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WORKING PAPER

Minimal rights based solidarity

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Abstract

In a model where individuals with different levels of skills exert different levels of effort, we propose to use individuals' minimal rights to divide an extra amount of income generated by a change in the skill profile. Priority is given to individuals with a positive minimal right. We characterize two families of Minimal Rights based Egalitarian mechanisms that implement this solidarity idea. One family guarantees each individual her claim when claims are feasible. The other family guarantees a non-negative income after redistribution for all individuals.

JEL Classification: D63.
Keywords: minimal rights, solidarity, compensation, claims.

1 Motivation

Suppose income inequalities are determined by unequal exerted effort levels and different innate skills. The goal of fair income redistribution is to guarantee an equal income for individuals exerting the same effort and to perform equal income transfers to individuals with equal skills. However, in many contexts, there does not exist a redistribution mechanism that simultaneously satisfies both requirements. We refer to Fleurbaey and Maniquet (2004) for an extensive survey of this compensation problem. Weakening one of both requirements leads to the proposition of different (families of) redistribution mechanisms. Many of these redistribution mechanisms rely on so called 'reference' income levels. Typically, these reference income levels are computed by replacing either skill levels or effort levels by some reference value. Within one family of redistribution mechanisms, different mechanisms are distinguished by different choices of the reference value. The literature on fair income redistribution has some strong

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similarities with the literature on bankruptcy problems and surplus sharing problems. This becomes clear when reference income levels are interpreted as claims. In a bankruptcy problem, a fixed amount of money must be allocated on the basis of monetary claims that sum up to more than can be divided. The objective is to design allocation mechanisms that associate with each claims problem a division of the amount available over the claimants. We refer to Thomson (2003) for an extensive survey of the literature on competing claims problems. In a surplus sharing problem, an amount of money that exceeds the total sum of claims must be divided over all claimants. An extensive survey on surplus sharing problems is Moulin (2002). Fair income redistribution problems can be interpreted as competing claims (surplus sharing) problems where the total sum of income before redistribution has to be divided over a population of claimants that have reference incomes as claims.

In this paper we focus on two families of fair income redistribution mechanisms: the family of Egalitarian Equivalent mechanisms due to Pazner and Schmeidler (1978) and Fleurbaey (1995) and the family of Proportionally Adjusted Equivalent mechanisms due to Iturbe (1997). These families are characterized in Fleurbaey and Maniquet (2004) using different strenghtenings of a compensation axiom called 'Solidarity', in combination with a weak responsibility axiom called 'Equal Transfer for Reference Skill'. The solidarity axioms describe solutions to 'the solidarity problem'. The solidarity problem is a competing claims/surplus sharing problem where a change in the skill profile generates an extra amount (or a loss) of pre-tax income that has to be divided over (taken away from) the population of claimants. Under an Egalitarian Equivalent mechanism these extra resources are divided equally and hence differences in exerted effort levels are not taken into account. Under a Proportionally Adjusted Equivalent mechanism differences in exerted effort levels are taken into account by dividing extra resources proportionally to claims. We propose to divide extra resources on the basis of the information contained in individuals' minimal rights. The minimal right of an individual equals the amount that remains from the total sum of pre-tax income when all other individuals receive their claim. Priority is given to individuals with a higher claim, which is due to higher exerted effort, once the total sum of pre-tax income exceeds threshold levels where individuals' minimal rights become positive. We characterize two families of Minimal Rights based Egalitarian mechanisms that implement this solidarity idea. One family also satisfies 'Equal Transfer for Reference Skill', while the other family guarantees a non-negative income after redistribution for all individuals.

In the next section we present the model, state the compensation problem, define the different families of fair income redistribution mechanisms, discuss their characterizations and illustrate the income distributions that result from these families. In section 3 we introduce the notion of minimal rights and propose to use minimal rights to solve the solidarity problem. We state the solidarity axioms of 'Symmetry' and 'Priority' that together constitute the central solidarity idea of this paper, Minimal Rights based Solidarity. We then characterize two families of Minimal Rights based Egalitarian mechanisms. For a given reference skill, we show that the family that guarantees all individuals a non-negative income redistributes income more equally than the family that satisfies 'Equal Transfer for Reference Skill'. This latter family is equivalent with the family of Egalitarian Equivalent mechanisms in rich economies where all minimal rights are positive, but redistributes income more equally in poorer economies where some or all minimal rights are zero. We show that the axioms used in the characterizations are independent. Section 4 summarizes our main conclusions.

2 Fair monetary compensation

2.1 The model

The fair monetary compensation model used in this paper is a one-dimensional version of the quasi-linear model in Fleurbaey and Maniquet (2004) which is due to Bossert (1995). Denote $N = \{1, \ldots, n\}$ the finite population of size $n \geq 2$. Let $x \in \mathbb{R}$ be an amount of transferable resource. The characteristic which elicits compensation, hereafter called 'skill', is $y \in Y$ and Y is an interval of $\mathbb{R}$. Denote $y_N = (y_1, \ldots, y_n)$ the skill profile in the population. The characteristic which does not elicit compensation, hereafter called 'effort', is $z \in Z$ and Z is an interval of $\mathbb{R}$. Effort is not influenced by redistribution as incentive issues are not taken up in the model. Denote $z_N = (z_1, \ldots, z_n)$ the effort profile in the population. Without loss of generality, we assume that individuals are ranked such that $z_1 \geq z_2 \geq \ldots \geq z_n$. An economy $e = (y_N, z_N)$ is the pair of characteristics' profiles. Denote $\mathcal{E}$ the set of economies.

We assume that utility functions are quasi-linear as follows:

$$u_i(x_i, y_i) = x_i + v(y_i, z_i).$$

In the context of this paper $u_i(x_i, y_i)$ measures a monetary outcome, namely final income after redistribution. The function $v : Y \times Z \to \mathbb{R}_{++} : (y_i, z_i) \to v(y_i, z_i)$ describes the pre-tax income function. We assume that v is continuous and strictly increasing in y and z. Furthermore, we assume that v is not additively separable in y and z, i.e. $v(y_i, z_i)$ cannot be written as $v_1(y_i) + v_2(z_i)$. Denote $R = \sum_{i \in N} v(y_i, z_i)$ the total sum of pre-tax income.

