. Any shock to the NAIRU, which may reflect changes in its underlying labour market institutions such as the benefit system or employment protection legislation, would have a permanent impact on the NAIRU and hence on the rate of unemployment. However, modelling the euro area NAIRU as a simple random walk yields to implausible results. The reason is that, contrary to the US, unemployment in Europe increased from the early 70s to the mid 80s and remained persistently high since then. In order to capture this increase, a random walk NAIRU would need to have a sequence of positive shocks which are not about to reversed within the sample. Thus the shocks driving the random walk NAIRU can hardly be described as a standard Gaussian white noise process with mean zero.[3] The trend in the rate of unemployment should be seen as a special feature for the time span analysed here rather than as characteristic for the rate of unemployment as such.[4] As argued by Fabiani and Mestre (2004) this upward trend is no longer present and might even about to be reversed. In NAIRU estimates for the euro area the trend in long-run unemployment is often modelled as a random walk implying that the NAIRU is an I(2) process (see e.g. Orlandi and Pichelmann, 2000; Laubach, 2001; Fabiani and Mestre, 2004). The advantage of the $AR(1)$ specification applied here is that the smoothness of the NAIRU is not imposed but estimated. Moreover, as long as $0 < \delta_1 < 1$ the NAIRU is I(1).

[3] An alternative to the drift might be to relax the assumption of Gaussian error terms. However, non-Gaussian state-space models are difficult to estimate since they cannot be estimated with standard maximum likelihood techniques.

[4] As the rate of unemployment as a bounded process it obviously cannot be an upward trending process in the long-run.

3.2 Output Decomposition

Following Watson (1986) and Clark (1987) output is modelled as the sum of two components, potential output $\overline{y}_t$ and cyclical output y_t^c.

$$y_t = \overline{y}_t + y_t^c \tag{5}$$

Potential output is specified as a random walk with drift

$$\overline{y}_t = \mu_i + \overline{y}_{t-1} + \eta_t^{\overline{y}}, \tag{6}$$

where μ_i is the drift of potential output, often referred to as output growth. The subscript i refers to different values of output growth with $i = 1, ..., m+1$, m denoting the number of structural breaks. In order to identify the number and the timing of structural breaks in GDP growth, the Bai and Perron (1998, 2001, 2003) (BP) multiple structural breaks test is used. The BP methodology has also been used by Rapach and Wohar (2005) to test for structural breaks in real interest rates and inflation rates and by Basistha (2007) to test for structural breaks in Canadian GDP growth within a bivariate UC model. Briefly, BP suggest to examine first two tests (the so called UD_{max} and WD_{max} tests) to check if there are any structural breaks. If these tests reject the null of no breaks use a sequential procedure to determine the number of breaks. According to the BP notation this means computing a sequence of $\text{Sup}F_T(l+1|l)$ statistics to test the null of l breaks against the alternative of $l+1$ breaks.[5]

Cyclical output, i.e. the output gap, is linked to the unemployment gap via Okun's Law. Okun (1970) showed that there is an empirical relation between output and unemployment. This relationship has been labelled Okun's Law and can be expressed as

$$y_t^c = \omega(L) u_t^c, \tag{7}$$

where ω is the Okun's Law parameter. It should be stressed that this relation does not come from a fully specified macroeconomic model but accounts for the negative correlation between the output and the unemployment gap.

[5] A detailed description of this test can be found in BP and Rapach and Wohar (2005).

3.3 The forward-looking Phillips curve

The new Keynesian Phillips curve (NKPC) states that the difference between actual inflation and its expected value is driven by marginal costs. Assuming that marginal costs are proportional to the output or unemployment gap, it takes the form

$$\pi_t = \mathrm{E}\left(\pi_{t+1}\right) + \alpha u_t^c, \tag{8}$$

where π_t is the inflation rate, and $\mathrm{E}\left(\cdot\right)$ is the expectation operator based on information up to time t. An obvious problem in the NKPC is that the expected value of future inflation is an unobservable variable. However the UC methodology offers a proxy for expected inflation. As shown by Beveridge and Nelson (1981) the inflation trend, resulting from a decomposition of inflation into a non-stationary trend and a stationary cycle

$$\pi_t = \pi_t^* + \pi_t^c, \tag{9}$$

where π_t^* is the trend of inflation

$$\pi_t^* = \pi_{t-1}^* + \tau_t + \eta_t^{\pi^*}, \tag{10}$$

$$\tau_t = \delta_2 \tau_{t-1} + \eta_t^{\tau}. \tag{11}$$

Similar to the NAIRU the trend in inflation is modelled as a unit-root process with some transitory dynamics. π_t^c denotes transitory inflation with mean zero. π_t^* can be interpreted as the long-run forecast of inflation since the long horizon forecast of transitory inflation is zero

$$\pi_t^* = \lim_{j \to \infty} \mathrm{E}\left(\pi_{t+j}\right) = \mathrm{E}_t(\pi_\infty). \tag{12}$$

