



**FACULTEIT ECONOMIE
EN BEDRIJFSKUNDE**

**HOVENIERSBERG 24
B-9000 GENT**

**Tel. : 32 - (0)9 - 264.34.61
Fax. : 32 - (0)9 - 264.35.92**

WORKING PAPER

Nash rationalizability of collective choice over lotteries

Thomas Demuynck ¹

Luc Lauwers ²

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¹ Sherppa, Ghent University, Hoveniersberg 24, 9000 Gent, Belgium; Thomas.Demuynck@ugent.be ; Fonds voor Wetenschappelijk onderzoek Vlaanderen (FWO), acknowledges financial support from the Interuniversity Attraction Poles Programme-Belgian Science Policy (contract P5/21).

² Center for Economic Studies, K.U. Leuven; Naamsestraat 69, 3000 Leuven, Belgium; Luc.Lauwers@econ.kuleuven.be

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Abstract. To test the joint hypothesis that players in a noncooperative game (allowing mixed strategies) maximize expected utilities and select a Nash equilibrium, it suffices to study the reaction of the revealed collective choice upon changes in the space of strategies available to the players. The joint hypothesis is supported if the revealed choices satisfy an extended version of Richter's congruence axiom together with a contraction-expansion axiom that models the noncooperative behavior. In addition, we provide sufficient and necessary conditions for a binary relation to have an independent ordering extension, and for individual choices over lotteries to be rationalizable.

Keywords and Phrases: independence condition, binary extensions, rationalizability, Nash equilibrium in mixed strategies

JEL Classification Numbers: C72, C92

1 Introduction

A recent track of research seeks to identify the testable restrictions of various theories of multi-agent decision making. Along these lines we set up a test to verify whether players are expected utility maximizers and select a Nash equilibrium. Let us start the exposition with an example.

Consider a noncooperative game between two players. Each player has two pure strategies: U (p) and D (own) for player 1, L (eft) and R (ight) for player 2. We do allow for mixed strategies. The strategy space for each player is the set

$$\Delta = \{(x_1, x_2) \mid 0 \leq x_1, x_2 \leq 1 \text{ and } x_1 + x_2 = 1\} \subset \mathbb{R}^2,$$

with x_1 the probability the player attaches to his first mentioned pure strategy (U for player 1, and L for player 2). Assume that player 2 is constrained to select strategy L with a probability between 0 and 0.35, and that player 1 can freely choose in the set Δ . Given this choice set, we observe that player 1 selects (0.4, 0.6) while player 2 selects (0.3, 0.7). Similar experiments generate the following data:

available strategies	\mapsto	selected strategies
$S = \Delta \times \{(x_1, 1 - x_1) \mid 0 \leq x_1 \leq 0.35\}$		$(0.4, 0.6) \times (0.3, 0.7),$
$S' = \Delta \times \{(x_1, 1 - x_1) \mid 0.4 \leq x_1 \leq 0.45\}$		$(0.4, 0.6) \times (0.42, 0.58),$
$S'' = \Delta \times \{(0.5, 0.5)\}$		$(0.5, 0.5) \times (0.5, 0.5).$

TABLE 1: OBSERVED DATA.

The following question pops up. Given such a data set, is it possible to check whether or not these players behave rational in the sense that they are expected utility maximizers and select a Nash equilibrium? In section 4 we return to this example and we will argue that the above data are not Nash rationalizable. The remaining part of this introduction positions this research in the literature and introduces our main results.

Many theories on behavior start from assumptions on the individual preference relations over the feasible set (e.g. transitivity, completeness). As soon as one accepts that binary relations are not observable while actual choices are observable; it is important to test whether the actual choices support or reject the assumptions. This issue has been discussed by, among others, Arrow (1959), Richter (1966), and Sen (1971).

There are at least two ways to tackle this problem. One approach (Sen, 1971) studies how the selection reacts upon particular changes in the set of feasible alternatives. Obviously, if the individual consults a transitive and complete preference relation, then he should not reconsider his selection when the choice set shrinks while his selected alternative remains feasible. Analogously, when he selects the same alternative from two different choice sets, then he should select again this alternative from the union of these two choice sets. As such, the hypothesis of rational behavior becomes testable. A second approach is offered through the theory of revealed preferences. If an alternative is chosen from a set, then it is top ranked according to the revealed preferences in this choice set. The transitive

closure of this revealed preference relation is called the indirect revealed preference relation. Richter's (1966) congruence axiom provides necessary and sufficient conditions for a choice correspondence to be rationalizable: if an alternative a is indirectly revealed preferred to b , then b should not be strictly revealed preferred to a .

Sprumont (2000) extends the problem of rationalizability to situations involving different and interacting individuals. He defines a joint choice function to be Nash rationalizable if there exists a profile of complete and transitive preference relations over the sets of actions, so that the observed outcomes coincide with the Nash equilibria based upon these preferences. In the spirit of Sen's approach, he characterizes Nash rationalizability through the combination of an expansion and a contraction property. Ray and Zhou (2001) perform a similar study for subgame perfect Nash equilibria.

We extend one of the results of Sprumont (2000) and tackle the Nash rationalizability of collective choice when individuals have mixed strategies at their disposal. We maintain the assumption that mixed strategies are observable. Table 1, for example, might result from an experiment. Following the tradition in game theory, we interpret the rational behavior of a player in terms of expected utility maximization. In particular, besides completeness and transitivity we impose an independency demand upon the preference relations of the (rational) players. This independency condition states that the relationships between two lotteries are not affected when they are mixed in the very same way with a third lottery. Myerson (1997, p11) discusses the strength of the independency axiom in the expected utility maximization theorem. In addition, he indicates some of the difficulties that arise in decision theory when independency is dropped.