Let the transferable resource x_i be an element of an allocation $x_N = (x_1, \ldots, x_n) \in \mathbb{R}^n$. We assume that the total amount to be distributed is 0, such that we are looking at a pure redistribution problem. An allocation for the economy $e \in \mathcal{E}$ is feasible when $\sum_{i \in N} x_i = 0$. The set of feasible allocations is denoted F. Notice that all feasible allocations are Pareto efficient since we ruled out free disposal in the definition of feasibility. The function $S : \mathcal{E} \to F : e \to S(e)$ is an allocation mechanism. Denote $\mathcal{S}$ the set of all allocation mechanisms.

Let $\tilde{y} \in Y$ be the reference skill. We assume throughout the paper that this constant parameter is exogenously determined by the social planner. Denote $v(\tilde{y}, z_i)$ the claim of individual i. It equals the pre-tax income that an individual would receive when exerting her effort level z_i but having skill $\tilde{y}$ instead of her own skill y_i. We bring all claims together in a vector ν and denote $C = \sum_{i \in N} v(\tilde{y}, z_i)$ the total sum of claims . Define the interval $Y_{cc} = \{\tilde{y} \in Y : C \geq R\}$. For parameter values of $\tilde{y}$ in Y_{cc}, the total sum of claims is at least as high as the total sum of pre-tax income (which can be redistributed) and a competing claims problem arises. A competing claims problem is a pair $(\nu, R) \in \mathbb{R}^n_{++} \times \mathbb{R}_{++}$, such that $C \geq R$. Define the interval $Y_{ss} = \{\tilde{y} \in Y : C < R\}$. For parameter values of $\tilde{y}$ in Y_{ss}, the total sum of claims is not as high as the total sum of pre-tax income and a surplus sharing problem arises. A surplus sharing problem is a pair $(\nu, R) \in \mathbb{R}^n_{++} \times \mathbb{R}_{++}$, such that $C < R$.

2.2 The compensation problem

We state the two key axioms that express the ethical goal of fair income redistribution, namely neutralizing income inequalities due to y while preserving income inequalities due to z.

The first axiom states that when two individuals only differ in skill levels, both should receive the same income after redistribution. An allocation mechanism S satisfies '**Equal Income for Equal Effort**' ($EIEE$, Fleurbaey (1994)) if:

$\forall e \in \mathcal{E}, x_N \in S(e), \forall i, j \in N,$

$$z_i = z_j \Rightarrow x_i + v(y_i, z_i) = x_j + v(y_j, z_j).$$

Denote $\mathcal{S}_{EIEE}$ the set of all allocation mechanisms that satisfy $EIEE$.

The second axiom states that when two individuals only differ in exerted effort levels, they should be equally affected by the performed redistribution. An allocation mechanism S satisfies '**Equal Transfer for Equal Skill**' ($ETES$, Fleurbaey (1994)) if:

$\forall e \in \mathcal{E}, x_N \in S(e), \forall i, j \in N,$

$$y_i = y_j \Rightarrow x_i = x_j.$$

Denote $\mathcal{S}_{ETES}$ the set of all allocation mechanisms that satisfy $ETES$.

The compensation problem states that there does not exist an allocation mechanism that satisfies $EIEE$ and $ETES$, i.e. $\mathcal{S}_{EIEE} \cap \mathcal{S}_{ETES} = \emptyset$. We refer to Fleurbaey and Maniquet (2004) for a proof.

2.3 Allocation mechanisms

Weakening $ETES$ or $EIEE$ leads to the proposal of a number of interesting allocation mechanisms. Two mechanisms that belong to $\mathcal{S}_{EIEE}$ (and thus weaken $ETES$) play an important role in this paper. We define them here. It concerns a) the family of Egalitarian Equivalent mechanisms due to Pazner and Schmeidler (1978) and Fleurbaey (1995) and b) the family of Proportionally Adjusted

Equivalent mechanisms due to Iturbe (1997). Denote $(x_i)_S$ the income transfer for an individual i from an allocation $(x_N)_S \in S(e)$ of an allocation mechanism S.

a) An $\tilde{y}$-Egalitarian Equivalent mechanism ($S_{\tilde{y}EE}$) allocates resources as follows:

$\forall e \in \mathcal{E}, \forall i \in N,$

$$(x_i)_{S_{\tilde{y}EE}} = -v(y_i, z_i) + v(\tilde{y}, z_i) + \tfrac{1}{n}(R - C).$$

b) An $\tilde{y}$-Proportionally Adjusted Equivalent mechanism ($S_{\tilde{y}PAE}$) allocates resources as follows:

$\forall e \in \mathcal{E}, \forall i \in N,$

$$(x_i)_{S_{\tilde{y}PAE}} = -v(y_i, z_i) + \tfrac{R}{C} v(\tilde{y}, z_i).$$

2.4 The characterizations of $S_{\tilde{y}EE}$ and $S_{\tilde{y}PAE}$

2.4.1 ETRS

The axiom of $ETES$ is weakened to only apply to economies where all skills are equal to the reference skill. In these economies no redistribution is performed. An allocation mechanism S satisfies '**Equal Transfer for Reference Skill**' ($ETRS$, Fleurbaey (1995)) if:

$\forall e \in \mathcal{E}, x_N \in S(e),$
$[y_i = \tilde{y} \ \ \forall i \in N] \Rightarrow [x_i = 0 \ \ \forall i \in N].$

Denote $\mathcal{S}_{ETRS}$ the set of all allocation mechanisms that satisfy $ETRS$. It is easy to check that $S_{\tilde{y}EE}$ and $S_{\tilde{y}PAE}$ belong to $\mathcal{S}_{ETRS}$.

2.4.2 Solidarity axioms

Solidarity axioms consider the effect of a change in one individual's skill on the allocation. Consider two skill profiles $y_N = (y_1, \dots, y_k, \dots, y_n)$ and $y'_N = (y'_1, \dots, y'_k, \dots, y'_n)$, where, for all j in $N \backslash \{k\}$, y_j equals y'_j. Let $e' = (y'_N, z_N)$ and $R' = \sum_{i \in N} v(y'_i, z_i)$. Denote the change in total pre-tax income $\Delta_R = R' - R$. Without loss of generality, we assume throughout the paper that e' yields more pre-tax income than e and hence $\Delta_R \in \mathbb{R}_{++}$. The solidarity problem is how Δ_R should be divided over the population. Note that, as $\tilde{y}$ is constant, a change in the skill profile does not alter individuals' claims, i.e. ν equals ν'.