Assuming that transitory inflation and cyclical unemployment are linearly related, i.e. $\pi_t^c = B(L)u_t^c + \eta_t^{\pi^c}$, equation (9) can be rewritten as

$$\pi_t = \mathrm{E}_t(\pi_\infty) + B(L)u_t^c + \eta_t^{\pi^c}. \tag{13}$$

This represents the expectation augmented Phillips-curve. Note that the slope of the Phillips-curve not only depends on $B(L)$ but is also affected by the expectation horizon. This point becomes clearer by taking the one-step ahead conditional expectation of equation (13)[6]

$$\mathrm{E}_t\left(\pi_{t+1}\right) = \mathrm{E}_t\left[\mathrm{E}_t\left(\pi_\infty\right) + B\left(L\right)u^c_{t+1} + \eta^{\pi^c}_{t+1}\right] \tag{14}$$

$$= \mathrm{E}_t\left(\pi_\infty\right) + B\left(L\right)\left(\phi_1 u^c_t + \phi_2 u^c_{t-1}\right) \tag{15}$$

Solving (15) for $\mathrm{E}_t\left(\pi_\infty\right)$ and combining it with (13) gives the one-step ahead forward-looking Phillips-curve

$$\pi_t = \mathrm{E}_t\left(\pi_{t+1}\right) + B\left(L\right)\left(1-\phi_1\right)u^c_t - B\left(L\right)\phi_2 u^c_{t-1} + \eta^{\pi^c}_t. \tag{16}$$

It must be stressed that this way of modelling the Phillips-curve, although not standard yet, is not entirely new. Nelson and Lee (2007) and Domenech and Gomez (2006) also use the trend in inflation as a measure for the expectation term in the NKPC.

3.4 State-Space form

The model can be written in a linear Gaussian state space representation of the the following form. The observation equation

$$y_t = \Gamma\alpha_t, \tag{17}$$

$$\begin{bmatrix} y_t \\ \pi_t \\ u_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \omega & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \overline{y}_t \\ \pi^*_t \\ \tau_t \\ \pi^c_t \\ u^*_t \\ \gamma_t \\ u^c_t \\ u^c_{t-1} \end{bmatrix}. \tag{18}$$

The states equations are given by

$$\alpha_{t+1} = \mu + S\alpha_t + \eta_t, \qquad \eta_t \sim N(0, Q) \tag{19}$$

[6]This derivation borrows heavily from Nelson and Lee (2007).

$$
\begin{bmatrix} \overline{y}_{t+1} \\ \pi^*_{t+1} \\ \tau_{t+1} \\ \pi^c_{t+1} \\ u^*_{t+1} \\ \gamma_{t+1} \\ u^c_{t+1} \\ u^c_t \end{bmatrix} = \begin{bmatrix} \mu_j \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \kappa_1 & \kappa_2 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \phi_1 & \phi_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \overline{y}_t \\ \pi^*_t \\ \tau_t \\ \pi^c_t \\ u^*_t \\ \gamma_t \\ u^c_t \\ u^c_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \\ \eta_{4t} \\ \eta_{5t} \\ \eta_{6t} \\ \eta_{7t} \\ 0 \end{bmatrix}. \tag{20}
$$

The variance-covariance matrix of the state innovations is given by

$$
Q = \begin{bmatrix} \sigma^2_{\eta_1} & 0 & 0 & 0 & 0 & 0 & \sigma_{\eta_1\eta_7} & 0 \\ 0 & \sigma^2_{\eta_2} & 0 & \sigma_{\eta_2\eta_4} & \sigma_{\eta_2\eta_5} & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\eta_3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\eta_2\eta_4} & 0 & \sigma^2_{\eta_4} & 0 & 0 & \sigma_{\eta_4\eta_7} & 0 \\ 0 & \sigma_{\eta_2\eta_5} & 0 & 0 & \sigma^2_{\eta_5} & 0 & \sigma_{\eta_5\eta_7} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{\eta_6} & 0 & 0 \\ \sigma_{\eta_1\eta_7} & 0 & 0 & \sigma_{\eta_4\eta_7} & \sigma_{\eta_5\eta_7} & 0 & \sigma^2_{\eta_7} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{21}
$$

The likelihood for the linear Gaussian state space model can be calculated by a routine application of the Kalman filter and maximised with respect to the unknown parameters using an iterative numerical procedure (see e.g. Harvey, 1989; Durbin and Koopman, 2001). The stationary state variables are initialised by drawing from their stationary distributions while a diffuse initialisation is used for the non-stationary state variables. Standard errors for the estimates are calculated by inverting the Hessian matrix.