Furthermore, in contrast to Sprumont (2000), we follow the track of revealed preferences. Where in the original setup it is sufficient to check the 'transitive' closure of the revealed preference relation, the present setting is more demanding. We modify Richter's axiom and require that the 'transitive and independent' closure of the revealed preference relation does not conflict with the strict revealed preference relation. Besides that, we need an axiom that connects the individual behavior to the collective behavior. We assume that a strategy profile belongs to the collective choice if each player keeps his selected strategy when he is assured that he is the only player allowed to deviate away. We refer to this condition as the collective choice being noncooperative. Later on, we will argue that this condition has some flavor of an expansion-contraction axiom. Our main result reads (see Theorem 3 in Section 4 for the exact formulation):

Theorem. *A collective choice correspondence is Nash rationalizable if and only if it is noncooperative and satisfies the modified version of Richter's congruence axiom.*

Let us highlight two intermediate results towards this theorem. *First*, we need a condition that is strong enough to guarantee that a binary relation extends to a transitive and independent relation. Here, we learn from Suzumura (1976), who showed that consistency is a sufficient and necessary condition for a relation to have an ordering extension. We shift

Suzumura’s result to a setup involving choices over lotteries, and we use the term ‘lottery-consistency’ as a reference. *Second*, we study the behavior of a single individual choosing from a set of lotteries (or mixed strategies). Here, we show that the extended version of Richter’s congruence axiom –restricted to one player– is sufficient and necessary for the individual choice function to be rationalizable.¹ Then, we broaden the setup from one individual to a finite number of interacting players. We apply the axiom of noncooperative behavior and conclude the above theorem.

This theorem can be used in an experimental setting to test whether players are expected utility maximizers and select a Nash equilibrium. In order to focus on the hypothesis that players select a Nash equilibrium, one can proceed in two steps. In a first experiment, the players are screened according to whether they are expected utility maximizers or not. This can be done, for instance, through some Allais-paradox test (e.g. Conlisk, 1989; MacDonald and Wall, 1989; and Oliver, 2003). This step filters out those individuals who violate the expected utility criterion. In the second step, one confronts the remaining players with a noncooperative game. As such, one can judge on the basis of observations whether in mixed strategies the Nash criterion is rejected or supported.

The next section introduces the notation and studies binary relations and their independent and transitive extensions. Section 3 introduces the concept of lottery-consistency as a test for the rational behavior of an individual choosing over lotteries. Section 4 extends the notation to collective choice and proves the main result. We return to the data in Table 1, and we briefly discuss the problem of observing mixed strategies. Section 5 closes the paper. Here, we link our result to the analysis of Sprumont (2000).

2 Independent ordering extensions

This section establishes the notation, introduces the concept of independency of a binary relation, and provides conditions for a relation to have an independent ordering extension.

Let $H \subset \mathbb{R}^n$ be the hyperplane of n -vectors the coordinates of which add up to 1, and let $\Delta = \Delta^n \subset H$ be the $(n - 1)$ -dimensional simplex. An element $x = (x_1, x_2, \dots, x_n)$ in Δ is an n -tuple of nonnegative real numbers adding up to 1, and is called a lottery. The i th coordinate x_i of the lottery x gives the probability that state i occurs.

Throughout, the set D refers to either Δ or H . A binary relation R in the set D is a subset of the cartesian product $D \times D$. The symmetric component $R \cap R^{-1}$ is denoted by I , the asymmetric part $R \setminus I$ by P , and the non-comparable part $D \times D \setminus (R \cup R^{-1})$ by N . For the binary relation R' we denote these induced relations by I' , P' , N' ; for R^* we use I^* , P^* , N^* ; etc. A reflexive and transitive relation is said to be a quasi-ordering. A complete quasi-ordering is said to be an ordering. The binary relation R' in D extends the relation

¹This result is similar to Theorem 3.1 of Kim (1996). See Section 3 for a discussion.

R if $R \subset R'$ and $P \subset P'$.²

Next, we introduce the notion of independency. This condition studies the behavior of a binary relation on compound vectors. For x and y in H and for α a nonnegative real number, the vector $[\alpha, x, y]$ denotes the linear combination $\alpha x + (1 - \alpha)y$. For α in between 0 and 1, the compound vector $[\alpha, x, y]$ is a convex combination of x and y . And, for $\alpha > 1$ the compound vector is a point on the ray starting in y and going through x and does not belong to the closed interval $[x, y]$.

A relation R in D is said to be independent if for each couple in R the composition with a third vector in D preserves the initial relationships. Formally, R is *independent* if for each x, y , and z in D , we have

$$\text{if } (x, y) \in R, \alpha \geq 0, [\alpha, x, z] \in D, \text{ and } [\alpha, y, z] \in D; \text{ then } ([\alpha, x, z], [\alpha, y, z]) \in R. \quad (1)$$

This condition implies the reflexivity of R (put $\alpha = 0$). Observe that α is allowed to take values larger than 1. As a consequence of this, an independent relation satisfies the ‘strict’ version of condition (1):

$$\text{if } (x, y) \in P, \alpha > 0, [\alpha, x, z] \in D, \text{ and } [\alpha, y, z] \in D; \text{ then } ([\alpha, x, z], [\alpha, y, z]) \in P.$$

Indeed, the opposite conclusion “ $([\alpha, y, z], [\alpha, x, z]) \in R$ ” implies

$$(y, x) = \left(\left[\frac{1}{\alpha}, [\alpha, y, z], z \right], \left[\frac{1}{\alpha}, [\alpha, x, z], z \right] \right) \in R,$$

and results in the contradiction $(y, x) \in R$ and $(x, y) \in P$. Note that α and $1/\alpha$ simultaneously occur (one of these values is larger than 1).

In case R happens to be a complete binary relation, a similar argument implies that R is independent if and only if R is reflexive and for each x, y, z in D , and each $\alpha, 0 < \alpha \leq 1$, we have

$$\text{if } (x, y) \in R \text{ (resp. } P), \text{ then } ([\alpha, x, z], [\alpha, y, z]) \in R \text{ (resp. } P). \quad (2)$$

Obviously, condition (1) entails condition (2). Let us check that (2) implies (1). Suppose the antecedent clause of (1) holds, and let $\alpha > 1$. Then, the opposite conclusion –in the assumption that R is complete– reads: “ $([\alpha, y, z], [\alpha, x, z]) \in P$ ”. Again, we obtain a contradiction: $(y, x) \in P$ while $(x, y) \in R$.