The axiom of $EIEE$ is strengthened to an axiom that says that a change in the skill profile should affect all agents' final incomes in the same direction.[1] An allocation mechanism S satisfies '**Solidarity**' (Fleurbaey and Maniquet (2004)) if:

[1] We refer to Fleurbaey and Maniquet (2004) for a proof.

$\forall e, e' \in \mathcal{E}, x_N \in S(e), x'_N \in S(e'),$

$$[x'_i + v(y'_i, z_i) \geq x_i + v(y_i, z_i) \ \ \forall i \in N].$$

The axiom of Solidarity is easily defensible. As differences in the skill profile elicit compensation, it is clear that changes in the skill profile should not make some individuals gain income while others lose income.

2.4.3 $S_{\tilde{y}EE}$

Solidarity can be strengthened by stating that all incomes should change equally due to a change in the skill profile. An allocation mechanism S satisfies '**Additive Solidarity**' (AS, Bossert (1995)) if:

$\forall e, e' \in \mathcal{E}, x_N \in S(e), x'_N \in S(e'),$

$$\left[x'_i + v(y'_i, z_i) - (x_i + v(y_i, z_i)) = x'_j + v(y'_j, z_j) - (x_j + v(y_j, z_j)) \ \ \forall i, j \in N\right].$$

An allocation mechanism S satisfies $ETRS$ and AS if and only if it is an $S_{\tilde{y}EE}$. We refer to Bossert and Fleurbaey (1996) for a proof.

When an $S_{\tilde{y}EE}$ is implemented and the skill profile changes, Δ_R is divided equally over all individuals. However, effort also determines Δ_R. Therefore, $S_{\tilde{y}EE}$ can be criticized for not taking differences in exerted effort into account when dividing Δ_R.

2.4.4 $S_{\tilde{y}PAE}$

Alternatively, Solidarity can be strengthened by requiring that all individuals' outcomes change proportionally. An allocation mechanism S satisfies '**Multiplicative Solidarity**' (MS, Iturbe (1997)) if:

$\forall e, e' \in \mathcal{E}, x_N \in S(e), x'_N \in S(e'),$

$$\left[(x'_i + v(y'_i, z_i))(x_j + v(y_j, z_j)) = (x_i + v(y_i, z_i))(x'_j + v(y'_j, z_j)) \ \ \forall i, j \in N\right].$$

An allocation mechanism S satisfies $ETRS$ and MS if and only if it is an $S_{\tilde{y}PAE}$. We refer to Iturbe (1997) for a proof.

When an $S_{\tilde{y}PAE}$ is implemented and the skill profile changes, differences in effort are taken into account by dividing Δ_R proportionally to individuals' claims. Indeed, the ratio of changes in individuals' incomes equals the ratio of their respective claims, i.e. $\frac{x'_i + v(y'_i, z_i) - (x_i + v(y_i, z_i))}{x'_j + v(y'_j, z_j) - (x_j + v(y_j, z_j))} = \frac{v(\tilde{y}, z_i)}{v(\tilde{y}, z_j)}$ for all i, j in N.[2]

In this paper we propose to reward effort in a different way by giving, in the division of Δ_R, priority to the highest claims once the total sum of pre-tax income exceeds a particular threshold level to be explained in section 3.

[2] We suppose that the denominator at the left hand side is different from zero. As such MS is a strengthening of a strict version of Solidarity with a strict inequality sign in the definition.

2.5 Income distributions under $S_{\tilde{y}EE}$ and $S_{\tilde{y}PAE}$

Figure 1 illustrates for every value of R the income distributions under an $S_{\tilde{y}EE}$ and an $S_{\tilde{y}PAE}$ for an economy with four individuals whose claims are in a ratio of 6:4:2:1. As both redistribution mechanisms satisfy the axiom of $ETRS$, income is redistributed such that every individual receives her claim when R equals C. As a consequence of their respective solidarity axioms, when R changes, the absolute income inequality remains constant under $S_{\tilde{y}EE}$ (full line), while the relative income inequality remains constant under $S_{\tilde{y}PAE}$ (dotted line).

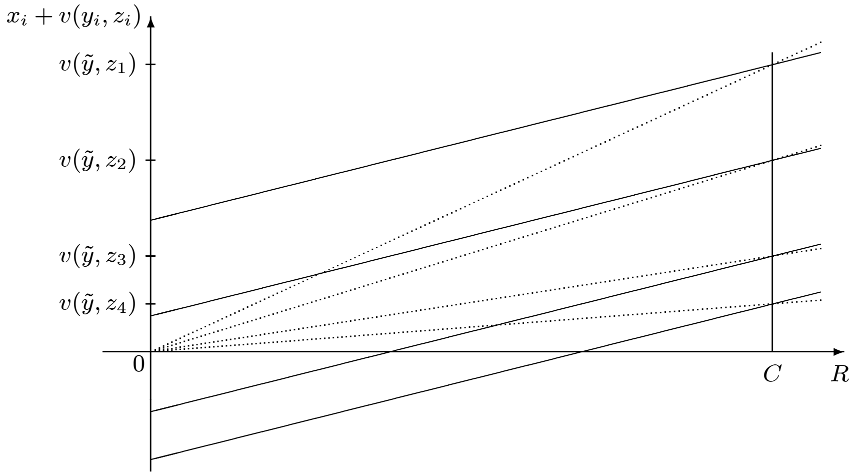

Figure 1: Income distributions under $S_{\tilde{y}EE}$ and $S_{\tilde{y}PAE}$

3 Minimal rights based solidarity

3.1 Minimal rights

Rather than to base the division of Δ_R on individuals' claims, we propose to use individuals' *minimal rights*, a concept often used in the competing claims literature originating from seminal contributions of O'Neill (1982) and Aumann and Maschler (1985).[3]

The minimal right of an individual equals the amount that remains from the total sum of pre-tax income when all other claimants have received their claim. However, the minimal right is not allowed to be negative or to exceed the individual's own claim. Formally, we define the minimal right of an individual i as follows:

[3]Minimal rights should not be confused with the concept of *equal rights* introduced in Maniquet (1998). In a model with production, an allocation mechanism guarantees an equal right when every individual weakly prefers her bundle over her best choice from a common opportunity set.

$$m_i(\nu, R) = \min(v(\tilde{y}, z_i), \max(R - C_{-i}, 0)),$$

where

$$C_{-i} = \sum_{j \in N \setminus \{i\}} v(\tilde{y}, z_j) \text{ for all } i \text{ in } N.$$

Figure 2 shows that as long as R is smaller than C_{-1}, all minimal rights are zero. As soon as R exceeds C_{-1}, the minimal right of the individual with effort level z_1 becomes positive. As soon as R exceeds C_{-2}, the minimal right of the individual with effort level z_1 exceeds $C_{-2} - C_{-1}$ and the minimal right of the individual with effort level z_2 becomes positive. As R increases, more and more individuals start to get a positive minimal right and as soon as R exceeds C_{-n} all minimal rights are positive. When R equals C (and thus claims are feasible) every individual has a minimal right equal to her claim. When R exceeds C minimal rights do not change anymore, so in all surplus sharing problems $m_i(\nu, R)$ equals $v(\tilde{y}, z_i)$ for all i in N.

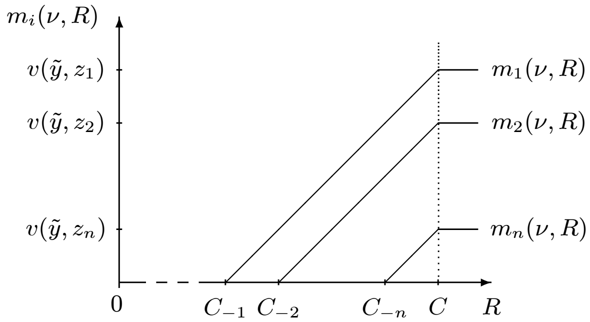

Figure 2: Aggregate resources, minimal rights and claims

Minimal rights could be given a 'democratic' interpretation. For a given $\tilde{y}$, suppose that R equals C_{-2}. Then all individuals in society agree that the individual with effort level z_1 deserves at least her minimal right, i.e. $C_{-2} - C_{-1}$, of R. Suppose that R equals C_{-3}. Then there is agreement that the amount $C_{-2} - C_{-1}$ should only go to the individual with effort level z_1, while the amount $C_{-3} - C_{-2}$ should be divided only over the individual with effort level z_1 and the individual with effort level z_2. Since both individuals deserve the amount $C_{-3} - C_{-2}$, it seems natural to divide the amount $C_{-3} - C_{-2}$ equally between both individuals.

3.2 Solidarity axioms

We exploit these ideas in the statement of our solidarity axioms. When the skill profile changes, it is clear that none, some or all minimal rights may change.

Denote $\Delta m_i(\nu, R, \Delta_R) = m_i(\nu, R+\Delta_R) - m_i(\nu, R)$ the change in the minimal right of individual i due to a change of total pre-tax income equal to Δ_R. These changes in minimal rights could be informative for the division of Δ_R. Suppose that R and R' are smaller than C_{-1}. Since all minimal rights are zero before and after the change in the skill profile, it seems reasonable to divide Δ_R equally over all individuals. Suppose now that R' further increases to C_{-2}. Then priority could be given to the individual with effort level z_1 who receives $C_{-2} - C_{-1}$ —an amount equal to the change in her minimal right— while the rest, $C_{-1} - R$, is divided equally over all individuals. Suppose now that R' further increases to C_{-3}. Then the amount $C_{-1} - R$ could be divided equally over all individuals, because nobody has a positive minimal right up to income level C_{-1}. Then priority could be given to the individual with effort level z_1 who additionally receives $C_{-2} - C_{-1}$ because there is no other individual with a positive minimal right from income level C_{-1} up to income level C_{-2}. Finally, the amount $C_{-3} - C_{-2}$ could be divided equally between the individual with effort level z_1 and the individual with effort level z_2 because they are the only individuals with a positive minimal right from income level C_{-2} up to income level C_{-3}. This iterative process can be continued as R' increases until R' exceeds C_{-n}. As all individuals have a positive minimal right after the income level C_{-n} and their minimal rights change equally between C_{-n} and R', it seems reasonable to divide $R' - C_{-n}$ again equally over all individuals. The following axioms express these ideas.

The first axiom weakens Additive Solidarity by requiring an equal treatment in the allocation of Δ_R only when minimal rights change equally. A redistribution mechanism S satisfies '**Symmetry**' if:[4]

$$\forall e, e' \in \mathcal{E}, x_N \in S(e), x'_N \in S(e'), \forall i, j \in N,$$
$$\Delta m_i(\nu, R, \Delta_R) = \Delta m_j(\nu, R, \Delta_R)$$
$$\Rightarrow x'_i + v(y'_i, z_i) - (x_i + v(y_i, z_i)) = x'_j + v(y'_j, z_j) - (x_j + v(y_j, z_j)).$$

The second axiom is inspired by the idea that individuals whose minimal rights change should be given priority over individuals whose minimal rights do not change in the allocation of Δ_R *once* not all minimal rights are equal to zero. Denote $N_1 = \{i \in N | \Delta m_i(\nu, R, \Delta_R) > 0\}$ the set of individuals whose minimal rights change due to a change of R. A redistribution mechanism S satisfies '**Priority**' if:

$$\forall e, e' \in \mathcal{E}, x_N \in S(e), x'_N \in S(e'),$$
if there exists $i \in N_1$, $\Delta m_i(\nu, R, \Delta_R) = \Delta_R$:
$$\textstyle\sum_{i\in N_1} (x'_i + v(y'_i, z_i) - x_i - v(y_i, z_i)) = \Delta_R.$$

[4]Note that, as minimal rights do not change anymore in surplus sharing problems, $ETRS$ and Symmetry characterize the $\tilde{y}$-Egalitarian Equivalent mechanism for $\tilde{y} \in Y_{ss}$ (see 2.4.3). In the surplus sharing literature, this mechanism is better known as the Equal Surplus Sharing mechanism: every claimant receives her claim plus an equal share of the surplus.