4 Estimation Results

4.1 Structural breaks on output growth

Table (1) presents the results of the BP tests on structural breaks in output growth.[7] Both, the WD_{max} and the UD_{max} test statistic clearly reject the null hypothesis of no structural breaks in GDP growth at conventional confidence levels. The sequential analysis also rejects the null of no breaks against the alternative hypothesis of one structural break. However more than one

[7]The results of the BP tests have been obtained by using the original GAUSS program from P. Perron available on his webpage.

Table 1: Test for structural breaks in GDP growth

WD_{max}	UD_{max}	$SupF_T(1\|0)$	$SupF_T(2\|1)$	$SupF_T(3\|2)$	$SupF_T(4\|3)$	$SupF_T(5\|4)$
23.21*	25.77*	23.31*	8.29	3.99	3.61	3.90

The maximum number of breaks is set to 5. The $*$ denotes significance at the 5% level. The 5% critical values are $UD_{max} = 9.52$, $WD_{max} = 10.39$, $SupF_T(1|0) = 9.1$, $SupF_T(2|1) = 10.55$, $SupF_T(3|2) = 11.36$, $SupF_T(4|3) = 12.35$, $SupF_T(5|4) = 12.97$.

structural break cannot be found. The detected break date is 1974:Q1. Although the focus here is not on explaining the source of the break one may recognise that this period is usually associated with a worldwide slowdown in productivity growth. Moreover, using the same methodology Basistha (2007) found a single break in Canadian GDP growth in 1973:Q4 and Perron and Wada (2005) also point to a single break in US GDP growth in 1973:Q1. Consequently the drift of potential output is modelled as a piecewise linear process taking the value μ_1 before 1974:Q1 and μ_2 thereafter.

4.2 UC model parameter estimates

The third column in Table (2) presents the parameter estimates of the baseline UC model as shown by equations (18), (20), and (21). In order to investigate the importance of the trend-cycle correlation and the structural break in output growth two alternative models are estimated. Model (b) restricts all covariances to zero and does not allow for a structural break in output growth. Model (c) considers a unit-root specification for output growth instead of a piecewise linear process.

In the baseline model, the estimated persistence of cyclical unemployment, as measured by the sum of the autoregressive parameters ϕ_1 and ϕ_2, is found to be very high. The result of strong persistence in cyclical unemployment is also found in previous studies. Laubach (2001), for instance, estimates NAIRU's for seven economies and finds that cyclical unemployment is very persistent, particularly in continental European countries. The point estimate of δ_1 implies strong persistence in the drift of the NAIRU. Thus, taking the parameter uncertainty into account, an I(2) behaviour for the NAIRU over the considered sample cannot be ruled out. This result confirms earlier NAIRU estimates for the euro area which imposed an I(2) NAIRU a priori (see e.g. Orlandi and Pichelmann, 2000; Laubach, 2001; Fabiani and Mestre, 2004).

In the output equation the piecewise linear trend is estimated to be 0.986 before 1974Q:1 and

Table 2: Parameter Estimates

		Baseline Model	Model (b)	Model (c)
Phillips curve	σ_{η_2}	0.575 (0.087)	0.076 (0.108)	0.500 (0.072)
	σ_{η_3}	0.048 (0.657)	0.446 (0.139)	0.047 (0.291)
	σ_{η_4}	0.888 (0.068)	1.169 (0.093)	0.938 (0.017)
	$\sigma_{\eta_2\eta_4}$	0.498 (0.082)	-	0.468 (0.074)
	$\sigma_{\eta_2\eta_5}$	-0.011 (0.014)	-	-0.002 (0.009)
	κ_1	-3.230 (1.366)	-4.568 (1.550)	-2.818 (1.031)
	κ_2	2.349 (1.293)	1.569 (1.166)	1.707 (1.066)
	δ_1	0.067 (0.107)	-0.083 (0.096)	0.086 (0.091)
Unemployment	σ_{η_5}	0.089 (0.006)	0.066 (0.010)	0.087 (0.006)
	σ_{η_6}	0.011 (0.005)	0.050 (0.016)	0.010 (0.006)
	σ_{η_7}	0.050 (0.007)	0.065 (0.014)	0.058 (0.009)
	$\sigma_{\eta_5\eta_7}$	0.000 (0.001)	-	-0.001 (0.001)
	$\sigma_{\eta_7\eta_4}$	0.021 (0.015)	-	0.022 (0.010)
	ϕ_1	1.893 (0.035)	1.672 (0.121)	1.886 (0.039)
	ϕ_2	-0.917 (0.035)	-0.686 (0.130)	-0.905 (0.039)
	δ_2	0.987 (0.013)	0.912 (0.052)	0.976 (0.013)
Output	σ_{η_1}	0.441 (0.032)	0.398 (0.049)	0.438 (0.031)
	σ_{η_μ}	-	-	0.011 (0.012)
	$\sigma_{\eta_1\eta_7}$	-0.018 (0.004)	-	-0.017 (0.004)
	ω	-1.590 (0.366)	-4.618(1.75)	-1.932 (0.359)
	μ_1	0.986 (0.120)	0.601 (0.066)*	-
	μ_2	0.547 (0.041)	-	-
log-likelihood		8.911	-5.563	-2.480