Condition (2) only considers convex combinations and is therefore, in the present setting, perhaps a more natural property.

There is an obvious relationship between the class of independent orderings on H and the class of independent orderings on Δ . If R is an independent ordering on H , then its restriction to Δ is an independent ordering on Δ . The next lemma looks at the reverse relationship.

²For two sets A and B , we write $A \subset B$ if each element in A belongs to B . The combination $A \subset B$ and $B \subset A$ is summarized as $A = B$.

Lemma 1. *An independent, transitive, and complete relation R in Δ , uniquely extends to an independent, transitive, and complete relation R' in H .*

Proof. Let x and y belong to H . Let $z \in \Delta$. Choose $\alpha > 0$ sufficiently close to 1, such that $x' = [\alpha, x, z]$ and $y' = [\alpha, y, z]$ belong to Δ . Let the ordering R' on $\{x, y\}$ agree with the ordering R on $\{x', y'\}$. The ordering R' on $\{x, y\}$ does not depend upon the choice of z and α . We show this by contradiction. Let $x'' = [\beta, x, u] \in \Delta$ and $y'' = [\beta, y, u] \in \Delta$ and assume that $(x', y') \in R$ while $(y'', x'') \in P$. By independency, we have

$$\left(\underbrace{\left[\frac{\beta}{\alpha + \beta}, x', y'' \right]}_{v_1}, \underbrace{\left[\frac{\beta}{\alpha + \beta}, y', y'' \right]}_{v_2} \right) \in R \quad \text{and} \quad \left(\underbrace{\left[\frac{\alpha}{\alpha + \beta}, y'', y' \right]}_{v_3}, \underbrace{\left[\frac{\alpha}{\alpha + \beta}, x'', y' \right]}_{v_4} \right) \in P.$$

Observe that v_2 and v_3 coincide. Transitivity of R implies that $(v_1, v_4) \in P$. One can write v_1 and v_4 in terms of x, y, z , and u and verify that $v_1 = v_4$. Hence, we obtain $(v_1, v_1) \in P$. This contradicts the definition of the asymmetric component P of the relation R . Therefore, R' is well defined. Transitivity and independency of R' follows from the definition of R' in combination with the transitivity and independency of R . \square

Now, we focus on conditions that are strong enough to guarantee that a binary relation has an extension that is complete, transitive, and independent.

Let us insert here a result of Suzumura (1976, Thm 3) who solved a similar exercise. Suzumura started from a relation R and looked for a complete and transitive relation R^* such that $R \subset R^*$ and $P \subset P^*$. A natural way to proceed is to check whether the transitive closure R_T of R respects the asymmetric part, i.e. $P \subset P_T$. Apparently, this provides sufficient (and necessary) conditions: R has an ordering extension if and only if

$$\text{for each } x, y, \text{ we have } (x, y) \in R_T \text{ implies } (y, x) \notin P.$$

Suzumura labelled this condition as consistency.

We proceed similarly. Let R be a relation in D . The independent order relation R^* in D is said to be an independent ordering extension of R if $R \subset R^*$ and $P \subset P^*$. The transitive and independent closure R_C of R is the smallest (for inclusion) relation in D that includes R , satisfies transitivity and independency. The relation R is said to be *lottery-consistent* if

$$\text{for each } x, y \text{ in } D, \text{ we have } (x, y) \in R_C \text{ implies } (y, x) \notin P.$$

Before we shift the result of Suzumura towards the present setting, we provide some further insight in the closure R_C of a relation R in H .

Lemma 2. *Let R be a relation in H . Then, (x, y) belongs to the transitive and independent closure R_C of R if and only if*

- either, (x, y) belongs to the ‘transitive’ closure of R ;

- or, $x - y = \sum_{i=1}^{\ell} \beta_i(x_i - y_i)$, with (x_i, y_i) in R and $\beta_i > 0$ for each i , and $\beta_j \neq 1$ for at least one j .

Proof. First, if $(x, y) \in R_C$, then R_C is able to rank x and y on the basis of a finite number of couples in R . Let x and y be linked through the finite sequence $x = x_1, x_2, \dots, x_{k+1} = y$; that is, the couples $(x_1, x_2), (x_2, x_3), \dots, (x_k, x_{k+1})$ belong to the independent closure of R . Hence, there exist z_1, z_2, \dots, z_k in H and $\alpha_1, \alpha_2, \dots, \alpha_k > 0$ such that

$$([\alpha_i, x_i, z_i], [\alpha_i, x_{i+1}, z_i]) \in R, \text{ with } i = 1, 2, \dots, k.$$

For each i we obtain $[\alpha_i, x_i, z_i] - [\alpha_i, x_{i+1}, z_i] = \alpha_i(x_i - x_{i+1})$. Multiply these equations by $1/\alpha_i > 0$, and add them up:

$$x - y = x_1 - x_{k+1} = \sum_{i=1}^k \frac{[\alpha_i, x_i, z_i] - [\alpha_i, x_{i+1}, z_i]}{\alpha_i}.$$

In case $\alpha_1 = \alpha_2 = \dots = \alpha_k = 1$, then (x, y) belongs to the transitive closure of R .

Next, assume $x - y = \sum_{i=1}^{\ell} \beta_i(x_i - y_i)$ with (x_i, y_i) in R , $\beta_i > 0$, and $\beta_j \neq 1$. We have to prove that $(x, y) \in R_C$. We proceed by induction on ℓ .

For $\ell = 1$, it suffices to observe that the vector $z = (x - \beta_1 x_1)/(1 - \beta_1)$ in H allows us to write $x = [\beta_1, x_1, z]$ and $y = [\beta_1, y_1, z]$.