Symmetry and Priority together express the paper's central solidarity idea of *Minimal Rights based Solidarity*, i.e. it explains how Δ_R should be divided over the population. When all minimal rights change equally, Symmetry implies that Δ_R is divided equally. As the pre-tax income function v is continuous and strictly increasing in y, there exist unique intermediary skill levels such that, when changes in minimal rights differ, Δ_R can be divided in specific subchanges. For each of these subchanges of Δ_R, the population is partitioned in two groups: (i) a group whose change in minimal rights equals the subchange of Δ_R and (ii) a group whose minimal rights do not change. Symmetry implies that within each group all individuals are treated in the same way, whereas Priority requires that the first group receives the subchange of Δ_R. Hence, the subchange of Δ_R is equally divided among the individuals of the first group. We state Minimal Rights based Solidarity formally in appendix.

3.3 Minimal Rights based Egalitarian mechanisms

In the previous subsection Minimal Rights based Solidarity described how to divide Δ_R. We call fair income redistribution mechanisms that satisfy Symmetry and Priority *Minimal Rights based Egalitarian mechanisms.* In order to characterize one particular family of Minimal Rights based Egalitarian mechanisms, it suffices to combine the solidarity axioms of Symmetry and Priority with an axiom that for one specific R implies one specific income distribution.

3.3.1 $S_{\tilde{y}MRE/E}$

The axiom of $ETRS$ states that when R equals C every individual should receive her claim. Combining Symmetry, Priority and $ETRS$ characterizes the following mechanism.

An $\tilde{y}$-Minimal Rights based Egalitarian mechanism $S_{\tilde{y}MRE/E}$ allocates resources as follows:

$\forall e \in \mathcal{E}$,

(1) when $C_{-n} \leq R$:

$$(x_i)_{S_{\tilde{y}MRE/E}} = -v(y_i, z_i) + v(\tilde{y}, z_i) + \tfrac{R-C}{n} \text{ for all } i \text{ in } N,$$

(2) when, for $k \leq n-1$, $C_{-k} \leq R < C_{-(k+1)}$:

$$(x_i)_{S_{\tilde{y}MRE/E}} = -v(y_i, z_i) + v(\tilde{y}, z_i) + \tfrac{C_{-n}-C}{n} - \sum_{h=k+1}^{n-1} \left(\tfrac{C_{-(h+1)}-C_{-h}}{h} \right) + \tfrac{R-C_{-(k+1)}}{k}$$

for all $i \in \{1, \dots, k\}$ and

$$(x_j)_{S_{\tilde{y}MRE/E}} = -v(y_j, z_j) + v(\tilde{y}, z_j) + \tfrac{C_{-n}-C}{n} - \sum_{h=j}^{n-1} \left(\tfrac{C_{-(h+1)}-C_{-h}}{h} \right)$$

for all $j \in \{k+1, \dots, n-1\}$ and

$$(x_n)_{S_{\tilde{y}MRE/E}} = -v(y_n, z_n) + v(\tilde{y}, z_n) + \tfrac{C_{-n} - C}{n},$$

(**3**) when $R < C_{-1}$:

$$(x_i)_{S_{\tilde{y}MRE/E}} = -v(y_i, z_i) + v(\tilde{y}, z_i) + \tfrac{C_{-n} - C}{n} - \sum_{h=i}^{n-1} \left(\tfrac{C_{-(h+1)} - C_{-h}}{h} \right) + \tfrac{R - C_{-1}}{n}$$

for all $i \in \{1, \dots, n-1\}$ and

$$(x_n)_{S_{\tilde{y}MRE/E}} = -v(y_n, z_n) + v(\tilde{y}, z_n) + \tfrac{C_{-n} - C}{n} + \tfrac{R - C_{-1}}{n}.$$

Proposition 1 *: $\forall e \in \mathcal{E} : S = S_{\tilde{y}MRE/E} \Leftrightarrow S$ satisfies Symmetry, Priority and ETRS.*

The proof of proposition 1 can be found in appendix.

Figure 3 depicts for every value of R the income distributions under an $S_{\tilde{y}MRE/E}$ for the same economy as in figure 1.

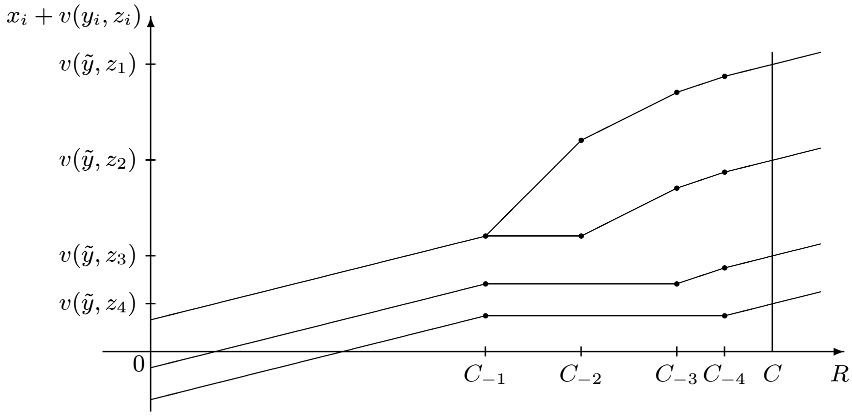

Figure 3: Income distributions under $S_{\tilde{y}MRE/E}$

As $S_{\tilde{y}MRE/E}$ also satisfies $ETRS$, income is redistributed such that every individual receives her claim when R equals C. When, due to a change in the skill profile, R becomes higher than C, every individual receives her claim plus an equal part of $R - C$. When R becomes lower than C, every individual receives an income that is lower than her claim, but the shortfall from the claim is never lower for individuals with a higher claim. As long as R is higher than C_{-4}, the loss of total pre-tax income is equally borne by all individuals. But, when R falls below C_{-4}, the income of the poorest individual is left constant, which brings about an extra loss of income for all other individuals. When R becomes smaller than C_{-3}, the incomes of the poorest and second poorest individuals remain constant and, when R falls below C_{-2}, the richest individual alone is saddled with the entire cost of keeping the incomes of all other individuals constant. When R becomes smaller than C_{-1}, the loss of total pre-tax income is again borne equally by all individuals.