Standard errors are in parentheses. Model (b) restricts all covariances to zero and does not allow for a structural break in potential output growth. Model (c) specifies potential output growth as a unit-root process. * μ_1 refers to potential output growth over the full sample.

0.547 thereafter which corresponds to an output growth rate of 3.94% respectively 2.19% p.a.. This sharp decrease highlights once again the need for a proper output growth specification. The Okun's Law parameter, ω, has the correct sign and a reasonable magnitude.[8] Turning to inflation, the slope of the Phillips-curve is found to be -0.881. At first sight, this might be surprising as previous studies estimated this parameter to range from -0.7 to zero. However, the way of modelling the Phillips curve here relies on a trend-cycle decomposition of inflation, implying that the slope of the Phillips-curve is equal to the trade-off between the cyclical components of inflation and unemployment. Using a similar specification of the Phillips-curve Nelson and Lee (2007)

[8] More dynamics in the Okun's Law equation, i.e. a lagged impact of cyclical unemployment on the output gap was found to be statistically insignificant.

estimated a slope of -0.9 for US data. The transitory dynamics around the trend in inflation are not significant, implying a simple unit-root behaviour for the core inflation rate. The estimated covariances not only ensures that the empirical model does not have unnecessary parameter restrictions, it also gives some economic insights. The covariance between the trend and cycle of the rate of unemployment can be interpreted as a test for possible hysteresis effects. The term hysteresis, originally stemming from physics, refers to a situation in which transitory unemployment translates into long-run unemployment. This idea has been introduced by Blanchard and Summers (1986) who argued hysteresis effects can arise from insider-outsider effects in wage formation. However, the results here do not show hysteresis effects in the rate of unemployment. Moreover, the covariance between shocks to the NAIRU and shocks to the core inflation rate are close to zero and statistically insignificant. Thus there is no trade-off between inflation and unemployment in the long-run as suggested by theory.

The parameter estimates of the two restricted alternative models show a somewhat mixed picture. In model (c), where trend output growth is a random walk process, the estimated parameters are close to those from the baseline model. In model (b), however, the point estimates of some parameters are not economically meaningful, e.g. the slope of the Phillips curve is estimated to be -2.999. Moreover, the Okun's Law parameter is found to be -4.618. A likelihood-ratio test between the baseline model and model (b) shows that the baseline model outperforms model (b). The test statistic of 28.948 which is distributed as chi-squared with six degrees of freedom leads to the rejection of the restrictions imposed in model (b) at standard significance levels.[9]

4.3 State estimates

Figure (1) shows the smoothed estimated states of the baseline model and their 95% confidence intervals together with the original data. The sharp increase in the rate of unemployment until the mid80s is explained by movements of the NAIRU. Cyclical unemployment, although very persistent, is rather small in magnitude. The output gap is symmetric to the unemployment gap since they are linked to each other via Okun's Law. The core inflation rate evolves smoothly, and thus transitory inflation appears to be very volatile. Overall, the estimated natural rates and the corresponding gaps are consistent with earlier studies mostly based on a backward-looking

[9] Note that a comparison between the baseline model and model (c) is not possible with a standard likelihood-ratio test as model (c) is a different model rather than a restricted model in which parameters are set to zero.

Phillips-curve (see e.g. Fabiani and Mestre, 2004). The shadowed area in the output gap graph indicate recessions as defined by the CEPR.[10] The estimated gap picks up the business cycle turning points quite accurately. Particularly the beginning of recessions is almost similar dated.

[10]The CEPR dating committee labels the decline in GDP from 2001 onwards a *prolonged pause in the growth of economic activity* rather than a recession but notes that this might be reversed as revised GDP statistics appear (see http://www.cepr.org/data/Dating/ for details).