Suppose the result holds up to ℓ . Consider a positive linear combination of length $\ell + 1$. Assume that $\beta_1 \neq 1$. Consider $(x' - y') = (1/\beta) \times \sum_{i=2}^{\ell+1} \beta_i(x_i - y_i)$, with $0 < \beta \neq 1$ such that at least one of the coefficients β_i/β differs from 1. From the induction basis, we obtain $(x', y') \in R_C$. Hence, we can write

$$x - y = \beta_1(x_1 - y_1) + \beta(x' - y'), \quad \text{with } 0 < \beta_1 \neq 1, 0 < \beta \neq 1.$$

Let z and \tilde{y} in H solve the equations $x_1 = [1/\beta_1, x, z]$ and $y_1 = [1/\beta_1, \tilde{y}, z]$. Independency implies $(x, \tilde{y}) \in R_C$. Next, let z' and y^* in H solve the equations $x' = [1/\beta, \tilde{y}, z']$ and $y' = [1/\beta, y^*, z']$. Then, $(\tilde{y}, y^*) \in R_C$. The transitivity of R_C implies $(x, y^*) \in R_C$. Finally, the equations $x_1 - y_1 = (x - \tilde{y})/\beta_1$ and $x' - y' = (\tilde{y} - y^*)/\beta$ imply that $y^* = y$. \square

Now, we can state and prove the main result of this section.

Theorem 1. *Let R be a relation in Δ . Then, R has an independent ordering extension if and only if R is lottery-consistent.*

Proof. Let R^* be an independent ordering extension of R . Then, $R \subset R_C \subset R^*$ and $P \subset P^*$. Hence, it cannot happen that $(x, y) \in R_C$ and $(y, x) \in P$; otherwise the combination $(x, y) \in R^*$ and $(y, x) \in P^*$ would occur. Conclude that R is lottery-consistent.

The proof of the reverse implication is more involved. Let R be a lottery-consistent relation in Δ . Let the set Ω collect all the independent and transitive extensions of R . We will apply Zorn's lemma upon Ω and show that each maximal element in Ω has the right properties.

First, we show that the closure R_C belongs to Ω (hence, Ω is nonempty). The closure R_C of R certainly includes R and is independent and transitive. Let us check, by contradiction, that $P \subset P_C$. Therefore, assume that $(y, x) \in P$ while $(y, x) \notin P_C$. This can only occur if $(x, y) \in I_C$. Conclude that $(x, y) \in R_C$ and $(y, x) \in P$. This conflicts with R being lottery-consistent. Thus, we obtain $R_C \in \Omega$.

Next, consider a chain $\dots \subset R_j \subset \dots \subset R_k \subset \dots$ in Ω ; the index runs over some (possibly infinite) set J . Let us verify that $\hat{R} = \cup_{j \in J} R_j$ belongs to Ω . The relation \hat{R} extends R , i.e. $R \subset \hat{R}$ and $P \subset \hat{P}$. Also, \hat{R} satisfies independency and transitivity (otherwise there exists a smallest relation R_j that violates independence or transitivity, and one ends up with a contradiction). Conclude that the relation \hat{R} in Ω is an upperbound for the chain. Hence, each chain in Ω has an upperbound in Ω . Application of Zorn's lemma results in the existence of a maximal (for inclusion) element in Ω .

Let R^* be a maximal element in Ω . As R^* belongs to Ω , the relation R^* extends R , is independent and transitive. We still have to show that R^* is complete.

We check, by contradiction, the completeness of R^* . Let $(\hat{x}, \hat{y}) \in N^*$. The relation $\hat{R} = R^* \cup \{(\hat{x}, \hat{y})\}$ certainly includes R^* (in the strict sense, $\hat{R} \supset R^*$ and $\hat{R} \neq R^*$). Furthermore, the relation \hat{R} extends R , i.e. $R \subset \hat{R}$ and $P \subset \hat{P}$. The closure \hat{R}_C is independent and transitive and includes R^* . The maximality of R^* implies that the condition $P \subset \hat{P}_C$ is violated. Hence, there is a pair x and y in Δ such that $(y, x) \in P$ and $(x, y) \in \hat{R}_C$. Now, consider the transitive and independent closure \hat{R}'_C of \hat{R} in the hyperplane H (Lemma 1). Application of Lemma 2 upon $(x, y) \in \hat{R}_C \subset \hat{R}'_C$ results in

$$x - y = \sum_{i=1}^{\ell} \beta_i (x_i - y_i), \quad \text{with } (x_i, y_i) \in \hat{R}, \text{ and } \beta_i > 0.$$

As R^* extends R and $(y, x) \in P$, the couple (\hat{x}, \hat{y}) must occur at the right hand side, say $(x_1, y_1) = (\hat{x}, \hat{y})$. Furthermore, as R^* is independent the values $\beta_2, \beta_3, \dots, \beta_{\ell}$ are manipulable. Conclude that the above equation can be solved for (\hat{x}, \hat{y}) :

$$\hat{y} - \hat{x} = \gamma(y - x) + \sum_{i=2}^{\ell} \gamma_i (x_i - y_i), \quad \text{with } \gamma, \gamma_i > 0.$$

From Lemma 2 we learn that the pair \hat{x}, \hat{y} is comparable for the relation R^* (extended to H). This conflicts with $(\hat{x}, \hat{y}) \in N^*$. \square

3 Rationalizability of choice over lotteries

This section extends Richter's result towards the rationalizability of individual choice over lotteries. At the end of this section we shortly discuss a similar study by Kim (1996).

Consider the $(n - 1)$ -dimensional simplex Δ and let \mathcal{S} be a collection of nonempty subsets of Δ . A choice correspondence C is a correspondence

$$C : \mathcal{S} \longrightarrow \Delta : S \longmapsto C(S) \subset S,$$

such that (at least) for each finite set S the choice set $C(S)$ is nonempty. Hence, the class of sets for which the choice correspondence is decisive certainly includes the class of finite

sets. The choice correspondence C is said to be rationalizable if there exists an independent ordering R^* in Δ such that for each S in \mathcal{S} the set $C(S)$ collects the maximizers of the restriction of R^* to S , i.e.

$$\text{for each } S \in \mathcal{S} : C(S) = M(R^*|S) = \{x \in S \mid \text{for all } y \text{ in } S : (x, y) \in R^*\}.$$

Observe that for a (rationalizable) choice correspondence the choice set $C(S)$ might be empty; e.g. if $S \subset \Delta$ is an open (in the Euclidean topology) set and if the ordering R^* happens to be continuous, then the set $M(R^*|S)$ of maximizers might be empty. As it is unclear what one should conclude on the basis of an empty choice set, we impose the choice correspondence to be decisive on \mathcal{S} , i.e. a set S for which $C(S) = \emptyset$ is excluded from \mathcal{S} .