Some general conclusions can be drawn from comparing figures 1 and 3. We say that, in the comparison of two income distributions A and B with the same mean, A is more equal than B when A is obtained from B by performing a series of (Pigou-Dalton) rich-to-poor transfers that do not entail rank reversals.

- In economies where every individual has a strictly positive minimal right ($C_{-n} < R$), an $S_{\tilde{y}MRE/E}$ redistributes income just like an $S_{\tilde{y}EE}$. Both mechanisms redistribute income more equally than an $S_{\tilde{y}PAE}$ as soon as R is larger than C.

- In economies where some but not all minimal rights are strictly positive ($C_{-1} < R \leq C_{-n}$), the income distribution under an $S_{\tilde{y}MRE/E}$ is more equal than the income distribution under an $S_{\tilde{y}EE}$.

- In economies where all minimal rights are zero ($R \leq C_{-1}$), absolute income inequalities remain constant under an $S_{\tilde{y}MRE/E}$ and under an $S_{\tilde{y}EE}$ when R changes, but incomes are more equally distributed under the former mechanism. Note that the incomes of the individuals with the highest and second highest effort level coincide under an $S_{\tilde{y}MRE/E}$ in these economies.

3.3.2 $S_{\tilde{y}MRE/P}$

Figures 1 and 3 show a debatable property of $S_{\tilde{y}EE}$ and $S_{\tilde{y}MRE/E}$: the poorest individuals might end up with a negative income after redistribution in poor societies (i.e. when R is sufficiently low). Our ethical intuition may lead us to consider a minimal amount of redistribution that we at least want to perform. Suppose that the poorest in society could not satisfy their basic needs when they receive a negative income after redistribution. Society wants to exclude this possibility in every situation by incorporating the requirement of a non-negative income after redistribution for all individuals in the construction of the redistribution mechanism. A redistribution mechanism S satisfies '**Participation**' (Maniquet (1998)) if:

$\forall e \in \mathcal{E}, x_N \in S(e)$,

$$x_i + v(y_i, z_i) \geq 0 \quad \forall i \in N.$$

An implication of Participation is that, when R converges to zero, all incomes should also converge to zero. Combining Participation with the solidarity axioms of Symmetry and Priority characterizes the following mechanism.

An $\tilde{y}$-Minimal Rights based Egalitarian mechanism $S_{\tilde{y}MRE/P}$ allocates resources as follows:

$\forall e \in \mathcal{E}$,

(**1**) when $R < C_{-1}$:

$(x_i)_{S_{\tilde{y}MRE/P}} = -v(y_i, z_i) + \frac{R}{n}$ for all i in N,

(**2**) when, for $k \leq n-1$, $C_{-k} \leq R < C_{-(k+1)}$:

$$(x_i)_{S_{\tilde{y}MRE/P}} = -v(y_i, z_i) + \frac{C_{-1}}{n} + \sum_{h=i}^{k} \left(\frac{C_{-(h+1)} - C_{-h}}{h} \right) + \frac{R - C_{-(k+1)}}{k}$$

for all $i \in \{1, \dots, k\}$ and

$(x_j)_{S_{\tilde{y}MRE/P}} = -v(y_j, z_j) + \frac{C_{-1}}{n}$ for all $j \in \{k+1, \dots, n\}$,

(**3**) when $C_{-n} \leq R$:

$$(x_i)_{S_{\tilde{y}MRE/P}} = -v(y_i, z_i) + \frac{C_{-1}}{n} + \sum_{h=i}^{n-1} \left(\frac{C_{-(h+1)} - C_{-h}}{h} \right) + \frac{R - C_{-n}}{n}$$

for all $i \in \{1, \dots, n-1\}$ and

$(x_n)_{S_{\tilde{y}MRE}} = -v(y_n, z_n) + \frac{C_{-1}}{n} + \frac{R - C_{-n}}{n}$.

Proposition 2 *: $\forall e \in \mathcal{E} : S = S_{\tilde{y}MRE/P} \Leftrightarrow S$ satisfies Symmetry, Priority and Participation.*

The proof of proposition 2 can be found in appendix.

Figure 4 illustrates, for the same economy as in figures 1 and 3, the income distributions under $S_{\tilde{y}MRE/P}$.

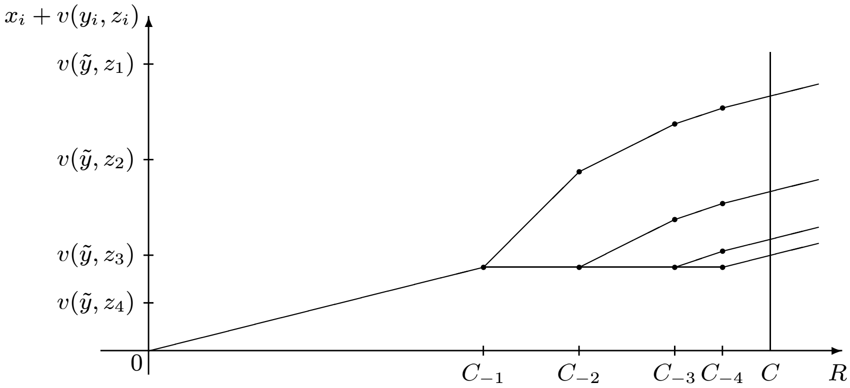

Figure 4: Income distributions under $S_{\tilde{y}MRE/P}$

An $S_{\tilde{y}MRE/P}$ redistributes incomes very equally. It is easy to check that, for every value of R, the income distribution under $S_{\tilde{y}MRE/P}$ is more equal than the income distribution under $S_{\tilde{y}MRE/E}$. An equal distribution of income prevails as long as all minimal rights are zero. More generally, when R is lower than or equal to C_{-i} for some i in N, all individuals with z lower than or equal to z_i receive the same income. However, one could argue that too much redistribution is performed. Figure 4 shows that when all individuals have y equal to $\tilde{y}$ and hence only differ with respect to their responsibility characteristic z (such that no redistribution is needed), Pigou-Dalton transfers are still performed. As $S_{\tilde{y}MRE/E}$ and $S_{\tilde{y}MRE/P}$ are different mechanisms, Participation is incompatible with $ETRS$ when Symmetry and Priority are imposed.