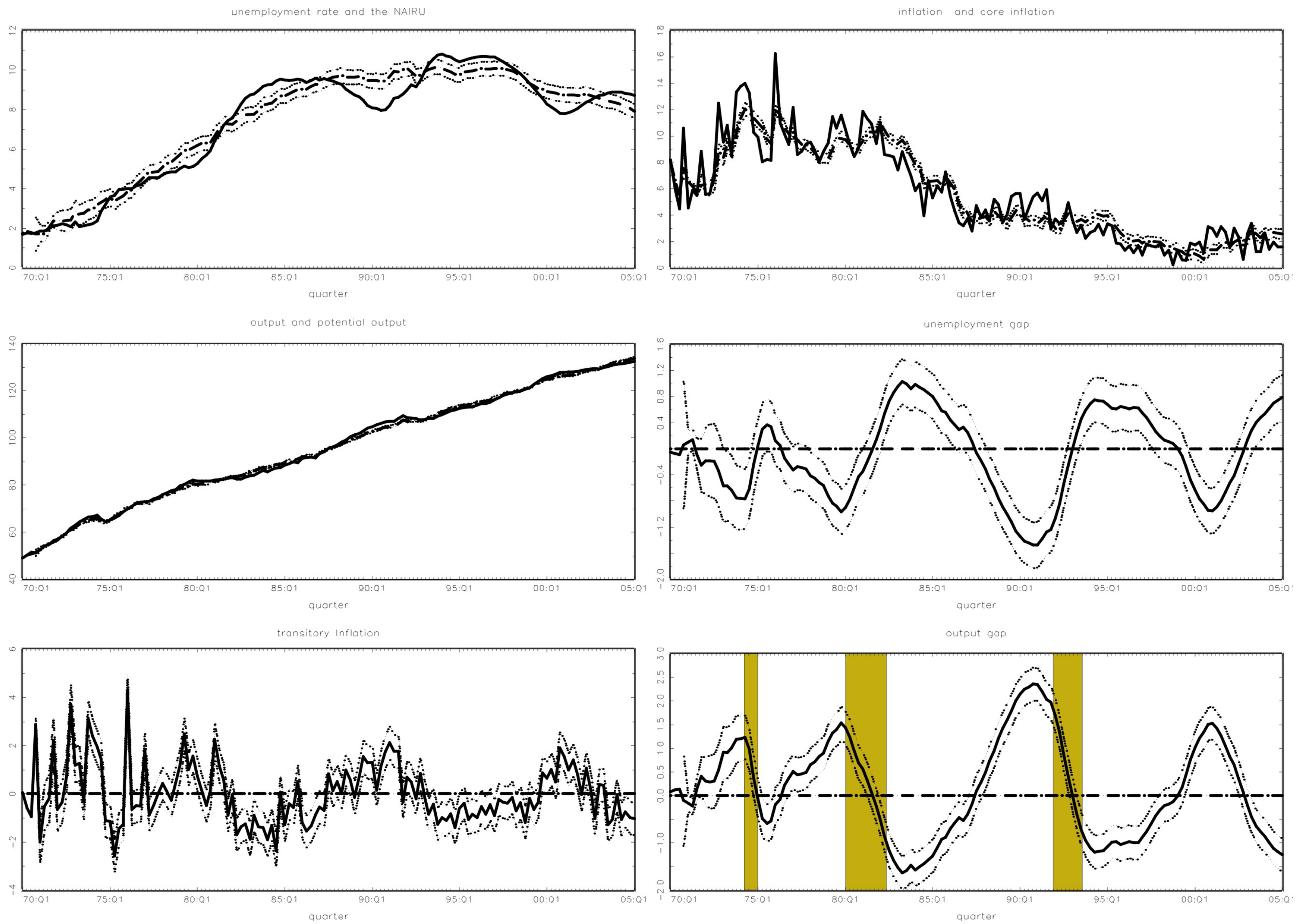

Figure 1: Smoothed states: baseline model

Figure (2) shows the trend in potential output for all three models. In model (b) this is a constant taking the value 0.601 per quarter. To the contrary, the random walk trend captures the sharp decrease in the early 70s but moves back to a level of around 0.8 at the end of the 70s. From then onwards it steadily decreases to approach μ_2 at the end of the sample. Thus, model (c) suggests that there was a sharp decrease in potential output growth at the beginning of the sample. The increase of more than 1% in 1975:Q1 is quite surprising from an economic perspective. However, if the true process is piecewise linear as in the baseline model, the random walk trend with Gaussian error terms is not an adequate alternative. The innovations of such a process with a single but large shift and only small changes before and after the break would not have a Gaussian distribution. Figure (3) shows the output gaps form the three models. Although the timing of the turning points is very similar in all models they show a very different picture. In model (b) actual output stays above potential output for the first half of the sample and below it for almost the complete second half of the sample. Model (c) suggests that there was a positive output gap of more than 3.5% around 1974. In the following years the output gap is close to zero, a finding that is not confirmed by the baseline model which shows a positive output gap in second half of the 70s.

Figure (4) and (5) in the Appendix show the smoothed states estimates of the two alternative models.