For a choice correspondence $C : \mathcal{S} \rightarrow \Delta$, the revealed preference relations \tilde{R} and $\tilde{\pi}$ in Δ are defined as follows. The couple (x, y) belongs to the revealed preference relation \tilde{R} if and only if there is a set S in \mathcal{S} such that $x \in C(S)$ and $y \in S$. Furthermore, the couple (x, y) belongs to the strict revealed preference relation $\tilde{\pi}$ if and only if there is a set S in \mathcal{S} such that $x \in C(S)$ while $y \in S \setminus C(S)$.

We extend the congruence axiom of Richter (1966). A choice correspondence $C : \mathcal{S} \rightarrow \Delta$ is said to satisfy *the congruence axiom* if and only if for each x and y in Δ we have

$$(x, y) \in \tilde{R}_C \quad \text{implies} \quad (y, x) \notin \tilde{\pi},$$

where \tilde{R}_C is the transitive and independent closure of the revealed preference relation \tilde{R} . We will show that this congruence axiom is strong enough to guarantee the choice correspondence to be rationalizable. The next lemma is a first step towards this result.

Lemma 3. *If the choice correspondence $C : \mathcal{S} \rightarrow \Delta$ satisfies the congruence axiom, then the asymmetric part \tilde{P} of the revealed preference relation \tilde{R} coincides with the strict revealed preference relation $\tilde{\pi}$.*

Proof. (i) : $\tilde{P} \subset \tilde{\pi}$. If $(x, y) \in \tilde{P}$, then $(x, y) \in \tilde{R}$ and $(y, x) \notin \tilde{R}$. Hence, there exists a set S such that $x \in C(S)$ and $y \in S$; and for each set T containing x and y , it holds that $y \notin C(T)$. Put $T = S$ and conclude that $x \in C(S)$ while $y \in S \setminus C(S)$, i.e. $(x, y) \in \tilde{\pi}$.

(ii) : $\tilde{\pi} \subset \tilde{P}$. If $(x, y) \in \tilde{\pi}$, then $(x, y) \in \tilde{R}$. In case also $(y, x) \in \tilde{R}$, the congruence axiom is violated: $(y, x) \in \tilde{R} \subset \tilde{R}_C$ and $(x, y) \in \tilde{\pi}$. Therefore, $(y, x) \notin \tilde{R}$ and $(x, y) \in \tilde{P}$. \square

As a corollary we obtain that if a choice correspondence satisfies the congruence axiom, then the revealed preference relation is lottery-consistent. The main result of this section reads:

Theorem 2. *Let the choice correspondence $C : \mathcal{S} \rightarrow \Delta$ be decisive on \mathcal{S} . Then, C is rationalizable if and only if it satisfies the congruence axiom.*

Proof. Let the independent ordering R^* in Δ rationalize the choice correspondence C . Obviously, R^* extends the revealed preference relation: $\tilde{R} \subset R^*$ and $\tilde{\pi} \subset P^*$. As R^* is transitive and independent, R^* includes the transitive and independent closure \tilde{R}_C of \tilde{R} . Suppose now that $(y, x) \in \tilde{\pi}$. Then, $(y, x) \in P^*$ and $(x, y) \notin R^*$. As a consequence, if $(y, x) \in \tilde{\pi}$, then $(x, y) \notin \tilde{R}_C$.

Let C satisfy the congruence axiom. By Lemma 3, the revealed preference relation is lottery-consistent. Apply Theorem 1 and extend the revealed preference relation \tilde{R} to an independent ordering R^* in Δ . Now, we have to verify whether $C(S) = M(R^*|S)$ holds each set S in \mathcal{S} . Let $x \in C(S)$. Hence, for each y in S we have $(x, y) \in \tilde{R} \subset R^*$, i.e. $x \in M(R^*|S)$. Next, let $x \in S \setminus C(S)$. By assumption, C is decisive on S : there exists a y in S such that $y \in C(S)$. It follows that $(y, x) \in \tilde{\pi} \subset P^*$. Conclude that $x \notin M(R^*|S)$. \square

The ultimate goal is to establish a test for the null hypothesis

H_0 : the individual choice correspondence $C : \mathcal{S} \longrightarrow \Delta$ is rationalizable.

Of course, one can extract the binary relation behind the choice correspondence (by checking all the pairs in Δ) and verify whether this relation is an independent ordering. In an empirical setting, however, this is impossible to manage. Theorem 2 allows us to test on the basis of a finite data set whether or not the null hypothesis should be rejected. As usual, not rejecting H_0 does not imply that H_0 is shown to hold. The next section returns to this issue.

We close this section by pointing out some differences with the work of Kim (1996), who also studied the preference relation on lotteries revealed through a choice correspondence. Although we both arrive at similar characterizations, we take different roads. Where Kim (1996, Appendix) uses a generalization of the theorem of the alternative, we follow the axiomatic approach and start from the theory of binary extensions. Furthermore, Kim (1996, Thm 3.1) restricts the attention to finite choice sets. We do not impose restrictions on the size of the choice set. However, recall from Theorem 2 that we need the choice correspondence to be decisive on the choice sets.

4 Nash rationalizability of collective choice

Assume an experimental setting with individuals playing a game in mixed strategies. For each individual, the experimenter (*i*) controls the set of pure strategies and the set of possible mixtures as well, and (*ii*) observes the lotteries the individuals select. In case we can extend the profile of revealed preferences to a profile of independent orderings such that the selection corresponds to a Nash equilibrium, then we say that the observations support the hypothesis of Nash rationalizable behavior. If the data reject this hypothesis, then either some player does not consult a complete, transitive, and independent binary relation, or the Nash equilibrium is not the right equilibrium concept. This section develops such a test procedure.