3.3.3 Discussion

We end this section by discussing that the incompatibility between Participation and $ETRS$ when Symmetry and Priority are imposed, is due to Symmetry rather than Priority. We also show that the axioms used in propositions 1 and 2 are independent.

When Priority is dropped, imposing Participation, Symmetry and $ETRS$ still leads to an incompatibility, except when the lowest $n-1$ responsibility characteristics are equal. This is most easily explained as follows. Start from an income distribution $(0, 0, \ldots, 0)$ when R converges to zero (Participation). Now suppose R' equals C. Then $ETRS$ requires that every individual receives her claim. Hence, the individual with effort level z_n should in that case receive $v(\tilde{y}, z_n)$. Symmetry requires that the subchanges of Δ_R for which all minimal rights change equally, i.e. C_{-1} and $C - C_{-n}$ $(= v(\tilde{y}, z_n))$, are divided equally over the entire population. Given our assumptions about the pre-tax income function v, the condition $\frac{1}{n}C_{-1} + \frac{1}{n}v(\tilde{y}, z_n) = v(\tilde{y}, z_n)$ can only hold when $z_2 = z_3 = \ldots = z_n$. Note the ineffectiveness of restricting the range of choices of $\tilde{y}$ the social planner can make, as the incompatibility between Symmetry, Participation and $ETRS$ holds for every value of $\tilde{y}$.

When Symmetry is dropped, Participation, Priority and $ETRS$ are compatible but do not characterize a unique redistribution mechanism. Figure 5 illustrates two different mechanisms that satisfy Participation, Priority and $ETRS$. When R falls below C both mechanisms first reduce the income of the poorest individual to zero. But, as soon as R falls below C_{-1}, the first mechanism (full line) in turn reduces the income of the second poorest, second richest and richest individual respectively to zero, whereas the second mechanism (dotted line) first equalizes the incomes of the three individuals and afterwards reduces their incomes by equal amounts.

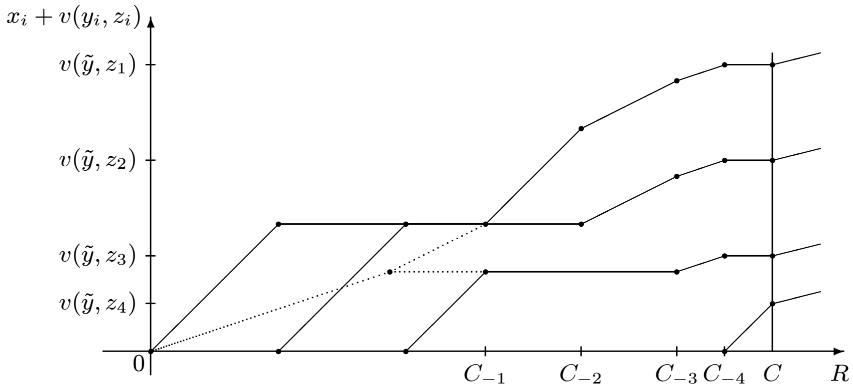

Figure 5: Two different mechanisms satisfying Participation, Priority and ETRS

The axioms used in the characterization of $S_{\tilde{y}MRE/E}$ are independent. For example, $S_{\tilde{y}EE}$ satisfies $ETRS$ and Symmetry but violates Priority. The two mechanisms of figure 5 satisfy $ETRS$ and Priority but violate Symmetry. $S_{\tilde{y}MRE/P}$ satisfies Symmetry and Priority but violates $ETRS$. The axioms used in the characterization of $S_{\tilde{y}MRE/P}$ are also independent. A straightforward example of a mechanism that satisfies Participation and Symmetry but violates Priority is the equal division of income. The two mechanisms of figure 5 satisfy Participation and Priority but violate Symmetry. Finally, $S_{\tilde{y}MRE/E}$ satisfies Symmetry and Priority but violates Participation.

4 Conclusion

The choice of a particular mechanism to divide the sum of total pre-tax income over a population of claimants inextricably brings about a particular way to solve the solidarity problem. We propose to use the information of individuals' minimal rights to divide an extra amount of income generated by a change in the skill profile. The idea is to give priority to individuals with a positive minimal right. We present Minimal Rights based Egalitarian mechanisms that implement this solidarity idea. We contrast Egalitarian Equivalent mechanisms ($S_{\tilde{y}EE}$) and Proportionally Adjusted Equivalent mechanisms ($S_{\tilde{y}PAE}$), two well known families of fair redistribution mechanisms presented in the literature, with two families of Minimal Rights based Egalitarian mechanisms ($S_{\tilde{y}MRE/E}$ and $S_{\tilde{y}MRE/P}$). The following table summarizes our axiomatic analysis into the properties of these mechanisms.

	$S_{\tilde{y}EE}$	$S_{\tilde{y}PAE}$	$S_{\tilde{y}MRE/E}$	$S_{\tilde{y}MRE/P}$
EIEE	+	+	+	+
ETES	−	−	−	−
ETRS	★	★	★	−
Solidarity	+	+	+	+
AS	★	−	−	−
MS	−	★	−	−
Symmetry	+	−	★	★
Priority	−	−	★	★
Participation	−	+	−	★

★: used in characterization of S
+: S satisfies the axiom (but not used in characterization)
−: S violates the axiom

The study into the income inequalities resulting from these mechanisms learns that, given $\tilde{y}$, an $S_{\tilde{y}MRE/P}$ always redistributes income more equally than an $S_{\tilde{y}MRE/E}$. The latter mechanism redistributes income more equally than an $S_{\tilde{y}EE}$ when none or some (but not all) minimal rights are positive. When all minimal rights are positive $S_{\tilde{y}EE}$ and $S_{\tilde{y}MRE/E}$ are equivalent. All three mechanisms redistribute income more equally than an $S_{\tilde{y}PAE}$ in surplus sharing problems.

Appendix: Proofs

Lemma 1 states formally the solidarity idea of Minimal Rights based Solidarity, i.e. how a change in the total sum of pre-tax income is divided over the population such that the axioms of Symmetry and Priority are satisfied.