5 Conclusion

This paper re-estimates potential output, the NAIRU, the core inflation rate, and the corresponding gaps for the euro area over the period 1970-2005. Output, inflation, and the rate of unemployment are decomposed into a non-stationary and a stationary component. An Okun type relation is used to link the output gap to the unemployment gap. In the Phillips curve, the core inflation rate is used as a proxy for the forward-looking expectations term. Different from previous studies in the field, the empirical model allows for large but infrequent shifts in the growth rate of output. Moreover, accounting for recent advances in unobserved components models, the shocks to the trend and the cyclical component of each series are allowed to be correlated. The results show that that there is an one-time large shift in the growth rate of output in 1974:Q1. This indicates that the conventional approach of modelling output growth as either deterministic or

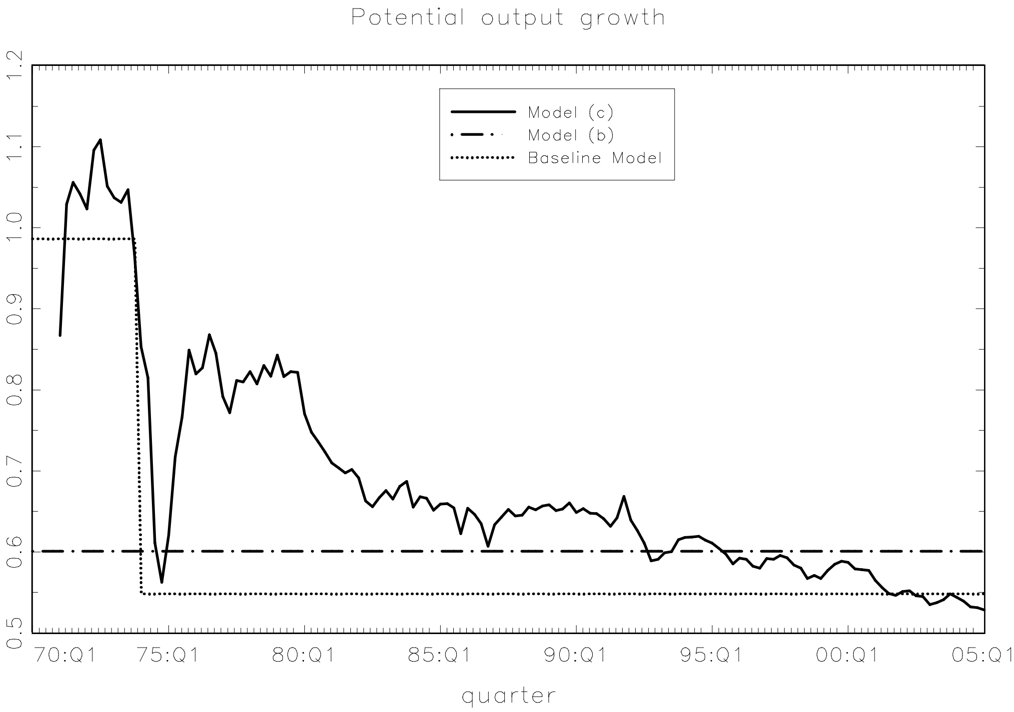

Figure 2: Potential Output Growth

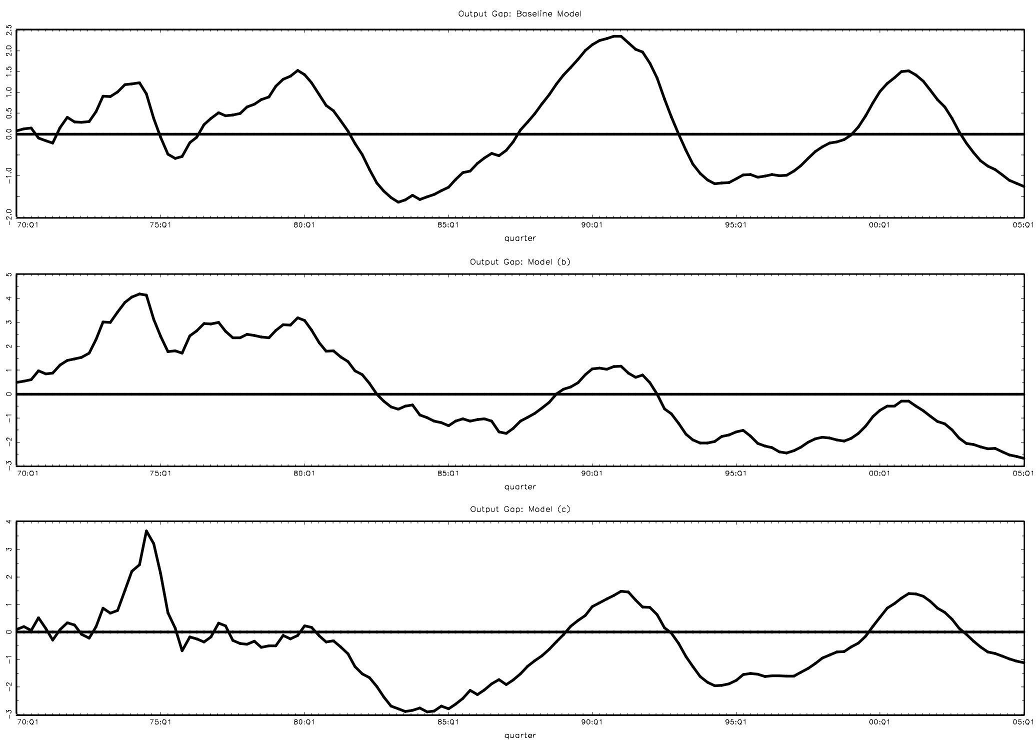

Figure 3: Output Gaps

a unit-root process with shocks occurring every period is inadequate of capturing this shift. By comparing the baseline model to an alternative model without a structural break and covariances that are set to zero, it is shown that these restrictions are not supported by the data. On the other hand, a model with a random walk drift in potential output suggests that after the large decrease in output growth in the early 70s there was a sharp increase in output growth. This result is more a statistical artifact as the shocks driving the unit-root trend are assumed to be Gaussian. Finally the output gap of the baseline model is compared to the business cycle dating of the CEPR and found to accord quite precisely with it.