We start by introducing some further notation. Let $J = \{1, 2, \dots, m\}$ be the set of players, $m \in \mathbb{N}$. Individual j has n_j pure strategies, his strategy space Δ_j is the $(n_j - 1)$ -dimensional simplex Δ^{n_j} . A strategy profile is a vector $x = (x_1, x_2, \dots, x_m)$ with x_j in Δ_j the strategy of player j . The cartesian product $\Delta = \Delta_1 \times \Delta_2 \times \dots \times \Delta_m$ collects all the strategy profiles:

$$\Delta = \{x = (x_1, x_2, \dots, x_m) \mid x_j = (x_{j1}, x_{j2}, \dots, x_{jn_j}) \in \Delta_j\}.$$

In order to distinguish the strategy x_j of player j from the strategies of his opponents, we denote the strategy profile x also by (x_j, x_{-j}) with $x_{-j} = (x_1, \dots, x_{j-1}, x_{j+1}, \dots, x_m)$ collecting the strategies of j 's opponents.

A choice set S is a cartesian product $S_1 \times S_2 \times \dots \times S_m$ with $S_j \subset \Delta_j$ and represents an experiment, i.e. each player is confronted with restrictions on his strategy space. For a choice set S , a strategy profile x in S , and a player j in J , we denote the cartesian product $S_j \times \{x_{-j}\}$ by S_j^x . In the choice set S_j^x the strategy space of player j is reduced to S_j while the opponents only have one option (opponent i selects x_i from his strategy space $\{x_i\}$).

Let \mathcal{S} be a collection of nonempty subsets of Δ . We assume that for each choice set S in \mathcal{S} , for each x in S , and for each j in J , the individual choice set S_j^x also belongs to \mathcal{S} . A joint choice correspondence C is a correspondence

$$C : \mathcal{S} \longrightarrow \Delta : S \longmapsto C(S) \subset S,$$

such that for each choice set of the form S_j^x we have $C(S_j^x) \neq \emptyset$. We refer to this assumption as C is individually decisive. When the choice of all but one players is limited to only one option, then we assume that this one player is able to select a strategy. Besides individual decisiveness, we keep the assumption that C is decisive on finite choice sets.

In contrast to the previous section, we do not equip the players with a preference relation on the set Δ of strategy profiles. Instead, we assume that the players have preferences over the probability distributions of pure strategy profiles (e.g. via the payoffs corresponding to the pure strategies). As each player j has n_j pure strategies, these pure strategies generate $n = n_1 n_2 \dots n_m$ pure strategy profiles.³ The $(n - 1)$ -dimensional simplex Δ^n collects all the distributions over these profiles. Let d denote the map that converts a strategy profile in Δ into a probability distribution in Δ^n :

$$d : \Delta \longrightarrow \Delta^n : x \longmapsto d(x), \text{ with } d_{i_1, i_2, \dots, i_m}(x) = x_{1 i_1} x_{2 i_2} \dots x_{m i_m},$$

where i_j runs over the pure strategies 1 to n_j of player j . Within this notation, we can define Nash rationalizability of choice over lotteries.

Definition. Let the joint choice correspondence $C : \mathcal{S} \rightarrow \Delta$ be individually decisive. Then, C is said to be Nash rationalizable as soon there exists a profile $(R_1^*, R_2^*, \dots, R_m^*)$ of independent orderings in Δ^n such that for each S in \mathcal{S} , we have

$$x \in C(S) \quad \text{if and only if} \quad d(x) \in M(R_j^* | d(S_j^x)) \quad \text{for each } j \text{ in } J.$$

In words, a joint choice correspondence is Nash rationalizable if each player consults an independent ordering to select his own strategy conditional upon his opponents' strategies.

For a Nash rationalizable choice correspondence it holds that $x \in C(S_i^x)$ if and only if $d(x) \in M(R_i^* | d(S_i^x))$. Hence, if C is Nash rationalizable, then $x \in C(S)$ if and only if $x \in$

³The example in the introduction exhibits four pure strategy profiles: (U, L) , (U, R) , (D, L) , and (D, R) .

$C(S_i^x)$ for each i in J . The noncooperative behavior of the players is clearly incorporated in the definition of Nash rationalizability: a joint strategy is chosen if no single player has an incentive to deviate away.

We modify the definitions of the revealed preference relations from the previous section towards the present setting. Let $a, b \in \Delta^n$.

We start with the revealed preference relations $\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_m$. We have $(a, b) \in \tilde{R}_j$ if there exist an S in \mathcal{S} and x, y in S such that $x \in C(S_j^x)$, $y \in S_j^x$, and $(a, b) = (d(x), d(y))$.

Next, we consider the strict revealed preference relations $\tilde{\pi}_1, \tilde{\pi}_2, \dots, \tilde{\pi}_m$. We have $(a, b) \in \tilde{\pi}_j$ if there exist an S in \mathcal{S} and x, y in S such that $x \in C(S_j^x)$, $y \in S_j^x \setminus C(S_j^x)$, and $(a, b) = (d(x), d(y))$.

These definitions imply that a player is only able to reveal preferences conditional upon a status quo of his opponents' strategies. A player is able to select a above b only if he has a and b at 'his' disposal, i.e. only if he is able to switch between a and b without the cooperation of any other player.

Similar to the previous section, we search for conditions upon the revealed preferences to guarantee the Nash rationalizability of a choice correspondence $C : \mathcal{S} \rightarrow \Delta$. The joint choice correspondence C is said to satisfy *the congruence axiom* if and only if for each a and b in Δ^n and for each j in J , we have

$$(a, b) \in \tilde{R}_{jC} \quad \text{implies} \quad (b, a) \notin \tilde{\pi}_j,$$

with \tilde{R}_{jC} the transitive and independent closure of the revealed preference relation \tilde{R}_j .