Lemma 1 (Minimal Rights based Solidarity): Consider $\Delta_R > 0$ due to a skill change from y_i to y_i' of an individual i in N. Denote $d_i = x_i' + v(y_i', z_i) - (x_i + v(y_i, z_i))$; $\sum_{i \in N} d_i = \Delta_R$. One of five possible situations occurs:

(**1**) when $R' \leq C_{-1}$ or $C_{-n} \leq R$:

$d_i = \frac{\Delta_R}{n}$ for all i in N,

(**2**) when, for $k \leq n-1$, $C_{-k} \leq R < R' < C_{-(k+1)}$:

$d_i = \frac{\Delta_R}{k}$ for all $i \in \{1, \dots, k\}$ and

$d_j = 0$ for all $j \in \{k+1, \dots, n\}$,

(**3**) when, for $k \leq n-1$, $C_{-k} \leq R < C_{-(k+1)}$ and, for $2 \leq l \leq n$, $C_{-l} \leq R' < C_{-(l+1)}$[5] and $k < l$:

$d_i = \frac{C_{-(k+1)} - R}{k} + \sum_{h=k+1}^{l-1} \left(\frac{C_{-(h+1)} - C_{-h}}{h} \right) + \frac{R' - C_{-l}}{l}$ for all $i \in \{1, \dots, k\}$ and

[5] When $l = n$, define $C_{-(l+1)} = +\infty$.

$d_j = \sum_{h=j}^{l-1} \left(\frac{C_{-(h+1)} - C_{-h}}{h} \right) + \frac{R' - C_{-l}}{l}$ for all $j \in \{k+1, \ldots, l-1\}$ and

$d_l = \frac{R' - C_{-l}}{l}$ and

$d_q = 0$ for all $q \in \{l+1, \ldots, n\}$,

(**4**) when $R \leq C_{-1}$ and, for $l \leq n-1$, $C_{-l} \leq R' < C_{-(l+1)}$:

$d_i = \frac{C_{-1} - R}{n} + \sum_{h=i}^{l} \left(\frac{C_{-(h+1)} - C_{-h}}{h} \right) + \frac{R' - C_{-(l+1)}}{l}$ for all $i \in \{1, \ldots, l\}$ and

$d_j = \frac{C_{-1} - R}{n}$ for all $j \in \{l+1, \ldots, n\}$,

(**5**) when $R \leq C_{-1}$ and $C_{-n} \leq R'$:

$d_i = \frac{C_{-1} - R}{n} + \sum_{h=i}^{n-1} \left(\frac{C_{-(h+1)} - C_{-h}}{h} \right) + \frac{R' - C_{-n}}{n}$ for all $i \in \{1, \ldots, n-1\}$ and

$d_n = \frac{C_{-1} - R}{n} + \frac{R' - C_{-n}}{n}$.

Proof. Suppose the antecedent of (1) is true. None of the minimal rights change and the division of Δ_R is obtained by Symmetry. Suppose the antecedent of (2) is true. There are two groups of individuals. For individuals 1 to k minimal rights change equally. For individuals $k+1$ to n minimal rights do not change. Symmetry implies that, within each group, all individuals are treated in the same way. Priority requires that the first group receives Δ_R. The division of Δ_R then follows straightforwardly. Suppose the antecedent of (3) is true. As v is continuous and strictly increasing in y, there exist for individual i unique intermediary skill levels $\hat{y}_i^k, \hat{y}_i^{k+1}, \ldots, \hat{y}_i^l$ such that, ceteris paribus, the total sum of pre-tax income equals $C_{-k}, C_{-(k+1)}, \ldots, C_{-l}$ respectively. Now consider skill changes from y_i to $\hat{y}_i^k$, $\hat{y}_i^k$ to $\hat{y}_i^{k+1}$,..., $\hat{y}_i^l$ to y_i' such that the total change of pre-tax income equals $C_{-k} - R, C_{-(k+1)} - C_{-k}, \ldots, R' - C_{-l}$ respectively. For each of these subchanges there are two groups of individuals: (i) a group whose change in minimal rights is equal to the subchange and (ii) a group whose minimal rights do not change. Symmetry implies that within each group all individuals are treated in the same way, whereas Priority requires that the first group receives the subchange. Hence, the subchange is equally divided among the individuals of the first group. The division of Δ_R is obtained from applying Symmetry and Priority to the division of these subchanges. The division of Δ_R under (4) and (5) is obtained by similar reasoning as in (3). ■

Proof of proposition 1

$\forall e \in \mathcal{E} : S = S_{\tilde{y}MRE/E} \Leftrightarrow S$ satisfies Symmetry, Priority and $ETRS$.

Proof. We only proof ($\Leftarrow$). Define an economy $\tilde{e} = ((\tilde{y}, \ldots, \tilde{y}), (z_1, \ldots, z_n))$. By $ETRS$, $\tilde{x}_i = 0$ for all i in N and individuals' final incomes are equal to their claims. Call this the 'initial income distribution'. Rather than successively considering n changes from $\tilde{y}$ to y_i for every i in N and using Lemma 1 successively to divide the intermediate subchanges in total pre-tax income (a process where in many cases previous subchanges in total pre-tax income would cancel out), we immediately use Lemma 1 to divide $\Delta_R = C - R$.[6] The transfers of (1) in the definition of $S_{\tilde{y}MRE/E}$ then follow from adding to the initial income distribution the transfers described in case (1) in lemma 1. The transfers of (2) then follow from subtracting of the initial income distribution the transfers described in case (3) with $l = n$ in lemma 1. The transfers of (3) then follow from subtracting of the initial income distribution the transfers described in case (5) in lemma 1. ■

Proof of proposition 2

$\forall e \in \mathcal{E} : S = S_{\tilde{y}MRE/P} \Leftrightarrow S$ satisfies Symmetry, Priority and Participation.

Proof. We only proof ($\Leftarrow$). Participation requires that, when R converges to zero, all incomes also converge to zero. Let $(0, 0, \dots, 0)$ be the initial income distribution. Now, use Lemma 1 to divide $\Delta_R = R - 0$. The transfers of (1) in the definition of $S_{\tilde{y}MRE/P}$ are then described in case (1) in lemma 1. The transfers of (2) are then described in case (4) in lemma 1. The transfers of (3) are then described in case (5) in lemma 1. ■

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