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Appendices

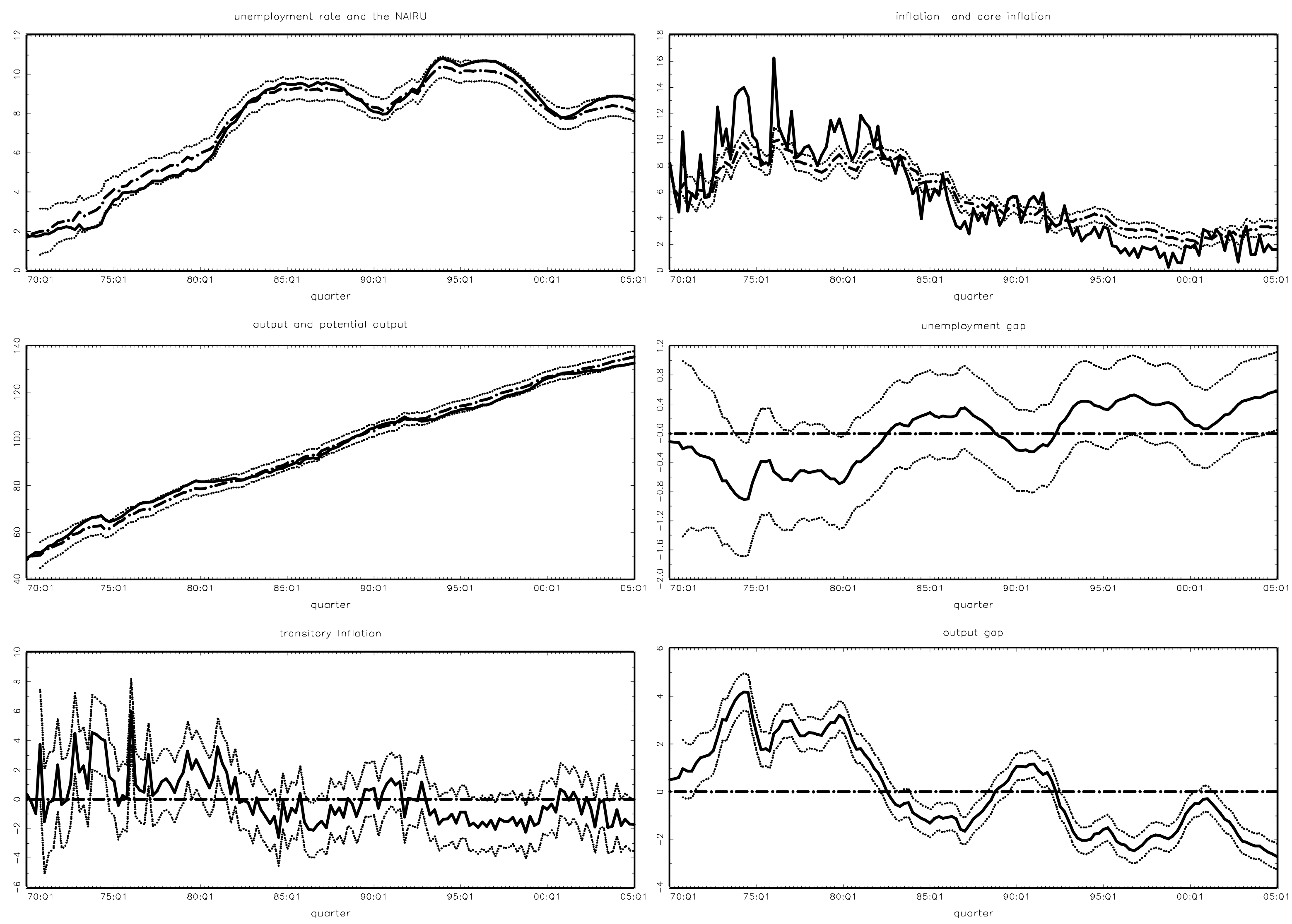