The next lemma states that if a joint choice correspondence satisfies the congruence axiom, then the revealed preference relations are lottery-consistent. Its proof only involves minor modifications of the proof of Lemma 3 and is omitted.

Lemma 4. *Let the joint choice correspondence $C : \mathcal{S} \rightarrow \Delta$ be individually decisive. If C satisfies the congruence axiom, then for each player j the asymmetric part \tilde{P}_j of the revealed preference relation \tilde{R}_j coincides with the strict revealed preference relation $\tilde{\pi}_j$.*

At this point we are ready to provide conditions for the rationalizability of the individual choice correspondences $S_j^x \mapsto C(S_j^x)$. In order to obtain rationalizability of the joint choice correspondence $\mathcal{S} \rightarrow \Delta$, we need some 'local-global' condition to link the collective choice from a set S with the individual choices from the sets S_j^x . Here, we return to the noncooperative nature of the Nash equilibrium. The correspondence $C : \mathcal{S} \rightarrow \Delta$ is said to be *noncooperative* if for each S in \mathcal{S} we have

$$x \in C(S) \quad \text{if and only if} \quad x \in C(S_j^x) \quad \text{for each } j \text{ in } J.$$

In words, if a strategy profile x is selected from S then each player j selects this profile when the choice set S contracts or shrinks into his individual choice set S_j^x . And, if the group of players jointly select x from the choice sets S_j^x , then the group of players jointly select x from the union $S = S_1^x \cup S_2^x \cup \dots \cup S_m^x$. As such, this axiom has some flavor of a

contraction-expansion property. The combination of noncooperation and the congruence axiom implies the rationalizability of the joint choice correspondence.

Theorem 3. *Let the joint choice correspondence $C : \mathcal{S} \rightarrow \Delta$ be individually decisive. Then, C is Nash rationalizable if and only if C is noncooperative and satisfies the congruence axiom.*

Proof. Let C be Nash rationalizable through the profile $(R_1^*, R_2^*, \dots, R_m^*)$ of independent orderings in Δ^n . To prove that C satisfies the congruence axiom, one can apply Theorem 2 upon the individual choice correspondences $C : \mathcal{S}_j \rightarrow \Delta$, where \mathcal{S}_j collects all the choice sets of the form S_j^x with S running through the collection \mathcal{S} . That C is noncooperative has been argued above (see Definition).

Now, suppose that C is noncooperative and satisfies the congruence axiom. Then, each revealed preference relation \tilde{R}_j is lottery-consistent and extends to an independent ordering R_j^* in Δ^n (use Theorem 2). We have to check whether for each S in \mathcal{S} , for each x in S , it holds that

$$x \in C(S) \quad \text{if and only if} \quad d(x) \in M(R_j^* | d(S_j^x)) \quad \text{for each } j \text{ in } J.$$

Let $x \in C(S)$. As C is noncooperative, it follows that $x \in C(S_j^x)$ for each j in J . Hence, for each y in S_j^x we have $(d(x), d(y)) \in \tilde{R}_j \subset R_j^*$. It follows that $d(x) \in M(R_j^* | S_j^x)$ for each j in J .

Finally, let $x \in S \setminus C(S)$ and assume that $d(x) \in d(S)$. As C is noncooperative, there exists at least one player i for which $x \notin C(S_i^x)$. Since C is individually decisive, there exists a y in S_i^x such that $y \in C(S_i^x)$. Therefore, $(d(y), d(x)) \in \tilde{\pi}_i \subset P_i^*$. It follows that for player i we have $d(x) \notin M(R_i^* | d(S_i^x))$. \square

This theorem establishes a rule to judge whether or not the hypothesis

H_0 : the collective choice correspondence $C : \mathcal{S} \rightarrow \Delta$ is Nash rationalizable

should be rejected. The test is exact in the sense that as soon as the observations conflict with the axioms of noncooperation or of congruence, the null hypothesis is false with certainty. The probability to reject the hypothesis when it is actually true is zero. Let us apply the test upon the data (Table 1) presented in Section 1.

Denote $x = C(S)$, $x' = C(S')$, and $x'' = C(S'')$. Let us list the four pure strategy profiles: (U, L) , (U, R) , (D, L) , and (D, R) . We have that $d(x) = (0.12, 0.28, 0.18, 0.42)$.

Use the axiom of noncooperation to conclude that player 1 reveals to (weakly) prefer $(0.4, 0.6)$ above any other strategy available to him, such as $(0.3, 0.7)$. Let us write $y = (0.3, 0.7) \times (0.3, 0.7)$, and $d(y) = (0.09, 0.21, 0.21, 0.49)$. As such we learn that $(d(x), d(y)) \in \tilde{R}_1$.

Similarly, $d(x') = (0.168, 0.232, 0.252, 0.348)$. Since also the strategy $(0.42, 0.58)$ is at the disposal of player 1, it follows (again, use the axiom of noncooperation) that $(d(x'), d(y')) \in \tilde{R}_1$, with $d(y') = (0.2205, 0.3045, 0.1995, 0.2755) \in \Delta^4$.

Finally, $d(x'') = (0.25, 0.25, 0.25, 0.25)$. The available strategy $(0.2, 0.8)$ leads to the distribution $d(y'') = (0.1, 0.1, 0.4, 0.4)$. The data imply $(d(x''), d(y'')) \in \tilde{\pi}_1$.

One can check that $2(d(x) - d(y)) + 4(d(x') - d(y')) + (d(x'') - d(y'')) = 0$. Solve this equation for $d(y'') - d(x'')$ and conclude (use Lemma 2) that $(d(y''), d(x''))$ belongs to the independent and transitive closure of \tilde{R}_1 . This contradicts our extended version of Richter's congruence axiom. Therefore, the data reject the hypothesis H_0 .

This exercise illuminates the need to observe the chosen distributions $(d(x), d(x'), \text{ and } d(x''))$ in Δ^4 over the pure strategy profiles. Although the problem of observability of mixed strategies is beyond the scope of this paper, we briefly discuss this issue. Hence, let us assume that only pure strategies (randomly drawn from the selected distributions) are observed.

One way to proceed, is to sufficiently often repeat the experiment and to perform the test upon the estimated distributions. This approach, however, is at the cost of exactness: the probability to reject a true hypothesis becomes strictly positive.

Alternatively, one can weaken the revealed preference relation. Suppose that the pure strategy a is observed from a choice set $S \subset \Delta$. This reveals the existence of a distribution in S with a positive probability on a that is preferred to each distribution in S with zero probability on a . Let S_a collect all the distributions in S the support of which contains a . Then, this scenario reveals that the set S_a contains an element ranked above each element in $S \setminus S_a$. One can look for conditions to rationalize such a choice correspondence. We expect, however, that the resulting restrictions on the revealed preference relations will turn out to be very weak.

5 Persistence axioms of Sprumont

In this section we show that the persistence conditions of Sprumont are equivalent to our conditions for Nash rationalizability (when restricted to the setting of pure strategies). As such we indicate that our Theorem 3 extends Theorem 2 of Sprumont (2000) to cases involving mixed strategies (or choices over lotteries).

Let A_j be the set of pure strategies available to player j and let $A = A_1 \times A_2 \times \dots \times A_m$ be the set of all joint pure strategies. When restricted to the pure strategies, the map d from the space Δ of strategy profiles to the space Δ^n of distributions over the pure strategy profiles becomes one-to-one. Observing a (degenerate) distribution in Δ^n boils down to observing the pure strategies selected by the players.

Let \mathcal{S} be a collection of nonempty subsets of A and let $C : \mathcal{S} \rightarrow A$ be a joint choice correspondence (assumed to be decisive on \mathcal{S}).

Then, C is said to be *persistent under expansion* if for each S and T in \mathcal{S} it holds that $C(S) \cap C(T) \subset C(S \vee T)$, with $S \vee T$ the smallest choice set in \mathcal{S} that includes S and T .

Furthermore, C is said to be *persistent under contraction* if (i) for each S and T in \mathcal{S} with $T \subset S$ it holds that $C(S) \cap T \subset C(T)$ and (ii) for each S and T in \mathcal{S} with $T \subset S_j^x$ and $C(S_j^x) \cap T \neq \emptyset$, it holds that $C(T) \subset C(S_j^x)$.

We express the equivalence as follows.

Proposition. *Let $C : \mathcal{S} \rightarrow A$ be a joint choice correspondence. Then, C is noncooperative and satisfies the congruence axiom (taking only the transitive closure into account) if and only if C is persistent under expansion and persistent under contraction.*

Proof. First, assume C is noncooperative and satisfies the congruence axiom. Let us check whether C is persistent under expansion. Let S and T in \mathcal{S} . If $a \in C(S) \cap C(T)$, then (use noncooperation) $a \in C(S_j^a) \cap C(T_j^a)$ for each j in J . Hence, the players reveal $(a, b) \in \tilde{R}_j$ for each b in $S_j^a \cup T_j^a$. If for player i in J we have $a \notin C((S \vee T)_i^a)$, then this player reveals to strictly prefer some action b (the decisiveness of C implies the existence of such an action) over a , i.e. $(b, a) \in \tilde{\pi}_i$. This contradicts the congruence axiom. Hence, $a \in C((S \vee T)_j^a)$ for each j in J . Noncooperation implies $a \in C(S \vee T)$.

We now verify persistence under contraction. Condition (i). Let $T \subset S$ and $a \in C(S) \cap T$. Noncooperation implies that each player j selects a from the individual choice set S_j^a . The congruence axiom implies that each player j selects a from the smaller choice set T_j^a . Conclude that $a \in C(T)$.

Contraction condition (ii). Let $T \subset S_j^x$, $b \in C(S_j^x) \cap T$, and $a \in C(T)$. As a consequence, $(a, b) \in \tilde{R}_j$. Hence, if this player does not select a from S_j^x , there exists a d in S_j^x such that $(d, a) \in \tilde{\pi}_j$. As $b \in C(S_j^x)$ and $d \in S_j^x$, it follows that $(b, d) \in \tilde{R}_j$. These observations contradict the congruence axiom: (a, d) belongs to the transitive closure of \tilde{R}_j , while $(d, a) \in \tilde{\pi}_j$.

Next, suppose that C satisfies the persistence axioms. Let us check the congruence axiom. Hence, assume (a, b) belongs to the transitive closure of \tilde{R}_j with j in J . Denote the sequence from a to b by $a = a_1, a_2, \dots, a_{k+1} = b$, i.e. we have $(a_1, a_2), (a_2, a_3), \dots, (a_k, a_{k+1}) \in \tilde{R}_j$. As player j is only able to reveal preferences conditional upon a status quo of his opponents, it must be the case that $a_1, a_2, \dots, a_{k+1} \in A_j \times \{a_{-j}\}$, remember that a_{-j} collects the strategies of j 's opponents. Persistence under contraction (part i) allows us to focus on the sets $S_\ell = \{a_1, a_2, \dots, a_\ell\}$ with $\ell = 2, 3, \dots, k+1$. One can check that $C(S_\ell) \cap S_{\ell-1} \neq \emptyset$. From persistence under contraction (part ii) it follows that $C(S_{\ell-1}) \subset C(S_\ell)$. Therefore, $a \in C(S_{k+1})$, and $a \in C(\{a, b\})$. Conclude that $(b, a) \notin \tilde{\pi}_j$ and $(a, b) \notin \tilde{R}_j$.

Finally, we check for noncooperation. Let $x \in C(S_j^x)$ for each j in J . Persistence under expansion implies $x \in C(S_1^x \vee S_2^x \vee \dots \vee S_m^x) = C(S)$. And, if $x \in C(S)$, then $x \in C(S_j^x)$ for each j (use persistence under contraction). \square

Of course, this proposition mutually supports our results and those of Sprumont. Besides that, it indicates that –when restricted to pure strategies– our Theorem 3 also extends Theorem 2 of Sprumont to cases involving infinitely many pure strategies.

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