Figure 4: Smoothed states: Model (b)

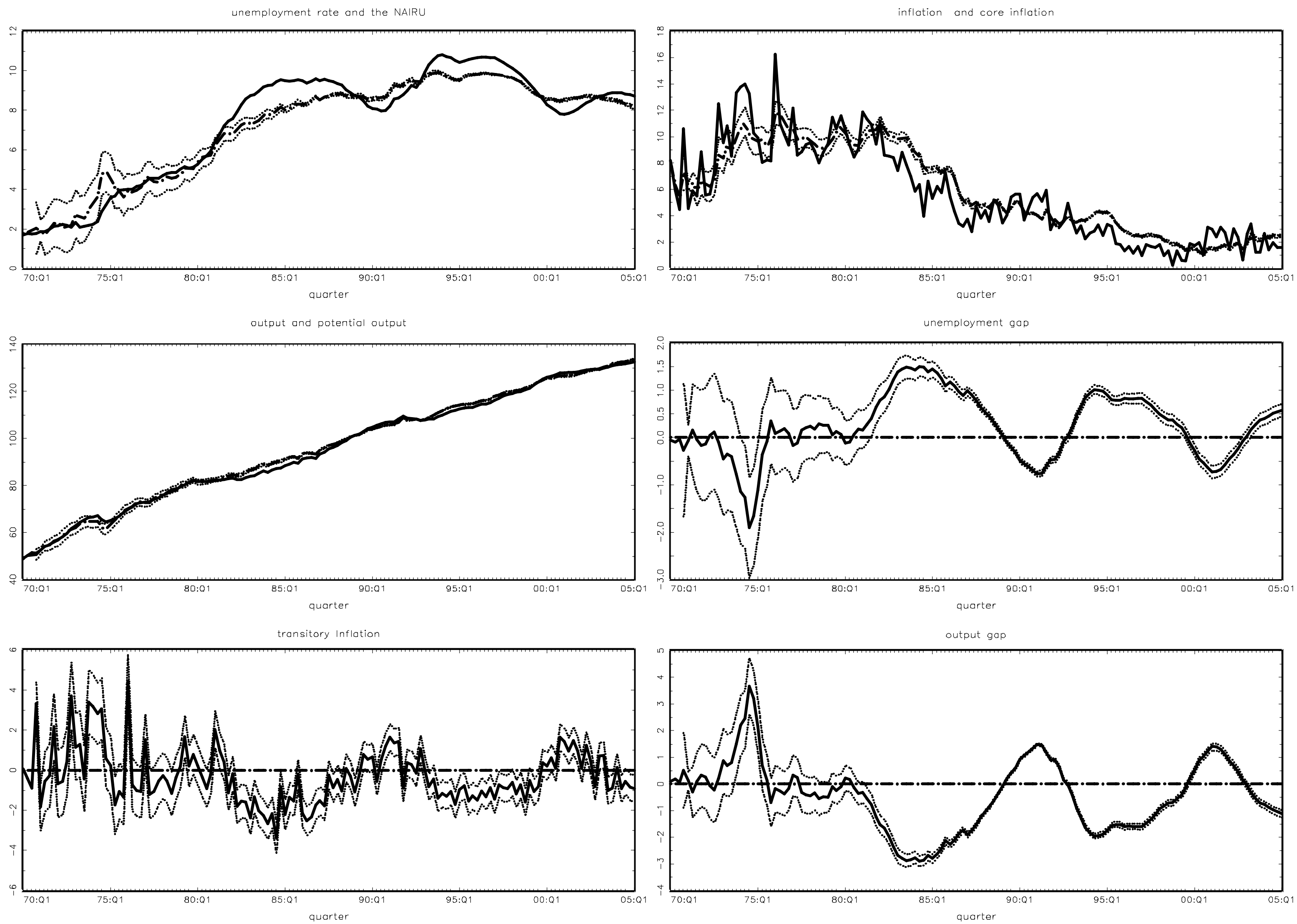

Figure 5: Smoothed states: Model (c)